# Chapter 6 Polynomials and Polynomial Functions

## Ex 6.5

### Answer 1e.

Consider the square root function $y = \sqrt{x}$ and the cube root function $y = \sqrt[3]{x}$.
We can see that these types of functions always contain a radical expression. Thus, the square root functions and the cube root functions are examples of radical functions.

### Answer 1gp.

First, take a set of random values for $x$, and then find the corresponding $y$-values.
Substitute 0 for $x$ in the given equation.

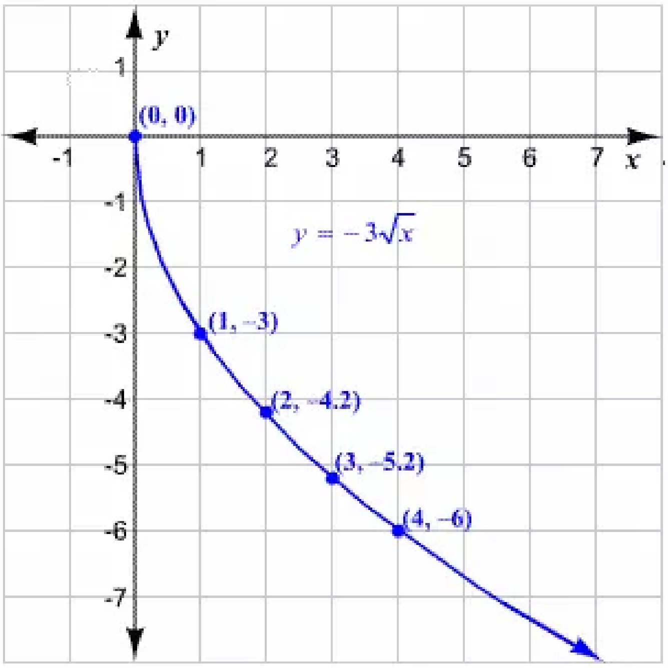
$$y = -3\sqrt{0}$$

$$y = 0$$

Similarly, find some other values for $y$.
List the values in a table.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | −3 | −4.2 | −5.2 | −6 |

Plot the points and join them using a smooth curve.




The radicand of a square root should be nonnegative. Thus, the domain is all real numbers greater than or equal to 0, or $x \geq 0$. The range is all real numbers less than or equal to zero, or $y \leq 0$.

## Answer 2e.

Graph of $y=\sqrt{x}$ is represented as graph of $y=a\sqrt{x-h}+k$ with $a=1$, $h=0$ and $k=0$

a. $a=-3$

$h=0$, $k=0$

The graph of $y=a\sqrt{x-h}+k$ when $a=-3$ is a vertical stretch of the graph of $y=\sqrt{x}$ by a factor of 3 followed by a reflection in the $x$-axis

b. $h=2$

$a=1$, $k=0$

The graph of $y=a\sqrt{x-h}+k$ when $h=2$ is a horizontal translation of the graph of $y=\sqrt{x}$ by a factor of 2 units on the right side

c. $k=4$

$a=1, h=0$

The graph of $y=a\sqrt{x-h}+k$ when $k=4$ is a vertical translation of the graph of $y=\sqrt{x}$ by a factor of 4 units to the up side

