

14.ELECTROSTATICS

Coulomb force between two point charges $F = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1q_2}{|r|^3} r = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1q_2}{|r|^2} r$

- The electric field intensity at any point is the force experienced by unit positive charge, given by $E = \frac{F}{q_0}$

Electric force on a charge 'q' at the position of electric field intensity E produced by some source charges is $F = qE$

Electric Potential

If $(W_{ext})_P$ is the work required in moving a point charge q from infinity to a point P , the electric potential of the point P is

$$V_P = \frac{(W_{ext})_P}{q} \quad \text{acc}=0$$

Potential Difference between two points A and B is

$$V_A - V_B$$

Formulae of E and potential V

(i) Point charge $E = \frac{Kq}{|r|^2} r = \frac{Kq}{r^3} r, V = \frac{Kq}{r}$

(ii) Infinitely long line charge $\frac{\lambda}{2\pi\epsilon_0 r} r = \frac{2K\lambda r}{r}$
 $V = \text{not defined}, V_B - V_A = 2K\lambda \ln(r_B/r_A)$

(iii) Infinite nonconducting thin sheet $\frac{\sigma}{2\epsilon_0} n$,

$$V = \text{not defined}, V_B - V_A = -\frac{\sigma}{2\epsilon_0} (r_B - r_A)$$

(iv) Uniformly charged ring

$$E_{ext} = \frac{KQx}{(R^2 + x^2)^{3/2}}, \quad E_{int} = 0$$

$$V_{ext} = \frac{KQ}{\sqrt{R^2 + x^2}}, \quad V_{int} = \frac{KQ}{R}$$

x is the distance from centre along axis.

(v) Infinitely large charged conducting sheet $\frac{\sigma}{\epsilon_0} n$

$$V = \text{not defined}, V_B - V_A = -\frac{\sigma}{\epsilon_0} (r_B - r_A)$$

(vi) Uniformly charged hollow conducting/ nonconducting /solid conducting sphere

(a) for $E = \frac{kQ}{|r|^2} r, r > R, V = \frac{KQ}{r}$

(b) $E = 0$ for $r < R, V = \frac{KQ}{R}$

(vii) Uniformly charged solid nonconducting sphere (insulating material)

$$(a) \quad E = \frac{kQ}{|r|^2} \text{ for } r > R, \quad V = \frac{kQ}{r}$$

$$(b) \quad E = \frac{KQr}{R^3} = \frac{\rho r}{3\epsilon_0} \text{ for } r < R, \quad V = \frac{\rho}{6\epsilon_0} (3R^2 - r^2)$$

(viii) thin uniformly charged disc (surface charge density is σ)

$$E_{\text{axis}} = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{x}{\sqrt{R^2 + x^2}} \right) \quad V_{\text{axis}} = \frac{\sigma}{2\epsilon_0} \left(\sqrt{R^2 + x^2} - x \right)$$

Work done by external agent in taking a charge q from A to B is

$$(W_{\text{ext}})_{AB} = q (V_B - V_A) \text{ or } (W_{\text{ext}})_{AB} = q (V_A - V_B).$$

The electrostatic potential energy of a point charge

$$U = qV$$

U = PE of the system =

$$\frac{U_1 + U_2 + \dots}{2} = (U_{12} + U_{13} + \dots + U_{1n}) + (U_{23} + U_{24} + \dots + U_{2n}) + (U_{34} + U_{35} + \dots + U_{3n}) \dots$$

$$\text{Energy Density} = \frac{1}{2} \epsilon E^2$$

$$\text{Self Energy of a uniformly charged shell} = U_{\text{self}} = \frac{KQ^2}{2R}$$

$$\text{Self Energy of a uniformly charged solid non-conducting sphere} = U_{\text{self}} = \frac{3KQ^2}{5R}$$

Electric Field Intensity Due to Dipole

$$(i) \text{ on the axis } E = \frac{2KP}{r^3}$$

$$(ii) \text{ on the equatorial position : } E = \frac{KP}{r^3}$$

(iii) Total electric field at general point O (r, θ) is

$$E_{\text{net}} = \frac{KP}{r^3} \sqrt{1 + 3\cos^2 \theta}$$

Potential Energy of an Electric Dipole in External Electric Field :

$$U = - \vec{p} \cdot \vec{E}$$

Electric Dipole in Uniform Electric Field :

$$\text{torque } \vec{\tau} = \vec{p} \times \vec{E}; \quad \vec{F} = 0$$

Electric Dipole in Nonuniform Electric Field:

$$\text{torque } \vec{\tau} = \vec{p} \times \vec{E}; \quad U = -p E, \text{ Net force } |F| = \left| p \frac{E}{r} \right|$$

Electric Potential Due to Dipole at General Point (r, θ) :

$$V = \frac{P \cos \theta}{4\pi\epsilon_0 r^2} = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$$

The electric flux over the whole area is given by

$$\phi_{\square} = \oint_S \vec{E} \cdot d\vec{S} = \int_S E_n dS$$

Flux using Gauss's law, Flux through a closed surface

$$\phi_{\square} = \oint_S \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}.$$

Electric field intensity near the conducting surface

$$= \frac{\sigma}{\epsilon_0} \hat{n}$$

Electric pressure : Electric pressure at the surface of a conductor is given by formula

$$P = \frac{\sigma^2}{2\epsilon_0} \text{ where } \sigma \text{ is the local surface charge density.}$$

Potential difference between points A and B

$$V_B - V_A = \int_A^B \vec{E} \cdot d\vec{r}$$

$$\begin{aligned} \vec{E} &= -i \frac{\partial V}{\partial x} - j \frac{\partial V}{\partial y} - k \frac{\partial V}{\partial z} = -i \frac{\partial V}{\partial x} - j \frac{\partial V}{\partial y} - k \frac{\partial V}{\partial z} \\ &= -\text{grad } V \end{aligned}$$