JEE (Main)-2025 (Online) Session-2

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

7 April 2025 Shift – 1

Time: 3 hrs. **M.M : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

SECTION-A

1. $\lim\limits_{x\to 0^+} \dfrac{\tan\left(5(x)^{\frac{1}{3}}\right)\log_e(1+3x^2)}{\left(\tan^{-1}3\sqrt{x}\right)^2\left(e^{5(x)^{\frac{4}{3}}}-1\right)}$ is equal to

(1) $\dfrac{1}{15}$ (2) 1

(3) $\dfrac{1}{3}$ (4) $\dfrac{5}{3}$

Ans. (3)

Sol.

$$\lim_{x\to 0^+}\left(\frac{\tan\left(5x^{1/3}\right)}{5x^{1/3}}\right).\left(\frac{\left(3\sqrt{x}\right)^2}{(\tan^{-1}3\sqrt{x})^2}\right)\left(\frac{\ell(1+3x^2)}{3x^2}\right)\left(\frac{5x^{4/3}}{e^{5x^{\frac{4}{3}}}-1}\right)\times\frac{5x^{1/3}.3x^2}{5x^{4/3}.9x}$$

$$=\frac{1}{3}$$

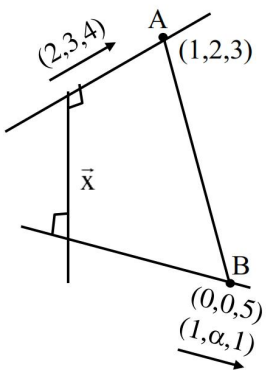
2. If the shortest distance between the lines $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and $\dfrac{x}{1}=\dfrac{y}{\alpha}=\dfrac{z-5}{1}$ is $\dfrac{5}{\sqrt{6}}$, then the sum of all possible values of α is

(1) $\dfrac{3}{2}$ (2) $-\dfrac{3}{2}$

(3) 3 (4) –3

Ans. (4)

Sol.



$$L_1:\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

$$L_1:\frac{x}{1}=\frac{y}{\alpha}=\frac{z-5}{1}$$

$$\vec{x}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&4\\1&\alpha&1\end{vmatrix}=\hat{i}(3-4\alpha)-\hat{j}(-2)+\hat{k}(2\alpha-3)$$

$$\text{S.D.}=\left|\frac{\overrightarrow{BA}.\vec{n}}{|\vec{n}|}\right|=\left|\frac{(\hat{i}+2\hat{j}-2\hat{k}).\vec{n}}{|\vec{n}|}\right|$$

$\Rightarrow 6(13-8\alpha)^2=25((4\alpha-3)^2+(2\alpha-3)^2+16)$

$6(64a^2-280\alpha+169)=25(20\alpha^2-36\alpha+34)$

$\Rightarrow 116\alpha^2+348\alpha-164=0$

$$\alpha_1+\alpha_2=\frac{-348}{116}=-3$$

3. Let x = –1 and x = 2 be the critical points of the function $f(x)=x^3+ax^2+b\log_e|x|+1$, $x\neq 0$. Let *m* and M respectively be the absolute minimum and the absolute maximum values of f in the interval $\left[-2,-\dfrac{1}{2}\right]$. Then |M + *m*| is equal to

(Take $\log_e 2 = 0.7$):

(1) 21.1 (2) 19.8

(3) 22.1 (4) 20.9

Ans. (1)

Sol. $f(x)=x^2+ax^2+b\ell n|x|+1, \quad x\neq 0$

$$f'(x)=3x^2+2ax+\frac{b}{x}$$

$f'(-1)=3-2a-b=0$

$$f'(-2)=12+4a-\frac{b}{2}=0$$

$$a=\frac{-9}{2},\ b=12$$

$$f'(x)=3x^2-9x+\frac{12}{x}=\frac{3(x+1)(x+2)^2}{x}$$

Max. at n = –1

$$f(x)=x^2-\frac{9}{2}x^2+12\ell n|x|+1$$

$f(-1) = -1 - \frac{9}{2} + 1 = -\frac{9}{2}$

$M = -4.5$

Min. value at $x = -2$

$f(-2) = -8 - 18 + 12\ell n2 + 1$

$m = -25 + 12\ell n2 = -16.6$

$|M + m| = 21.1$

4. The remainder when $\left((64)^{(64)}\right)^{(64)}$ is divided by 7 is equal to

(1) 4 (2) 1

(3) 3 (4) 6

Ans. (2)

Sol. Let $N = \left((64)^{64}\right)^{64}$

$\mathbf{N} = (64)^{64^2}$

$\mathbf{N} = (1+63)^{64^2}$, let $64^2 = n$

Expanding by binomial

$N = (1 + 63)^n = 1 + {}^nC_1\, 63 + {}^nC_2(63)^2 + \ldots..$

$= 1 + 63\lambda = 1 + 7\,(9\lambda)$

Remainder when divided by 7 is 1

5. Let P be the parabola, whose focus is (–2, 1) and directrix is $2x + y + 2 = 0$. Then the sum of the ordinates of the points on P, whose abscissa is –2, is

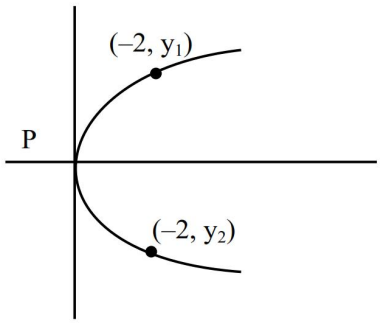
(1) $\frac{3}{2}$ (2) $\frac{5}{2}$

(3) $\frac{1}{4}$ (4) $\frac{3}{4}$

Ans. (1)

Sol. Equation of parabola

$$(x + 2)^2 + (y - 1)^2 = \left(\frac{2x + y + 2}{\sqrt{5}}\right)^2$$



$5[(x + 2)^2 + (y - 1)^2] = (2x + y + 2)^2$

Put $x = -2$, $5(y - 1)^2 = (y - 2)^2$

$5(y^2 - 2y + 1) = y^2 - 4y + 4$

$\Rightarrow 4y^2 - 6y + 1 = 0 \Rightarrow y_1 + y_2 = \frac{3}{2}$

6. Let $y = y(x)$ be the solution curve of the differential equation $x(x^2 + e^x)dy + (e^x(x - 2)y - x^3)dx = 0$, $x > 0$, passing through the point (1, 0). Then $y(2)$ is equal to :

(1) $\frac{4}{4 - e^2}$ (2) $\frac{2}{2 + e^2}$

(3) $\frac{2}{2 - e^2}$ (4) $\frac{4}{4 + e^2}$

Ans. (4)

Sol. $x(x^2 + e^x)\, dy + (e^x\,(x - 2)\, y - x^3)\, dx = 0$

$x(x^2 + e^x)\, \frac{dy}{dx} + e^x(x - 2)\, y = x^3$

$\frac{dy}{dx} + \frac{e^x(x-2)}{x(x^2+e^x)}\, y = \frac{x^2}{x^2 + e^x}$

$\text{I.F.} = e^{\int \frac{e^x(x-2)}{x(x^2+e^x)}\,dx} = e^{\int \frac{e^x\left(\frac{1}{x^2} - \frac{2}{x^2}\right)dx}{\left(1+\frac{e^x}{x^2}\right)}\,dx}$

Let $1+\frac{e^x}{x^2}=t \Rightarrow \frac{x^2e^x-e^x2x}{x^4}\,dx=dt$

$\Rightarrow$ I.F. $e^{\ell n\left(1+\frac{e^x}{x^2}\right)}=1+\frac{e^x}{x^2}$

Now $y\left(1+\frac{e^x}{x^2}\right)=\int\frac{x^2}{x^2+e^x}\cdot\frac{x^2+e^x}{x^2}\,dx+C$

$y\left(1+\frac{e^x}{x^2}\right)=x+C$

Passing through (1, 0)

$\Rightarrow C=-1$

$y=\frac{x-1}{1+\frac{e^x}{x^2}}$

$y(2)=\frac{1}{1+\frac{e^2}{4}}=\frac{4}{4+e^2}$

7. From a group of 7 batsmen and 6 bowlers, 10 players are to be chosen for a team, which should include atleast 4 batsmen and atleast 4 bowlers. One batsmen and one bowler who are captain and vice-captain respectively of the team should be included. Then the total number of ways such a selection can be made, is

(1) 165 (2) 155

(3) 145 (4) 135

Ans. (2)

Sol. 7 Batsmen & 6 Bowlers

To select 10 players including atleast

4 Batsmen & 4 Bowlers

Captain & vice-captain already selected

No. of ways $= {}^6C_5\times{}^5C_3+{}^6C_4\times{}^5C_4+{}^6C_3\times{}^5C_5$

$= 6\times10+15\times5+20\times1$

$= 60+75+20=155$

8. If for $\theta\in\left[-\frac{\pi}{3},0\right]$, the points

$(x,y)=\left(3\tan\left(\theta+\frac{\pi}{3}\right),2\tan\left(\theta+\frac{\pi}{6}\right)\right)$ lie on

$xy+\alpha x+\beta y+\gamma=0$, then $\alpha^2+\beta^2+\gamma^2$ is equal to :

(1) 80 (2) 72

(3) 96 (4) 75

Ans. (4)

Sol. $x=3\left(\frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}\right)$

$x-\sqrt{3}\tan\theta=3\tan\theta+3\sqrt{3}$

$\tan\theta=\frac{x-3\sqrt{3}}{3+\sqrt{3}x}$ ...(1)

$2\left(\frac{\tan\theta+\frac{1}{\sqrt{3}}}{1-\frac{\tan\theta}{\sqrt{3}}}=y\right)$

$2\left(\sqrt{3}\tan\theta+1\right)=y\left(\sqrt{3}-\tan\theta\right)....(2)$

using (1) and (2)

$2\left(\frac{x-3\sqrt{3}}{\sqrt{3}+x}+1\right)=y\left(\sqrt{3}-\frac{\left(x-3\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}+x\right)}\right)$

$2\sqrt{3}\left(x-3\sqrt{3}+x+\sqrt{3}\right)=y\left(3\left(\sqrt{3}+x\right)-x+3\sqrt{3}\right)$

