Chapter 10

Members Carrying Combined Axial Load and Moments

10.1 Introduction

- It is more common to have members subjected to axial load and moments rather than members subjected either to pure axial load or pure moments.
- The most convenient way to analyse a member subjected to both axial load and moment is to analyse the member subjected to axial load and moment separately. After that the results arrived at are compared with their respective developed strengths and stability limit states and then the separately arrived at results are combined in some suitable manner. (Limit State Approach).
- Another situation which is often practically encountered is that the axis of axial load rarely coincides with the geometrical axis of the member. There is always present some minimum amount of eccentricity.
- In tension members, with increase of axial load, the geometric axis of the member tries to coincide with the load line but in compression members, as the load increases, the geometric axis moves further away from the load line thereby increasing the eccentricity further which ultimately leads to failure of compression member due to buckling.

10.2 Order of Moments

(a) **Primary Moments:** Members subjected to transverse loads make the members to deflect and are subjected to moments called as primary moments.

(b) **Secondary Moments:** The members are often subjected to axial loads along with transverse loads. The presence of axial load causes additional deflection in the member which is in addition to the deflection caused by transverse load. The additional moments produced by axial load is referred to as secondary moments.

- Now, in order to find out the total deflection due to transverse and axial load, one will be tempted to add the deflections produced by transverse and axial load individually. But this is not true because the problem of finding out the total deflection is non-linear and principle of superposition cannot be applied and thus without knowing the deflection, moments cannot be calculated.

(c) **First order analysis:** In ordinary structural analysis, we do not consider the displaced geometry of the structure i.e. the deflections produced by the loads are considered to be very small in magnitude and this type of analysis is called as first order analysis

(d) **Second order analysis:** When deflections produced by the loads are not small and cannot be ignored then we have to take into account the displaced geometry of the structure and this type of analysis does not give explicit equations but the implicit equations which cannot be solved directly are required to be solved by iterative procedure often involving the use of numerical techniques like finite difference method, finite element method etc. This type of analysis is called as second order analysis.

NOTE

In routine design of structures, we normally go for first order analysis since the second order analysis requires a lot of calculations which are quite cumbersome and are usually done with softwares. IS 800 allows amplifying the moments which involves computing the maximum flexural moment due to transverse loads using first order analysis followed by amplifying the moment by a moment magnification factor called as interaction ratio (K) to take into account secondary moments.

(e) **Member effect:** In a framed structure, a beam-column may be braced or un-braced. When a beam-column is subjected to a moment along its un-braced length, it will get displaced laterally in the bending plane. For side-sway restrained member, the maximum secondary moment will be $P\delta$ which is called as member effect (Fig. 10.1). This member effect ($P\delta$) is added to maximum moment arrived at by first order analysis.

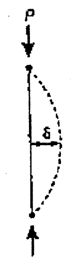


Fig. 10.1 *Member Effect (Pδ)*

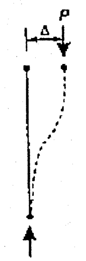


Fig. 10.2 *Structure Effect (PΔ)*

(f) **Structure effect:** For un-braced frame, there will be a side-sway where the ends of the column move laterally relative to each other. This induces additional secondary moment with maximum value $P\Delta$ which is called as structure effect (Fig. 10.2). Structure effect represents an amplification of the end moment. This moment can either be caused by lateral loads or unbalanced gravity loads or both.

NOTE

$P\delta$ effect is to be accounted for whether the column is braced or un-braced against side-sway while $P\Delta$ effect is accounted only for column un-braced against side-sway. Design of beam-column requires magnification of primary moments due to $P\delta$ and $P\Delta$ effects.

- Member restrained against side-sway i.e. whose ends cannot displace, when subjected to equal end moments, will produce a single curvature bending curve (Fig. 10.3). For this case, maximum primary moment and the maximum amplification occur at mid span/mid-height where the deflection is the largest.

- Now for the case of equal end moments, the moment is constant along the length of the member and thus the primary and the secondary moment are additive (being of the same sign).
- In case end moments are unequal but one of the moment is clockwise and other anti-clockwise, then even in this case, the member bends in single curvature and maximum primary and secondary moments occur near each other.

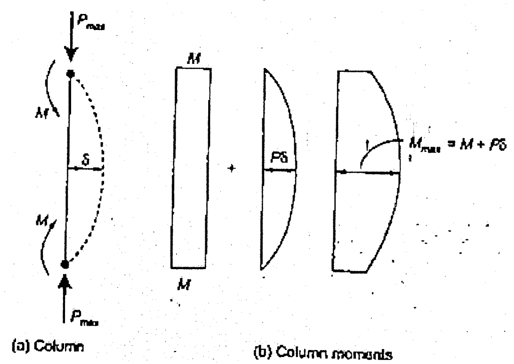


Fig. 10.3 Moment magnification in column bending in single curvature

- In case, end moments are such that member bends in reverse curvature (Fig. 10.4), the maximum primary moment will be at one end and maximum amplification of moment will occur somewhere between the two ends. Depending on the magnitude of factored axial compressive load P, the magnified moment can either be larger or smaller than the end moment.

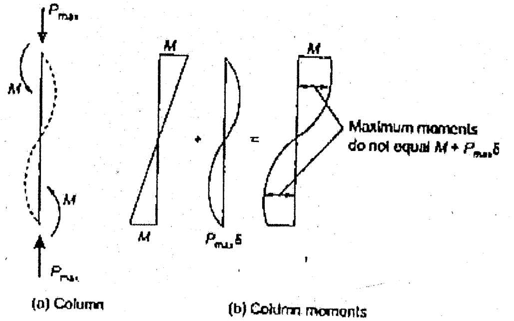


Fig. 10.4 Moment magnification in column bending in single curvature

(g) **Moment reduction factor:** In a beam-column, the maximum moment depends on the variation of bending moment along the member. This distribution of moment is taken into account by a factor called as equivalent moment factor or modification factor or moment reduction factor (C_m).

