

Previous Years Paper

27 May 2023 - (Shift 1)

- Q1.** $f(x) = x^3 + px^2 + qx + 10$ has a maximum at $x = -3$ and a minimum at $x = 1$. The values of p and q are:

- (a) $p = -3, q = -9$
 (b) $p = 3, q = 9$
 (c) $p = -3, q = 9$
 (d) $p = 3, q = -9$

- Q2.** Area of the region bounded by ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

- (a) πab sq. units
 (b) ab sq. units
 (c) πa sq. units
 (d) πb sq. units

- Q3.** If the feasible region of a system of linear inequalities can be enclosed within a circle then it is called

- (a) Shaded area
 (b) Bounded region
 (c) Circular region
 (d) Unbounded region

- Q4.** $y = \log_e \left(\frac{1-x^2}{1+x^2} \right)$, then $\frac{dy}{dx}$ is equal to:

- (a) $\frac{4x^3}{1-x^4}$
 (b) $\frac{-4x}{1-x^4}$
 (c) $\frac{1}{4-x^4}$
 (d) $\frac{-4x^3}{1-x^4}$

- Q5.** If $x = t^4, y = t$, then $\frac{d^2y}{dx^2}$ is given by _____.

- (a) $-\frac{3}{16t^7}$
 (b) $\frac{3}{16t^7}$
 (c) $-\frac{3}{4t^4}$
 (d) $\frac{3}{16t^4}$

- Q6.** The value of the determinant $\begin{vmatrix} 66 & 18 & 36 \\ 1 & 3 & 4 \\ 11 & 3 & 6 \end{vmatrix}$ is:

- (a) -1
 (b) 1
 (c) 0
 (d) 2

- Q7.** Points at which normal to the curve $y = x^3 - 3x$ is parallel to y-axis are:

- (a) (1, 2) and (1, -2)
 (b) (1, -2) and (-1, 2)
 (c) (1, 2) and (-1, -2)
 (d) (-1, 2) and (1, 2)

- Q8.** One root of the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ is:

- (a) $\frac{8}{3}$
 (b) $\frac{2}{3}$

(c) $\frac{1}{3}$

(d) $\frac{16}{3}$

- Q9.** The probability distribution of a random variable X is given by

X	0	1	2	3	4
P(X)	0.1	k	2k	2k	k

The value of k is:

- (a) 0
 (b) 0.1
 (c) 1
 (d) $\frac{3}{20}$

- Q10.** The inverse of the matrix $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ is:

- (a) $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$
 (b) $\begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$
 (c) $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$
 (d) $\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

- Q11.** $\int_0^2 (x^2 + 1)dx$ is equal to

- (a) $\frac{14}{3}$
 (b) 4
 (c) 10
 (d) 8

- Q12.** $\int \frac{x}{x^2+x-12} dx$ is equal to

- (a) $\frac{3}{7} \log|x-3| + \frac{4}{7} \log|x+4| + C$
 (b) $-\frac{3}{7} \log|x-3| + \frac{4}{7} \log|x+4| + C$
 (c) $\frac{4}{7} \log|x-3| + \frac{3}{7} \log|x+4| + C$
 (d) $\frac{4}{7} \log|x-3| - \frac{3}{7} \log|x+4| + C$

- Q13.** There are 4 bulbs out of which 2 are defective. Each bulb is tested in random order till both the defective bulbs are identified. The probability that only two tests are required to identify the defective bulb is:

- (a) $\frac{1}{2}$
 (b) $\frac{1}{3}$
 (c) $\frac{1}{6}$
 (d) $\frac{1}{4}$

- Q14.** Corner points of the feasible region for a linear programming problem are (0, 2), (3, 0), (6, 0) and (0,

- 5). Let $F = 4x + 6y$ be the objective function. The minimum value of F occurs at:
- (a) (0, 2) only
(b) (3, 0) only

- (c) Any point on the line joining the points (0, 2) and (3, 0) only
(d) The mid - point of the segment joining the points (0, 2) and (3, 0) only.

Q15. Match List I with List II

LIST I		LIST II	
A.	$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{\frac{3}{2}}$	I.	order + degree = 2
B.	$2\left(\frac{d^3y}{dx^3}\right)^2 + 3\left(\frac{d^2y}{dx^2}\right)y\left(\frac{dy}{dx}\right)^2 = e^x$		order + degree = 3
C.	$\frac{dy}{dx} + \frac{1}{\frac{dy}{dx}} = 3$	III.	order + degree = 4
D.	$\frac{dy}{dx} + x^2 = 5$	IV.	order + degree = 5

Choose the correct answer from the options given below:

- (a) A-III, B-IV, C-II, D-I
(b) A-III, B-IV, C-I, D-II
(c) A-IV, B-III, C-I, D-II
(d) A-IV, B-III, C-II, D-I

Q16. For $x \in [-2, 2]$, let $f(x) = x^2 + 2$

- A. $f(x)$ is continuous in $[-2, 2]$
B. $f(x)$ is differentiable in $(-2, 2)$
C. $f(-2) = f(2) = 6$
D. $f'(1) = 0$

Choose the correct answer from the options given below:

- (a) A, D only
(b) B, C, D only
(c) A, B, C, D only
(d) A, B and C only

Q17. The area of region bounded by two curves $y^2 = x$ and $x^2 = y$ is:

- (a) 1 sq. unit
(b) $\frac{1}{2}$ sq. unit
(c) $\frac{1}{3}$ sq. unit
(d) $\frac{2}{3}$ sq. unit