$4\sqrt{3}x-12=y\left(2x+6\sqrt{3}\right)$

$xy-2\sqrt{3}x+3\sqrt{3}y-6=0$

$\Rightarrow \alpha=-2\sqrt{3}\,,\ \beta=3\sqrt{3}\,,\ \gamma=-6$

$\alpha^2+\beta^2+\gamma^2=12+27+36=75$

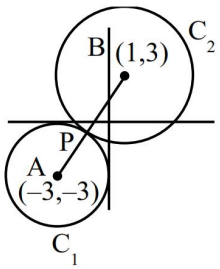
9. Let C_1 be the circle in the third quadrant of radius 3, that touches both coordinate axes. Let C_2 be the circle with centre (1, 3) that touches C_1 externally at the point (α,β). If $(\beta-\alpha)^2=\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to :

(1) 9 (2) 13

(3) 22 (4) 31

Ans. (3)

Sol. $C_1: (x+3)^2+(y+3)^2=3^2$



Let C_1 and C_2 has centres
$A(-3_1, -3)$ and $B(1, 3)$

$AB = \sqrt{16+36} = 2\sqrt{13}$

$r_1 = 3$ and $r_2 = 2\sqrt{13} - 3$

$P(\alpha, \beta),\ \alpha = \dfrac{r_1(1)+r_2(-3)}{r_1+r_2}, \beta = \dfrac{r_1(3)+r_2(-3)}{r_1+r_2}$

$\alpha = \dfrac{3-3\left(2\sqrt{13}-3\right)}{2\sqrt{13}}, \beta = \dfrac{18-6\sqrt{13}}{2\sqrt{13}},$

$(\beta-\alpha)^2 = \left(\dfrac{6}{2\sqrt{13}}\right)^2$

$(\beta-\alpha)^2 = \left(\dfrac{6}{2\sqrt{13}}\right)^2,\quad m+n = 22$

10. The integral $\int_0^{\pi} \dfrac{(x+3)\sin x}{1+3\cos^2 x}\,dx$ is equal to :

(1) $\dfrac{\pi}{\sqrt{3}}(\pi+1)$ (2) $\dfrac{\pi}{\sqrt{3}}(\pi+2)$

(3) $\dfrac{\pi}{3\sqrt{3}}(\pi+6)$ (4) $\dfrac{\pi}{2\sqrt{3}}(\pi+4)$

Ans. (3)

Sol. $I = \int_0^{\pi} \dfrac{(x+3)\sin x}{1+3\cos^2 x}dx$

$I = \int_0^{\pi} \dfrac{(\pi-x+3)\sin x}{(1+3\cos^2 x)}dx$

$2I = \int_0^{\pi} \dfrac{(\pi+6)\sin x.dx}{(1+3\cos^2 x)} = 2\int_0^{\pi/2} \dfrac{(\pi+6)\sin x}{(1+3\cos^2 x)}$

$I = \int_0^{\pi/2} \dfrac{(\pi+6)\sin x.dx}{(1+3\cos^2 x)} = \dfrac{\pi}{3\sqrt{3}}(\pi+6)$

$\sqrt{3}\cos x = t$

$\sqrt{3}\sin x = dt$

11. Among the statements

(S1) **:** The set $\{z \in \mathbb{C} - \{-i\} : |z| = 1$ and $\dfrac{z-i}{z+i}$ is purely real$\}$ contains exactly two elements, and

(S2) **:** The set $\{z \in \mathbb{C} - \{-1\} : |z| = 1$ and $\dfrac{z-1}{z+1}$ is purely imaginary$\}$ contains infinitely many elements.

(1) both are incorrect (2) only (S1) is correct
(3) only (S2) is correct (4) both are correct

Ans. (3)

Sol. $S_1 : |z| = 1,\ \dfrac{z-i}{z+i} = \dfrac{\overline{z}+i}{\overline{z}-i}$

$\Rightarrow (z-i)(\overline{z}-i) = (z+i)(\overline{z}+i)$

$|z|^2 - i(z+\overline{z}) - 1 = |z|^2 + i(z+\overline{z}) - 1$

$i(z+\overline{z}) = 0$

$z+\overline{z} = 2\cos\theta = 0 \Rightarrow \cos\theta = 0$

$z = 0 + 0i,\ |z| \neq 1$

$S_1 : \dfrac{z-1}{z+1} + \dfrac{\overline{z}-1}{\overline{z}+1} = 0$

$(z-1)(\overline{z}+1) + (z+1)(\overline{z}-1) = 0$

$\Rightarrow |z|^2 + (z-\overline{z}) - 1 + |z|^2 + (z-\overline{z}) - 1 = 0$

$|z|^2 = 1$

12. The mean and standard deviation of 100 observations are 40 and 5.1, respectively, By mistake one observation is taken as 50 instead of 40. If the correct mean and the correct standard deviation are μ and σ respectively, then $10(\mu + \sigma)$ is equal to

(1) 445 (2) 451
(3) 447 (4) 449

Ans. (4)

Sol. Actual means $= \mu = \dfrac{100(40)-50+40}{100}$

$\mu = 40 - \dfrac{1}{10} = 39.9$

Incorrect variance

$$(5.1)^2 = \frac{\sum x_i^2}{100} - (\bar{x})^2$$

$$\sum x_i^2 = 100 \times (40^2) + 100(5.1)^2$$

$$\sum x_i^2 = 16 \times 10^4 + (5.1)^2 \times 100 = 162601$$

$$\sigma^2 = \frac{\sum x_i^2 - 50^2 + 40^2}{100} - (\mu)^2$$

$$\sigma^2 = 1617.01 - (39.9)^2 = 25$$

$$\sigma = 5$$

$$10(\mu + \sigma) = 10\ (39.9 + 5)$$

$$= 10 \times 44.9 = 449$$

13. Let x_1, x_2, x_3, x_4 be in a geometric progression. If 2, 7, 9, 5 are subtracted respectively from x_1, x_2, x_3, x_4 then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(x_1\ x_2\ x_3\ x_4)$ is :

(1) 72 (2) 18

(3) 36 (4) 216

Ans. (4)

Sol. $x_1, x_2, x_3, x_4 \rightarrow$ G.P.

Let a, ar, ar^2, $ar^3 \rightarrow$ G.P.

Now $a - 2, ar - 7, ar^2 - 9, ar^3 - 5 \rightarrow$ A.P.

$2(ar - 7) = a - 2 + ar^2 - 9$ …. (i)

$2(ar^2 - 9) = ar - 7 + ar^3 - 5$ … (ii)

Solving r = 2, a = – 3

$\therefore$ Product = $x_1, x_2, x_3, x_4 = a^4r^6 = 81 \times 64$

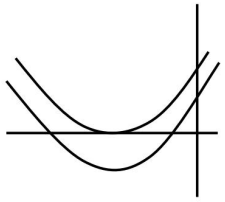
14. Let the set of all values of $p \in \mathbb{R}$, for which both the roots of the equation $x^2 - (p + 2)x + (2p + 9) = 0$ are negative real numbers, be the interval $(\alpha, \beta]$. Then $\beta - 2\alpha$ is equal to

(1) 0 (2) 9

(3) 5 (4) 20

Ans. (3)

Sol. Using location of roots :



(i) $D \geq 0$

(ii) $\frac{-b}{2a} < 0$

(iii) a. $f(0) > 0$

$(p + 2)^2 - 4(2p + 9) \geq 0$

$(p + 4)(p - 8) \geq 0$ $\quad p + 2 < 0$ $\quad 2p + 9 > 0$

Intersection $p \in \left(-\frac{9}{2}, -4\right]$

$\therefore \beta - 2\alpha = -4 + 9 = 5$

15. Let A be a 3×3 matrix such that

$|adj\ (adj\ (adj\ A))| = 81$. If

$$S = \left\{ n \in \mathbb{Z} : \left(|adj(adj\ A)|\right)^{\frac{(n-1)^2}{2}} = |A|^{\left(3n^2 - 5n - 4\right)} \right\}$$

, then $\sum_{n \in S} \left| A^{\left(n^2 + n\right)} \right|$ is equal to

(1) 866 (2) 750

(3) 820 (4) 732

Ans. (4)

Sol. $|adj(adj)(adjA)| = 81$

$\Rightarrow |adjA|^4 = 81$

$\Rightarrow |adjA| = 3$

$\Rightarrow |A|^2 = 3$

$\Rightarrow |A| = \sqrt{3}$

$$\left(|A|^4\right)^{\frac{(n-1)^2}{2}} = |A|^{3n^2 - 5n - 4}$$

$\Rightarrow 2(n - 1)^2 = 3n^2 - 5n - 4$

$\Rightarrow 2n^2 - 4n + 2 = 3n^2 - 5n - 4$

$\Rightarrow n^2 - n - 6 = 0$

$\Rightarrow (n - 3)(n + 2) = 0$

$\Rightarrow n = 3, -2$

$$\sum_{n \in s} |A^{n^2 + n}|$$

$= |A^2| + |A^{12}|$

$= 3 + 36 = 3 + 729 = 732$

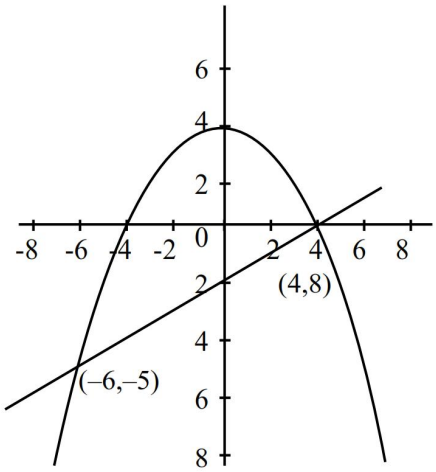
16. If the area of the region bounded by the curves $y = 4 - \frac{x^2}{4}$ and $y = \frac{x-4}{2}$ is equal to α, then 6α equals

(1) 250 (2) 210

(3) 240 (4) 220

Ans. **(1)**

Sol.