The significance of C_m factor is that it can convert any end moment configuration into an equivalent uniform moment along the length of the member. This C_m factor is a function of another factor ψ which is the ratio of end moments M_A/M_B ($M_B > M_A$) and axial load ratio P_u/P_{cr} where P_u is the factored axial load and P_{cr} is the critical Euler's buckling load. The value of ψ is positive for opposite moments at the ends (single curvature bending), and negative for same kind of moments (double curvature bending) (Fig. 10.5).

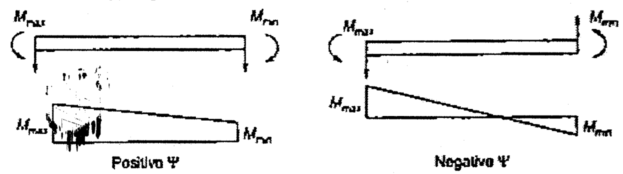
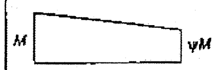
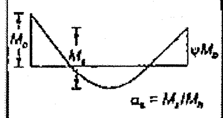
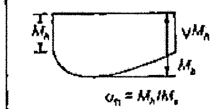


Fig. 10.5 Non uniform flexural moment distribution

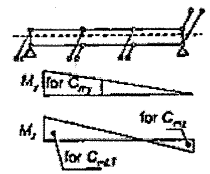
For different types of situations, the corresponding C_m values are given in Table 10.1.

Table 10.1 Moment reduction factor (C_m)

Bending Moment Diagram	Range		C_{my}, C_{mz}, C_{mLT} Uniform Loading	C_{my}, C_{mz}, C_{mLT} Concentrated Load
	$-1 \le \psi \le 1$		$0.6 + 0.4\psi \ge 0.4$	
 $\alpha_s = M_s/M_h$	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0.2 + 0.8\alpha_s \ge 0.4$	$0.2 + 0.8\alpha_s \ge 0.4$
	$-1 \le \alpha_s \le 0$	$0 \le \psi \le 1$	$0.1 - 0.8\alpha_s \ge 0.4$	$-0.8\alpha_s \ge 0.4$
		$-1 \le \psi \le 0$	$0.1(1-\psi) - 0.8\alpha_s \ge 0.4$	$0.2(1-\psi) - 0.8\alpha_s \ge 0.4$
 $\alpha_h = M_h/M_s$	$0 \le \alpha_s \le 1$	$1 \le \psi \le 1$	$0.095 + 0.05\alpha_h$	$0.90 + 0.10\alpha_h$
	$-1 \le \alpha_s \le 0$	$0 \le \psi \le 1$	$0.095 + 0.05\alpha_h$	$0.90 + 0.10\alpha_h$
		$-1 \le \psi \le 0$	$0.95 + 0.05\alpha_h(1 + 2\psi)$	$0.90 + 0.05\alpha_h(1 + 2\psi)$
For members with sway buckling mode, the equivalent uniform factor $C_{my} = C_{mz} = 0.9$				

C_{my}, C_{mz}, C_{mLT} shall be obtained according to the bending moment diagram betwnn the relevant braced points.

Moment factor	*Bending axis*	*Point braced in direction*
C_{my}	z-z	y-y
C_{mz}	y-y	z-z
C_{mLT}	z-z	z-z



Important: If moment reduction factor is not applied then one will get the same total moment for the situations as depicted in Figs. 10.4 and 10.5. C_m factor is based on rotational restraint at the ends of the member and moment gradients (i.e. variation of moment along the length) in the member. For the case of equal end moments as shown in Fig. 10.4, no modification in the moment is required and moment modification C_m is unity.

- Secondary moments produced due to side-sway are taken into account by amplification factor for side-sway moment given by $1/[1 - \Sigma P_u(\Delta/\Sigma HL)]$ or by $1/[1-(\Sigma P_u/\Sigma P_{cr})]$ where ΣP_u is the summation of all the factored loads on all the columns in the storey under consideration, ΣH is the summation of all the horizontal forces causing lateral drift Δ of the storey under consideration, L is the storey height and ΣP_{cr} is the summation of critical Euler buckling loads $(=\Sigma(\pi^2 EI/(KL)^2))$ for all the columns in the storey under consideration.

10.3 Beam-Column Behaviour

- Beam-column is the most general case as an element in a structure.
- As the flexural moment in the beam-column approaches zero, the unit will act like a column and when the axial load approaches zero, the unit will act like a beam.
- Thus all the factors that affect the behavior of beam and column individually will definitely affect the behavior of beam-column unit.
- Let there be a compact or plastic I-section beam-column. This may fail either by flexure, yielding or torsional-flexural buckling. Of course, the actual mode of failure will depend on the magnitude of axial load, its eccentricity and the slenderness ratio. Let there is no possibility of local buckling and lateral-torsional buckling. The load deflection (moment-rotation) curve *OABC* for such a beam column is shown in Fig. 10.6.
- The curve *OABC* is non-linear from the beginning itself because of P-δ effect. The secondary moment induced due to P-δ effect becomes more significant as the applied end moments increase. Due to the combined effect of primary and secondary moments, the most severely stressed fiber of the cross-section may yield as represented by point *A* in Fig. 10.6.
- Yielding (point *A*) decreases the stiffness of the member and slope of the curve after point A decreases.
- On further increasing the moments, secondary moments increase which induce plasticity into the section. Thus plastic hinge rotation gets developed at point *B* in Fig. 10.6.
- On further increasing the moments beyond point *B*, plasticity spreads a short distance along the length of beam-column and finally the moment capacity of the section gets used up completely at point *C* in Fig. 10.6.
- Slender beam-columns with open type of cross-section (like channel section) are torsionally weak and are prone to lateral torsional buckling. This lateral torsional buckling may occur either in elastic range (curve *A* of Fig. 10.6) or inelastic range (curve *B* of Fig. 10.6) depending on the slenderness ratio.