Q18. The value of $\begin{vmatrix} \sin 15^\circ & \cos 15^\circ & 2 \sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ & \sin 45^\circ \\ \cos 75^\circ & \sin 75^\circ & \sin 90^\circ \end{vmatrix}$ is:

- (a) 1
(b) 0
(c) $\frac{1}{2}$
(d) $\frac{\sqrt{3}}{2}$

Q19. The area of the parallelogram of which $\hat{i}$ and $\hat{i} + \hat{j}$ are adjacent sides is _____

- (a) 1
(b) 2
(c) $\frac{1}{2}$
(d) $\sqrt{2}$

Q20. Match List I with List II

LIST I	LIST II
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A.	$\int \sin x \cos x \, dx$	I.	$\log(1 + \sin^2 x) + C$
B.	$\int \frac{dx}{\sin^2 \cos^2 x}$	II.	$\tan \frac{x}{2} + C$
C.	$\int \frac{\sin 2x}{1 + \sin^2 x} dx$	III.	$-\frac{1}{4} \cos 2x + C$
D.	$\int (1 - \cos x) \operatorname{cosec}^2 x \, dx$	IV.	$\tan x - \cot x + C$

Choose the correct answer from the options given below:

- (a) A-III, B-IV, C-I, D-II
(b) A-IV, B-I, C-II, D-III
(c) A-III, B-II, C-IV, D-I
(d) A-II, B-III, C-IV, D-I

Q21. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is:

- (a) $\frac{1}{3}$
(b) $\frac{4}{13}$
(c) $\frac{1}{4}$
(d) $\frac{1}{2}$

Q22. The integrating factor of the differential equation $x \frac{dy}{dx} - y = \sin x$ is:

- (a) $-\frac{1}{x}$
(b) x
(c) $e^{\log x}$
(d) $\frac{1}{x}$

Q23. Area of the region bounded by the curve $y = \frac{1}{2} \cos x$ and x -axis between $x = 0$ and $x = 2\pi$ is:

- (a) 2 sq. units
(b) 4 sq. units
(c) 3 sq. units
(d) 1 sq. units

Q24. The number of all possible matrices of order 2×2 with each entry 0, 1, 2, and 3 is:

- (a) 16
(b) 64
(c) 256
(d) 1024

Q25. Match List I with List II

LIST I		LIST II	
A.	If $4 \sin^{-1} x + \cos^{-1} x = \pi$, then x equal to	I.	$\frac{\pi}{2}$
B.	The value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$	II.	$\frac{1}{2}$
C.	If $x + \frac{1}{x} = 2$, then principal value of $\sin^{-1} x$ is	III.	$\frac{3\pi}{4}$
D.	Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$, then third angle is	IV.	$\frac{\sqrt{3}}{2}$

Choose the correct answer from the options given below:

- (a) A-II, B-IV, C-I, D-III
(b) A-III, B-I, C-IV, D-II
(c) A-I, B-II, C-III, D-IV
(d) A-IV, B-III, C-II, D-I

Q26. The value of k for which the following system of equations does not possess a unique solution is:

$$\begin{aligned} kx + 3y - z &= 1 \\ x + 2y + z &= 2 \\ -kx + y + 2z &= -1 \end{aligned}$$

- (a) $-\frac{7}{2}$
(b) $\frac{7}{2}$
(c) 7
(d) $7-$

Q27. The direction cosines of the line

$$x - y + z - 5 = 0 = x - 3y - 6 = 0$$

- (a) 2, -4, 1
(b) 3, 1, -2
(c) $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$
(d) $\frac{3}{\sqrt{14}}, \frac{-4}{\sqrt{14}}, \frac{1}{\sqrt{14}}$

Q28. The image of the point (3, -2, 1) in the plane $3x - y + 4z = 2$ is:

- (a) (3, -1, 4)
(b) (0, -1, -3)
(c) $(\frac{3}{2}, -\frac{5}{2}, -1)$
(d) (0, -1, 3)

Q29. $\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\cos^4 x + \sin^4 x} dx =$

- (a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) π
(d) 0

Q30. If $y = A \sin x + B \cos x$, then which of the following is correct?

- (a) $\frac{d^2 y}{dx^2} - y = 0$

(b) $\frac{d^2 y}{dx^2} + y = 0$

(c) $\frac{d^2 y}{dx^2} = \frac{dy}{dx}$

(d) $\frac{d^2 y}{dx^2} = \frac{-dy}{dx}$

Q31. The feasible region of the constraints $x \geq 0$, $x + y \leq 1$ and $x - y \leq 1$ is situated in:

- (a) I and II quadrant
(b) II and III quadrant
(c) I and IV quadrant
(d) I, II, III and IV quadrants

Q32. Match List I with List II

LIST I		LIST II	
A.	$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3x^2}$	I.	2
B.	$\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$	II.	8
C.	$\lim_{x \rightarrow 0} \frac{\sin ax + 4x}{ax + \sin 4x}$	III.	1
D.	$\lim_{z \rightarrow 1} \frac{\frac{1}{z^3} - 1}{\frac{1}{z^6} - 1}$	IV.	$\frac{2}{3}$

Choose the correct answer from the options given below:

- (a) A-I, B-II, C-II, D-IV
(b) A-IV, B-II, C-III, D-I
(c) A-IV, B-II, C-I, D-III
(d) A-III, B-I, C-IV, D-II

Q33. Match List I with List II

LIST I		LIST II	
A.	Range of $\operatorname{cosec}^{-1} x$	I.	$(0, \pi)$
B.	Range of $\tan^{-1} x$	II.	$[0, \pi] - \{\frac{\pi}{2}\}$
C.	Range of $\sec^{-1} x$	III.	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
D.	Range of $\cot^{-1} x$	IV.	$(-\frac{\pi}{2}, \frac{\pi}{2})$