$$\text{Area} = \int_{-6}^{4}\left\{\left(4-\frac{x^2}{4}\right)-\left(\frac{x-4}{2}\right)\right\}dx$$

$$= \int_{-6}^{4}\left\{-\frac{x^2}{4}-\frac{x-6}{2}\right\}dx$$

$$\alpha = -\frac{x^3}{12}-\frac{x^2}{4}+6x\Big|_{-6}^{4} = \frac{125}{3}$$

$\therefore 6\alpha = 250$

17. Let the system of equations :

$2x + 3y + 5z = 9$,

$7x + 3y - 2z = 8$,

$12x + 3y - (4 + \lambda)z = 16 - \mu$,

have infinitely many solutions. Then the radius of the circle centred at (λ, μ) and touching the line $4x = 3y$ is

(1) $\frac{17}{5}$ (2) $\frac{7}{5}$

(3) 7 (4) $\frac{21}{5}$

Ans. **(2)**

Sol. $\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(\lambda+4) \end{vmatrix} = 0$

$\Rightarrow 12(-21) - 3(-39) - (\lambda + 4)(-15) = 0$

$\Rightarrow -252 + 117 + 15(1 + 4) = 0$

$\Rightarrow 15\lambda + 177 - 252 = 0$

$\Rightarrow 15\lambda - 75 = 0 \Rightarrow \lambda = 5$

$$\begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 7 \\ \mu-8 & 0 & 7 \\ 16-\mu & 3 & -9 \end{vmatrix} = 0$$

$\Rightarrow 7 - 7(\mu - 8) = 0 \Rightarrow 1 - (\mu - 8) = 0 \Rightarrow \mu = 9$

$\Rightarrow$ centre of circle (5, 9)

radius = length of $\perp$ from centre (5, 9) = $\left|\frac{20-27}{5}\right| = \frac{7}{5}$

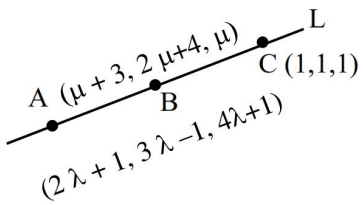
18. Let the line L pass through (1, 1, 1) and intersect the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z}{1}$. Then, which of the following points lies on the line L?

(1) (4, 22, 7) (2) (5, 4, 3)

(3) (10, –29, –50) (4) (7, 15, 13)

Ans. **(4)**

Sol.



Dr's of AC $\Rightarrow 2\lambda, 3\lambda - 2, 4\lambda$

Dr's of BC $\Rightarrow \mu + 2, 2\mu + 3, \mu - 1$

$\Rightarrow \frac{\mu+2}{2\lambda} = \frac{2\mu+3}{3\lambda-2} = \frac{\mu-1}{4\lambda}$

$\Rightarrow 2(\mu + 2) = \mu - 1 \Rightarrow \mu = -5$

$\Rightarrow$ Dr's of BC $\Rightarrow 3, 7, 6$

$\Rightarrow$ equation of L $\Rightarrow \frac{x-1}{3} = \frac{y-1}{7} = \frac{z-1}{6}$

(7, 15, 13) satisfies.

19. Let the angle $\theta, 0 < \theta < \frac{\pi}{2}$ between two unit vectors $\hat{a}$ and $\hat{b}$ be $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$. If the vector $\vec{c} = 3\hat{a} + 6\hat{b} + 9(\hat{a} \times \hat{b})$, then the value of $9(\vec{c} \cdot \hat{a}) - 3(\vec{c} \cdot \hat{b})$ is

(1) 31 (2) 27

(3) 29 (4) 24

Ans. (3)

Sol. $\vec{c} = 3\vec{a} + 6\vec{b} + 9\left(\vec{a} \times \vec{b}\right)$

$\sin^{-1}\left(\frac{\sqrt{65}}{9}\right) \Rightarrow \sin\theta = \frac{\sqrt{65}}{9} \Rightarrow \cos\theta = \frac{4}{9}$

$\vec{c}.\vec{a} = 3|\vec{a}|^2 + 6\vec{a}.\vec{b} = 3 + \frac{6.4}{9} = \frac{51}{9}$

$\vec{c}.\vec{a} = 3\vec{a}.\vec{b} + 6|\vec{b}|^2 = \frac{3.4}{9} + 6 = \frac{22}{3}$

$\therefore\ 9\left(\vec{c}.\vec{a}\right) - 3\left(\vec{c}.\vec{b}\right) = 51 - 22 = 29$

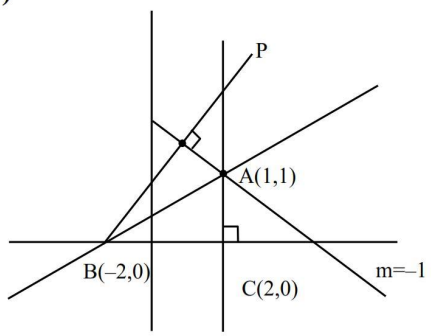
20. Let ABC be the triangle such that the equations of lines AB and AC be $3y - x = 2$ and $x + y = 2$, respectively, and the points B and C lie on x-axis. If P is the orthocentre of the triangle ABC, then the area of the triangle PBC is equal to

(1) 4 (2) 10

(3) 8 (4) 6

Ans. (4)

Sol.



Equation of Altitude AP : $x = 1$

Equation of Altitude BP : $y - 0 = 1(x + 2)$

$\Rightarrow x = 1$ &

$x - y + 2 = 0$

P(1, 3)

Area of $\Delta PBC = \frac{1}{2} \times 4 \times 3 = 6$

SECTION-B

21. The number of points of discontinuity of the function $f(x) = \left[\frac{x^2}{2}\right] - \left[\sqrt{x}\right], x \in [0,4]$, where $[\cdot]$ denotes the greatest integer function is _____

Ans. (8)

Sol. Check for $\left[\frac{x^2}{2}\right]$ and $\left[\sqrt{x}\right]$ becomes integers.

$\{0, 1, \sqrt{2}, 2, \sqrt{6}, \sqrt{8}, \sqrt{10}, \sqrt{12}, \sqrt{14}, 4\}$

Continuous at 0^+, continuous at 4^-

$\left[\frac{x^2}{2}\right] = \left[\sqrt{x}\right]$, occurs at $x = \sqrt{2}$

$\Rightarrow$ Not continuous

$\therefore$ function is discontinuous at 8 points.

22. The number of relations on the set A = {1, 2, 3} containing at most 6 elements including (1, 2), which are reflexive and transitive but not symmetric, is _____

Ans. (5)

Sol. A = {1,2,3}

(1,1), (2,2), (3,3), (1,2) $\in$ R

Remaining elements are

(2,1), (2,3), (1,3), (3,1), (3,2)

(1) If relation contains exactly 4 elements = 1 way

(2) if relation contains exactly 5 elements

It can be (1, 3), (3, 2) $\Rightarrow$ 2 ways

(3) If relation contain exactly 6 elements

It can be

((2, 3), (1,3)), ((1,3), (3, 2)), ((3,1), (3,2))

$\Rightarrow$ 3 ways.

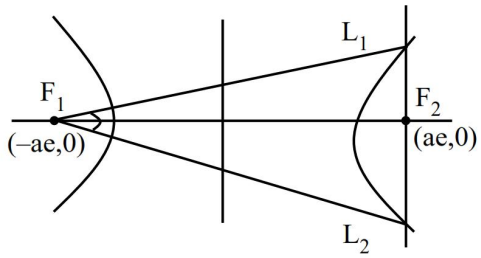
Total = 6 ways

23. Consider the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having one of its focus at P(–3, 0). If the latus ractum through its other focus subtends a right angle at P and $a^2b^2=\alpha\sqrt{2}-\beta,\alpha,\beta\in\mathbb{N}$.

Ans. (1944)

Sol. $f_1\equiv(-ae,0)\equiv P(-3,0)$

$\Rightarrow ae=3$



$\tan 45^o=\frac{b^2/a}{2ae}$

$2ae=\frac{b^2}{a}$

$b^2=6a$

Also $a^2e^2=a^2+b^2$

$9=a^2+6a$

$a^2+6a-9=0$

$a=-3\pm3\sqrt{2}=-3\left(1\pm\sqrt{2}\right)$

$\therefore\ a^2b^2=a^2.6a=6a^3$

$=6(135\sqrt{2}-189)$

$\alpha=810$ and $\beta=1134$

$\therefore\ \alpha+\beta=1944$

24. The number of singular matrices of order 2, whose elements are from the set {2, 3, 6, 9} is

Ans. (36)

Sol. $\begin{vmatrix}a & d\\ b & c\end{vmatrix}=ad-bc\Rightarrow ad=bc$

Case-I Exactly 1 no. is used

$\Rightarrow$ All singular $={}^4C_1$

Case-II Exactly 2 no. is used

$\Rightarrow {}^4C_2\times2\times2$

Case-III Exactly 3 no. is used

None will be singular

Case-IV Exactly 4 No. is used

ad = bc

$\Rightarrow 2\times9=3\times6$

$\begin{vmatrix}9 & -\\ - & 2\end{vmatrix}\Rightarrow {}^4C_1\times21$

Total = 36

25. For $n\geq2$, let S_n denote the set of all subsets of {1, 2.....,*n*}with **no** two consecutive numbers. For example $\{1,3,5\}\in S_6$, but $\{1,2,4\}\notin S_6$. Then $n(S_5)$ is equal to _____

Ans. (13)

Sol. A = {1, 2, 3, 4, 5.....n}

No. of subsets having r elements such that no two are consecutive is $={}^{n-r+1}C_r$

for n = 5, no. of ways $={}^{6-r}C_r$

Subsets having no element = 1

Subsets having exactly 1 element $={}^5C_1=5$

Subsets having exactly 2 element $={}^4C_2=6$

Subsets having exactly 3 element $={}^3C_3=1$

$\Rightarrow 5+6+1+1=13$

SECTION-A

26. Two harmonic waves moving in the same direction superimpose to form a wave x = a cos (1.5t) cos (50.5t) where t is in seconds. Find the period with which they beat (close to nearest integer)

(1) 6 s (2) 4 s

(3) 1 s (4) 2 s

Ans. **(4)**

Sol. The given equation can be written as

$$x = \frac{a}{2}\cos[1.5+50.5]t + \frac{a}{2}\cos[50.5-1.5]$$

$$x = \frac{a}{2}\cos[52t] + \frac{a}{2}\cos[49t]$$

Here, $2\pi f_1$ & $2\pi f_2 = 49$

$$f_1 = \frac{52}{2\pi},\ f_2 = \frac{49}{2\pi}$$

$$\therefore\ f_{Beat} = f_1 - f_2 = \frac{3}{2\pi}\text{Hz}$$

$$\therefore\ T_{Beat} = \frac{1}{f_{Beat}} = \frac{2\pi}{3}\text{sec}$$

$= 2.09 \text{ sec} \approx 2 \text{ sec}$

27. Two plane polarized light waves combine at a certain point whose electric field components are

$E_1 = E_0 \sin \omega t$

$E_2 = E_0 \sin(\omega t + \frac{\pi}{3})$

Find the amplitude of the resultant wave.