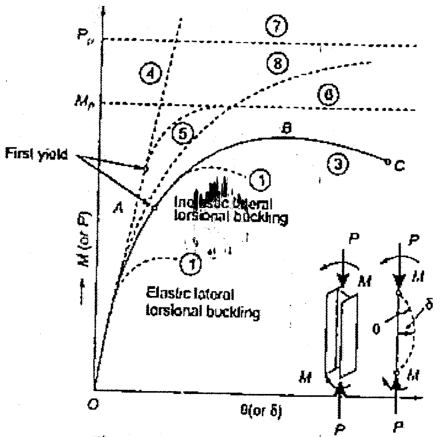


Fig. 10.6 Beam-column behavior

(i) Members having large slenderness ratio will fail by lateral elastic torsional buckling.

(ii) Members having intermediate slenderness ratio will fail by inelastic lateral torsional buckling.

- Slender columns subjected to combined axial compressive load and bending may fail under different modes of instability or material failure.

10.3.1 Classification of Beam-column Behavior

(a) **Case 1:** Short column subjected to axial load and uniaxial bending about either axis or biaxial bending. Here the failure usually occurs when plastic capacity of the section is reached.

(b) **Case 2:** Long column subjected to axial load and uniaxial bending about major axis. If long column is supported against lateral buckling about the minor axis, the column fails by buckling about the major axis. In case of small axial loads or if the column is not very long, a plastic hinge gets developed at the end or at the point of maximum moment.

(c) **Case 3:** Long column subjected to axial load and uniaxial bending about minor axis. If long column is supported against lateral buckling about the major axis, the column fails by buckling about the minor axis. At very low axial load, the column reaches its bending capacity about the minor axis.

(d) **Case 4:** Long column subjected to axial load and uniaxial bending about major axis. If long column is not supported against lateral buckling about the minor axis, it fails due to the combined effect of column buckling about minor axis and lateral torsional buckling. In this case, column section twists as well as deflects in the major and minor planes.

(e) **Case 5:** Long column subjected to axial load and biaxial bending. It is a general loading case. In this case, the failure is same as that of Case 4 above but effect of minor axis buckling will be quite significant.

Thus beam-column may fail either by reaching their ultimate strength (for small axial load and short members) or by reaching their buckling strength (as governed by lateral torsional buckling or weak axis buckling).

10.4 Strength of Beam-Column

- Predicting the exact behaviour of beam-column is quite complex. Nevertheless simplified design equations are available to evaluate the strength of the members though bit conservatively. The inelastic analysis to determine the interaction between axial load and flexural moment for beam-column consists of the following:

1. **Cross-section analysis:** The behavior of cross-section under the combined effect of axial load and flexural moment will be the most critical at the point of greatest flexural moment and axial load. The resulting capacity of the section is controlled by local buckling or yielding.
2. **Member analysis:** The member is divided into a number of small segments where the equilibrium and compatibility conditions along the length of the member at each division point are applied to give a set of loadings and deflections. Numerical techniques are required in case elasto-plastic analysis is done which makes the analysis and design complicated and tedious.

   So beam-column must be checked for local capacity at the point of greatest flexural moment and axial load. These points are usually at the ends of the member. But most of the times, lateral loads are also present and for that points, the greatest flexural moment and axial load are somewhere along the length of the member. Thus member must be checked for overall buckling also.

NOTE: The design equations as given in IS 800 in order to check local section failure and overall stability of beam-column are conservative simplifications of the too complex non-linear failure.

10.5 Check for Local Section Capacity

In case of short and stocky beam-column with relatively small axial compression ratio and beam-column being bent in reverse curvature, there the local section failure mostly occurs. The strength of end section achieved under combined axial load and flexural moment governs the failure.

The local section capacity of beam-column as given by IS 800 is as given below:

10.5.1 Plastic and Compact Sections

IS 800 recommends the following non-linear interaction relation for plastic and compact I-sections:

$$\left(\frac{M_y}{M_{ndy}}\right)^{\alpha_1}+\left(\frac{M_z}{M_{ndz}}\right)^{\alpha_2} \le 1 \quad ...(10.1)$$

The following equation can also be used in a more conservative manner:

$$\frac{N}{N_d}+\frac{M_y}{M_{dy}}+\frac{M_z}{M_{dz}} \le 1 \quad ...(10.2)$$

Where α_1 and α_2 are constants as given in Table 10.2.