Choose the correct answer from the options given below:

- (a) A-II, B-IV, C-III, D-I
(b) A-III, B-IV, C-II, D-I
(c) A-III, B-IV, C-I, D-II
(d) A-IV, B-III, C-I, D-II

Q34. If $f: \mathbb{R} \rightarrow A$ given by $f(x) = x^2 - 6x + 12$ is a surjective function, then the set A is:

- (a) $(3, \infty)$
(b) $(-\infty, 3)$
(c) $[3, \infty)$
(d) $(-\infty, 3]$

Q35. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$ where $b_{ij} = 2^{i+j} a_{ij} \forall 1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of Q is:

- (a) 2^{13}
(b) 2^{12}
(c) 2^{11}
(d) 2^{10}

Q36. Suppose 10% of men and 0.5% of women have grey hair. A grey hair person is selected at random. If there are equal number of males and females, then the probability of the selected person being a female is

- (a) $\frac{20}{21}$
 (b) $\frac{1}{21}$
 (c) $\frac{1}{200}$
 (d) $\frac{1}{5}$

Q37. For $x < \frac{1}{2}$, derivative of $\tan^{-1}\left(\frac{1+2x}{1-2x}\right)$ with respect to $\sqrt{1+4x^2}$ is:

- (a) $\frac{2x}{1+4x^2}$
 (b) $\frac{1}{2x\sqrt{1+4x^2}}$
 (c) $\frac{1}{x\sqrt{1+4x^2}}$
 (d) $2x\sqrt{1+4x^2}$

Q38. Radius of a spherical balloon is increasing at the rate of 0.5 cm/sec; then the rate of increase of its volume when radius is 10 cm is

- (a) 150π cm³/sec
 (b) 200π cm³/sec
 (c) 400π cm³/sec
 (d) 300π cm³/sec

Q39. If the relation $R: A \rightarrow B$, where $A = \{2, 3, 4\}$ and $B = \{3, 4, 5\}$ is defined by $R = \{(x, y) : x < y, x \in A, y \in B\}$ then

- (a) $R = \{(2, 3), (2, 4), (3, 4), (3, 5), (4, 4)\}$
 (b) $R = \{(2, 3), (2, 4), (2, 5), (3, 5), (4, 5)\}$
 (c) $R = \{(2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}$
 (d) $R = \{(2, 3), (2, 5), (3, 3), (3, 5), (4, 4), (4, 5)\}$

Q40. The slope of the normal to the curve $y = x^3 - 4\sin x$ at $x = 0$ is:

- (a) -4
 (b) $\frac{1}{4}$
 (c) 4
 (d) $-\frac{1}{4}$

Q41. If the difference of two unit vectors is again a unit vector, then the angle between them is _____.

- (a) 30°
 (b) 45°
 (c) 60°
 (d) 90°

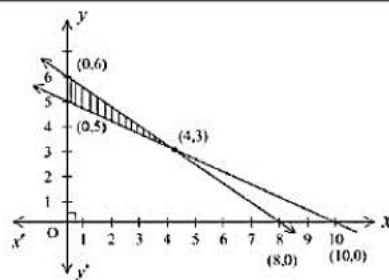
Q42. The function $f(x) = \cos x$ is strictly decreasing in:

- (a) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
 (b) $(0, 2\pi)$
 (c) $(\pi, 2\pi)$
 (d) $(0, \pi)$

Q43. If order of matrices A, B and C are $4 \times 3, 5 \times 4$ and 3×7 respectively, then order of $C' \times (A' \times B')$ is

- (a) 7×5
 (b) 4×5
 (c) 4×3
 (d) 5×7

Q44. According to the graph drawn here, identify the constraints of the associated linear programming problem:



- (a) $x, y \geq 0, x + 2y \geq 10, 3x + 4y \geq 24$
 (b) $x, y \geq 0, x + 2y \leq 10, 3x + 4y \leq 24$
 (c) $x, y \geq 0, x + 2y \leq 10, 3x + 4y \geq 24$
 (d) $x, y \geq 0, x + 2y \geq 10, 3x + 4y \leq 24$

Q45. Match List I with List II

	LIST I		LIST II
A.	If $A^{-1} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$, then A is	I.	$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$
B.	If $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$, then AA^T is	II.	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
C.	If $A^{-1} = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$, then $(A^T)^{-1}$ is	III.	$\begin{bmatrix} -3 & 2 \\ 5 & 3 \\ 2 & -2 \end{bmatrix}$
D.	If $A = [a_{ij}]_{2 \times 2}$ where $a_{ij} = (i - j)^2$, then A is	IV.	$\begin{bmatrix} 5 & 5 \\ 5 & 10 \end{bmatrix}$

Choose the correct answer from the options given below:

- (a) A-II, B-IV, C-III, D-I
 (b) A-IV, B-III, C-I, D-II
 (c) A-III, B-IV, C-I, D-II
 (d) A-I, B-IV, C-III, D-II

Q46. The maximum number of equivalence relations on the set $A = \{3, 5, 7\}$

- (a) 1
 (b) 2
 (c) 3
 (d) 5

Q47. The maximum value of $\frac{\log x^3}{3x}$ occurs at $x =$ _____

- (a) e
 (b) $\frac{1}{e}$
 (c) $3e$
 (d) $\frac{3}{e}$

Q48. The function $f(x) = \frac{x}{(x-2)(x-3)(x-5)}$, $x \in \mathbb{R}$ is

- (a) continuous everywhere
 (b) not continuous anywhere
 (c) discontinuous at $x = 2, 3, 5$
 (d) discontinuous at $x = 0$