(1) 0.9 E (2) E_0

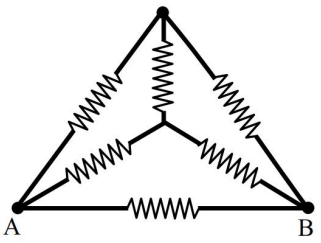
(3) 1.7 E_0 (4) 3.4 E_0

Ans. **(3)**

Sol. $$E = \sqrt{(E_0)^2 + (E_0)^2 + 2(E_0)(E_0)\cos\frac{\pi}{3}}$$

$$E = \sqrt{2E_0^2 + E_0^2} = \sqrt{3}E_0 = 1.73E_0$$

28. A wire of resistance R is bent into a triangular pyramid as shown in figure with each segment having same length. The resistance between points A and B is R/n. The value of n is :

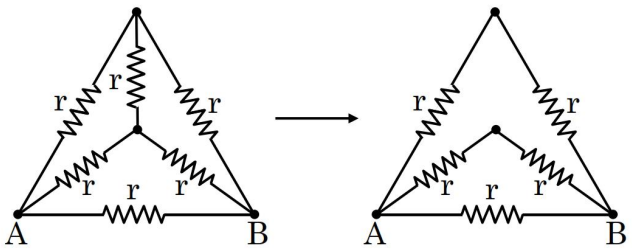


(1) 16 (2) 14

(3) 10 (4) 12

Ans. **(4)**

Sol. As $r = \frac{R}{6}$



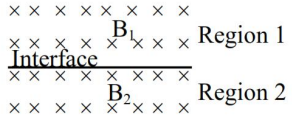
(As balanced wheat stone bridge is formed)

Now, Equivalent resistance between A and B can be written as

$$\frac{1}{R_{AB}} = \frac{1}{2r} + \frac{1}{2r} + \frac{1}{r} = \frac{2}{r}$$

$$R_{AB} = \frac{R}{12}$$

29. Uniform magnetic fields of different strengths (B_1 and B_2), both normal to the plane of the paper exist as shown in the figure. A charged particle of mass m and charge q, at the interface at an instant, moves into the region 2 with velocity v and returns to the interface. It continues to move into region 1 and finally reaches the interface. What is the displacement of the particle during this movement along the interface ?



(Consider the velocity of the particle to be normal to the magnetic field and $B_2 > B_1$)

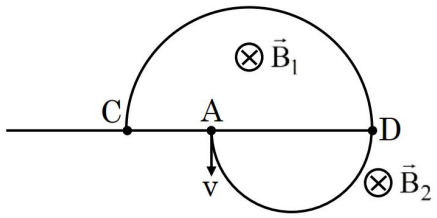
(1) $\frac{mv}{qB_1}\left(1-\frac{B_2}{B_1}\right)\times 2$ (2) $\frac{mv}{qB_1}\left(1-\frac{B_1}{B_2}\right)$

(3) $\frac{mv}{qB_1}\left(1-\frac{B_2}{B_1}\right)$ (4) $\frac{mv}{qB_1}\left(1-\frac{B_1}{B_2}\right)\times 2$

Ans. (4)

Sol. As $\vec{v}$ is $\perp$ to $\vec{B}$, so charge particle will move in circular path, whose radius is given by

$$R = \frac{mv}{qB}$$



Starting point → A

Ending point → C

∴ Net displacement = AC

AC = CD – AD

$$AC = \frac{2mv}{qB_1} - \frac{2mv}{qB_2}$$

$$AC = \frac{2mv}{qB_1}\left[1-\frac{B_1}{B_2}\right]$$

30. If $\in_0$ denotes the permittivity of free space and Φ_E is the flux of the electric field through the area bounded by the closed surface, then dimension of $\left(\in_0 \frac{d\phi_E}{dt}\right)$ are that of :

(1) Electric field (2) Electric potential

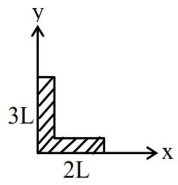
(3) Electric charge (4) Electric current

Ans. (4)

Sol. We know that formula for displacement current is given by

$$id = \varepsilon_0 \frac{d\phi_\varepsilon}{dt}$$

31. A rod of length 5 L is bent right angle keeping one side length as 2 L.



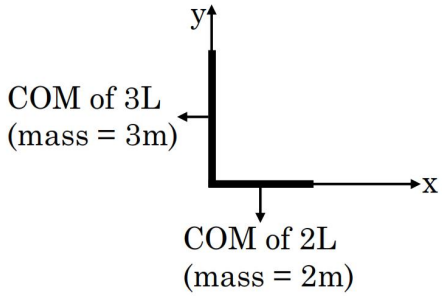
The position of the centre of mass of the system:
(Consider L = 10 cm)

(1) $2\hat{i} + 3\hat{j}$ (2) $3\hat{i} + 7\hat{j}$

(3) $5\hat{i} + 8\hat{j}$ (4) $4\hat{i} + 9\hat{j}$

Ans. (4)

Sol.



$$x_{com} = \frac{2m(10)+3m(0)}{5m} = 4cm$$

$$y_{com} = \frac{2m(0)+3m(15)}{5m} = 9cm$$

$$\vec{r}_{com} = 4\hat{i} + 9\hat{j}$$

32. The percentage increase in magnetic field (B) when space within a current carrying solenoid is filled with magnesium (magnetic susceptibility $\chi_{mg} = 1.2 \times 10^{-5}$) is :

(1) $\frac{6}{5}\times10^{-3}\%$ (2) $\frac{5}{6}\times10^{-5}\%$

(3) $\frac{5}{6}\times10^{-4}\%$ (4) $\frac{5}{3}\times10^{-5}\%$

Ans. (1)

Sol. $\%$ change in $B = \frac{B_{new} - B_{old}}{B_{old}} \times 100\%$

$= \frac{\mu ni - \mu_0 ni}{\mu_0 ni} \times 100\% = \frac{(\mu - \mu_0)}{\mu_0} \times 100\%$

$= \frac{(\mu_0\mu_r - \mu_0)}{\mu_0} \times 100\%$

$= (\mu_r - 1) \times 100\%$

$= \chi_n \times 100\%$

$= 1.2 \times 10^{-3}\ \%$

33. A lens having refractive index 1.6 has focal length of 12 cm, when it is in air. Find the focal length of the lens when it is placed in water.
(Take refractive index of water as 1.28)

(1) 355 mm (2) 288 mm

(3) 555 mm (4) 655 mm

Ans. (2)

Sol. As we know,

$$\frac{1}{f} = \left[\frac{\mu_L}{\mu_m} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

For air $\mu_m = 1$

$$\frac{1}{12} = [1.6 - 1]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{12} = \frac{6}{10}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{10}{72}$$

For water

$$\frac{1}{f} = \left[\frac{1.6}{1.28} - 1\right]\left[\frac{10}{72}\right] = \frac{32}{128} \times \frac{10}{72}$$

$$\frac{1}{f} = \frac{1}{4} \times \frac{10}{72}$$

$f = 28.8$ cm

$f = 288$ mm

34. An ac current is represented as

$$i = 5\sqrt{2} + 10\cos\left(650\pi t + \frac{\pi}{6}\right) \text{Amp}$$

The r.m.s value of the current is

(1) 50 Amp (2) 100 Amp

(3) 10 Amp (4) $5\sqrt{2}$ Amp

Ans. (3)

Sol. $$i = 5\sqrt{2} + 10\cos\left(650\pi t + \frac{\pi}{6}\right)$$

$$i^2 = 50 + 100\cos^2\left(650\pi t + \frac{\pi}{6}\right)$$

$$+(2)(5\sqrt{2})(10)\cos\left(650\pi t + \frac{\pi}{6}\right)$$

$$< i^2 >= 50 + \frac{100}{2} + 0$$

$< i^2 > = 100$

$< i > = 10$ Amp.