Where, N = Factored axial load (tensile or compressive)

N_d = Design tensile strength (T_d) or design compressive strength due to yielding and is given by,

$$N_d = \frac{A_g f_y}{\gamma_{m0}}$$

A_g = Gross cross-sectional area of the section

f_y = Yield stress

γ_{m0} = Partial factor of safety in yielding

M_z, M_y = Factored flexural moments about major and minor axes respectively

M_{dz}, M_{dy} = Design flexural strength due to corresponding moment acting individually i.e. alone

M_{ndy}, M_{ndz} are the design reduced flexural strength under combined axial load and the respective uniaxial moment acting alone which is as computed below:

Table 10.2: Values of constants α_1 and α_2

S.No.	Section	α_1	α_2
1	I, channel	$5n \ge 1$	2
2	Circular tube	2	2
3	Rectangular tube	$1.66/(1-1.13n^2) \le 6$	$1.66/(1-1.13n^2) \le 6$
4	Solid rectangle	$1.73 + 1.8\,n^3$	$1.73 + 1.8\,n^3$

For sections without bolt holes, the following approximate relationship may be used for estimating M_{ndy} and M_{ndz}:

(a) Plates

$$M_{nd} = M_d(1-n^2) \quad ...(10.3)$$

(b) Welded I or H sections

$$M_{nd} = M_{ndy}\left[1-\left(\frac{n-a}{1-a}\right)^2\right] \le M_{dy} \text{ where } n \ge a \quad ...(10.4)$$

$$M_{ndz} = M_{dz}\frac{(1-n)}{(1-0.5a)} \le M_{dz} \quad ...(10.5)$$

where, $n = \dfrac{N}{N_d}$, $a = \dfrac{(A-2bt_f)}{A} \le 0.5$

(c) Standard I or H sections

For $n \le 0.2$ $\quad M_{ndy} = M_{dy}$ ...(10.6)

For $n > 0.2$ $\quad M_{ndy} = 1.56\,M_{dy}(1-n)(n+0.6)$ ...(10.7)

$M_{ndz} = 1.11\,M_{dz}(1-n) \le M_{dz}$ ...(10.8)

(d) Rectangular hollow sections and welded box sections

For symmetric section about both the axes and without bolt holes,

$$M_{ndy} = M_{dy}\left(\frac{1-n}{1-0.5a_f}\right) \le M_{dy} \quad ...(10.9)$$

$$M_{ndz} = M_{dz}\left(\frac{1-n}{1-0.5a_w}\right) \le M_{dz} \quad ...(10.10)$$

Where, $a_w = \left(\dfrac{A-2bt_f}{A}\right) \le 0.5$, $a_f = \left(\dfrac{A-2ht_w}{A}\right) \le 0.5$

(e) Circular hollow tubes without bolt holes

$$M_{nd} = 1.04M_d\left(1-n^{1.7}\right) \le M_d \quad ...(10.11)$$

10.5.2 Semi-Compact Section

Design of semi-compact section is quite satisfactory under the effect of combined axial load and flexural moment in the absence of high shear force provided maximum longitudinal stress under the combined axial load and flexural bending (f_x) satisfies the following criteria:

$$f_x \le \frac{f_y}{\gamma_{m0}}$$

For checking the local capacity of a beam-column, Eq. (10.2) can be used.

10.6 Check for Overall Strength of Member

- Overall stability of beam-column may not be adequate enough in case the member is subjected to large compressive force and single curvature bending occurs about the minor axis.

- It may also occur in a member which is not very long but subjected to axial compression and subjected to single curvature bending about the major axis.
- The member fails on account of reaching the strength of the member at a section over the length of the member under combined axial compression and magnified flexural moment. In long members bending about the weak axis, failure may occur due to weak axis buckling or failure of the maximum moment section under combined axial load and flexural moment.
- Section may also fail due to elastic or plastic buckling depending on slenderness ratio of plate elements.
- In long members subjected to large compression and uniaxial bending about the major axis or biaxial bending, the overall instability occurs due to flexural-torsional buckling.
- The overall strength of tension and compression members subjected to axial load and flexural moments is checked as per the following procedure:

10.6.1 Checking the Overall Stability of Tension Member

The reduced effective moment (M_{eff}) must not exceed the flexural strength (M_d) due to torsional buckling i.e.

$$M_{eff} \leq M_d$$

where,

$$M_{eff} = M - \frac{\psi T Z_{ec}}{A} \qquad \text{...(10.12)}$$

where, M = Factored flexural moment, T = Factored tensile force

A = Cross-sectional area of the section

Z_{ec} = Elastic sectional modulus w.r.t. extreme compression fiber

Ψ = 0.8 if T and M can vary independently

= 1 otherwise

$M_d = \beta_b Z_p f_{bd}$ (for laterally unsupported beam section)

10.6.2 Checking the Overall Stability of Compression Member

- In general the moments applied to beam-columns in frames are of opposite nature.
- Thus it is necessary to first arrive at a uniform moment that is equivalent to any combination of applied end moments followed by arriving at a suitable modification factor that will adequately account for the effects of axially imposed loads on this equivalent uniform moment.
- Members subjected to combined axial compressive load and biaxial bending should satisfy the following interaction relationship as given by IS 800:

$$\frac{P}{P_{dy}} + K_y \frac{C_{my} M_y}{M_{dy}} + K_{LT} \frac{M_z}{M_{dz}} \leq 1 \qquad \text{...(10.13)}$$

$$\frac{P}{P_{dz}} + 0.6 K_y \frac{C_{my} M_y}{M_{dy}} + K_z \frac{C_{mz} M_z}{M_{dz}} \leq 1 \qquad \text{...(10.14)}$$

where,

C_{my}, C_{mz} = Equivalent uniform moment factor (reduction factor) which accounts for unequal end moments i.e. the effect of moment gradient (as given in Table 10.1).

