CBSE Test Paper 05

CH-05 Complex & Quadratic

1. If ω is a cube root of unity , then the linear factors of $x^3 + y^3$ in complex numbers are
   a. $(x+y)(x+y\omega)\left(x+y\omega^2\right)$
   b. $(x-y)(x+y\omega)\left(x+y\omega^2\right)$
   c. $(x+y)(x-y\omega)\left(x-y\omega^2\right)$
   d. $(x+y)(x+y\omega)\left(x-y\omega^2\right)$
2. If (x + iy) (3 – 4i) = (5 + 12i) , then $\sqrt{x^2+y^2} =$
   a. 16
   b. 13/5
   c. 65
   d. 5/13
3. Multiplicative inverse of the non zero complex number x + iy $(x, y \in R,)$
   a. none of these
   b. $\frac{x}{x+y} - \frac{y}{x+y}i$
   c. $\frac{x}{x^2+y^2} - \frac{y}{x^2+y^2}i$
   d. $-\frac{x}{x^2+y^2} + \frac{y}{x^2+y^2}i$
4. The value of $(-1+\sqrt{-3})^2 + (-1-\sqrt{-3})^2$ is
   a. 8
   b. -4
   c. -2
   d. 4
5. The value of $\left(\frac{1+\omega}{\omega^2}\right)^3$ is
   a. 1
   b. -1
   c. none of these
   d. 0
6. Fill in the blanks:

   The unique value of θ such that $-\pi < \theta \leq \pi$ is called the ________ value fo the argument.

7. Fill in the blanks:

   The value of $\sqrt{-25}$ + 3$\sqrt{-4}$ + 2$\sqrt{-9}$ is _______.

8. If | z | = 1, then find the value of $\frac{1+z}{1+\bar{z}}$

9. Find the sum of the complex number (5 + i3), (- 4 - i).

10. Find the multiplicative inverse of the complex numbers -i

11. If z_1 and z_2 are complex numbers, then prove that Re (z_1 z_2) = Re (Z_1) Re (z_2) - Im (z_1) Im (z_2).

12. Solve $\sqrt{2}$ x^2 + x + $\sqrt{2}$ = 0.

13. If z_1 = 3 + 5i and z_2 = 2 - 3i, then verify that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$.

14. If z_1 and z_2 are two complex numbers such that $|z_1|$ = $|z_2|$ and arg (z_1) + arg (z_2) = π, then prove that z_2 = $\overline{-z_1}$

15. If Z_1 is a complex number other than -1 such that $|z_1|$ = 1 and z_2 = $\frac{z_1-1}{z_1+1}$, then show that the real parts of z_2 is zero.

Solution

1. (a) $(x+y)(x+y\omega)\left(x+y\omega^2\right)$

   Explanation:
   We have $x^3+y^3=(x+y)\left(x^2-xy+y^2\right)$

   Now consider $x^2-yx+y^2=0$ which is a quadratic equation in x with a=1,b=-y and c=y^2

   Hence the roots are

   $x=\frac{-(-y)\pm\sqrt{(-y)^2-4.1.y^2}}{2.1}=\frac{y\pm\sqrt{-3y^2}}{2}=\frac{y\pm iy\sqrt{3}}{2}=\frac{y(1\pm i\sqrt{3})}{2}$
   $\Rightarrow x=y\omega \quad or \quad y\omega^2$

   Hence $x^3+y^3=(x+y)\left(x^2-xy+y^2\right)=(x+y)(x+y\omega)\left(x+y\omega^2\right)$

2. (b) 13/5

   Explanation:

   Given (x + iy) (3 – 4i) = (5 + 12i)
   $\Rightarrow x+iy=\frac{5+12i}{3-4i}=\frac{(5+12i)(3+4i)}{(3-4i)(3+4i)}$
   $\frac{(15-48)+i(20+36)}{9+16}=\frac{-33+56i}{25}$
   $\Rightarrow x=\frac{-33}{25},y=\frac{56}{25}$
   $\therefore\sqrt{x^2+y^2}=\sqrt{\left(\frac{-33}{25}\right)^2+\left(\frac{56}{25}\right)^2}$
   $=\sqrt{\frac{1089+3136}{625}}=\sqrt{\frac{4225}{625}}=\frac{65}{25}=\frac{13}{5}$

3. (c) $\frac{x}{x^2+y^2}-\frac{y}{x^2+y^2}i$

   Explanation:

   Multiplicative inverse of the complex number x + iy =

$$\frac{1}{x+iy}=\frac{1}{x+iy}\cdot\frac{x-iy}{x-iy}=\frac{x-iy}{x^2-(iy)^2}=\frac{x-iy}{x^2+y^2} \qquad [\because i^2=-1]$$

4. (b) -4

   Explanation:

   We have $\sqrt{-3}=\sqrt{-1.3}=\sqrt{-1}\sqrt{3}=\sqrt{3}i$

   $\therefore -1+\sqrt{-3}=-1+i\sqrt{3}=2\omega \quad and \quad -1-\sqrt{-3}=-1-i\sqrt{3}=2\omega^2$

   $(-1+\sqrt{-3})^2+(-1-\sqrt{-3})^2=(2\omega)^2+\left(2\omega^2\right)^2=4\omega^2+4\omega^4=4\left(\omega^2+\omega^3.\omega\right)$

5. (b) -1

   Explanation:

   $\left(\frac{1+\omega}{\omega^2}\right)^3=\left(\frac{-\omega^2}{\omega^2}\right)^3=(-1)^3=-1 \qquad [\because \quad 1+\omega+\omega^2=0]$

6. principal

7. 17i

8. Given, $|z|$ = 1

   $\Rightarrow |z|^2$ = 1 $\Rightarrow z\overline{z}$ = 1

   Now, $\frac{1+z}{1+\overline{z}}=\frac{z\overline{z}+z}{1+\overline{z}}$ [$\because$ 1 = $z\overline{z}$]

   = $\frac{z(\overline{z}+1)}{(\overline{z}+1)}$ = z = 1

9. $(5+i3)+(-4-i)$ = (5 - 4) + i (3 - 1) = 1 + i2

10. M.I. of $-i=\frac{1}{-i}=\frac{i}{-i^2}=\frac{i}{-(i)}=i$

11. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

    Then, $z_1 z_2 = (x_1 x_2 - y_1 y_2) + i (x_1 y_2 + y_1 x_2)$

    $\therefore$ Re $(z_1 z_2) = x_1 x_2 - y_1 y_2$

    = Re (z_1) Re (z_2) - Im (z_1) Im (Z_2)

    Hence proved.

