

Chapter 9

Project Planning and Network Analysis

CHAPTER HIGHLIGHTS

- Project planning and network analysis
- Planning terminologies
- Network
- Critical path
- Characteristics of critical path
- Programme evaluation and review technique (PERT)

PROJECT PLANNING AND NETWORK ANALYSIS

Planning is defined as drawing up a method or scheme of acting was done, proceeding, making, etc., developed well in advance.

The activities involved in construction planning are:

1. Defining the scope of work
2. Identifying activities
3. Establishing project duration
4. Defining procedures for controlling and assigning resources
5. Developing appropriate interfaces
6. Updating and revising plans

PLANNING TERMINOLOGIES

Event and Activity

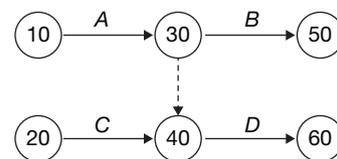
The event is a point in time when certain conditions have been fulfilled, such as the start or completion of one or more activities. Event consumes neither time nor any other resource



Laying of concrete floor where i and j are events

Dummy Activity

This activity does not involve the consumption of resources and, therefore, does not need any time to be completed. It is used to define interdependence between activities and included in a network for logical and mathematical reasons.



where F (30, 40) is a dummy activity

NETWORK

A network consists of nodes and arrows showing logical dependence between them. It can be of two types:

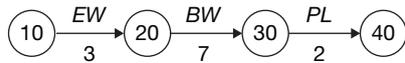
1. Activity on Arrow (AoA)
2. Activity on Node (AoN)

Let us learn more about the networks:

Activity on Arrow (AoA)

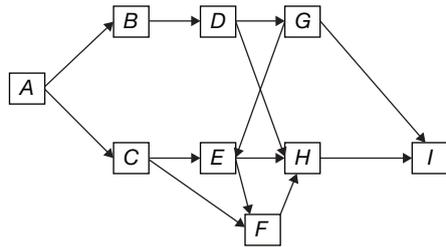
Considering a simple project, it can be broken down into activities, such as earthwork, brickwork and plastering which necessarily follow in order. In this model, each activity is defined in terms of two nodes, such that i and

j represent the start and the end of the activity. The start and end timings of an activity (i, j) need to be calculated separately.



Activity on Node (AoN)

In this type of network, the activities are denoted by circles or boxes called 'nodes', and this immediate predecessor relationship between the two activities is shown by an arrow connecting the two nodes.

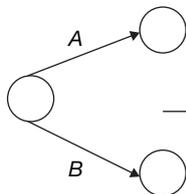


Precedence

This is the logical relationship implying that an activity needs one activity to be completed, before the activity can start. For example, in order to start plastering, the brickwork needs to have been completed, i.e., brickwork precedes plastering.

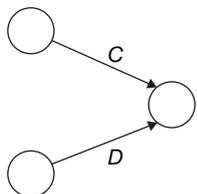
Network Logic

There are many logics which can be used for preparing a network.



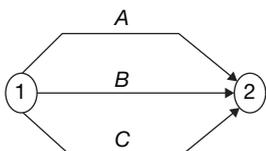
It is the situation wherein two activities A and B are starting in parallel

Example of burst situation



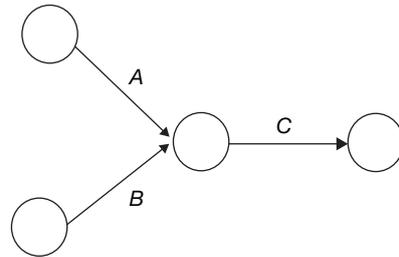
It is the situation wherein two activities c and b get completed together

Example of merge situation

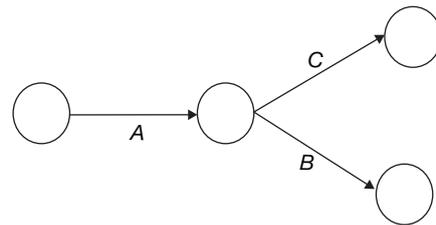


It is the situation where 3 activities are parallel having same initial node and final node, it is an incorrect representation