## Answer 2gp.

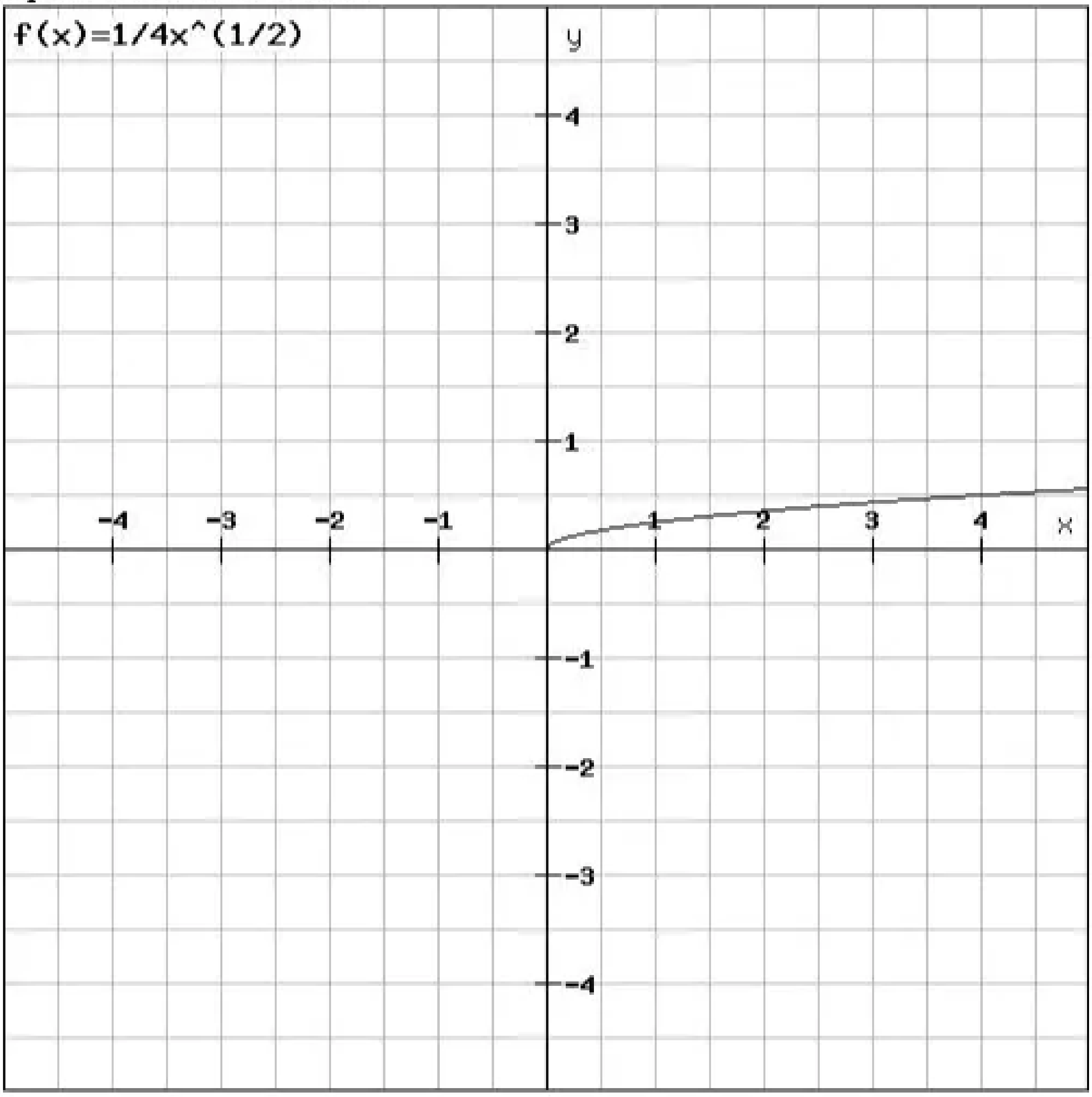
Given function $f(x)=\frac{1}{4}\sqrt{x}$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 0.25 | 0.354 | 0.433 | 0.5 | 0.559 |

The graph is obtained as follows




The square root function exists if and only if the radicand of the square root must be nonnegative, so x should be nonnegative real number.

Therefore domain of the function is $x \geq 0$

From the graph, range of the function is $f(x) \geq 0$ for all $x \geq 0$ values.

Domain: $\boxed{x \geq 0}$

Range: $\boxed{f(x) \geq 0}$

## Answer 3e.

First, take a set of random values for $x$, and then find the corresponding $y$-values.
Substitute 0 for $x$ in the given equation.