P = Factored axial compressive load

M_y, M_z = Maximum factored flexural moment about y and z axis of the member

P_{dy}, P_{dz} = Design strength of the member under axial compression as governed by buckling about minor (y) and major (z) axes respectively

M_{dy}, M_{dz} = Design flexural strength about minor (y) and major (z) axes respectively considering laterally unsupported length of the cross-section

$$K_y = 1 + (\lambda_y - 0.2) n_y \leq 1 + 0.8\, n_y$$

$$K_z = 1 + (\lambda_z - 0.2) n_z \leq 1 + 0.8\, n_z$$

$$K_{LT} = \frac{1 - 0.1 \lambda_{LT} n_y}{(C_{mLT} - 0.25)} \geq 1 - \frac{0.1 n_y}{(C_{mLT} - 0.25)}$$

K_y, K_z, K_{LT} = Interaction factors (moment magnification factors) to account for moment amplification due to P-δ effect and simplify the equivalent uniform moment ($C_m M$). The interaction factors K have to account for the effects of combined action leading to specific K factors for various modes of buckling of beam-column. K_y and K_z are used for flexural buckling and K_{LT} for flexural-torsional buckling.

n_y, n_z = Ratio of actual applied axial force to the design axial strength for buckling about minor (y) and major (z) axes respectively.

$$= \frac{P}{P_{dy}} \text{ and } \frac{P}{P_{dz}}$$

C_{mLT} = Equivalent uniform moment factor for lateral torsional buckling (Table 10.1) corresponding to actual moment gradient between lateral supports against torsional deformations in the critical region under consideration. In general, C_{mLT} is taken equal to C_{mz}.

λ_y, λ_z = Non dimensional slenderness ratio about the minor and major axes respectively

10.7 Procedure for the Design of Beam Columns

- The design procedure for a beam column is a trial and error one and involves iterations
- A trial section is selected first based on some rationale or experience and is checked for local capacity of the section along with overall capacity of the member.
- The checking of the section and the whole member is not only a lengthy procedure but calculations are also tedious and involve a lot of effort to arrive at an economical and of course a safe section.

Step-1. Determine the factored loads and the moments.

Step-2. A trial section is selected arbitrarily. The design axial stress for the beam-column is determined. With this value, the area required for axial compression is arrived at. The area so determined is increased suitably to account for bending moment.

Step-3. Another trial section with this area is selected from the steel tables or otherwise and relevant properties of the section are noted down.

Step-4. The section so selected is classified. Plastic and compact sections are preferred over semi-compact sections because semi-compact sections fail by yielding.

Step-5. The most critical section (i.e. heavily loaded) is checked for local capacity as,

$$\left(\frac{M_y}{M_{ndy}}\right)^{\alpha_1}+\left(\frac{M_z}{M_{ndz}}\right)^{\alpha_2} \le 1, \text{ for plastic and compact sections}$$

and

$$\frac{N}{N_d}+\frac{M_y}{M_{dy}}+\frac{M_z}{M_{dz}} \le 1, \text{ for semi-compact sections.}$$

Step-6. The member is checked for its buckling resistance in compression.

Step-7. The member is checked for its buckling resistance in flexure as per the following procedure:

The lateral-torsional buckling moment is determined by the following relations:

$$M_{cr} = c_1\sqrt{\frac{\pi^2 EI_y}{(KL)^2}\left(GI_t+\frac{\pi^2 EI_w}{(KL)^2}\right)} \quad \ldots(10.15)$$

or by using,

$$M_{cr} = \beta_b Z_p f_{cr,b} \quad \ldots(10.16)$$

Where,

$$f_{cr,b} = \frac{1.1\pi^2 E}{(L_{LT}/r_y)^2}\left[1+\frac{1}{20}\left(\frac{L_{LT}/r_y}{h_f/t_f}\right)^2\right]^{1/2}$$

or by,

$$M_{cr} = c_1\frac{\pi^2 EI_y h_f}{2L_{LT}^2}\left[1+\frac{1}{20}\left(\frac{L_{LT}/r_y}{h_f/t_f}\right)^2\right]^{1/2} \quad \ldots(10.17)$$

where the factor c_1 is used to account for different loading and support conditions. Eq. (10.16) can be used for prismatic members made of standard rolled I-sections or welded doubly symmetric I-sections.

Step-8. Once the value of M_{cr} has been determined, the design flexural strength and design flexural stress can be determined.

Step-9. The member is then checked for overall resistance for combined axial bending and axial compression as per Eqs. (10.13) and (10.14).

10.8 Procedure for the Design of Tension Members Subjected to Bending Moment

- Often a tension member is subjected to bending moment along with direct tension due to one or more of the following reasons:
  - (a) The member may not be perfectly straight.
  - (b) The member may not be vertical which in fact gives rise to flexural stresses due its self-weight.
  - (c) Eccentric connections induce flexural moments and thus flexural stresses.
- However it is quite advantageous that flexural stress in a tension member is not quite significant since direct pull of a tension member tends to decrease the lateral deflection caused by bending.
- But a member subjected to both axial tension and flexure must be proportioned for lateral-torsional buckling under reduced effective moment M_{eff} due to tension and flexure.

For the design of tension member subjected to axial load and flexure, the following procedure is adopted:

Step-1. The factored axial tension and bending moment on the member are computed.

Step-2. The net area required to carry the factored axial tension T is given by,

$$A_n = \frac{T}{0.9(f_u/\gamma_{m1})}$$

Where, T = Factored design tensile force
f_u = Ultimate strength of the material
γ_{m1} = 1.25

Step-3. The net area so computed above is increased by 50 to 100% roughly to account for flexural moment and further by 25 to 40% to compute the gross sectional area.