Q49. The order and degree of differential equation

$$\sqrt[3]{2y + \frac{dy}{dx}} = \left(x + \frac{d^3y}{dx^3}\right) \text{ respectively are:}$$

- (a) 3, 1
 (b) 3, 3
 (c) 1, 1
 (d) 2, 2

Q50. The projection of $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ is:

- (a) $\sqrt{2}$
 (b) $\frac{1}{\sqrt{2}}$
 (c) 2
 (d) 0

SOLUTIONS

S1. Ans. (d)

Sol. Given function

$$f(x) = x^3 + px^2 + qx + 10$$

$$f'(x) = 3x^2 + 2px + q$$

$f(x) = x^3 + px^2 + qx + 10$ has a maximum at $x = -3$ and a minimum at $x = 1$.

$$3(-3)^2 + 2p(-3) + q = 0 \Rightarrow 27 - 6p + q = 0 \Rightarrow$$

$$6p - q = 27 \dots\dots\dots (i)$$

$$3(1)^2 + 2p(1) + q = 0 \Rightarrow 3 + 2p + q = 0 \Rightarrow 2p + q = -3 \dots\dots\dots (ii)$$

Adding eq. (i) & (ii)

$$8p = 24 \Rightarrow p = 3$$

From eq. (ii)

$$2(3) + q = -3 \Rightarrow p = -9$$

S2. Ans. (a)

Sol. Given equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = \frac{b}{a} \sqrt{a^2 - x^2}$$

$$\text{Area bounded by curve} = 4 \int_0^a y dx =$$

$$4 \int_0^a \frac{b}{a} \sqrt{a^2 - x^2} dx$$

$$= 4 \frac{b}{a} \left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right\}_0^a = 4 \frac{b}{a} \times$$

$$\frac{a^2}{2} \sin^{-1} \left(\frac{a}{a} \right) = 4 \frac{ab}{2} \times \frac{\pi}{2} = \pi ab \text{ sq. unit}$$

S3. Ans. (b)

Sol. If the feasible region of a system of linear inequalities can be enclosed within a circle then, it is called Bounded region

S4. Ans. (b)

Sol. We have

$$y = \log_e \left(\frac{1-x^2}{1+x^2} \right) = \log(1-x^2) - \log(1+x^2)$$

$$\frac{dy}{dx} = \frac{1}{1-x^2} \times -2x - \frac{1}{1+x^2} \times 2x =$$

$$-2x \left(\frac{1+x^2+1-x^2}{(1-x^2)(1+x^2)} \right) = \frac{-4x}{1-x^4}$$

S5. Ans. (a)

Sol. Given

$$x = t^4, y = t$$

$$\frac{dx}{dt} = 4t^3, \frac{dy}{dt} = 1$$

$$\frac{dx}{dy} = \frac{1}{4t^3}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{4} t^{-4} \frac{dt}{dx} = -\frac{3}{4t^4} \times \frac{1}{4t^3} = -\frac{3}{16t^7}$$

S6. Ans. (c)

Sol. We have

$$\begin{vmatrix} 66 & 18 & 36 \\ 1 & 3 & 4 \\ 11 & 3 & 6 \end{vmatrix} = 6 \begin{vmatrix} 11 & 3 & 6 \\ 1 & 3 & 4 \\ 11 & 3 & 6 \end{vmatrix} = 6 \times 3 \times$$

$$2 \begin{vmatrix} 11 & 1 & 3 \\ 1 & 1 & 2 \\ 11 & 1 & 3 \end{vmatrix}$$

$$= 36\{11(1) - 1(-19) + 3(-10)\}$$

$$36(30 - 30) = 0$$

S7. Ans. (b)

Sol. Given curve

$$y = x^3 - 3x$$

$$\frac{dy}{dx} = 3x^2 - 3$$

If normal to the curve is parallel to the y-axis, then tangent to the curve is parallel to x-axis

$$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow 3x^2 - 3 = 0 \Rightarrow x = \pm 1$$

$$x = 1 \Rightarrow y = -2$$

$$x = -1 \Rightarrow y = 2$$

So, required points are (1, -2) & (-1, 2)

S8. Ans. (b)

Sol. We have

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\begin{vmatrix} 3x-2 & 3x-2 & 3x-2 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

$$(3x-2) \begin{vmatrix} 1 & 1 & 1 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\text{One root is } x = \frac{2}{3}$$

S9. Ans. (d)

Sol. We have

$$P(0) + P(1) + P(2) + P(3) + P(4) = 1$$

$$0.1 + k + 2k + 2k + k = 1$$

$$6k = 0.9 \Rightarrow k = \frac{0.9}{6} = \frac{9}{60} = \frac{3}{20}$$

S10. Ans. (d)

Sol. Given

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

$$|A| = 1$$

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

S11. Ans. (a)

Sol. We have

$$\int_0^2 (x^2 + 1) dx = \left\{ \frac{x^3}{3} + x \right\}_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$$

S12. Ans. (a)

Sol. We have

$$\int \frac{x}{x^2+x-12} dx$$

$$\frac{x}{x^2+x-12} = \frac{x}{(x-3)(x+4)}$$

Let

$$\frac{x}{(x-3)(x+4)} = \frac{A}{(x-3)} + \frac{B}{(x+4)}$$

$$x = A(x+4) + B(x-3)$$

Put $x = 3$, we get

$$A = \frac{3}{7}$$

Put $x = -4$, we get

$$B = \frac{4}{7}$$

Now

$$\int \frac{x}{x^2+x-12} dx = \frac{3}{7} \int \frac{dx}{x-3} + \frac{4}{7} \int \frac{dx}{x+4}$$

$$= \frac{3}{7} \log|x-3| + \frac{4}{7} \log|x+4| + C$$

S13. Ans. (c)