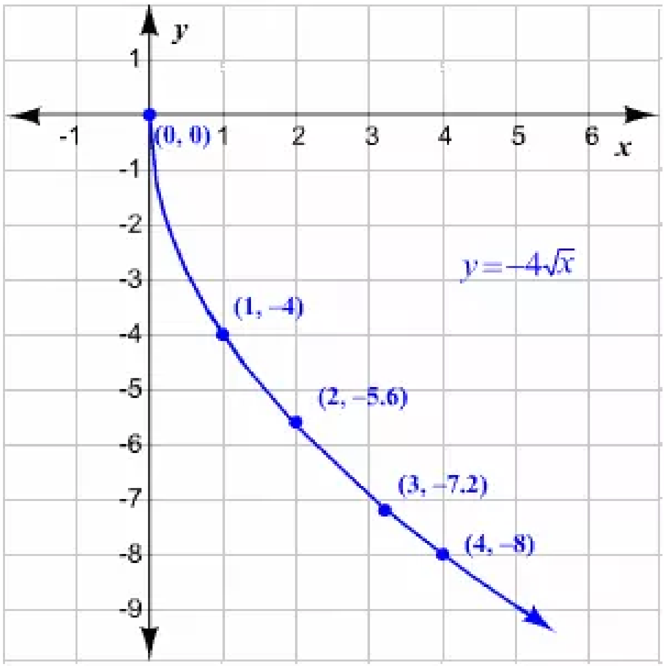
$$y = -4\sqrt{0}$$

$$y = 0$$

Similarly, find some other values for $y$.
List the values in a table.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | −4 | −5.6 | −7.2 | −8 |

Plot the points and join them using a smooth curve.




The radicand of square root should be nonnegative. Thus, the domain is all real numbers greater than or equal to 0, or $x \geq 0$. The range is all real numbers less than or equal to zero, or $y \leq 0$.

## Answer 3gp.

First, take a set of random values for $x$, and then find the corresponding $y$-values.
Substitute 0 for $x$ in the given equation.

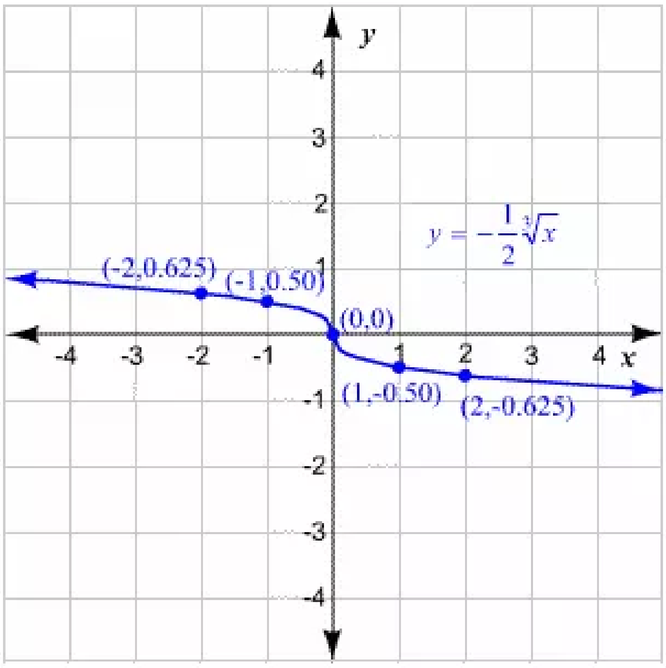
$$y = -\frac{1}{2}\sqrt[3]{0}$$

$$y = 0$$

Similarly, find some other values for $y$.
List the values in a table.

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 0.625 | 0.5 | 0 | −0.5 | −0.625 |

Plot the points and join them using a smooth curve.




The domain and the range are the set of all real numbers.

