CBSE Test Paper 05

Chapter 2 Inverse Trigonometric Functions

1. If $x+y=\frac{\pi}{4}$ then (1 + tanx)(1 + tany) is equal to
   a. 2
   b. 1
   c. -1
   d. none of these
2. The relation $\cos ec^{-1}\left(\frac{x^2+1}{2x}\right)=2\cot^{-1}x$ is valid for
   a. $x \geqslant 0$
   b. $|x| \geqslant 1$
   c. $x \geqslant 1$
   d. None of these.
3. $\sin\left(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}\right)$ equals
   a. $-\frac{1}{4}$
   b. 0
   c. $\frac{1}{2}$
   d. 1
4. The number of solutions of the equation $\sin^{-1}x-\cos^{-1}x=\sin^{-1}\left(\frac{1}{2}\right)$ is
   a. 2
   b. 1
   c. 3
   d. Infinite.
5. $\sin\left(\cot^{-1}x\right)$ is equal to
   a. None of these
   b. $\frac{x}{\sqrt{1+x^2}}$

c. $\frac{1}{\sqrt{1+x^2}}$

d. $\sqrt{1+x^2}$

6. The value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ is ________.
7. The principle value branch of cosec^{-1}x is ________.
8. The value of $\sin^{-1}\left(cos\left(\frac{33\pi}{5}\right)\right)$ is ________.
9. Find cosec^{-1}(2)
10. Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.
11. Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.
12. Find sin(tan^{-1}x) =. **(2)**
13. Find the value of $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$.
14. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = ?$
15. Prove that: $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$.
16. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$.
17. Prove that cot^{-1}7 + cot^{-1}8 + cot^{-1}18 = cot^{-1}3. **(4)**
18. Prove that $\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$.

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Solution

1. a. 2, **Explanation:** If $x + y = \frac{\pi}{4}$ then (1 + tanx)(1 + tany)
   $\Rightarrow (1+\tan x)(1+\tan(\frac{\pi}{4}-x))$
   $\Rightarrow (1+\tan x)\left(1+\frac{1-\tan x}{1+tanx}\right)$
   $\Rightarrow (1+\tan x)\left(\frac{2}{1+\tan x}\right) = 2$
2. c. $x \geqslant 1$, **Explanation:** The relation is true for all real values of x greater than or equal to 1.
3. d. 1, **Explanation:** We know that $sin^{-1}x + cos^{-1}x = \frac{\pi}{2}$
   $\therefore \sin(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}) = \sin(\frac{\pi}{2}) = 1$
4. b. 1, **Explanation:** $\sin^{-1}x - \cos^{-1}x = \sin^{-1}\frac{1}{2}$
   $\Rightarrow \sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$
   $\Rightarrow \sin^{-1}x - (\frac{\pi}{2} - \sin^{-1}x) = \frac{\pi}{6}$
   $\Rightarrow 2\sin^{-1}x = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$
   $\Rightarrow \sin^{-1}x = \frac{\pi}{3} \Rightarrow x = \frac{\sqrt{3}}{2}$
   Hence, there is only one solution
5. c. $\frac{1}{\sqrt{1+x^2}}$, **Explanation:** $\cot^{-1}x = \theta \Rightarrow x = \cot\theta \Rightarrow \cot\theta = \frac{x}{1}$
   $sin(cot^{-1}x) = \sin\theta = \frac{Perp.}{Hyp.} = \frac{1}{\sqrt{x^2+1}}$.
6. $\frac{2\pi}{5}$
7. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
8. $\frac{-\pi}{10}$
9. Let cosec^{-1}(2) = y
   $\Rightarrow$ cosec y = 2
   $\Rightarrow cosecy = cosec\frac{\pi}{6}$
   Since, the principal value branch of cosec-1 is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.
   Therefore, principal value of cosec^{-1}(2) is $\frac{\pi}{6}$.
10. $\sin^{-1}\left(\sin\frac{3\pi}{5}\right) = ?$

$\sin^{-1}\left(\sin\frac{3\pi}{5}\right) = \sin^{-1}\left[\sin\left(\pi - \frac{3\pi}{5}\right)\right]$

=$\sin^{-1}$(sin $\frac{2\pi}{5}$)

$[\because \sin^{-1}(\sin\theta)] = \theta$

When $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ = $\frac{2\pi}{5}$

11. $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$
$= \cos^{-1}\left(\cos\frac{12\pi+\pi}{6}\right)$
$= \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right]$
$= \cos^{-1}\left(\cos\frac{\pi}{6}\right)$
$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}$

12. Let $\tan^{-1}x = \theta$
$\frac{x}{1} = \tan\theta$
$\sin\theta = \frac{x}{\sqrt{1+x^2}}$
$\theta = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$
$\Rightarrow \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$
$\sin(\tan^{-1}x) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{\sqrt{1+x^2}}$

13. We have, $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\tan\left(\pi - \frac{\pi}{3}\right)$
$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right)\ [\because \tan^{-1}(-x) = -\tan^{-1}x]$
$= \tan^{-1}\tan\ \left(\frac{-\pi}{3}\right) = -\frac{\pi}{3}\ \left[\because \tan^{-1}(\tan x) = x,\ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$
Note: Remember that, $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$
Since, $\tan^{-1}$(tan x) = x, if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\frac{2\pi}{3} \notin \left(\frac{-\pi}{2},\ \frac{\pi}{2}\right)$

14. $\tan^{-1}\left(\tan\frac{3\pi}{\pi}\right) \neq \frac{3\pi}{4}$ as $\frac{3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$\because \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$
$= \tan^{-1}\left[-\tan\left(\frac{\pi}{4}\right)\right] = -\frac{\pi}{4}$

15. We know that $1 + \sin x = \cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\cos\frac{x}{2}\sin\frac{x}{2} = \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2$
Again, $1 - \sin x = \cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\cos\frac{x}{2}\sin\frac{x}{2}$
$= \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2$
$= \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$$= \cot^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$$
$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$$
$$= \cot^{-1}\cot\frac{x}{2} = \frac{x}{2}$$

16. Put $x = \cos 2\theta$

$$\text{L.H.S} = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$
$$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$
$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$
$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]$$
$$= \frac{\pi}{4} - \theta \Rightarrow \frac{\pi}{4} - \frac{1}{2}\cos^{-1}$$

17. We have $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18$

$$= \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} \left(\text{since } \cot^{-1}x = \tan^{-1}\frac{1}{x},\ \text{if } x > 0\right)$$
$$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right) + \tan^{-1}\frac{1}{18} \left(\text{since } x, y = \frac{1}{7}\cdot\frac{1}{8} < 1\right)$$
$$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18} = \tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\times\frac{1}{18}}\right) \text{ (since xy < 1)}$$
$$= \tan^{-1}\frac{65}{195} = \tan^{-1}\frac{1}{3} = \cot^{-1}3$$

18. Put $\cos^{-1}x = \theta \Rightarrow x = \cos\theta$

$$\cos^{-1}(\cos\theta) = 2\sin^{-1}\sqrt{\frac{1-\cos\theta}{2}} = 2\cos^{-1}\sqrt{\frac{1+\cos\theta}{2}}$$
$$\theta = 2\sin^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2}} = 2\cos^{-1}\sqrt{\frac{2\cos^2\frac{\theta}{2}}{2}}$$
$$\theta = 2\sin^{-1}\left(\sin\frac{\theta}{2}\right) = 2\cos^{-1}\left(\cos\frac{\theta}{2}\right)$$
$$\theta = 2.\frac{\theta}{2} = 2.\frac{\theta}{2}$$

$\theta = \theta = \theta$ **Hence Proved.**