Sol. Total bulbs = 4

Defective bulbs = 2

Probability of first be defective = $\frac{2}{4} = \frac{1}{2}$
 Probability of second bulb to be defective = $\frac{1}{3}$
 Required probability = $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

S14. Ans. (c)

Sol. We have

Corner points of the feasible region for a linear programming problem are (0, 2), (3, 0), (6, 0) and (0, 5)

$$\& F = 4x + 6y$$

$$\text{At } (0, 2), F = 12$$

$$\text{At } (3, 0), F = 12$$

$$\text{At } (6, 0), F = 24$$

$$\text{At } (0, 5), F = 30$$

The minimum value of F occurs at any point on the line joining the points (0, 2) and (3, 0) only

S15. Ans. (a)

Sol. (A) $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^3$

Squaring both sides

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left(\frac{dy}{dx}\right)^6$$

Order = 2 & degree = 2 $\Rightarrow$ Order + degree = 4

(B) $2\left(\frac{d^3y}{dx^3}\right)^2 + 3\left(\frac{d^2y}{dx^2}\right)y\left(\frac{dy}{dx}\right)^2 = e^x$

Order + degree = 3 + 2 = 5

(C) We have

$$\frac{dy}{dx} + \frac{1}{dy} = 3 \Rightarrow \left(\frac{dy}{dx}\right)^2 + 1 = 3\left(\frac{dy}{dx}\right)$$

Order + degree = 1 + 2 = 3

(D) We have

$$\frac{dy}{dx} + x^2 = 5$$

Order + degree = 1 + 1 = 2

S16. Ans. (d)

Sol. $f(x)$ is continuous in $[-2, 2]$

$f(x)$ is differentiable in $(-2, 2)$

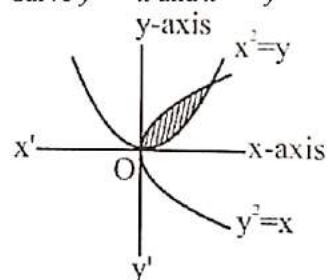
$$f(-2) = f(2) = 6$$

But

$$f'(1) \neq 0, 1 \in (-2, 2)$$

S17. Ans. (c)

Sol. Curve $y^2 = x$ and $x^2 = y$



$$\text{then, Area} = \int_0^1 (\sqrt{x} - x^2) dx$$

$$= \frac{2}{3} [x^{\frac{3}{2}}]_0^1 - \left[\frac{x^3}{3} \right]_0^1$$

$$= \frac{2}{3} (1 - 0) - \left(\frac{1}{3} - 0 \right)$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

S18. Ans. (b)

Sol. We have

$$\begin{vmatrix} \sin 15^\circ & \cos 15^\circ & 2 \sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ & \sin 45^\circ \\ \cos 75^\circ & \sin 75^\circ & \sin 90^\circ \end{vmatrix} =$$

$$\begin{vmatrix} \sin(45^\circ - 30^\circ) & \cos(45^\circ - 30^\circ) & 2 \times \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ \cos(45^\circ + 30^\circ) & \sin(45^\circ + 30^\circ) & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\sqrt{3}-1}{2\sqrt{2}} & \frac{\sqrt{3}+1}{2\sqrt{2}} & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{3}-1}{2\sqrt{2}} & \frac{\sqrt{3}+1}{2\sqrt{2}} & 1 \end{vmatrix} = 0 \quad \{\text{since } R_1 \text{ \& } R_3 \text{ are same}\}$$

S19. Ans. (a)

Sol. The area of the parallelogram of which $\hat{i}$ and $\hat{i} + \hat{j}$ are adjacent sides is

$$|\hat{i} \times (\hat{i} + \hat{j})| = |\hat{k}| = 1 \text{ sq. unit}$$

S20. Ans. (a)

Sol. (A) $\int \sin x \cos x dx = \frac{1}{2} \int 2 \sin x \cos x dx =$

$$\frac{1}{2} \int \sin 2x dx = \frac{1}{2} \left\{ \frac{-\cos 2x}{2} \right\} = -\frac{1}{4} \cos 2x + C$$

(B) $\int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \sec^2 x dx +$

$$\int \operatorname{cosec}^2 x dx = \tan x - \cot x + C$$

(C) $\int \frac{\sin 2x}{1 + \sin^2 x} dx = \int \frac{2 \sin x \cos x}{1 + \sin^2 x} dx$
 Put $\sin x = t \Rightarrow \cos x dx = dt$
 $= \int \frac{2t dt}{1 + t^2}$

$$\text{Put } 1 + t^2 = u \Rightarrow 2t dt = du$$

$$= \int \frac{du}{u} = \log u = \log(1 + t^2) = \log(1 + \sin^2 x)$$

(D) $\int (1 - \cos x) \operatorname{cosec}^2 x dx = \int (\operatorname{cosec}^2 x - \cos x \operatorname{cosec}^2 x) dx$
 $= \int \operatorname{cosec}^2 x dx - \int \frac{\cos x}{\sin^2 x} dx$
 Put $\sin x = t \Rightarrow \cos x dx = dt$
 $= -\cot x - \int \frac{dt}{t^2} = -\cot x - \frac{t^{-1}}{-1} + C = -\cot x +$

$$\frac{1}{\sin x} + C$$

$$\frac{1}{\sin x} - \frac{\cos x}{\sin x} + C = \frac{1 - \cos x}{\sin x} + C$$

$$= \frac{1 - (1 - 2 \sin^2 \frac{x}{2})}{2 \sin \frac{x}{2} \cos \frac{x}{2}} + C$$

$$= \tan \frac{x}{2} + C$$

S21. Ans. (c)