## Answer 4e.

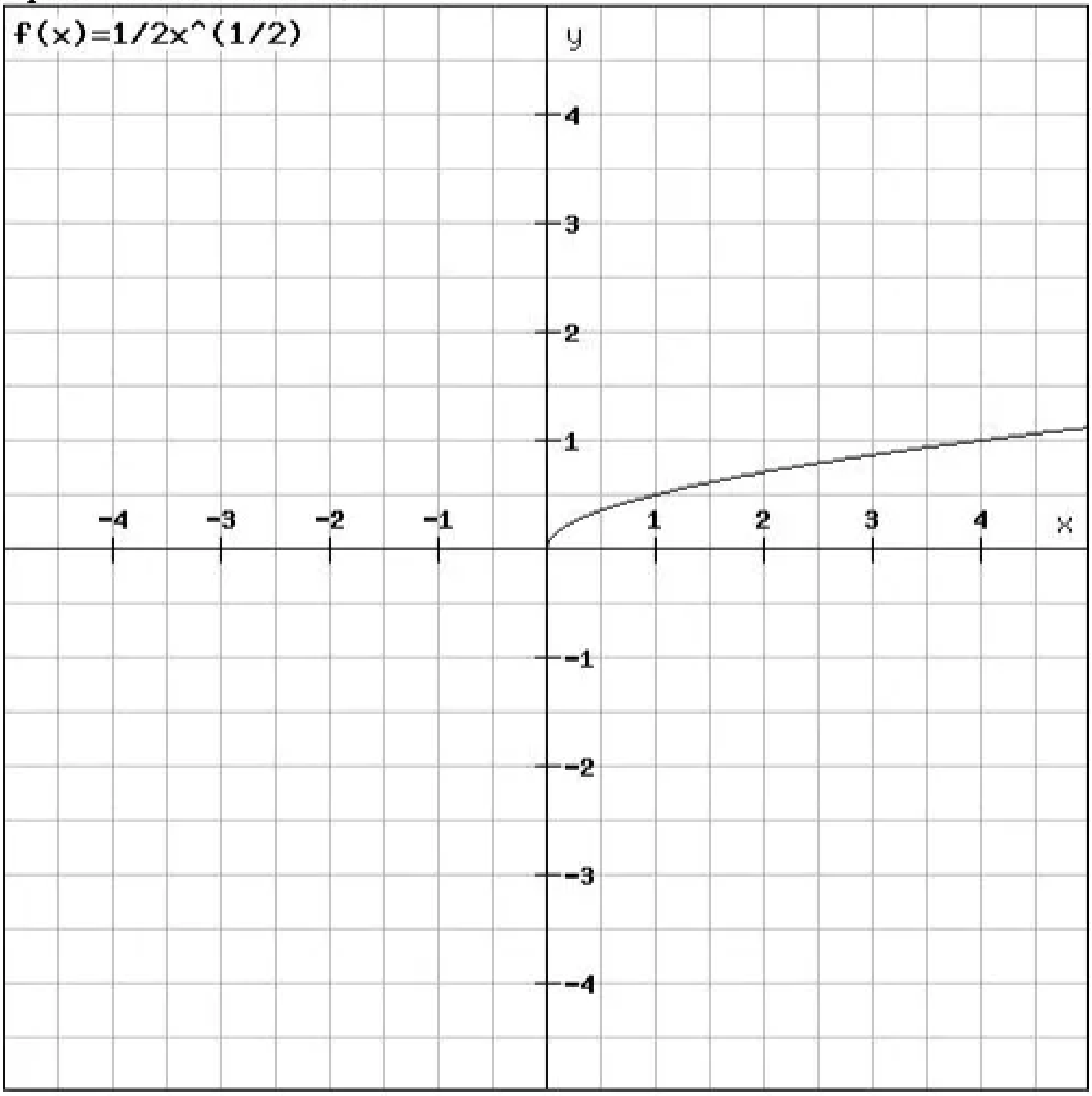
Given function $f(x) = \frac{1}{2}\sqrt{x}$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 0.5 | 0.707 | 0.866 | 1 | 1.118 |

The graph is obtained as follows




The square root function exists if and only if the radicand of the square root must be nonnegative, so x should be nonnegative real number.

Therefore domain of the function is $x \geq 0$

From the graph, range of the function is $f(x) \geq 0$ for all $x \geq 0$ values.