The gross sectional area can also be determined from the yield strength of the gross section by,

$$A_g = \frac{T}{(f_y/\gamma_{m0})}$$

f_y = Yield strength of the material, γ_{m0} = 1.1

The gross area so computed is increased by 50 to 100% to account for bending moment.

Step-4. From the steel tables or otherwise, a suitable section or a built up section is selected with cross-sectional area equal to or slightly more than the gross area required computed above.

Step-5. The reduced effective moment is computed using the expression as given below:

$$M_{eff} = M - \frac{\psi T Z_{ec}}{A}$$

Where M = Factored flexural moment
T = Factored tensile force
A = Cross-sectional area
Z_{ec} = Elastic sectional modulus w.r.t. extreme compression fiber
ψ = 0.8 if T and M can vary independently
= 1 otherwise

Step-6. The reduced effective moment obtained above must be less than the bending strength due to lateral-torsional buckling (M_d).

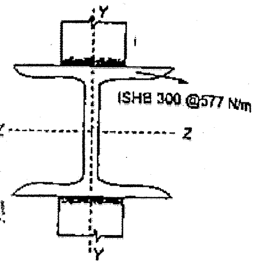
$$M_d = \beta_b Z_p f_{bd}$$

Where, f_{bd} = Design bending compressive stress
Z_e, Z_p = Elastic and plastic section moduli respectively w.r.t. to extreme compression fiber
β_b = 1 for plastic and compact sections
= Z_e/Z_p for semi compact sections

Example 10.1 ISHB 300 @ 577 N/m in a framed structure acts as a column and supports two beams as shown at its top. The beams are welded to the column flanges and transfer a reaction of 220 kN and 365 kN. In addition to that, column carries an axial head of 665 kN coming from the top floors. The bottom end of the column also carries a similar beam-column arrangement and loading also. The effective length of the column is 3.05 m w.r.t. both the axes. Check the adequacy of the column. Assume loads as factored loads. Assume Fe410 steel.

Solution:

For steel of grade Fe410,



$$f_u = 410 \text{ N/mm}^2$$
$$f_y = 250 \text{ N/mm}^2$$

Partial factor of safety for materials, $\gamma_{m0} = 1.1$

For rolled I-sections, the relevant bucking curve is '*a*' and corresponding imperfection factor $(\alpha_{LT}) = 0.21$

The relevant section properties of ISMB 300 @ 577 N/m are as follows:

Cross-sectional area, $A = 7425 \text{ mm}^2$

Depth of section, $h = 300$ mm

Flange width, $b_f = 250$ mm

Flange thickness, $t_f = 10.6$ mm

Web thickness, $t_w = 7.6$ mm

MOI about *Z-Z*, $I_z = 12545.2 \times 10^4 \text{ mm}^4$

MOI about *Y-Y*, $I_y = 2193.6 \times 10^4 \text{ mm}^4$

Radius of gyration about *Z-Z*, $\lambda_z = 129.5$ mm

MOI about *Y-Y*, $\lambda_y = 54.1$ mm

Section modulus about *Z-Z*, $Z_z = 836.3 \times 10^3 \text{ mm}^3$

Section modulus about *Y-Y*, $Z_y = 175.5 \times 10^3 \text{ mm}^3$

Radius of root, $r_1 = 11$ mm

Plastic section modulus, $Z_p = 924.7 \times 10^3 \text{ mm}^3$

∴ Depth of web, $d = h - 2(t_f \times r_1) = 300 - 2(10.6 + 11) = 256.8$ mm

Design Forces

Total maximum factored compressive load

$$= 665 + 220 + 365 = 1250 \text{ kN}$$

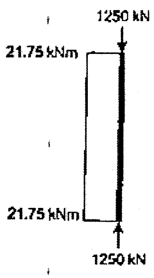
Assuming reaction from beam gets transferred to column at the location of column flange.

Total maximum factored bending moment

$$= \left(365 \times \frac{300}{2} - 220 \times \frac{300}{2}\right) \text{ kN.mm} = 21.75 \text{ kNm}$$

Because of identical beam arrangements and loading conditions at the top and bottom of column ends, the bending moment will also be same at the column bottom.

Classification of Section



$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

Flange outstand, $b = \frac{b_f}{2} = \frac{250}{2} = 125$ mm

$$\frac{b}{t_f} = \frac{125}{10.6} = 11.792$$
$$< 15.7\,\epsilon\ (= 15.7)$$
$$> 10.5\,\epsilon\ (= 10.5)$$

Thus flange is semi-compact.

$$\frac{d}{t_w} = \frac{256.8}{7.6} = 33.789 < 87\epsilon\ (= 84)$$

Thus web is plastic

∴ Section is semi-compact.

Check for adequacy of the section in local capacity

Factored axial compressive load $(N) = 1250$ kN.

Factored bending moment $(M) = 21.75$ kNm

Design compressive strength due to yielding (N_d)

$$= \frac{A_g f_y}{\gamma_{m0}} = \frac{7485 \times 250}{1.1} N = 1701.14 \text{ kN}$$

Design bending strength under corresponding moment acting alone about *z*-axis.

$$M_{dz} = \beta_b Z_{pz} \frac{f_y}{\gamma_{m0}}$$

For semi-compact sections, $\beta_b = \frac{Z_{ez}}{Z_{pz}}$

$$\therefore \quad M_{dz} = \frac{Z_{ez}}{Z_{pz}} Z_{pz} \frac{f_y}{\gamma_{m0}} = \frac{Z_{ez} f_y}{\gamma_{m0}} = \frac{836.3 \times 10^3 \times 250}{1.1} \text{ N.mm}$$
$$= 190.068 \text{ kNm}$$

$$\therefore \quad \frac{N}{N_d} + \frac{M_z}{M_{dz}} \le 1$$

$$\Rightarrow \quad \frac{1250}{1701.14} + \frac{21.75}{190.068} \le 1$$

$$\Rightarrow \quad 0.7348 + 0.1144 \le 1$$

$$\Rightarrow \quad 0.8492 \le 1 \text{ which is true}$$

Thus section is safe in local capacity criterion.