35. Two thin convex lenses of focal lengths 30 cm and 10 cm are placed coaxially, 10 cm apart. The power of this combination is :

(1) 5 D (2) 1 D

(3) 20 D (4) 10 D

Ans. (4)

Sol. $f_1 = 30$ cm, $f_2 = 10$ cm

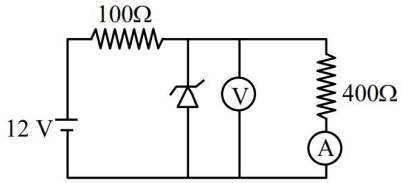
$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$, d = distance between lens

$$\frac{1}{f_{eq}} = \frac{1}{0.3} + \frac{1}{0.1} - \frac{0.1}{(0.3)(0.1)}$$

$$\frac{1}{f_{eq}} = \frac{1}{0.1}$$

$$\text{Power} = \frac{1}{f_{eq}} = 10\,\text{D}$$

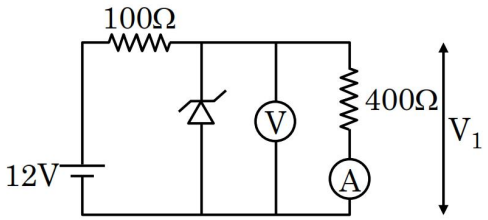
36. In the following circuit, the reading of the ammeter will be (Take Zener breakdown voltage = 4 V)



(1) 24 mA (2) 80 mA
(3) 10 mA (4) 60 mA

Ans. (3)

Sol.



$$V_1 = \frac{400}{100+400} \times 12V = \frac{4}{5} \times 12 = \frac{48}{5} V$$

here, $V_1 > V_z$, (V_z = Zener Voltage)

So, Zener breakdown will be take place

So, voltage across 400Ω will be 4V

$$I = \frac{4}{400} A = \frac{1}{100A} = 10mA$$

37. Two projectiles are fired from ground with same initial speeds from same point at angles $(45^\circ + \alpha)$ and $(45^\circ - \alpha)$ with horizontal direction. The ratio of their times of flights is

(1) 1 (2) $\frac{1-\tan\alpha}{1+\tan\alpha}$

(3) $\frac{1+\sin 2\alpha}{1-\sin 2\alpha}$ (4) $\frac{1+\tan\alpha}{1-\tan\alpha}$

Ans. (4)

Sol. $\theta_1 = 45 + \alpha$; $\theta_2 = 45 - \alpha$

Time of flight, $T = \frac{2v\sin\theta}{g}$

$$\frac{T_1}{T_2} = \frac{\sin(45+\alpha)}{\sin(45-\alpha)}$$

$$\frac{T_1}{T_2} = \frac{\frac{1}{\sqrt{2}}\cos\alpha + \frac{1}{\sqrt{2}}\sin\alpha}{\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{\sqrt{2}}\sin\alpha}$$

$$\frac{T_1}{T_2} = \frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

38. Match the **List-I** with **List-II**

List-I		List-II	
A.	Triatomic rigid gas	I.	$\frac{C_P}{C_V} = \frac{5}{3}$
B.	Diatomic non-rigid gas	II.	$\frac{C_P}{C_V} = \frac{7}{5}$
C.	Monoatomic gas	III.	$\frac{C_P}{C_V} = \frac{4}{3}$
D.	Diatomic rigid gas	IV	$\frac{C_P}{C_V} = \frac{9}{7}$

Choose the **correct** answer from the options given below :

(1) A-III, B-IV, C-I, D-II

(2) A-III, B-II, C-IV, D-I

(3) A-II, B-IV, C-I, D-III

(4) A-IV, B-II, C-III, D-I

Ans. (1)

Sol. $\gamma = 1 + \frac{2}{f}$

f = 6, Triatomic rigid gas

f = 7, Diatomic non-rigid gas

f = 5, Diatomic rigid gas

f = 3, monoatomic rigid gas

$\gamma = 1 + \frac{2}{6} = \frac{4}{3}$ (Triatomic)

$\gamma = 1 + \frac{2}{7} = \frac{9}{7}$ (Diatomic, non-rigid)

$\gamma = 1 + \frac{2}{5} = \frac{7}{5}$ (Diatomic, rigid)

$\gamma = 1 + \frac{2}{3} = \frac{5}{3}$ (Monoatomic, rigid)

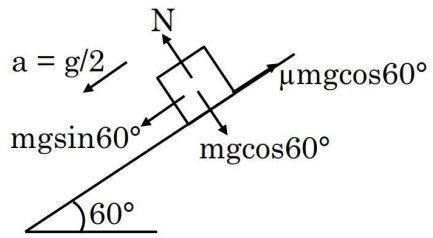
A-III, B-IV, C-I, D-II

39. A cubic block of mass m is sliding down on an inclined plane at 60° with an acceleration of $\frac{g}{2}$, the value of coefficient of kinetic friction is

(1) $\sqrt{3}-1$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{2}}{3}$ (4) $1-\frac{\sqrt{3}}{2}$

Ans. (1)

Sol.



$mg\sin60° - \mu mg\cos60° = ma$

$$g\sin 60 - \mu g\cos 60 = \frac{g}{2}$$

$$\frac{\sqrt{3}}{2} - \frac{\mu}{2} = \frac{1}{2}$$

$$\mu = \sqrt{3} - 1$$

40. In a hydrogen like ion, the energy difference between the 2nd excitation energy state and ground is 108.8 eV. The atomic number of the ion is

(1) 4 (2) 2 (3) 1 (4) 3

Ans. (2)

Sol. $$\Delta E = 13.6z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$(13.6)z^2\left[\frac{1}{1} - \frac{1}{9}\right] = 108.8$$

$$\frac{(13.6)(8)}{9}(z^2) = 108.8$$

$$z = 3$$

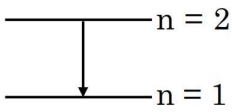
41. For a hydrogen atom, the ratio of the largest wavelength of Lyman series to that of the Balmer series is.

(1) 5 : 36 (2) 5 : 27

(3) 3 : 4 (4) 27 : 5

Ans. (2)

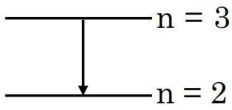
Sol. **Lyman**



$$\frac{1}{\lambda_1} = R\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3R}{4}$$

$$\lambda_1 = \frac{4}{3R} \quad ...(1)$$

and **Balmer**



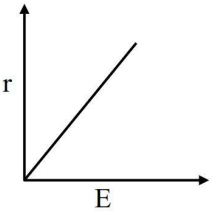
$$\frac{1}{\lambda_2} = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5R}{36}$$

$$\lambda_2 = \frac{36}{5R}$$

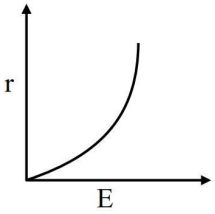
Then, $\frac{\lambda_1}{\lambda_2} = \frac{5}{27}$

42. A particle of charge q, mass m and kinetic energy E enters in magnetic field perpendicular to its velocity and undergoes a circular arc of radius(r). Which of the following curves represents the variation of r with E?

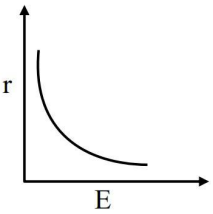
(1)



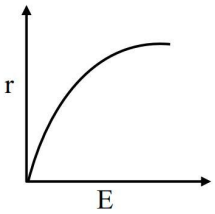
(2)



(3)

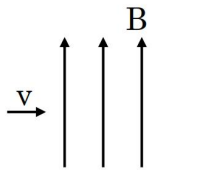


(4)



Ans. (4)

Sol.



$$\frac{mv^2}{r} = qvB$$

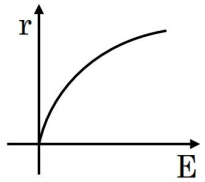
$$mv = qBr$$

$$E = \frac{1}{2}mv^2$$

$$E = \frac{1}{2}m\left(\frac{q^2B^2r^2}{m^2}\right) = \frac{q^2B^2r^2}{2m}$$

$$E = \left(\frac{q^2B^2}{2m}\right)r^2$$

$$r^2 \propto E$$



43. An object of mass 1000 g experiences a time dependent force $\vec{F} = \left(2t\hat{i} + 3t^2\hat{j}\right)N$. The power generated by the force at time t is :

(1) $(2t^2 + 3t^3)$ W (2) $(2t^2 + 18t^3)$ W

(3) $(3t^3 + 5t^5)$ W (4) $(2t^3 + 3t^5)$ W

Ans. (4)

Sol. $\vec{F} = (2t\hat{i} + 3t\hat{j})N$

$m = 1000\ gm = 1\ kg$

$\vec{F} = m\vec{a}$, $\vec{a} = 2t\hat{i} + 3t^2\hat{j}$

$$\frac{d\vec{v}}{dt} = 2t\hat{i} + 3t^2\hat{j}$$

$$\vec{v} = t^2\hat{i} + t^3\hat{j}$$

Power, $P = \vec{F}\cdot\vec{v}$

$$P = (2t\hat{i} + 3t^2\hat{j})\cdot(t^2\hat{i} + t^3\hat{j})$$

$$P = (2t^3 + 3t^5)W$$

44. Two wires A and B are made of same material having ratio of lengths $\frac{L_A}{L_B} = \frac{1}{3}$ and their diameters ratio $\frac{d_A}{d_B} = 2$. If both the wires are stretched using same force, what would be the ratio of their respective elongations?

(1) 1 : 6 (2) 1 : 12

(3) 3 : 4 (4) 1 : 3

Ans. (2)

Sol. $\frac{L_A}{L_B} = \frac{1}{3}$ and $\frac{d_A}{d_B} = 2$

$$\Delta L_A = \frac{F_A L_A}{A_A Y_A} \text{ and } \Delta L_B = \frac{F_B L_B}{A_B Y_B}$$

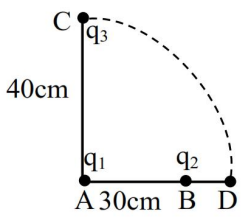
Given, $F_A = F_B$ and $Y_A = Y_B$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{\frac{F_A L_A}{A_A Y_A}}{\frac{F_B L_B}{A_B Y_B}} = \left(\frac{L_A}{L_B}\right)\left(\frac{A_B}{A_A}\right)$$

$$\frac{\Delta L_A}{\Delta L_B} = \left(\frac{L_A}{L_B}\right)\left(\frac{\frac{\pi}{4}d_B^2}{\frac{\pi}{4}d_A^2}\right) = \left(\frac{L_A}{L_B}\right)\left(\frac{d_B}{d_A}\right)^2$$

$$\frac{\Delta L_A}{\Delta L_B} = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{12}$$

45. Two charges q_1 and q_2 are separated by a distance of 30 cm. A third charge q_3 initially at 'C' as shown in the figure, is moved along the circular path of radius 40 cm from C to D. If the difference in potential energy due to movement of q_3 from C to D is given by $\frac{q_3 K}{4\pi\epsilon_0}$, the value of K is :



(1) $8q_2$ (2) $6q_2$

(3) $8q_1$ (4) $6q_1$

Ans. (1)

Sol. Potential at C

$$V_C = \frac{kq_1}{0.4} + \frac{kq_2}{0.5}$$

Potential at D

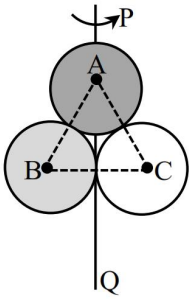
$$V_D = \frac{kq_1}{0.4} + \frac{kq_2}{0.1}$$

$$\Delta U = (V_D - V_C)(q_3) = \left(\frac{kq_2}{0.1} - \frac{kq_2}{0.5}\right)(q_3)$$

$$\Delta U = 8kq_2q_3 = \frac{8q_2q_3}{4\pi\varepsilon_0}$$

SECTION-B

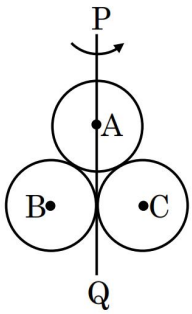
46. A,B and C are disc, solid sphere and spherical shell respectively with same radii and masses. These masses are placed as shown in figure.