Domain: $\boxed{x \geq 0}$

Range: $\boxed{f(x) \geq 0}$

## Answer 4gp.

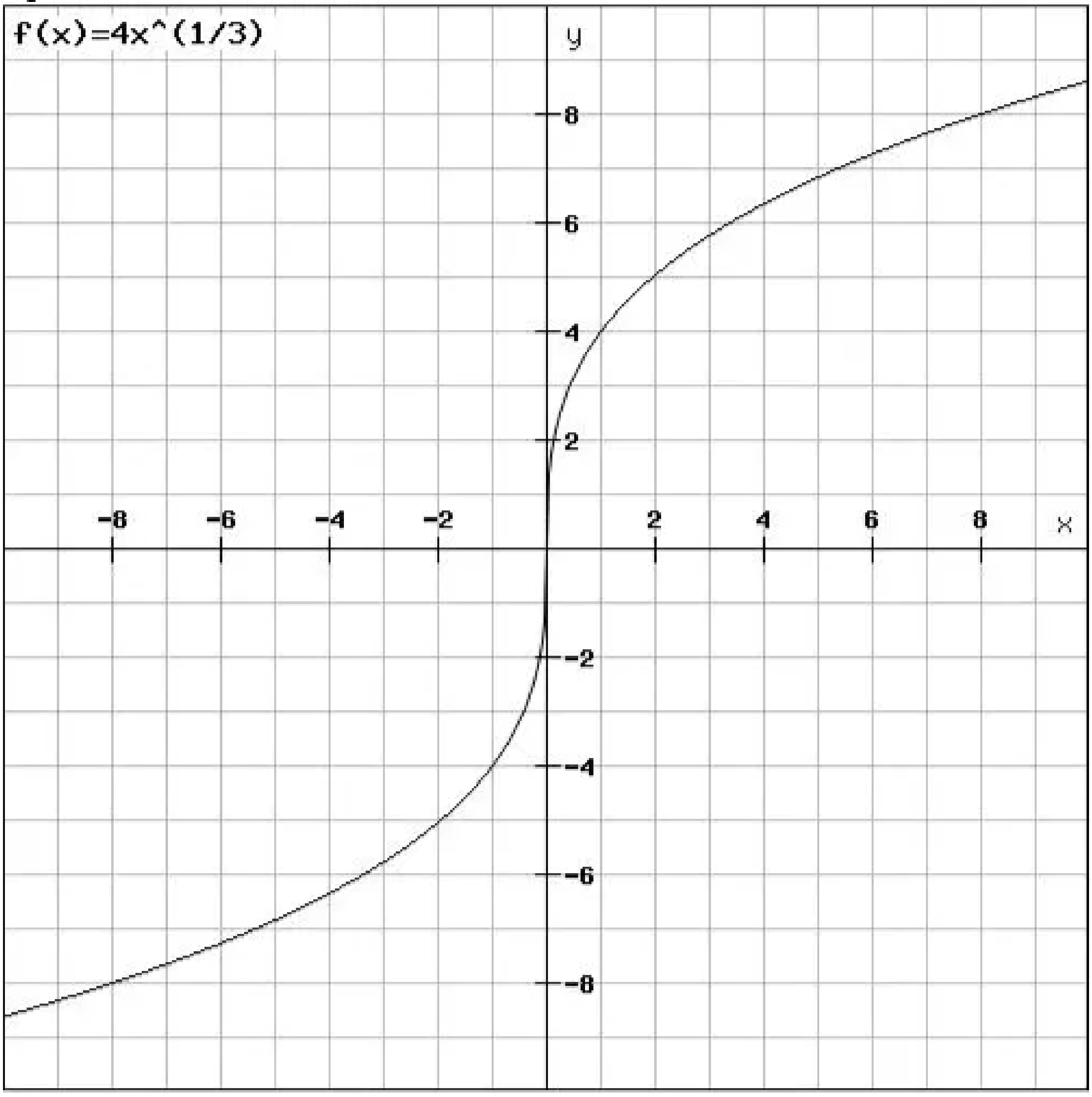
Given function $g(x) = 4\sqrt[3]{x}$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | -10 | -8 | -6 | -4 | -2 | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $g(x)$ | -8.61 | -8 | -7.26 | -6.35 | -5.04 | 0 | 5.04 | 6.35 | 7.26 | 8 | 8.61 |

The graph is obtained as follows




The function $4\sqrt[3]{x}$ exists for all real numbers
Therefore domain of the function is all real numbers
Therefore range of the function is all real numbers

## Answer 5e.

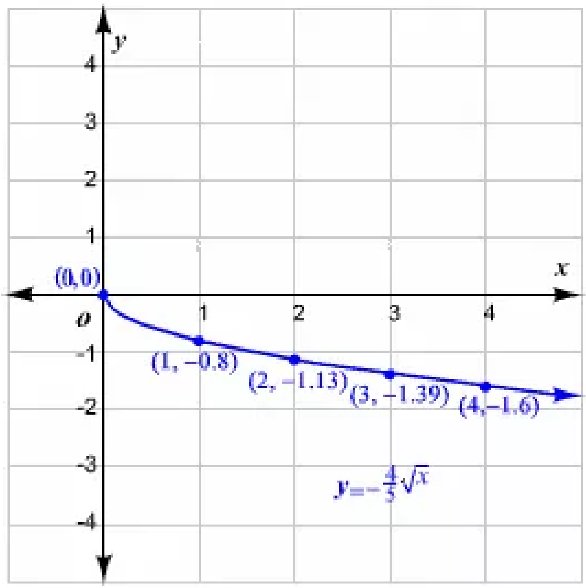
First, take a set of random values for $x$ and then find the corresponding $y$-values.
Substitute 0 for $x$ in the given equation.

$$\begin{aligned} y &= -\frac{4}{5}\sqrt{0} \\ &= 0 \end{aligned}$$

Similarly, find some other values for $y$.
List the values in a table.