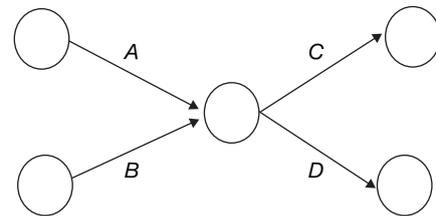
Example of incorrect representation



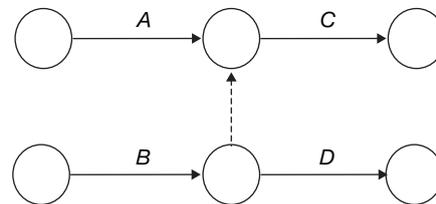
Activity C has A and B as predecessors



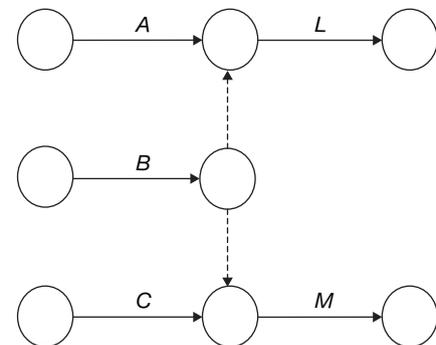
Activities B and C having predecessor A



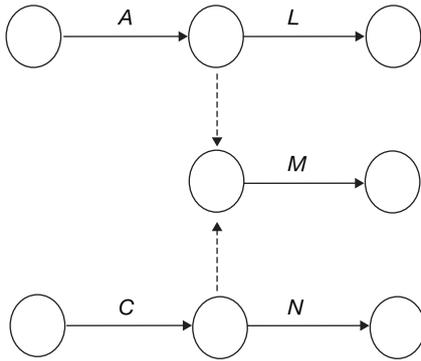
Activities C and D have predecessor activities A and B



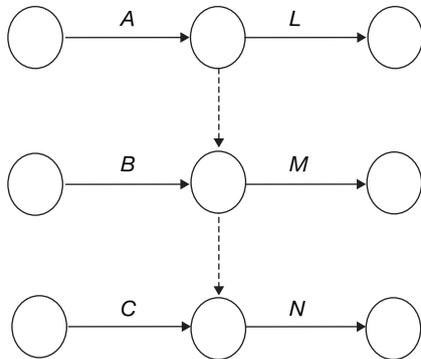
Activity C controlled by activities A and B and activity D controlled by B alone



Activities A and B controls L, and activities B and C control M



Activity *A* controls activities *L* and *M* and activity *C* controls *M* and *N*



L, *M* and *N* are controlled by *A*; *M* and *N* are controlled by *B*; *N* is controlled by *C*

Duration of an Activity

Duration of an activity (i, j) is denoted by $D(i, j)$. This is the duration of time required to carry out an activity (i, j) from the beginning to end. Depending on the activity and the level of detail, the duration may be expressed in days, weeks, or months.

- **Start and finish time:** An activity can be started as soon as the ground work involved has been completed, but the client or contractor may wait for sometime before starting the activity without affecting the overall project completion. In terms of start and finish time, the following terms are considered.
- **Earliest start time of an activity:** It is the earliest, that the activity (i, j) can be started, i.e., all the necessary preconditions are met. Earliest start time is denoted by $EST(i, j)$.
- **Earliest finish time of an activity:** It is the earliest, that the activity (i, j) can be completed. Earliest finish time is denoted by $EFT(i, j)$

$$EFT(i, j) = EST(i, j) + D(i, j)$$

- **Latest finish time of an activity:** This is the time that an activity needs to be completed in order that there is no delay in the project completion. It is, generally, denoted by $LFT(i, j)$.

- **Latest start time of an activity:** This is the ‘latest time’ when an activity must be started so that there is no delay in project completion. Latest start time of an activity (i, j) is denoted by $LST(i, j)$.

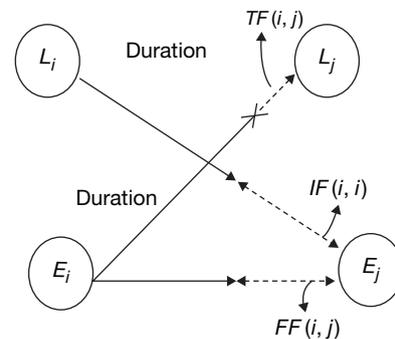
$$LST(i, j) = LFT(i, j) - D(i, j)$$

Float or Slack Time

It is the delay in the start time of the project without affecting the completion of project. There are different types of floats:

Total Float in an Activity

It is the amount of time by which the start of an activity may be delayed without causing a delay in the completion of the project. This is calculated as follows.



Where

E_i and L_i = Early and late occurrence times of event i
 E_j and L_j = Early and late occurrence times of event j

$$TF(i, j) = LST(i, j)$$

or

$$TF(i, j) = LFT(i, j) - EFT(i, j)$$

Free Float

Free float is the amount of time by which the start of an activity may be delayed without delaying the start of the following activity. Free float is defined as the earliest occurrence time E_j of the following event minus earliest occurrence time E_i of the preceding event minus duration of the activity defined between these events.

$$FF(i, j) = E_j - E_i - D(i, j)$$

Independent Float

It is the amount of time by which the start of an activity may be delayed without affecting the preceding or following activity.

$$IF(i, j) = E_j - L_i - D(i, j)$$

Interference Float

It is defined as the difference in total float and free float. In other words:

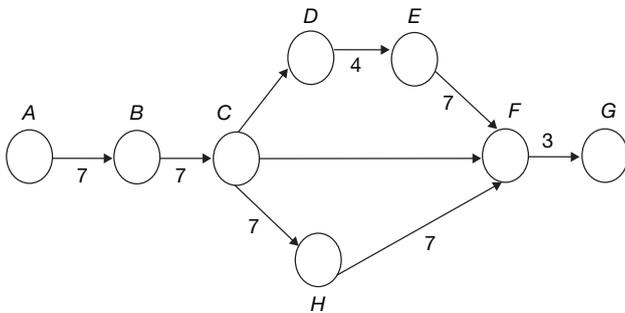
$$IF = TF(i, j) - FF(i, j)$$

- Projects in which activity times can be estimated with sufficient certainty. In these type of projects, time can be estimated using CPM, i.e., critical path method.

CRITICAL PATH

A network consists of chains of activities called ‘paths of network’. The addition of durations of activities on any path gives the duration of the path. It has the longest duration, the overall duration of the project and the activities on the critical path are known as critical activities.

Example:



CHARACTERISTICS OF CRITICAL PATH

- Every network has critical path.
- It is possible to have more than one critical path.
- A critical path is one of the connecting links between the first and final event.
- A critical path may have lesser number of activities compared non-critical paths.
- A critical path may run out through a dummy activity.

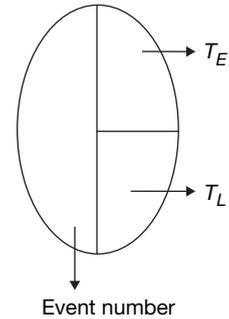
As the number of activities increases, it becomes difficult and time consuming to find the critical path by complete enumeration or inspection. Therefore, for larger networks, a more systematic procedure is needed to determine the critical path.

The most commonly used method employs the following two sets of calculations.

- Forward pass computation
- Backward pass computation

Forward Pass Computation

The convention used in recording of the events time is given as:



Where

- T_E = Earliest time of the event
- T_L = Latest time of the event

The steps involved in forward pass are:

Step 1: The first activity of the project can be started as soon as the project is undertaken which implies that earliest event time of the initial event is zero $T_E = 0$.

Step 2: Each activity of the network is completed as soon as the activity preceding to it is completed.

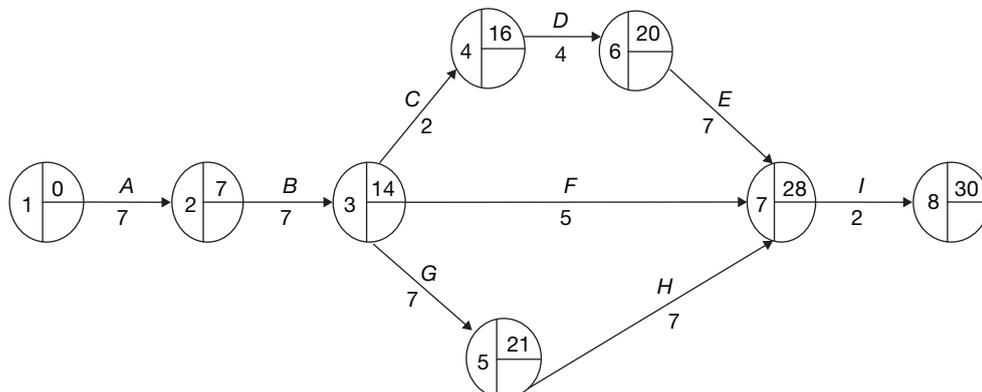
That is, $TE_j = TE_i + t_{ij}$

Where

- TE_j = The earliest event time of event j
- TE_i = The earliest event preceding to event i
- t_{ij} = Duration of the activity from event i to j

Step 3: The earliest event time ‘TE’ of a merge event equals largest of the sum of TE of the preceding event plus duration of the activity from the preceding event.

Example:

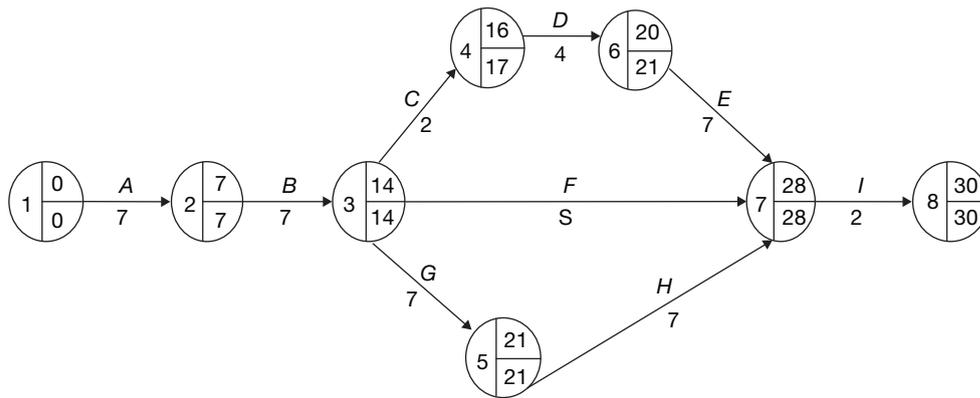


Applying the above rules, we get:

$$\begin{aligned}
 TE_1 &= 0 \text{ (starting event set at zero)} \\
 TE_2 &= TE_1 + t_{23} = 0 + 7 = 7 \\
 TE_3 &= TE_2 + t_{23} = 7 + 7 = 14 \\
 TE_4 &= TE_3 + t_{34} = 14 + 2 = 16 \\
 TE_5 &= TE_3 + t_{35} = 14 + 7 = 21 \\
 TE_6 &= TE_4 + t_{46} = 16 + 4 = 20 \\
 TE_7 &= \max[TE_3 + t_{37}, TE_5 + t_{57}, TE_6 + t_{67}] \\
 &= \max[14 + 5, 21 + 7, 20 + 7] \\
 &= \max(17, 28, 27) = 28 \\
 TE_8 &= TE_7 + t_{78} = 28 + 2 = 30
 \end{aligned}$$

Backward Pass Computation

1. Latest event time of the end event equals to its earliest event time, otherwise the project duration will be affected.



Latest event time of the event 8 (TL_8) = Earliest event time, since it is the end event (TE_8).

$$\begin{aligned}
 TL_7 &= TL_8 - t_{78} = 30 - 2 = 28 \\
 TL_6 &= TL_7 - t_{67} = 28 - 7 = 21 \\
 TL_5 &= TL_7 - t_{57} = 28 - 7 = 21 \\
 TL_4 &= TL_6 - t_{46} = 21 - 4 = 17 \\
 TL_3 &= \min[TL_4 - t_{34}, TL_7 - t_{37}, TL_5 - t_{35}] \\
 &= \min(15, 23, 14) = 14 \\
 TL_2 &= TL_3 - t_{23} = 14 - 7 = 7 \\
 TL_1 &= TL_2 - t_{12} = 7 - 7 = 0
 \end{aligned}$$

PROGRAMME EVALUATION AND REVIEW TECHNIQUE (PERT)

The concept of multiple times (three time estimates) is evolved in PERT to reduce the incidence of uncertainty in project planning. The three time estimates are as follows:

1. **Optimistic time (a):** This indicates the minimum time that an activity can take if everything goes smoothly without any interruptions.

2. The latest event time of a tail event equals the latest time of the succeeding event minus the duration of the activity converging on the head event.

$$TL_i = TL_j - t_{ij}$$

Where

TL_i = The latest allowable time of event i .

TL_j = The latest allowable time of the event j towards which activity (i, j) is headed.

t_{ij} = Duration of the activity from event i to event j .

3. The latest event time of a burst event equals the smallest of the difference between the latest event times of the head event less duration of the activity.

Example:

2. **Pessimistic time (b):** This indicates the maximum time an activity can take under adverse conditions.
3. **Most likely time (m):** This indicates the time an activity can take most often if it is repeated, again and again, under the same conditions. Three time estimates are not directly entered into the network. They are transformed into an expected time using the following relation.

$$\frac{a + 4m + b}{6}$$

Difference Between a PERT Network and CPM Network

PERT	CPM
Event oriented	Activity oriented
Use of three time estimates	Use of one time estimate
Complete and difficult to implement	Easy to use and update
Very good for simulating alternative plan, resource allocations	Very good for scheduling highly sequential work with parallel and independent chains
Most expensive system to maintain	Easy to maintain
It is not related to cost	Activity times in CPM technique are related to cost

EXERCISES

1. Match List I (Description of activity floats) with List II (Names of the floats) and select the correct answer using the codes given the lists:

List I	List II
a. Earliest start time of successor activity minus earliest activity in question minus the duration	1. Total
b. Time available for an activity performance minus the duration of the activity	2. Free
c. Excess of minimum available time over the required activity duration	3. Interfering
d. Difference between total and free float of an activity	4. Independent float

Codes:

- | | |
|-------------|-------------|
| a b c d | a b c d |
| (A) 1 2 3 4 | (B) 1 2 4 3 |
| (C) 2 1 3 4 | (D) 2 1 4 3 |

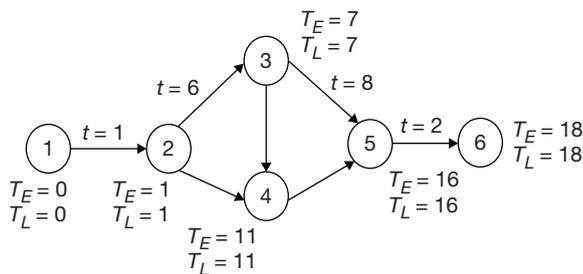
2. Match List I (Items) with List II (Characteristics) and select the correct answer using the codes given below the lists:

List I	List II
a. Activity	1. Resourceless element
b. Event	2. Resource consuming element
c. Dummy	3. Spare time
d. Float	4. Instantaneous stage

Codes:

- | | |
|-------------|-------------|
| a b c d | a b c d |
| (A) 1 4 3 2 | (B) 2 1 4 3 |
| (C) 2 4 1 3 | (D) 3 4 1 2 |

3. The following diagram shows the details necessary for the CPM network analysis:

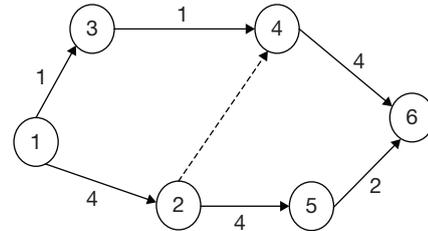


The critical path will be

- (A) 1-2-3-5-6 (B) 1-2-3-4-5-6
 (C) 1-2-4-5-6 (D) 1-2-4-3-5-6
4. The optimistic, most likely, and pessimistic estimates of time for an activity are 4 days, 11 days and 12 days respectively. The expected completion time of this activity is

- (A) 8 days (B) 9 days
 (C) 10 days (D) 11 days

5. From the network shown in the figure below (the number on each arrow denotes the time duration of activity in days), the earliest start time, in days for activity 5-6 is



- (A) 8 (B) 7
 (C) 9 (D) 11

6. The time estimate obtained from four contractors P, Q, R and S for executing a particular job are as under:

Contractor	Optimistic Time, t_o	Most Likely Time, t_L	Pessimistic Time, t_p
P	5	10	13
Q	6	9	12
R	5	10	14
S	4	10	13

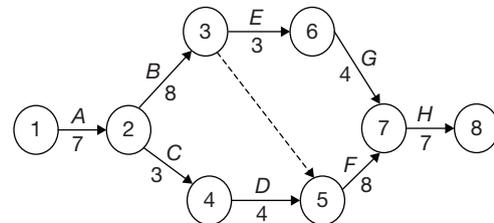
Which one of these contractors is more certain about completing the job in time?

- (A) P (B) Q
 (C) R (D) S

7. The probability distribution taken to represent the completion time in PERT analysis is

- (A) gamma distribution
 (B) normal distribution
 (C) beta distribution
 (D) log-normal distribution

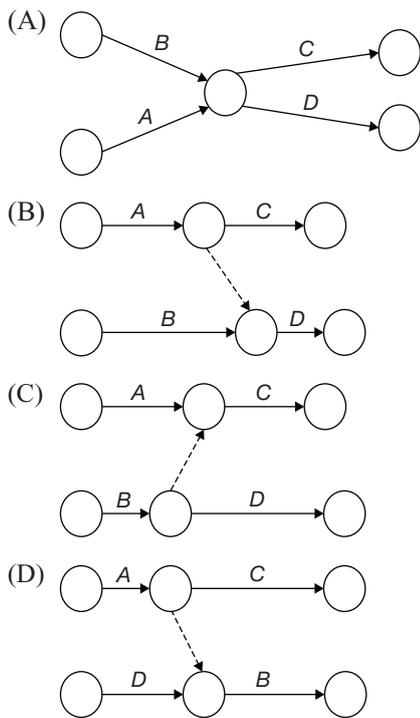
8. The flow net of the activities of a project are shown in the network given below indicating the duration of the activities along their arrows.



The critical path of the activities is along

- (A) 1 → 2 → 4 → 5 → 7 → 8
 (B) 1 → 2 → 3 → 6 → 7 → 8
 (C) 1 → 2 → 3 → 5 → 7 → 8
 (D) 1 → 2 → 4 → 5 → 3 → 6 → 7 → 8

9. Activity 'C' follows activity 'A' and activity 'D' follows activities 'A' and 'B'. The correct network for the project is



10. The optimistic, most likely and pessimistic time estimates of an activity are 5, 10, and 21 days respectively. What are the expected time and standard deviation?
 (A) 12, 3 (B) 11, 4
 (C) 11, 2.67 (D) 10, 16

11. Which one of the following techniques is most suitable in case of research and development type of activity?
 (A) Critical path method
 (B) Project evaluation and review technique
 (C) Bar chart
 (D) Graphical evaluation and review technique

12. PERT calculation yield a project length of 60 weeks with a variance of 9 weeks. Probability factor corresponding to 90% probability is 1.647; then the number of weeks required to complete the project with a probability of 90% is
 (A) 60.94 (B) 62.94
 (C) 64.94 (D) 66.94