The moment of inertia of the given system about PQ is $\frac{x}{15}I$, where I is the moment of inertia of the disc about its diameter. The value of x is_____.

Ans. **(199)**

Sol.



All bodies have same mass and same radius.

A → Disc

B → Solid sphere

C → Spherical shell

and, $I = \frac{MR^2}{4}$

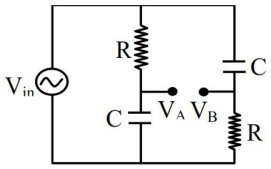
$$I_{PQ} = \frac{MR^2}{4} + \left(\frac{2}{5}MR^2 + MR^2\right) + \left(\frac{2}{3}MR^2 + MR^2\right)$$

$$I_{PQ} = \frac{15MR^2 + 24MR^2 + 60MR^2 + 40MR^2 + 60MR^2}{60}$$

$$I_{PQ} = \frac{199}{60}MR^2 = \frac{199}{15}\left(\frac{MR^2}{4}\right)$$

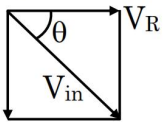
$$= \frac{199}{15}I$$

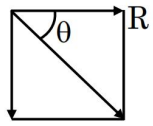
47. For ac circuit shown in figure, R = 100 kΩ and C = 100 pF and the phase difference between V_{in} and $(V_B - V_A)$ is 90°. The input signal frequency is 10^x rad/sec, where 'x' is _____



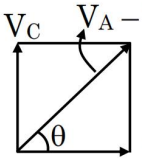
Ans. **(5)**

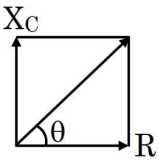
Sol. Input voltage





$V_A - V_B$:





$\theta + \theta = 90°$; $\theta = 45°$

$$\tan\theta = \frac{X_C}{R}$$

$$X_C = R \Rightarrow \frac{1}{W_C} = R$$

$$W = \frac{1}{R_C} = \frac{1}{100\times10^3\times100\times10^{-12}}$$

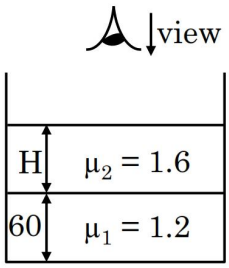
$$= \frac{10^{12}}{10^7} = 10^5$$

48. A container contains a liquid with refractive index of 1.2 up to a height of 60 cm and another liquid having refractive index 1.6 is added to height H above first liquid. If viewed from above, the apparent shift in the position of bottom of container is 40 cm. The value of H is___cm.

(Consider liquids are immisible)

Ans. (80)

Sol.



y = apparent depth of bottom

$$\frac{y}{1} = \frac{H}{1.6} + \frac{60}{1.2}$$

Shift = 40

$H + 60 - y = 40$

$$H + 60 - \frac{H}{1.6} - \frac{60}{1.2} = 40$$

$$\frac{6}{16}H = 30$$

H = 80 cm

49. A wire of length 10 cm and diameter 0.5 mm is used in a bulb. The temperature of the wire is 1727°C and power radiated by the wire is 94.2 W. Its emissivity is $\frac{x}{8}$ where x =_____

(Given $\sigma = 6.0 \times 10^{-8}$ W m^{-2} K^{-4}, $\pi = 3.14$ and assume that the emissivity of wire material is same at all wavelength.)

Ans. (5)

Sol. L = 10 cm, d = 0.5 mm, T = 1727°C = 2000 K

Power, P = 94.2 W

$P = \varepsilon\sigma AT^4$

$94.2 = \varepsilon \times (6 \times 10^{-8})(\pi dL)(2000)^4$

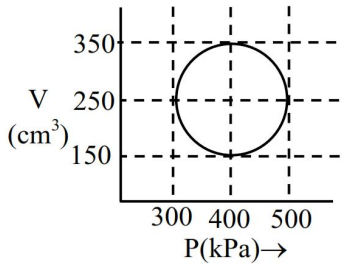
$94.2 = \varepsilon \times (6 \times 10^{-8})\ (3.14)\ (0.5)\ (10^{-3})$

$(10 \times 10^{-2})(2000)^4$

$$\varepsilon = \frac{94.2}{(94.2)(16)} = \frac{5}{8}$$

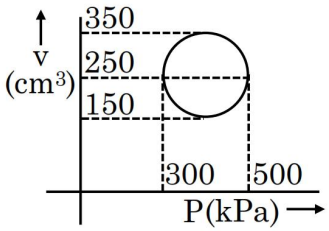
50. An ideal gas has undergone through the cyclic process as shown in the figure. Work done by the gas in the entire cycle is_____ $\times 10^{-1}$ J.

(Take $\pi = 3.14$)



Ans. (314)

Sol.



Area of circle, $W = \frac{\pi}{4}d_1d_2$

$$W = \frac{\pi}{4}(500-300)\times10^3(350-150)\times10^{-6}$$

W = 31.4 Joule

W = 314 $\times 10^{-1}$ Joule

SECTION-A

51. Given below are two statements :

Statement I : Ozonolysis followed by treatment with Zn, H_2O of cis-2-butene gives ethanal.

Statement II : The production obtained by ozonolysis followed by treatment with Zn, H_2O of 3, 6-dimethyloct-4-ene has no chiral carbon atom.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Both Statement I and Statement II are true

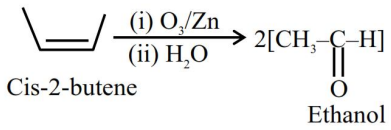
(2) Statement I is false but Statement II are true

(3) Statement I is true but Statement II is false

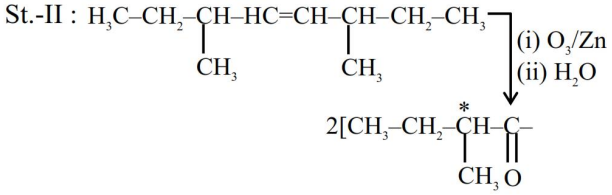
(4) Both Statement I and Statement II are false

Ans. (3)

Sol. St.-I :



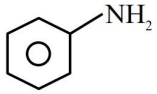
St.-II :



St-I : Correct statement

St-II : In correct statement because product has chiral centre.

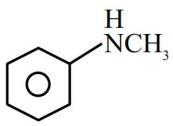
52. Which of the following amine (s) show (s) positive carbylamines test ?

A. 

B. $(CH_3)_2NH$

C. CH_3NH_2

D. $(CH_3)_3N$

E. 

Choose the **correct** answer from the options given below :

(1) A and E Only

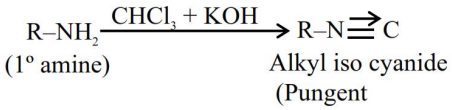
(2) C Only

(3) A and C Only

(4) B, C and D Only

Ans. (3)

Sol. Only 1° or primary amines gives positive carbylamines test.



Option (A) and (C) are primary amine and given ⊕ve carbyl amine test

53. Reaction $A(g) \rightarrow 2B(g) + C(g)$ is a first order reaction. It was started with pure A

t/min	Pressure of system at time t/mm Hg
10	160
∞	240

Which of the following option is <u>incorrect</u> ?

(1) Initial pressure of A is 80 mm Hg

(2) The reaction never goes to completion

(3) Rate constant of the reaction is 1.693 min^{-1}

(4) Partial pressure of A after 10 minute is 40 mm Hg

Ans. (3)

Sol. $A(g) \longrightarrow 2B(g) + C(g)$

$t = 0 \quad P_0$

$t \to \infty \quad 0 \quad 2P_0 \quad P_0$

$P_\infty = 3P_0 = 240$

$P_0 = 80$ mm of Hg

$$Kt = \ell n\left(\frac{P_\infty - P_0}{P_\infty - Pt}\right)$$

$$K \times 10 = \ell n\left(\frac{240-80}{240-160}\right)$$

$$K = \frac{\ell n 2}{10} = 0.0693 \text{ min}^{-1}$$

Option (3) is incorrect

54. Total enthalpy change for freezing of 1 mol of water at 10°C to ice at –10°C is _____

(Given : $\Delta_{fus}H = x$ kJ/mol

$C_p[H_2O(l)] = y$ J mol^{-1} K^{-1}

$C_p[H_2O(s)] = z$ J mol^{-1} K^{-1}

(1) $-x - 10y - 10z$

(2) $-10(100x + y + z)$

(3) $10(100x + y + z)$

(4) $x - 10y - 10z$

Ans. (2)

Sol.

$$\begin{array}{ccc} H_2O_{(\ell)} & \longrightarrow & H_2O_{(s)} \\ 10°C & & 10°C \\ \downarrow nC_{P(\ell)}\Delta T & & \uparrow nC_{P(s)}\Delta T \\ H_2O_{(\ell)} & \xrightarrow{-\Delta H_{fusion}} & H_2O_{(s)} \\ 0°C & & 0°C \end{array}$$

$\Delta H = 1 \times y(0 - 10) - x \times 1000 + 1 \times z(-10° - 0°)$

$\Delta H = -10(100x + y + z)$ Joule.

55. An aqueous solution of HCl with pH 1.0 is diluted by adding equal volume of water (ignoring dissociation of water). The pH of HCl solution would

(Given log 2 = 0.30)

(1) reduce to 0.5

(2) increase to 1.3

(3) remain same

(4) increase to 2

Ans. (2)

Sol. $HCl_{(aq)}$ pH = 1; $[H^+] = 10^{-1}$

If equal volume of water is added concentration will become half

$$[H^+]_{sol} = \frac{10^{-1}}{2}$$

pH = 1.3

56. Given below are two statements :

Statement I : Dimethyl ether is completely soluble in water. However, diethyl ether is soluble in water to a very small extent.