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 |
| $y$ | 0 | −0.8 | −1.13 | −1.39 | −1.6 |

Plot the points and join them using a smooth curve.



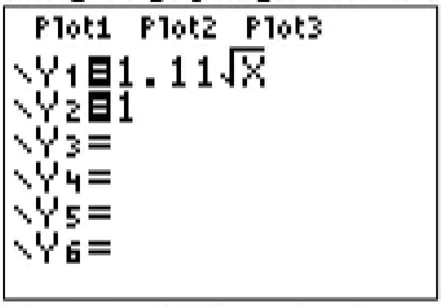

The radicand of a square root must be nonnegative. Thus, the domain is $x \geq 0$.
The range is $y \leq 0$.

## Answer 5gp.

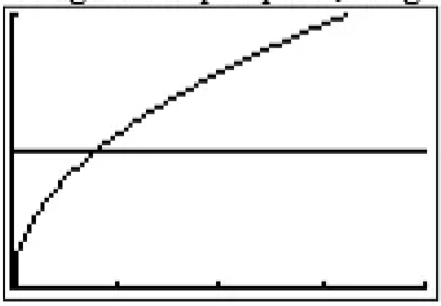
In a pendulum, the relation between period of pendulum $T$ (in seconds) and length of pendulum $l$ (in feet) is given by $T = 1.11\sqrt{l}$
Period of pendulum $T = 1\text{sec}$

To solve the given equation $T = 1.11\sqrt{l}$ for length for the given period, using the graphing calculator and intersect feature will ease the process

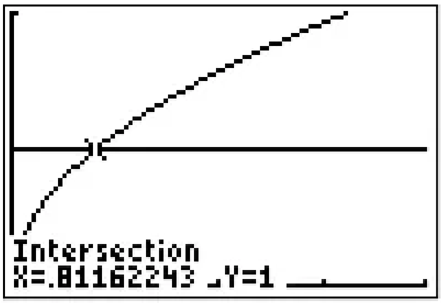
Using the graphing calculator and enter the functions as shown below




Using the Graph option, the graphs for the entered functions are obtained as shown below



Using the Intersect option, it is showed that the both graphs intersect at $x = 0.8116$ and $y = 1$