13. A father notes that when his teenage daughter uses the telephone she takes no less than 5 minutes for a call but sometimes as much as an hour. 15 minutes calls are more frequent than calls of any other duration. If the daughter's calls were to be represented as an activity in PERT project, the expected duration of each phone call is
 (A) $14\frac{5}{6}$ minutes (B) $16\frac{5}{6}$ minutes
 (C) $18\frac{5}{6}$ minutes (D) $20\frac{5}{6}$ minutes

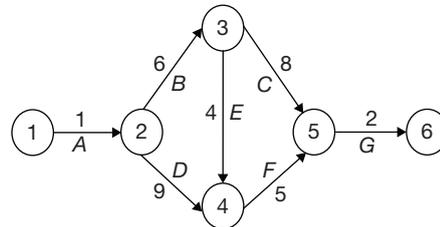
14. Critical path moves along the activities having total float as
 (A) positive
 (B) negative
 (C) zero
 (D) unity

15. The probability distribution taken to represent the completion time in PERT analysis is _____.
 (A) gamma distribution
 (B) normal distribution
 (C) beta distribution
 (D) log normal distribution

16. The probability of completion of any activity within its expected time is _____.
 (A) 50% (B) 84.1%
 (C) 99.9% (D) 100%

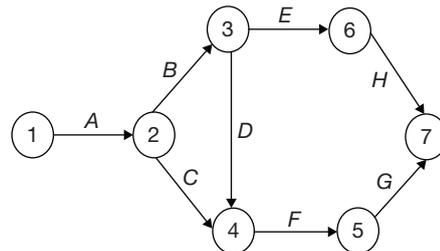
17. Negative slack occurs when
 (A) dummy activities are large in number.
 (B) events stick to their schedule.
 (C) dummy activities do not exist.
 (D) deficiency of resources occurs.

18.



- Duration of the project shown in the network is _____.
 (Note: Durations given above are in days)
 (A) 18 (B) 17
 (C) 16 (D) 15

19. In the network shown in the figure, the activity 'F' can be started only when



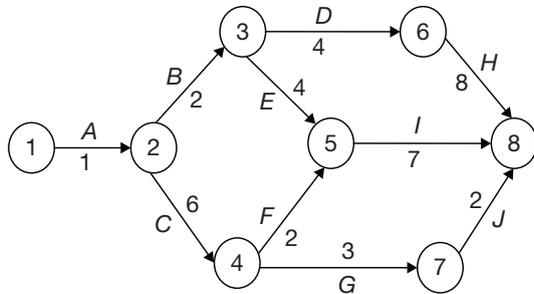
- (A) activity B is completed.
 (B) activity C is completed.
 (C) activity D is completed.
 (D) activity C and D both are completed.
20. In drawing AOA network and making time computations, following processes are involved.
 I. Activity listing
 II. Work breakdown structure
 III. Activity time allotment
 IV. Consideration of available resources for each activity

- V. Activity dependencies
- VI. Float computations
- VII. Backward path computation
- VIII. Project duration
- IX. Forward path computation.

What is the correct sequence of the process given above.

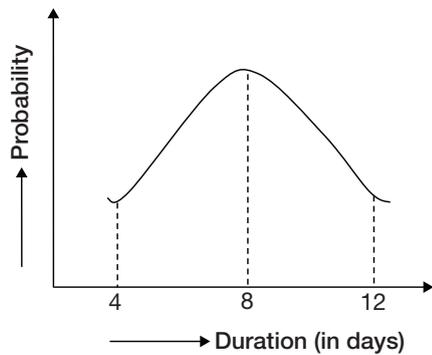
- (A) I, II, IX, III, IX, VII, V, VI, VIII
- (B) IV, I, III, II, IX, V, VII, VIII, VI
- (C) II, I, IX, III, V, IX, VIII, VII, VI
- (D) I, III, II, IV, V, IX, VII, VIII, VI

21. A building project consist of 10 activities, represented by the network shown below. Find the critical path of the network.



- (A) 1-2-4-7-8
- (B) 1-2-3-5-8
- (C) 1-2-4-5-8
- (D) 1-2-3-6-8

22. For a given probability distribution curve,

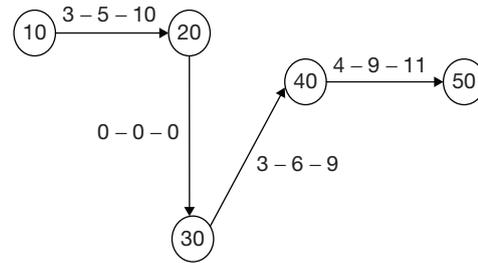


Match the following:

- | | |
|----------------------|--------|
| I. Optimistic time | (a) 12 |
| II. Most likely time | (b) 4 |
| III. Expected time | (c) 8 |
| IV. Pessimistic time | |

- (A) I-b, II-b, III-c, IV-a
- (B) I-a, II-b, III-b, IV-c
- (C) I-a, II-c, III-c, IV-b
- (D) I-b, II-c, III-c, IV-a

23. For the path of a certain network shown below, the expected time and standard deviation will be _____.



- (A) 15 and 1.75
- (B) 20 and 1.93
- (C) 15 and 1.5
- (D) 20 and 1.75

24. For an activity $i-j$, the EST, EFT, LST and LFT are given as are 5, 24, 9, and 29, respectively.

Activity duration is 6. Then, Match the List I, with List II.