Check for buckling resistance in compression.

$$\frac{h}{b_f} = \frac{300}{250} = 1.2 \le 1.2$$
$$t_f = 10.6 \text{ mm} \le 100 \text{ mm}$$

Thus from concepts of chapter 5, the curve applicable for bending about z-z axis is curve '*b*' and the bending about *y-y* axis, the applicable curve is curve '*c*'.

Given effective length of member is same about both the axes.

Also $r_y < r_z$

∴ effective slenderness ratio $= \frac{KL_y}{r_y} = \frac{3.05 \times 1000}{54.1} = 56.377$

Thus for $\frac{KL_y}{r_y} = 56.377$

and $\quad f_y = 250\ \text{N/m}^2$

and bucking curve 'c', the design compressive stress,

$$f_{cd} = 173.4345\ \text{N/m}^2$$

∴ Design compressive strength

$$P_{dy} = A_e \cdot f_{cd}$$

$$= 7485 \times 173.4345\ \text{N} = 1298.16\ \text{kN} > 1250\ \text{kN}$$

$$\frac{KL_z}{r_z} = \frac{3.05 \times 1000}{129.5} = 23.552$$

and $\quad f_y = 250\ \text{N/mm}^2$

and for buckling curve '*b*', design compressive stress (f_{cd})

$$= 221.8032\ \text{N/mm}^2$$

∴ Design compressive strength in *Z-Z*,

$$P_{dz} = A_e f_{cd} = 7485 \times 221.8032\ \text{N}$$

$$= 1660.2\ \text{kN} > 1250\ \text{kN} \qquad \text{(OK)}$$

Check for member buckling resistance in bending

Lateral-torsional moment, $\quad M_{cr} = C_1\sqrt{\frac{\pi^2 EI_y}{L_{LT}^2}\left(GI_t + \frac{\pi^2 EI_w}{L_{LT}^2}\right)}$

Because of equal moments at both the column ends,

$$C_1 = 1.0$$

Effective span $\quad L_{LT} = KL = 3050\ \text{mm}$

$$E = 2 \times 10^5\ \text{N/mm}^2$$

$$G = \frac{mE}{2(m+1)}$$

where $\quad \frac{1}{m} = \mu = 0.3$ (say)

∴ $$G = \frac{E}{2\left(1+\frac{1}{m}\right)} = \frac{2 \times 10^5}{2(1+0.3)} = 76.92 \times 10^3\ \text{N/mm}^2$$

St. Vanent's equation

$$I_t = \sum \frac{b_i t_i^3}{3} = 2 \times \frac{b_f t_f^3}{3} + \frac{(h-t_f)t_w^3}{3}$$

$$= \frac{2 \times 250 \times 10.6^3}{3} + \frac{(300-10.6)7.6^3}{3} = 24.085 \times 10^4\ \text{mm}^4$$

Warping constant (I_w) $= (1-\beta_f)\beta_f I_y h_f^2$

where $\quad h_f$ = center-to-center distance between the flanges

$$= 300 - \frac{10.6}{2} - \frac{10.6}{2} = 289.4\ \text{mm}$$

$$\beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} = 0.5 \qquad (\because I_{fc} = I_{ft})$$

∴ $\quad I_w = (1-0.5)\,0.5\,(2193.6 \times 10^4)(289.4)^2 = 4.593 \times 10^{11}\ \text{mm}^6$

∴ $$M_e = C_1\sqrt{\frac{\pi^2 EI_y}{L_{LT}^2}\left(GI_t + \frac{\pi^2 EI_w}{L_{LT}^2}\right)}$$

$$= 1 \times \sqrt{\frac{\pi^2 \times 2 \times 10^5 \times 2193.6 \times 10^4}{(3050)^2}\left(76.92 \times 10^3 \times 24.085 \times 10^4 + \frac{\pi^2 \times 2 \times 10^5 \times 4.523 \times 10^{11}}{(3050)^2}\right)}$$

$$= 2157.\text{[illegible]}\sqrt{1.8526 \times 10^{10} + 9.746 \times 10^{10}} = 7347.6 \times 10^5\ \text{N.mm}$$

Non-dimensional slenderness ratio,

$$\lambda_{LT} = \sqrt{\frac{\beta_b z_{pz} f_y}{M_e}}$$

For semi-compact section, $\quad \beta_b = \frac{z_{ez}}{z_{pz}}$

∴ $$\lambda_{LT} = \sqrt{\frac{z_{ez} f_y}{M_e}} = \sqrt{\frac{836.3 \times 10^3 \times 250}{7347.6 \times 10^5}} = 0.53343 > 0.4$$

$$\lambda_{LT} > 0.4$$

Effect of lateral torsional buckling need to be considered.