12. We have, $\sqrt{2}\ x^2 + x + \sqrt{2} = 0$...(i)

On comparing Eq. (i) with $ax^2 + bx + c = 0$, we get

$a = \sqrt{2}$, $b = 1$ and $c = \sqrt{2}$

Then, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{(1)^2 - 4 \times \sqrt{2} \times \sqrt{2}}}{2 \times \sqrt{2}}$

$= \frac{-1 \pm \sqrt{1-8}}{2\sqrt{2}}$

$= \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}}$

$= \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$

$x = \frac{-1+\sqrt{7}i}{2\sqrt{2}}$ or $x = \frac{-1-\sqrt{7}i}{2\sqrt{2}}$

Hence, the roots of the given equation are $\frac{-1+\sqrt{7}i}{2\sqrt{2}}$ and $\frac{-1-\sqrt{7}i}{2\sqrt{2}}$.

13. Now, $\frac{z_1}{z_2} = \frac{3+5i}{2-3i} = \frac{3+5i}{2-3i} \times \frac{2+3i}{2+3i}$

[multiplying numerator and denominator by conjugate of 2 - 3i i.e., 2 + 3i]

$= \frac{6+9i+10i+15i^2}{4-9i^2} = \frac{6+19i-15}{4+9}$ [$\because i^2 = -1$]

$= \frac{-9+19i}{13} = \overline{\frac{-9}{13} + \frac{19}{13}i}$

LHS $= \overline{\frac{z_1}{z_2}} = \overline{\frac{-9}{13} + \frac{19}{13}i} = \frac{-9}{13} - \frac{19}{13}i$

RHS $= \frac{\overline{z_1}}{\overline{z_2}} = \frac{\overline{3+5i}}{\overline{2-3i}} = \frac{3-5i}{2+3i} = \frac{3-5i}{2+3i} \times \frac{2-3i}{2-3i}$

[multiplying numerator and denominator by conjugate of 2 + 3i i.e., 2 - 3i]

$= \frac{6-9i-10i+15i^2}{4-9i^2} = \frac{6-19i-15}{4+9}$

$= \frac{-9-19i}{13} = \frac{-9}{13} - \frac{19}{13}i$

$\therefore \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

Hence verified.

14. Let, $z_1 = r_1 (\cos\theta_1 + i \sin\theta_1)$

and $z_2 = r_2 (\cos\theta_2 + i \sin\theta_2)$

Since, $|z_2| = |z_1|$

$\therefore r_2 = r_1 = r$ [say]

Also, $\arg (z_1) + \arg (z_2) = \pi$

$\therefore$ arg (z_2) = π - arg (z_1)

$\Rightarrow$ arg (z_2) = π - θ_1

$\therefore$ z_2 = r {cos (π - θ_1) + i sin (π - θ_1)}

= r (- cosθ_1 + i sinθ_1)

= - r (cosθ_1 - i sinθ_1) = $-\overline{z}_1$

$\Rightarrow$ z_2 = $-\overline{z}_1$

Hence proved.

15. Let, $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

Given: $|z_1| = 1$

$\therefore x_1^2 + y_1^2 = 1$

Also, $z_2 = \frac{z_1 - 1}{z_1 + 1}$

$\therefore x_2 + iy_2 = \frac{x_1 + iy_1 - 1}{x_1 + iy_1 + 1}$

$\Rightarrow x_2 + iy_2 = \frac{x_1 - 1 + iy_1}{x_1 + 1 + iy_1}$

$\Rightarrow x_2 + iy_2 = \frac{(x_1 - 1 + iy_1)(x_1 + 1 - iy_1)}{(x_1 + 1 + y_1)(x_1 + 1 - iy_1)}$ [Rationalizing the denominator]

$\Rightarrow x_2 + iy_2 = \frac{(x_1 - 1)(x_1 + 1) - iy_1(x_1 - 1) + iy_1(x_1 + 1) + y_1^2}{(x_1 + 1)^2 - (iy_1)^2}$

$\Rightarrow x_2 + iy_2 = \frac{x_1^2 - 1 + y_1^2 - iy_1 x_1 + iy_1 + iy_1 x_1 + iy_1}{(x_1 + 1)^2 - (iy_1)^2}$

$\Rightarrow x_2 + iy_2 = \frac{x_1^2 + y_1^2 - 1 + 2iy_1}{(x_1 + 1)^2 - (iy_1)^2}$

$\Rightarrow x_2 + iy_2 = \frac{1 - 1 + 2iy_1}{(x_1 + 1)^2 - (iy_1)^2} \left[\because x_1^2 + y_1^2 = 1\right]$

$\Rightarrow x_2 + iy_2 = \frac{2iy_1}{(x_1 + 1)^2 - (iy_1)^2} \left[\because x_1^2 + y_1^2 = 1\right]$

Since there is no real in the RHS, therefore $x_2 = 0$.

Thus, the real part of the $z_2 = 0$.