List I	List II
a. Free float	1. 5
b. Total float	2. 9
c. Interfering float	3. 13
d. Independent float	4. 18

Codes:

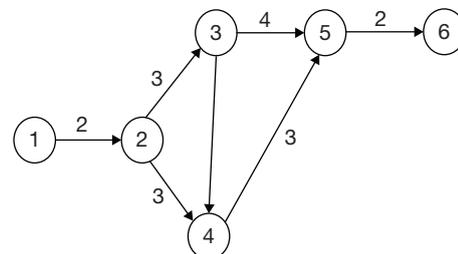
- | | |
|-------------|-------------|
| a b c d | a b c d |
| (A) 4 3 1 2 | (B) 3 4 1 2 |
| (C) 3 4 2 1 | (D) 4 3 2 1 |

25. Match the List I with List II.

List I	List II
a. PERT	1. Activity based
b. Node	2. Event oriented
c. Dummy	3. End of job
d. CPM	4. Imaginary activity

- (A) a-1, b-4, c-3, d-2
- (B) a-2, b-4, c-3, d-1
- (C) a-2, b-3, c-4, d-1
- (D) a-1, b-3, c-4, d-2

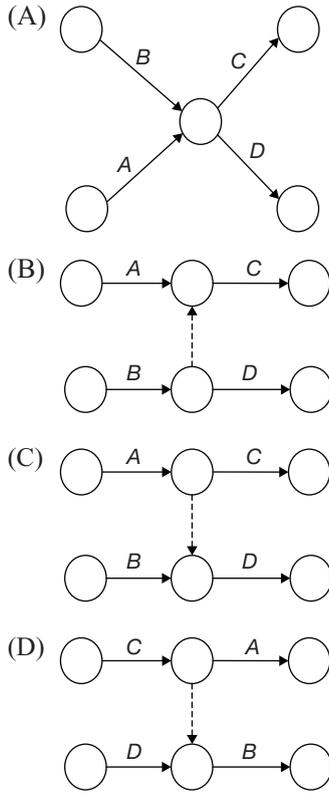
- 26.



In the network shown, total float for the activities 2-4 and 3-5 are respectively

- (A) 0 and 0
- (B) 2 and 2
- (C) 2 and 1
- (D) 1 and 1

27. Activity 'C' follows activity 'A' and activity 'D' follows activities 'A' and 'B'. The correct network for the project is



28. Consider the following statements
- I. A dummy activity is artificially introduced in a network.
 - II. A dummy activity is represented by a dotted arrow.
 - III. The critical activity is represented by a thick arrow in network.
 - IV. A dummy activity must/necessarily be introduced in every network.
- Which of the statements given are correct?
- (A) I, II only (B) I, II, III only
 (C) II, III, IV only (D) I, II, III and IV

29. The time estimates obtained from four contractors P, Q, R and S for executing a particular job are as under:

Contractor	Optimistic Time	Most Likely Time	Pessimistic Time
P	5	10	13
Q	6	9	12
R	5	10	14
S	4	10	13

- Which one of these contractors is more certain about completing the job in time?
- (A) P (B) Q
 (C) R (D) S

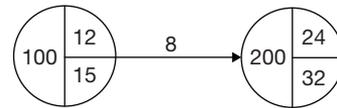
30. Consider the following statements.
- I. PERT is activity-oriented and adopts probabilistic approach
 - II. CPM is event-oriented and adopts deterministic approach

- III. PERT is event-oriented and adopts probabilistic approach.
- IV. CPM is activity-oriented and adopts deterministic approach.

- Which of these statements is/are correct?
- (A) I and II (B) II and III
 (C) III and IV (D) IV and I

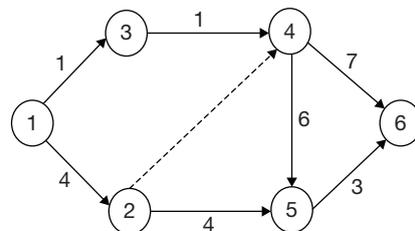
31. PERT calculations yield a project length of 60 weeks with a variance of 9 weeks. Probability factor corresponding to 95% probability is 1.647; then the number of weeks required to complete the project with a probability of 95% is ____.
- (A) 64.94 (B) 74.83
 (C) 45.17 (D) 55.06

32. An activity 100–200 in an AOA network is given below:



Between free float, interference float and independent float, which of the following is the correct sequence in the decreasing magnitude order?