Hence, it is shown that the solution from the graph is $x = 0.8116$

The length of pendulum for the given period of 1 second is

$$\boxed{l = 0.8116 \text{ feet}}$$

## Answer 6e.

Given function $y = -6\sqrt{x}$

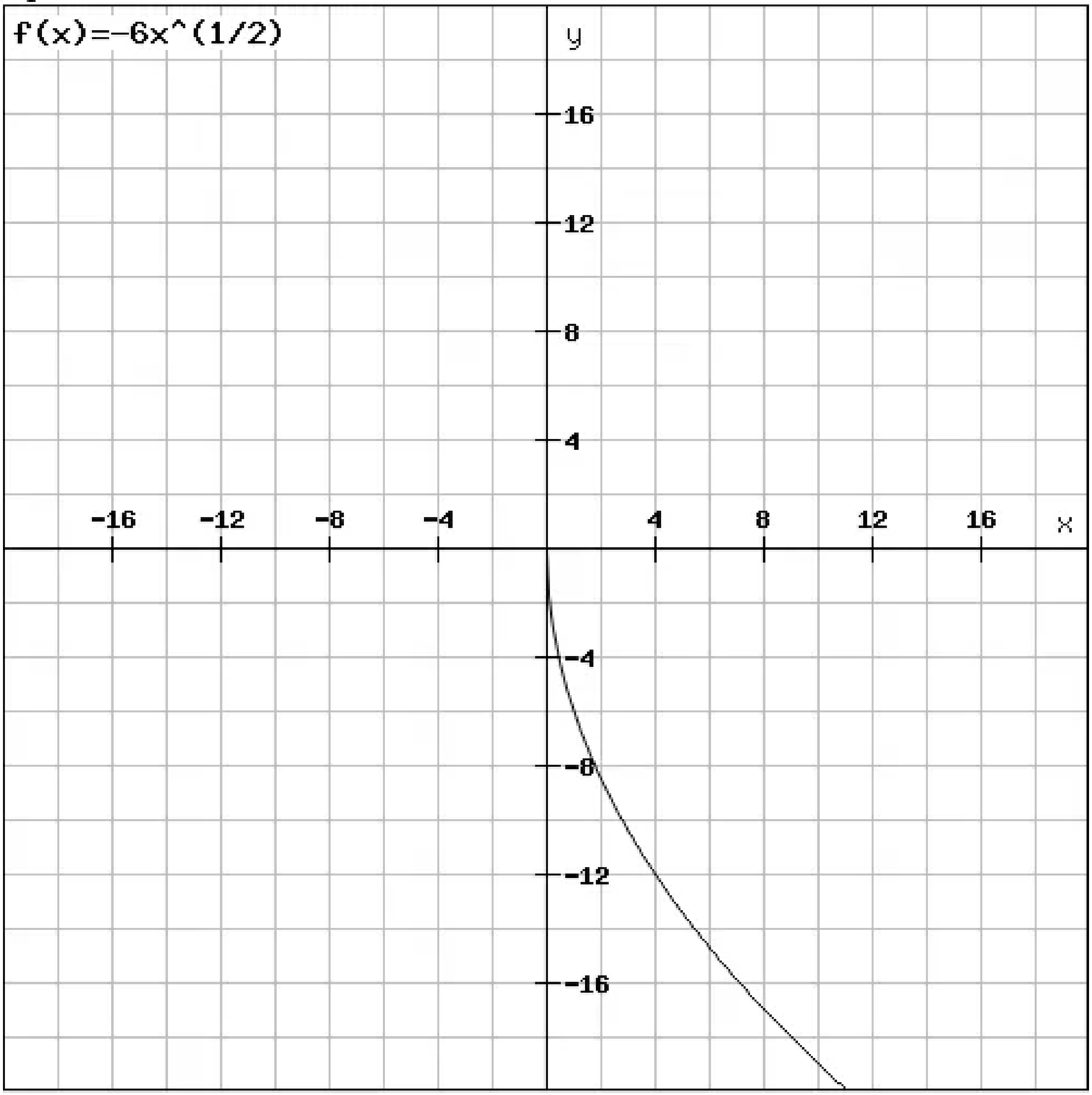
We need to sketch the graph and state the domain and range of the function.

For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | 0 | 2 | 4 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|
| $y$ | 0 | -8.485 | -12 | -16.971 | -18.974 | -20.785 |

The graph is obtained as follows




The square root function exists if and only if the radicand of the square root must be nonnegative, so x should be nonnegative real number.
Therefore domain of the function is $x \geq 0$
From the graph, range of the function is $y \leq 0$ for all $x \geq 0$ values.