∴ $$\phi_{LT} = 0.5\left[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2\right]$$

$$= 0.5[1 + 0.21\,(0.53343 - 0.2) + 0.53343^2] = 0.67728$$

∴ $\quad$ Strength reduction factor $\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}}$

$$= \frac{1}{0.67728 + \sqrt{0.67728^2 - 0.53343^2}} = 0.91357$$

∴ Design flexural compressive stress (f_{bd})

$$= \chi_{LT} \cdot \frac{f_y}{\gamma_{m0}} = 0.91357 \times \frac{250}{1.1}\ \text{N/mm}^2 = 207.629\ \text{N/mm}^2$$

∴ Design flexural capacity in compression,

$$M_{dz} = \beta_b z_{pz} f_{bd} = z_{ez} \cdot f_{bd}$$

$$(\because \beta_b = z_{ez}/z_{pz} \text{ for semi-compact section})$$

$$= 836.3 \times 10^3 \times 207.629\ \text{Nmm}$$

$$= 173.64\ \text{kNm} > 21.75\ \text{kNm} \qquad \text{(OK)}$$

Thus section is safe in flexural buckling.

Check for overall buckling resistance for combined axial compression and bending

$$\frac{P}{P_{dz}} + 0.6K_y \frac{C_{my} M_y}{M_{dy}} + K_z \frac{C_{mz} M_z}{M_{dz}} \le 1.0$$

Here $M_y = 0, P = 1250$ kN

$$P_{dz} = 1660.2 \text{ kN (As computed above)}$$

$$K_z = 1 + (\lambda_z - 0.2)\, n_z \le 1 + 0.8\, n_z$$

where $$n_z = \frac{P}{P_{dz}} = \frac{1250}{1660.2} = 0.7529$$

Non-dimensional slenderness ratio

$$(\lambda_z) = \sqrt{\frac{f_y}{f_{cc,z}}}$$

$$f_{cc} = \frac{\pi^2 E}{(KL_z/r_z)^2} = \frac{\pi^2 \times 2 \times 10^5}{(23.552)^2} = 3558.56 \text{ N/mm}^2$$

$\therefore$ $$\lambda_z = \sqrt{\frac{250}{3558.56}} = 0.2651$$

$\therefore$ $$K_z = 1 + (\lambda_z + 0.2)n_z$$

$$= 1 + (0.2651 + 0.2)\, 0.7529 = 1.35$$

$$1 + 0.8\, n_z = 1 + 0.8\,(0.7529) = 1.602$$

$\therefore$ $$K_z < 1 + 0.8\, n_z \text{ (O.K.)}$$

Equivalent uniform moment factor

$$C_{mz} = 0.6 + 0.4\, \psi_z$$

where $$\psi_z = \frac{M_z}{M_1} = \frac{21.75}{21.75} = 1$$

$\therefore$ $$C_{mz} = 0.6 + 0.4(1) = 1.0 \ge 0.4$$

$\therefore$ $$\frac{P}{P_{dz}} + K_z \frac{C_{mz} M_z}{M_{dz}} = \frac{1250}{1660.2} + 1.35 \frac{(1)21.75}{173.64}$$

$$= 0.7529 + 0.1692 = 0.922 < 1 \qquad \text{(OK)}$$

Thus, overall column section is safe.

Objective Brain Teasers

Q.1 Secondary moments arise due to:
(i) Member effect δ
(ii) Structure effect Δ
For the design of member carrying combined axial load and moment, the secondary moments to be considered in design are:
(a) δ only (b) Δ only
(c) δ − Δ (d) δ + Δ

Q.2 A beam column bent in double curvature as compared to beam column bent in single curvature:
(a) will have less secondary moments
(b) more secondary moments
(c) equal secondary moments
(d) data insufficient

Q.3 Failure of a beam column occurs due to:
(i) Local failure of the section
(ii) Overall instability failure due to flexural yielding
(iii) Overall instability due to flexural torsional buckling
Of the above statements, the correct ones are:
(a) (i) and (iii) (b) (i) and (ii)
(c) (ii) and (iii) (d) (i), (ii) and (iii)

Q.4 In a typical framed structure, the columns of the frame should be designed as:
(a) Compression member
(b) Tension member
(c) Beam column
(d) Beam

Q.5 The interaction equation for beam-column as given in IS Code, takes into account:
(i) the moment amplification factor
(ii) additional stresses produced due to secondary moments
(iii) the second order analysis
Of the above statements, the correct ones are:
(a) (i) and (ii) (b) (i) and (iii)
(c) (ii) and (iii) (d) (i), (ii) and (iii)

Q.6 A typical beam-column member in a framed structure is subjected to:
(a) Axial compressive force only
(b) Axial tension and bending
(c) Axial compression and bending
(d) Axial tension and shear

Q.7 The factor C_m which takes into account the distribution of bending moment in a beam column is known as:
(a) Modification factor
(b) Moment reduction factor
(c) Equivalent moment factor
(d) All of the above

Answers

1. (d) 2. (a) 3. (d) 4. (c) 5. (d)
6. (c) 7. (d)

Conventional Practice Questions

Q.1 Design a beam-column of a moment resisting braced frame for the following data:
Effective height of the column = 4.2 m
Max. axial compressive load = 950 kN
Max. moments:
At top end, M_z = 180 kNm
M_y = 65 kNm
At bottom end, M_z = −180 kNm
M_y = 55 kNm

Q.2 Design a beam-column of effective length 5 m carrying factor loads as follows:
Axial compressive load = 800 kN
M_{Ztop} = 70 kNm
$M_{Zbottom}$ = 70 kNm

Q.3 Design a pinned end column of lengths 5.2 m carrying a load of 550 kN at an eccentricity of 135 mm from the centroidal axis of the column through web.

Q.4 The bottom chord tension member of a roof truss is 3.2 m long and carries a factored axial transfers load of 30 kN at its mid span. Apart from that it also carries an axial force of 380 kN. Design the tension member.

■■■■