Statement II : Sodium metal can be used to dry diethyl ether and not ethyl alcohol.

In the light of given statements, choose the ***correct*** answer from the options given below

(1) Statement I is false but Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

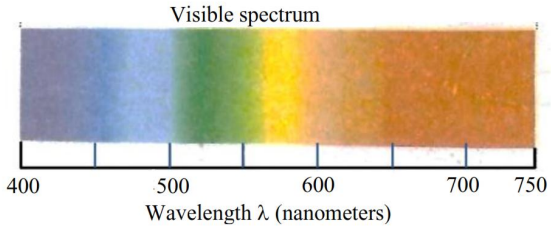
Ans. (4)

Sol. **St-I** – St-I is correct because both given ether are soluble in water → Di ethyl ether and butan-1-ol are miscible to almost same extent i.e., 7.5 and

9 gm per 100 ml water due to H-bonding

St-II : - St. II is also correct because sodium metal is not used with ethyl alcohol as H_2 gas release with ethyl a below

57.



Which of the following statements are correct, if the threshold frequency of caesium is 5.16×10^{14} Hz ?

A. When Cs is placed inside a vacuum chamber with an ammeter connected to it and yellow light is focused on Cs the ammeter shows the presence of current.

B. When the brightness of the yellow light is dimmed, the value of the current in the ammeter is reduced.

C. When a red light is used instead to the yellow light, the current produced is higher with respect to the yellow light.

D. When a blue light is used, the ammeter shows the formation of current.

E. When a white light is used, the ammeter shows formation of current.

Choose the correct answer from the options given below :

(1) A, D and E Only

(2) B, C and D Only

(3) A, C, D and E Only

(4) A, B, D and E Only

Ans. (4)

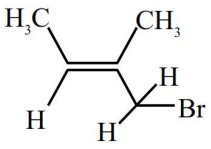
Sol. $\lambda = \frac{C}{\nu} = \frac{3\times10}{5.16\times10^{14}}$

$\lambda = 581.39$ nm

* λ_{Photon} is near & below yellow light it can show photoelectric effect.
* If intensity of light decreases photocurrent decreases.
* Red light will not produce photoelectric effect.
* $\nu_{Blue} > \nu_{yellow}$ so photoelectric current will be produced.
* White light contain all frequencies so it will show photo electric current.

Correct statements are ABD & E.

58. Which of the following is the correct IUPAC name of given organic compound (X) ?



(X)

(1) 2-Bromo-2-methylbut-2-ene

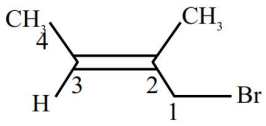
(2) 3-Bromo-3-methylprop-2-ene

(3) 1-Bromo-2-methylbut-2-ene

(4) 4-Bromo-3-methylbut-2-ene

Ans. (3)

Sol.



1-Bromo-2-methyl but-2-ene

59. At the sea level, the dry air mass percentage composition is given as nitrogen gas : 70.0, oxygen gas : 27.0 and argon gas : 3.0. If total pressure is 1.15 atm, then calculate the ratio of followings respectively :

(i) partial pressure of nitrogen gas to partial pressure of oxygen gas

(ii) partial pressure of oxygen gas to partial pressure of argon gas

(Given : Molar mass of N, O and Ar are 14, 16, and 40 g mol^{-1} respectively)

(1) 4.26, 19.3 (2) 2.59, 11.85

(3) 5.46, 17.8 (4) 2.96, 11.2

Ans. (4)

Sol. $\frac{P_{N_2}}{Po_2} = \frac{x_{N_2}.P_T}{x_{O_2}.P_T} = \frac{n_{N_2}}{n_{O_2}}$ {using Dalton's law of partial pressure}

$= \frac{70/28}{27/32} = 2.96$

$\frac{P_{O_2}}{P_{Ar}} = \frac{n_{O_2}}{n_{Ar}} = \frac{27/32}{3/40} = 11.25$

60. Given below are two statements :

Statement I : Mohr's salt is composed of only three types of ions-ferrous, ammonium and sulphate.

Statement II : If the molar conductance at infinite dilution of ferrous, ammonium and sulphate ions are x_1, x_2 and x_3 S cm^2 mol^{-1}, respectively then the molar conductance for Mohr's salt solution at infinite dilution would be given by $x_1 + x_2 + 2x_3$

In the light of the given statements, choose the **correct** answer from the options given below :

(1) Both statements I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II are false

(4) Both statements I and Statement II are true

Ans. (3)

Sol. Mohr's salt : $FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$

Using Kohlrousch law

λ_m^∞ (Mohr's salt) = $x_1 + 2x_2 + 2x_3$

61. The number of valence electrons present in the metal among Cr, Co, Fe and Ni which has the lowest enthalpy of atomisation is

(1) 8

(2) 9

(3) 6

(4) 10

Ans. (3)

Sol. Out of Cr, Co, Fe and Ni

Chromium has lowest heat of atomisation.

Cr = $[Ar]3d^5 4s^1$

$\because$ Total six valence e^- in Cr.

62. When a salt is treated with sodium hydroxide solution it gives gas X. On passing gas X through reagent Y a brown coloured precipitate is formed. X and Y respectively, are

(1) X = NH_3 and Y = HgO

(2) X = NH_3 and Y = K_2HgI_4 + KOH

(3) X = NH_4Cl and Y = KOH

(4) X = HCl and Y = NH_4Cl

Ans. (2)

Sol. $NH_4^+ + NaOH \longrightarrow H_2O + NH_3\uparrow$

$$2[HgI_4]^{2-} + NH_3 + 3OH^- \longrightarrow \underbrace{HgO.Hg\begin{matrix}NH_2\\ I\end{matrix}}_{\text{Brown ppt}} + 7I^- + 2H_2O$$

NH_3 is identify by $K_2[HgI_4]$ + KOH

63. The group 14 elements A and B have the first ionisation enthalpy values of 708 and 715 kJ mol^{-1} respectively. The above values are lowest among their group members. The nature of their ions A^{2+} B^{4+} respectively is

(1) both reducing (2) both oxidising

(3) reducing and oxidising (4) oxidising and reducing

Ans. (3)

Sol. As per given information of ionisation energy

A = Sn & B = Pb

$A^{+2} = Sn^{2+}$ = Reducing agent

$B^{+4} = Pb^{+4}$ = Oxidising agent

64. The first transition series metal 'M' has the highest enthalpy of atomisation in its series. One of its aquated ion (M^{n+}) exists in green colour. The nature of the oxide formed by the above M^{n-} ion is :

(1) neutral

(2) acidic

(3) basic

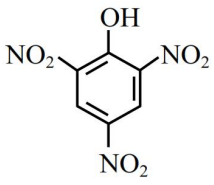
(4) amphoteric

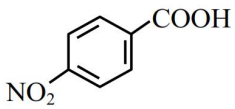
Ans. (3)

Sol. * In 3d series Vanadium has highest enthalpy of atomization and colour of V^{+3} is green.

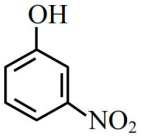
* Oxide form by V^{+3} is V_2O_3 (Basic oxide)

65. Which of the following compounds is least likely to give effervescence of CO_2 in presence of aq. $NaHCO_3$?

(1) 

(2) 

(3) $Ph - \overset{(+)}{N}H_3\ \overset{(-)}{Cl}$

(4) 

Ans. (4)

Sol. Concept – Those compounds which are more acidic than H_2CO_3 can gives effervescence of CO_2 with aq. $NaHCO_3$.

Release $CO_2\uparrow$ gas with aq. NaHCO3

$\Rightarrow [A.S.]_{Comp.} > [A.S.]_{H_2CO_3}$

$\rightarrow$ Option 1, 2 and 3 gives effervescence of CO_2 gas with $NaHCO_3$

$\rightarrow$ Option (4) Not gives CO_2 gas with $NaHCO_3$.

66. Match the **LIST-I** with **LIST-II**.

LIST-I Molecule/ion		**LIST-II Bond pair : lone pair (on the central atom)**	
A.	ICl_2^-	I.	4 : 2
B.	H_2O	II.	4 : 1
C.	SO_2	III.	2 : 3
D.	XeF_4	IV.	2 : 2

Choose the ***correct*** answer from the options given below :

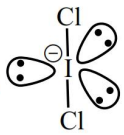
(1) A-IV, B-III, C-II, D-I

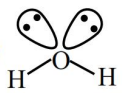
(2) A-III, B-IV, C-II, D-I

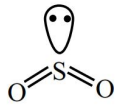
(3) A-III, B-IV, C-I, D-II

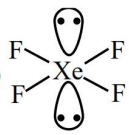
(4) A-II, B-I, C-IV, D-III

Ans. (2)

Sol. (A)  B.P : L.P = 2 : 3

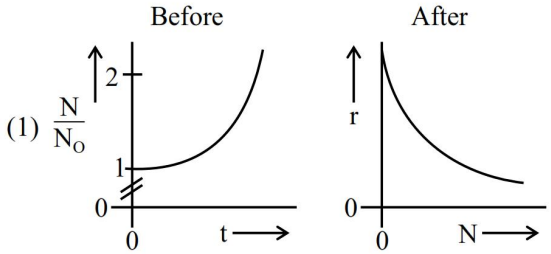
(B)  B.P : L.P = 2 : 2

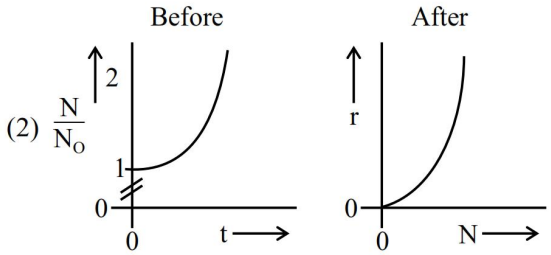
(C)  B.P : L.P = 4 : 1

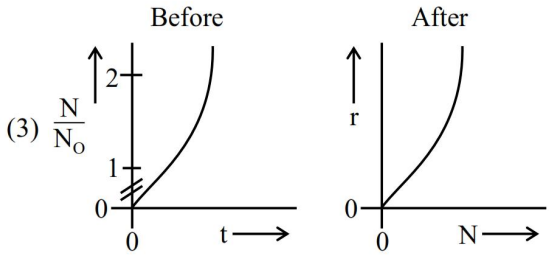
(D)  B.P : L.P = 4 : 2

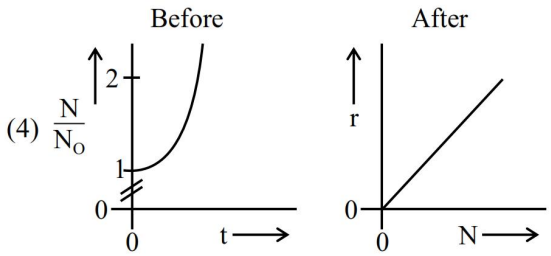
67. A person's wound was exposed to some bacteria and then bacteria growth started to happen at the same place. The wound was later treated with some antibacterial medicine and the rate of bacterial decay (r) was found to be proportional with the square of the existing number of bacteria at any instance. Which of the following set of graphs correctly represents the 'before' and 'after' situation of the application of the medicine?