- (A) Interference float, independent float, free float
 - (B) Interference float, free float, independent float
 - (C) Independent float, free float, Interference float
 - (D) Free float, independent float, interference float
33. While doing time computation on a CPM network, the correct sequence followed would be ____.
- I. FP: Forward pass
 - II. BP: Backward pass
 - III. LET: Late event time
 - IV. EET: Early event time
 - V. TF: Total float
 - VI. PD: Project duration
 - VII. AD: Activity duration
- (A) VII, I, IV, VI, II, III, V (B) VII, IV, I, II, VI, III, V
 (C) I, IV, VII, II, III, VI, V (D) IV, I, VII, III, II, V, VI
34. The earliest start time, for an activity 4–6 and 5–6 in the given network is ____.

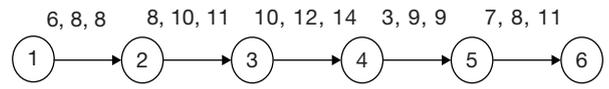


- (A) 2 and 8
 (B) 4 and 10
 (C) 4 and 8
 (D) 2 and 10

Select the correct statements from the given.

- (A) I and II
- (B) I and IV
- (C) II and III
- (D) III and IV

44. Consider a series of activities A, B, C, D and E comprising project where in t_o, t_m, t_p of each activity are indicated. What is the probability of completing the project in 45 days? The area under the normal probability curve is indicated in the table by adopting standard notations.

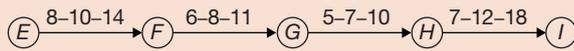


Z	1	0.8	0.6	0.4	0.2	0
A	0.841	0.788	0.726	0.655	0.579	0.5

- (A) 0.20
- (B) 0.23
- (C) 0.26
- (D) 0.29

PREVIOUS YEARS' QUESTIONS

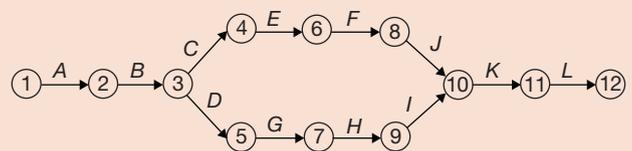
1. The Optimistic Time (O), Most likely Time (M) and Pessimistic Time (P) (in days) of the activities in the critical path are given below in the format $O-M-P$.
[GATE, 2016]



The expected completion time (in days) of the project is _____.

2. A construction project consists of twelve activities. The estimated duration (in days) required to complete each of the activities along with the corresponding network diagram is shown in the following table.
[GATE, 2016]

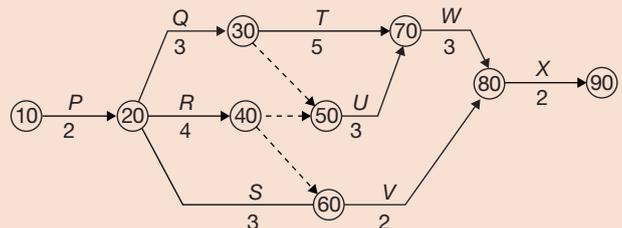
Activity	Duration (days)	Activity	Duration (days)
A. Inauguration	1	G Flooring	25
B. Foundation work	7	H Electrification	7
C. Structural construction-1	30	I Plumbing	7
D. Structural construction-2	30	J Wood work	7
E. Brick masonry work	25	K Coloring	3
F. Plastering	7	L Handing over function	1



Total floats (in days) for the activities 5-7 and 11-12 for the project are, respectively,

- (A) 25 and 1
- (B) 1 and 1
- (C) 0 and 0
- (D) 81 and 0

3. The activity-on-arrow network of activities for a construction project is shown in the figure. The durations (expressed in days) of the activities are mentioned below the arrows.
[GATE, 2016]



The critical duration for this construction project is

- (A) 13 days
- (B) 14 days
- (C) 15 days
- (D) 16 days

ANSWER KEYS

Exercises

- 1. D 2. C 3. B 4. C 5. C 6. B 7. B 8. C 9. B 10. C
- 11. B 12. C 13. D 14. C 15. C 16. A 17. D 18. A 19. D 20. C
- 21. C 22. D 23. B 24. B 25. C 26. C 27. C 28. B 29. B 30. C
- 31. A 32. B 33. A 34. B 35. C 36. A 37. C 38. C 39. D 40. D
- 41. C 42. C 43. B 44. D

Previous Years' Questions

- 1. 37.83 2. C 3. C