Sol. Total cards = 52

Cards of queen = 4

Cards of spade = 13

E: Spade cards

F: Cards of queen

$$n(E) = 13, n(F) = 4, n(E \cap F) = 1$$

$$P(E \cap F) = \frac{1}{52} \& P(F) = \frac{4}{52}$$

$$\text{Required probability } P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{52}}{\frac{4}{52}} = \frac{1}{4}$$

S22. Ans. (d)

Sol. The differential equation is

$$x \frac{dy}{dx} - y = \sin x$$

$$\frac{dy}{dx} - \frac{y}{x} = \frac{\sin x}{x}$$

$$P = -\frac{1}{x}$$

$$\text{Integrating factor} = e^{\int P dx} = e^{\int -\frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$$

S23. Ans. (a)

Sol. Given curve

$$y = \frac{1}{2} \cos x$$

$$\text{Area of bounded curve} = \int_0^{2\pi} \frac{1}{2} \cos x dx =$$

$$2 \int_0^{\pi} \frac{1}{2} \cos x dx = 4 \times \frac{1}{2} \int_0^{\pi} \cos x dx = 2[\sin x]_0^{\pi} = 2 \times 1 = 2 \text{ sq. unit}$$

S24. Ans. (c)

Sol. Order of matrix = 2×2

Number of elements = 4

Number of entries = 4

Number of possible matrices = $4^4 = 256$

S25. Ans. (a)

Sol. (A) $4 \sin^{-1} x + \cos^{-1} x = \pi$
 $3 \sin^{-1} x + \sin^{-1} x + \cos^{-1} x = \pi$

$$3 \sin^{-1} x + \frac{\pi}{2} = \pi$$

$$3 \sin^{-1} x = \frac{\pi}{2}$$

$$\sin^{-1} x = \frac{\pi}{6}$$

$$x = \frac{1}{2}$$

(B) We have

$$\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \cos 2(15^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

(C) We have

$$x + \frac{1}{x} = 2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0$$

$$x = 1$$

$$\text{Now principal value of } \sin^{-1} x = \sin^{-1}(1) = \frac{\pi}{2}$$

(D) Let

$$\cot^{-1} 2 = \alpha, \cot^{-1} 3 = \beta$$

$$\cot \alpha = 2 \Rightarrow \tan \alpha = \frac{1}{2}$$

$$\cot \beta = 3 \Rightarrow \tan \beta = \frac{1}{3}$$

$$\tan(\alpha + \beta) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\alpha + \beta = \frac{\pi}{4}$$

Suppose third angle be θ

$$\alpha + \beta + \theta = \pi$$

$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

S26. Ans. (a)

Sol. Given

$$kx + 3y - z = 1$$

$$x + 2y + z = 2$$

$$-kx + y + 2z = -1$$

Here

$$A = \begin{bmatrix} k & 3 & -1 \\ 1 & 2 & 1 \\ -k & 1 & 2 \end{bmatrix}$$

If $|A| = 0$, then the system of equations does not have unique solution

$$\begin{vmatrix} k & 3 & -1 \\ 1 & 2 & 1 \\ -k & 1 & 2 \end{vmatrix} = 0$$

$$k(3) - 3(2 + k) - 1(1 + 2k) = 0$$

$$3k - 6 - 3k - 1 - 2k = 0$$

$$-7 - 2k = 0$$

$$k = -\frac{7}{2}$$

S27. Ans. (c)

Sol. Given

$$x - y + z - 5 = 0 \quad \dots\dots\dots (i)$$

$$x - 3y - 6 = 0 \quad \dots\dots\dots (ii)$$

$$\text{Direction ratios of line } (\hat{i} - \hat{j} + \hat{k}) \times (\hat{i} - 3\hat{j}) = 3\hat{i} + \hat{j} - 2\hat{k}$$

Direction ratios are 3, 1, -2

So, direction cosines are

$$\frac{3}{\sqrt{(3)^2 + (1)^2 + (-2)^2}}, \frac{1}{\sqrt{(3)^2 + (1)^2 + (-2)^2}}, -\frac{2}{\sqrt{(3)^2 + (1)^2 + (-2)^2}}$$

i.e.

$$\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}$$

S28. Ans. (b)

Sol. Given equation of plane is

$$3x - y + 4z = 2 \text{ \& given points are } (3, -2, 1)$$

If image of (α, β, γ) with respect to the plane $ax + by + cz + d = 0$ is (x, y, z) , then

$$\frac{x - \alpha}{a} = \frac{y - \beta}{b} = \frac{z - \gamma}{c} = \frac{-2(a\alpha + b\beta + c\gamma + d)}{a^2 + b^2 + c^2}$$

i.e.

$$\frac{x - 3}{3} = \frac{y + 2}{-1} = \frac{z - 1}{4} = \frac{-2(3a - b + 4\gamma + 2)}{(3)^2 + (-1)^2 + (4)^2}$$

$$\frac{x - 3}{3} = \frac{y + 2}{-1} = \frac{z - 1}{4} = \frac{-26}{26}$$

$$\frac{x - 3}{3} = \frac{y + 2}{-1} = \frac{z - 1}{4} = -1$$

$$x = 0, y = -1, z = -3$$

S29. Ans. (b)