[Given : N = No. of bacteria, t = time, bacterial growth follows I^{st} order kinetics.]

(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. *Before applying medicine

$\frac{dA}{dt} = K[A]$ (First order growth) (Rate law)

$\frac{A}{A_0} = \frac{N}{N_0} = e^{Kt}$

*After applying medicine

Active Bacteria $\rightarrow$ Inactive Bacteria

(A) (I)

$r = -\frac{dA}{dt} = K[A]^2$ (Rate law)

$\boxed{y = Kx^2}$ Parabola

68. Given below are two statements :

Statement I: D-(+)-glucose + D-(+) fructose $\xrightarrow{-H_2O}$ sucrose

sucrose $\xrightarrow{\text{Hydrolysis}}$ D-(+)-glucose + D-(+) fructose

Statement II : Invert sugar is formed during sucrose hydrolysis.

In the light of the above statements, choose the ***correct*** answer from the options given below -

(1) Both Statement I and Statement II are true.

(2) Statement I is false but Statement II are true.

(3) Statement I is true but Statement II is false.

(4) Both Statement I and Statement II are false.

Ans. (2)

Sol. On hydrolysis of sucrose gives D-(+)-glucose and D-(–)-fructose while in St. (1) D-(+)-fructose is given evince St-(1) is incorrect.

St. II – It is correct because sucrose on hydrolysis gives invert sugar

69. An octahedral complex having molecular composition $Co.5NH_3.Cl.SO_4$ has two isomers A and B. The solution of A gives a white precipitate with $AgNO_3$ solution and the solution of B gives white precipitate with $BaCl_2$ solution. The type of isomerism exhibited by the complex is,

(1) Co-ordinate isomerism

(2) Linkage isomerism

(3) Ionisation isomerism

(4) Geometrical isomerism

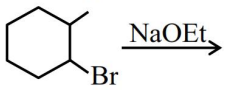
Ans. (3)

Sol. (A) complex is $[Co(NH_3)_5(SO_4)]Cl$

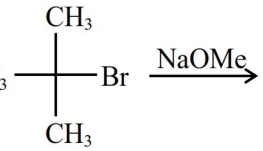
(B) complex is $[Co(NH_3)_5Cl]SO_4$

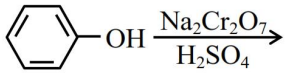
Both (A) and (B) are Ionisation isomers.

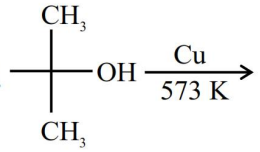
70. The reactions which cannot be applied to prepare an alkene by elimination, are

A. 

B. $CH_3 - CH_2 - \underset{\underset{Br}{|}}{CH} - CH_3 \xrightarrow{KOH\ (aq.)}$

C. 

D. 

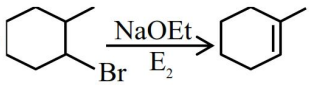
E. 

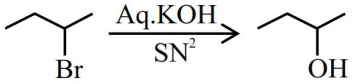
Choose the ***correct*** answer from the option given below :

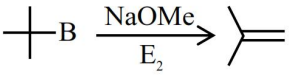
(1) B & E Only (2) B, C & D Only

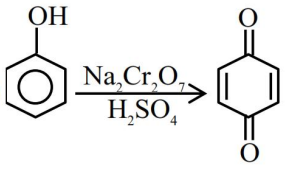
(3) A, C & D Only (4) B & D Only

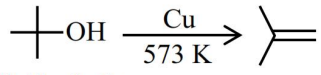
Ans. (4)

Sol. (A) 

(B) 

(C) 

(D) 

(E) 

(3° Alcohol)

Option (B) and (D) reaction are not able to form alkene as a product.

SECTION-B

71. An organic compound weighing 500 mg, produced 220 mg of CO_2. on complete combustion. The percentage composition of carbon in the compound is ..... %. (nearest integer)
(Given molar mass in g mol^{-1} of C : 12, O : 16)

Ans. **(12)**

Sol. Organic compound $\xrightarrow[\Delta]{CuO} CO_2 + H_2O$

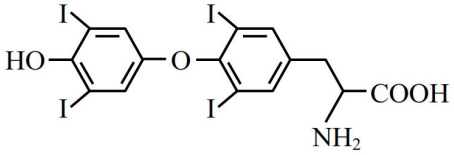
$$n_{CO_2} = \frac{220\times10^{-3}}{44} = 5\times10^{-3} \text{ moles}$$

$m_C = 5 \times 10^{-3} \times 12$

$$\%m \text{ carbon} = \frac{5\times10^{-3}\times12}{500\times10^{-3}}\times100 = 12\%$$

Correct answer is 12

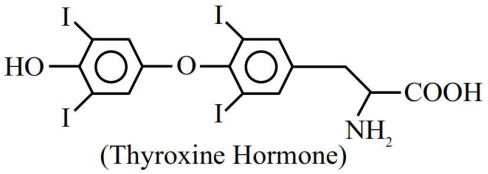
72. Thyroxine, the hormone has given below structure



The percentage of iodine in thyroxine is ..... %. (nearest integer)
(Given molar mass in g mol^{-1} C:12. H:1, O:16, N:14, I:127)

Ans. **(65)**

Sol.



(Thyroxine Hormone)

→ Molecular formula of Thyroxine ⇒ $C_{15}H_{11}O_4NI_4$

→ Molecular mass of Thyroxine –

C → 15 × 12 = 180

H → 11 × 1 = 11

O → 16 × 4 = 64

N → 14 × 1 = 14

I → 127 × 4 = 508

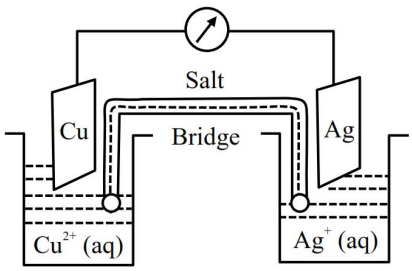
→ Molecular mass of Thyroxine ⇒ 777

$$\rightarrow \%\text{ of Iodine} = \frac{508}{777}\times100$$

= 65.38 %

Nearest integer = 65

73. 1 Faraday electricity was passed through Cu^{2+} (1.5 M, 1 L)/Cu and 0.1 Faraday was passed through Ag^+ (0.2 M, 1 L)/Ag electrolytic cells. After this the two cells were connected as shown below to make an electrochemical cell. The emf of the cell thus formed at 298 K is-



Given : $E^{\circ}_{Cu^{2+}/Cu} = 0.34$ V

$E^{\circ}_{Ag^+/Ag} = 0.8$ V

$$\frac{2.303RT}{F} = 0.06 \text{ V}$$

Ans. **(400)**

Sol. * $Cu^{+2} + 2e^- \longrightarrow Cu$

(1 faraday = charge on 1 mole electron)

t = 0 1.5 1 mole

t = t 1 – 0.5 mole

$[Cu^{+2}]$ = 1 M after electrolysis

* $Ag\oplus + e^- \longrightarrow Ag$

t = 0 0.2 0.1 mole

t = t 0.1 – –

$[Ag^+]$ = 0.1 M after electrolysis

Cell reaction $Cu_{(s)} + 2Ag^+_{(aq)} \rightarrow Cu^{+2}_{(aq)} + 2Ag_{(s)}$

$$E = E^{\circ} - \frac{0.06}{n}\log\frac{[Cu^{+2}]}{[Ag^+]^2}$$

$$E = (0.8 - 0.34) - \frac{0.06}{2}\log\frac{1}{(0.1)^2} = 0.4V$$

Correct answer = 400 mV

74. The percentage dissociation of a salt (MX_3) solution at given temperature (van't Hoff factor i = 2) is..... % (Nearest integer)

Ans. **(33)**

Sol. $MX_3 \rightarrow M^{+3} + 3X^{\ominus}$

$i = 1 + (n - 1)\alpha$

$i = 1 + (4 - 1)\alpha = 2$

$\alpha = \frac{1}{3} = 33.33\% \approx 33\%$

75. The number of paramagnetic complex among $[FeF_6]^{3-}$, $[Fe(CN)_6]^{3-}$, $[Mn(CN)_6]^{3-}$, $[Co(C_2O_4)_3]^{3-}$, $[MnCl_6]^{3-}$ and $[CoF_6]^{3-}$, which involved d^2sp^3 hybridization is .....

Ans. **(2)**

Sol.

$[FeF_6]^{3-}$	Paramagnetic	sp^3d^2
$[Fe(CN)_6]^{3-}$	Paramagnetic	d^2sp^3
$[Mn(CN)_6]^{3-}$	Paramagnetic	d^2sp^3
$[Co(C_2O_4)_3]^{3-}$	Diamagnetic	d^2sp^3
$[MnCl_6]^{3-}$	Paramagnetic	sp^3d^2
$[CoF_6]^{3-}$	Paramagnetic	sp^3d^2

Only $[Fe(CN)_6]^{3-}$ and $[Mn(CN)_6]^{3-}$ are paramagnetic and d^2sp^3 hybridisation of metal.