Electrostatic Potential And Capacitance

Electrostatic Potential energy

- Work done by an external force in bringing a charge q from a point R to a point P in electric field of a certain charge configuration is UP-UR, which is the difference in potential energy of charge q between the final and initial points.
- Potential energy at a point is the work done by an external force in moving a charge from infinity to that point.

Electrostatic Potential

- Electrostatic potential at any point in a region of electrostatic field is the minimum work done in carrying a unit positive charge (without acceleration) from infinity to that point.
- Electric potential due to a point charge of magnitude *q* at a distance r from the charge is given as V=q4πε0r
- Potential difference between two points P and R can be written as VP-VR =UP-URq

Equipotential Surfaces

An equipotential surface is that surface at every point of which, the electric potential is same.

No work is done in moving a test charge from one point of the equipotential surface to the other.

The equipotential surface for a point charge are concentric spherical surfaces centered at the charge.

Potential due to a System of Charges

- For a system of point charges q_1, q_2, q_3, … q_n at distances r_1, r_2, r_3, … r_n respectively from the point P,
- The potential at a point P is given by the superposition principle

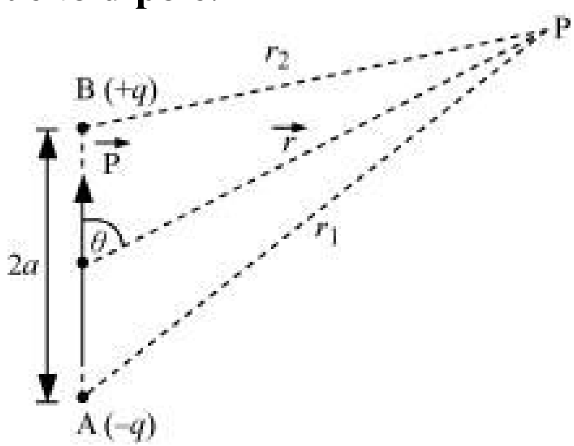
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_n}{r_n}\right)$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

Dipole

- A dipole is a system of two charges of equal magnitude q and opposite polarity separated by a distance $2a$ .The dipole moment of the dipole is $\vec{p}$ with magnitude p=q×2a and direction - q to + q.

Potential due to dipole.



- Potential at point P due to charge at point A is given as:

$$V_1 = \frac{-q}{4\pi\varepsilon_0 r_1}$$

- Potential at point P due to charge at point B is given as:

$$V_2 = \frac{q}{4\pi\varepsilon_0 r_2}$$

- Total Potential at point P due to dipole

$$V = V_1 + V_2$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

V at position vector r can be expressed as

$$\boxed{V = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}} \qquad \text{for}\,(r >> a)$$

Potential Energy of a Single Charge

It is the work done in bringing a charge *q* from infinity to a point *P* whose position vector is r→and V(r→) is potential due to external field there.

The magnitude of work done $= q\cdot V(\vec{r})$.

Potential Energy of a System of Two Charges in an External Field

- It is the sum of the work done in bringing q_1 and q_2 from infinity to r1⇀ and r2→ respectively and assembling the charges at their respective locations.

$$U = W_1 + W_2 + W_3$$

$$\boxed{U = q_1\cdot V(\vec{r}_1) + q_2\cdot V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}}$$

Potential Energy of an Electric Dipole, When Placed in a Uniform Electric Field

- The potential energy of an electric dipole in a uniform electric field is given as $U = -\ \vec{p}\cdot\vec{E}$

where $\vec{p}$ = dipole moment of the dipole
$\vec{E}$ = strength of external electric field

Electrostatics of conductors

- Inside a conductor, the electric field is zero.
- The interior of a conductor can have no excess charge in static situation.
- The electric field on the surface of a charged conductor is perpendicular to the surface of the conductor at every point.
- Electrostatic potential is constant throughout the volume of the conductor, and has the same value as on its surface.

Non-Polar Dielectrics

- When the Non-polar dielectric is placed in an external electric field the two centres of positive and negative charges in the molecule are separated and the non-polar molecule gets polarised.

Polar Dielectrics

When an external electric field is applied, the individual dipole moments tend to align with the field.
A net dipole moment in the direction of the external field is developed
Induced dipole moment P acquired by the molecule may be written as

$$P = \alpha\varepsilon_0 E_0$$

When a dielectric is placed in an external electic field $\vec{E}_0$ due to polarisation there is developmentof an electric field $\vec{E}_P$ opposite to the $\vec{E}_0$

∴ Effective electric field in a polarised dielectric = $E = E_0 - E_P$

Capacitor

- A capacitor is a system of two conductors separated by an insulator.
- Its capacitance, $C = Q/V$, where Q *and* $-Q$ are the charges on the two conductors and V is the potential difference between them.
- C is determined purely geometrically, by the shapes, sizes and relative positions of the two conductors. For a parallel plate capacitor (with vacuum between the plates),

C=ε0Ad

Here, *A* is the area of each plate and *d* is the separation between them.

- If the medium between the plates of a capacitor is filled with an insulating substance (dielectric), the electric field due to the charged plates induces a net dipole moment in the dielectric. This effect is called polarisation and it gives rise to a field in the opposite direction.
- The net electric field inside the dielectric, and hence the potential difference between the plates, is thus reduced. Consequently, the capacitance *C* increases from its value C_0 {when there is no medium (vacuum)} to $C = KC_0$. Here, *K* is the dielectric constant of the insulating substance.
- Capacitors are of the following types:
  - Parallel plate capacitor
  - Cylindrical capacitor
  - Spherical capacitor
- Capacitance of cylindrical capacitor without dielectric is given as C=2πεollnba
- Capacitance of spherical capacitor without dielectric is given as C=4πεoabb-a

Capacitors in Series

- In series connection the potential difference applied across the combination is the sum of the resulting potential differences across each capacitor.
- In series combination the charge in all of the capacitors is same.
- Total capacitance in a series combnination of the capcitors is given as:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in parallel

- In parallel connection the potential difference is same across each capacitor.
- Total capacitance in parallel combination of the capacitors is given as:

$$C_P = C_1 + C_2 + C_3$$

The Energy U stored in a capacitor of capacitance *C*, with charge *Q* and voltage *V* is

- $U = \frac{Q^2}{2C}$

- $$U = \frac{1}{2}CV^2$$

Van de Graff generator

It works on the principle that charge given to a hollow conductor is transferred to the outer surface and is distributed uniformly on it.

It is a device used for building up high potential differences of the order of a few million volts.

It consists of a large spherical conducting shell. By means of a moving belt and suitable brushes,charge is continuously transferred to the shell and potential difference of the order of several million volts is built up.

It is used as a particle accelerator.