Sol. We have

$$\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\cos^4 x + \sin^4 x} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{1 - \frac{1}{2} \times 4 \sin^2 x \cos^2 x} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{1 - \frac{1}{2} (\sin 2x)^2} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{1 - \frac{1}{2} \sin^2 2x} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{1 - \frac{1}{2}(1 - \cos^2 2x)} dx = \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\frac{1}{2}(1 + \cos^2 2x)} dx$$

$$= 2 \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{(1 + \cos^2 2x)} dx$$

Put $\cos 2x = t \Rightarrow -2 \sin 2x dx = dt \Rightarrow 2 \sin 2x dx = -dt$

If $x = 0$, then $t = 1$

If $x = \frac{\pi}{4}$, then $t = 0$

$$= - \int_1^0 \frac{dt}{1 + t^2} = -\{\tan^{-1} t\}_1^0 = \frac{\pi}{4}$$

S30. Ans. (b)

Sol. We have

$$y = A \sin x + B \cos x$$

$$\frac{dy}{dx} = A \cos x - B \sin x$$

$$\frac{d^2 y}{dx^2} = -A \sin x - B \cos x$$

$$\frac{d^2 y}{dx^2} + y = -A \sin x - B \cos x + A \sin x + B \cos x = 0$$

S31. Ans. (c)

Sol. Given constraints

$$x \geq 0, x + y \leq 1 \text{ and } x - y \leq 1$$

$$x + y = 1 \quad \dots\dots\dots (i)$$

$$x - y = 1 \quad \dots\dots\dots (ii)$$

From eq. (i), we get

$$(1, 0) \text{ \& } (0, 1)$$

From eq. (ii), we get

$$(1, 0) \text{ \& } (0, -1)$$

Feasible region is situated in I and IV quadrant

S32. Ans. (b)

Sol. (A) We have

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{3x^2} \quad \{\text{form } \frac{0}{0}\}$$

Using L' Hospital rule

$$\lim_{x \rightarrow 0} \frac{2 \sin 2x}{6x} \quad \{\text{form } \frac{0}{0}\}$$

Again, Using L' Hospital rule

$$\lim_{x \rightarrow 0} \frac{4 \cos 2x}{6} = \frac{4}{6} = \frac{2}{3}$$

(B) We have

$$\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4} = \lim_{x \rightarrow 4} \frac{(x + 4)(x - 4)}{x - 4} = 4 + 4 = 8$$

(C) We have

$$\lim_{x \rightarrow 0} \frac{\sin ax + 4x}{ax + \sin 4x} \quad \{\text{form } \frac{0}{0}\}$$

$$\lim_{x \rightarrow 0} \frac{a \cos ax + 4}{a + 4 \cos 4x} = \frac{a + 4}{a + 4} = 1$$

(D) We have

$$\lim_{z \rightarrow 1} \frac{\frac{1}{z^3} - 1}{\frac{1}{z^6} - 1}$$

$$\text{Put } z^{\frac{1}{6}} = x \quad z \rightarrow 1 \text{ as } x \rightarrow 1$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x + 1)(x - 1)}{x - 1} = 2$$

S33. Ans. (b)

Sol. (A) Range of $\operatorname{cosec}^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(B) Range of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) Range of $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

(D) Range of $\cot^{-1} x$ is $(0, \pi)$

S34. Ans. (c)

Sol. Given

$$f: \mathbb{R} \rightarrow A \text{ given by } f(x) = x^2 - 6x + 12$$

Let

$$y = x^2 - 6x + 12 = x^2 - 6x + 9 + 3 = (x - 3)^2 + 3$$

If $f: \mathbb{R} \rightarrow A$ given by $f(x) = x^2 - 6x + 12$ is a surjective function, then the set A is $[3, \infty)$.

S35. Ans. (a)

Sol. Let

$$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$Q = [b_{ij}], \text{ where } b_{ij} = 2^{i+j} a_{ij} \text{ for } 1 \leq i, j \leq 3.$$

$$Q = \begin{bmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{bmatrix}$$

$$|Q| = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix} = 2^2 \times 2^3 \times$$

$$2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= 2^9 \times 2 \times 2^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2^{12} \times |P| = 2^{12} \times 2 = 2^{13} \quad \{\text{since } |P| = 2\}$$

S36. Ans. (b)

Sol. Let

M: Male

F: Female

G: Grey hair person

Probability of the selected person being a female is

$$P(F|G) = \frac{P(F)P(G|F)}{P(M)P(G|M) + P(F)P(G|F)}$$

Here

$$P(M) = \frac{1}{2}, P(G) = \frac{1}{2}, P(G|M) = \frac{10}{100} = \frac{1}{10} \text{ \& } P(G|F) = \frac{0.5}{100}$$

$$P(F|G) = \frac{\frac{1}{2} \times \frac{0.5}{100}}{\frac{1}{2} \times \frac{1}{10} + \frac{1}{2} \times \frac{0.5}{100}} = \frac{\frac{0.5}{200}}{\frac{1}{20} + \frac{0.5}{200}} = \frac{1}{21}$$

S37. Ans. (b)

Sol. Let

$$u = \tan^{-1} \left(\frac{1+2x}{1-2x} \right) = \tan^{-1}(1) + \tan^{-1} 2x = \frac{\pi}{4} + \tan^{-1} 2x$$

$$v = \sqrt{1 + 4x^2}$$

$$\frac{du}{dx} = \frac{1}{1 + 4x^2} \times 2 = \frac{2}{1 + 4x^2}$$

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1 + 4x^2}} \times 8x = \frac{4x}{\sqrt{1 + 4x^2}}$$

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{2}{1 + 4x^2}}{\frac{4x}{\sqrt{1 + 4x^2}}} = \frac{1}{2x\sqrt{1 + 4x^2}}$$

S38. Ans. (b)

Sol. Radius of a spherical balloon is increasing at the rate of 0.5 cm/sec; then the rate of increase of its volume when radius is 10 cm is

Let r be the radius of spherical balloon, then

$$\text{Volume} = \frac{4}{3}\pi r^3$$

Given

$$\frac{dr}{dt} = 0.5 \text{ cm/sec} \text{ \& } r = 10 \text{ cm}$$

$$\frac{dV}{dt} = \frac{d}{dt} \left(\frac{4}{3}\pi r^3 \right) = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt} = 4\pi \times (10)^2 \times 0.5 = 200\pi \text{ cm}^3/\text{sec}.$$

S39. Ans. (c)

Sol. Given

Relation $R: A \rightarrow B$, where $A = \{2, 3, 4\}$ and $B = \{3, 4, 5\}$ is defined by $R \{(x, y): x < y, x \in A, y \in B\}$ then $R = \{(2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)\}$

S40. Ans. (b)

Sol. Given curve

$$y = x^3 - 4\sin x$$

$$\frac{dy}{dx} = 3x^2 - 4\cos x$$

Slope of tangent at $x = 0$ is

$$\left. \frac{dy}{dx} \right|_{x=0} = -4$$

$$\text{Slope of normal} = \frac{1}{4}$$

S41. Ans. (c)

Sol. Let $|\vec{a}|$ & $|\vec{b}|$ be two unit vectors such that $|\vec{a} - \vec{b}| = 1$

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

$$1^2 = 1^2 + 1^2 - 2|\vec{a}| |\vec{b}| \cos \theta$$

$$1 = 2 - 2\cos \theta \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

S42. Ans. (d)

Sol. Given function is

$$f(x) = \cos x$$

$$f'(x) = -\sin x$$

$$\sin x > 0 \text{ in } (0, \pi).$$

$$\text{So, } -\sin x < 0 \text{ in } (0, \pi)$$

i.e. $f(x) = \cos x$ is strictly decreasing in $(0, \pi)$.

S43. Ans. (a)

Sol. Given order of matrices A, B and C are $4 \times 3, 5 \times 4$ and 3×7 respectively, then

Order of matrices A', B' and C' are $3 \times 4, 4 \times 5$ and 7×3 .

Now

Order of $C' \times (A' \times B')$ is 7×5 .

S44. Ans. (d)

Sol. $x, y \geq 0, x + 2y \geq 10, 3x + 4y \leq 24$

$$x + 2y = 10 \quad \dots\dots\dots (i)$$

$$3x + 4y = 24 \quad \dots\dots\dots (ii)$$

From eq. (i)

$$(10, 0) \text{ \& } (0, 5)$$

From eq. (ii)

$$(8, 0) \text{ \& } (0, 6)$$

From eq. (i) & (ii), we get

$x = 4$ & $y = 3$ are the intersection point.

S45. Ans. (c)

Sol. (A) Given

$$A^{-1} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$(A^{-1})^{-1} = A = \begin{bmatrix} -3 & 2 \\ \frac{5}{2} & -\frac{3}{2} \end{bmatrix}$$

(B) Given

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 5 & 10 \end{bmatrix}$$

(C) Given

$$A^{-1} = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$$

We have

$$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$$

(D) Given

$$a_{ij} = (i - j)^2$$

Then

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

S46. Ans. (d)

Sol. Given set is $A = \{3, 5, 7\}$

Equivalence relations on A are

$$\{(3, 3), (5, 5), (7, 7)\}$$

$$\{(3, 3), (5, 5), (7, 7), (3, 5), (5, 3)\}$$

$$\{(3, 3), (5, 5), (7, 7), (5, 7), (7, 5)\}$$

$$\{(3, 3), (5, 5), (7, 7), (3, 7), (7, 3)\}$$

$$\{(3, 3), (5, 5), (7, 7), (3, 5), (5, 3),$$

$$(5, 7), (7, 5), (3, 7), (7, 3)\}$$

There are 5 equivalence relations.

S47. Ans. (a)

Sol. Given

$$f(x) = \frac{\log x^3}{3x}$$

$$f'(x) = \frac{3x \times \frac{1}{x^3} \times 3x^2 - \log x^3 \times 3}{9x^2} = \frac{9 - 3 \log x^3}{9x^2}$$

For maxima or minima

$$f'(x) = 0$$

$$\frac{9 - 3 \log x^3}{9x^2} = 0$$

$$\log x^3 = 3$$

$$x^3 = e^3$$

$$x = e$$

$$\frac{9 - 3 \log x^3}{9x^2} \text{ has maximum value at } x = e$$

S48. Ans. (c)

Sol. $f(x) = \frac{x}{(x-2)(x-3)(x-5)}$ is not exist at $x = 2, 3, 5$. So, $f(x)$ is discontinuous at $x = 2, 3, 5$.

S49. Ans. (b)

Sol. Given differential equation is

$$\sqrt[3]{2y + \frac{dy}{dx}} = \left(x + \frac{d^3y}{dx^3} \right)$$

$$\left(2y + \frac{dy}{dx} \right)^{\frac{1}{3}} = \left(x + \frac{d^3y}{dx^3} \right)$$

Taking power '3' both sides

$$\left(2y + \frac{dy}{dx} \right) = \left(x + \frac{d^3y}{dx^3} \right)^3$$

Order = 3 & degree = 3

S50. Ans. (d)

Sol. The projection of $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ is

$$\frac{(\hat{i} - \hat{j}) \cdot (\hat{i} + \hat{j})}{|\hat{i} + \hat{j}|} = \frac{1 - 1}{\sqrt{2}} = 0$$