Domain: $\boxed{x \geq 0}$

Range: $\boxed{y \leq 0}$

## Answer 6gp.

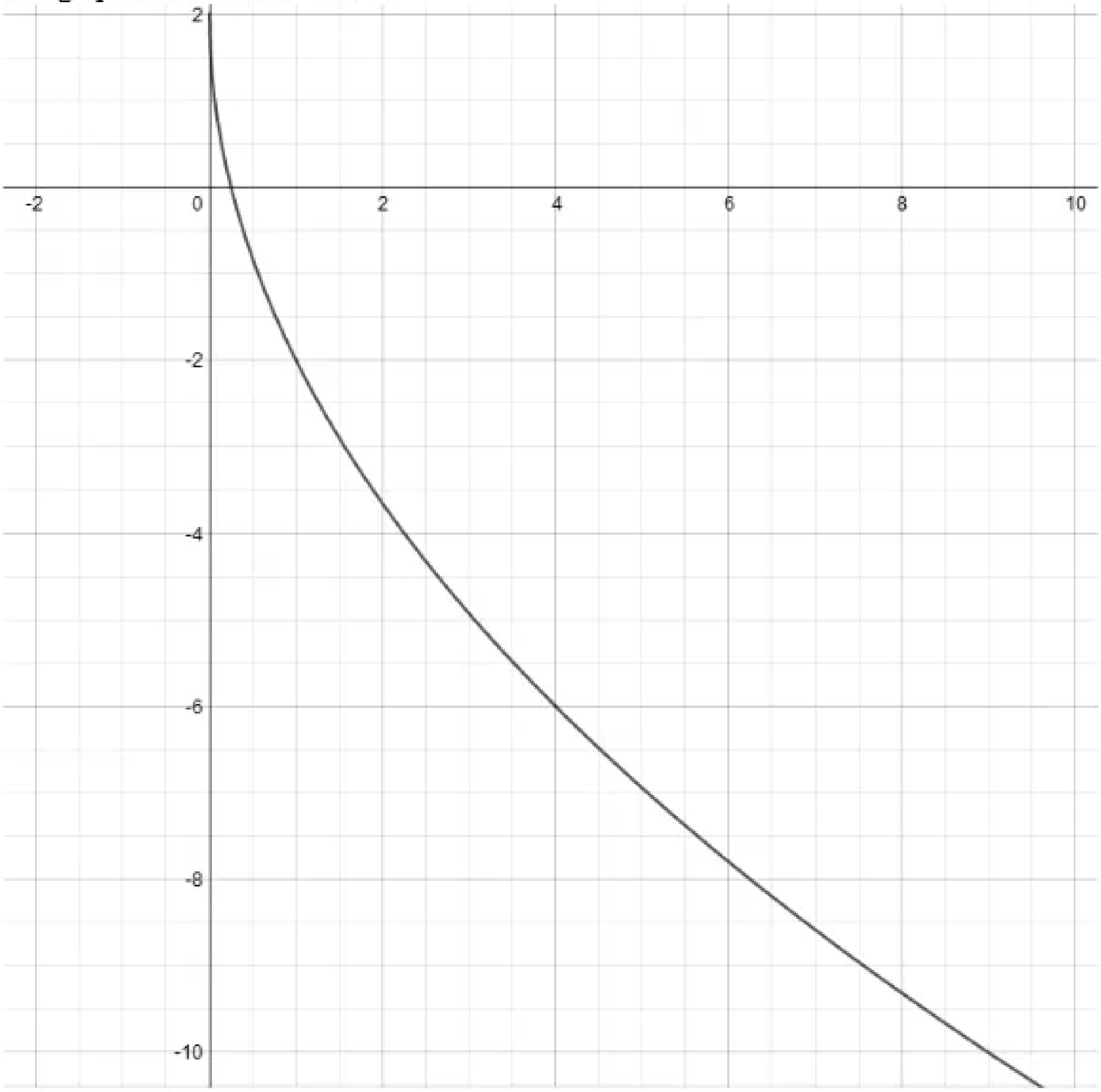
Given function $y = -4\sqrt{x} + 2$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $y$ | 2 | -3.657 | -6 | -7.798 | -9.314 | -10.649 |

The graph is obtained as follows




The square root function exists if and only if the radicand of the square root must be nonnegative, so x should be nonnegative real number.

Therefore domain of the function is $x \geq 0$

From the graph, range of the function is $y \leq 2$ for all $x \geq 0$ values.

Domain: $\boxed{x \geq 0}$

Range: $\boxed{y \leq 2}$

## Answer 7e.

First, take a set of random values for $x$, and then find the corresponding $y$-values.
Substitute 0 for $x$ in the given equation.

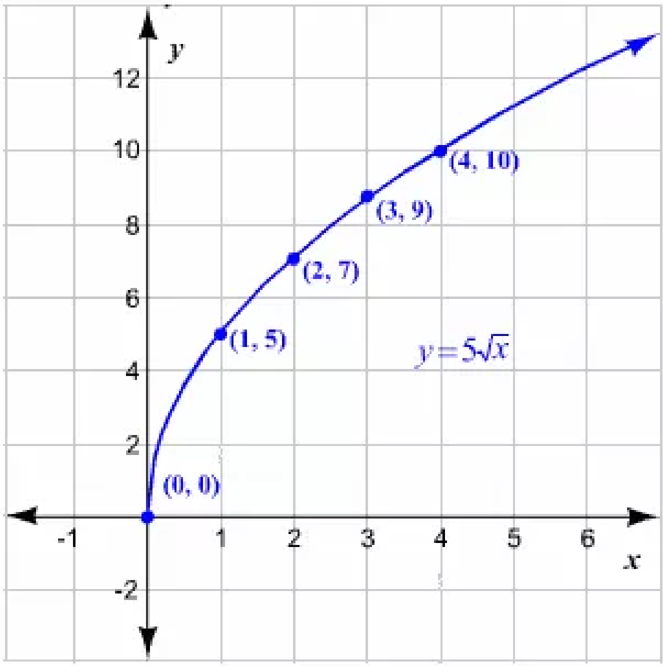
$$y = 5\sqrt{0}$$

$$y = 0$$

Similarly, find some other values for $y$.
List the values in a table.

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 |
| $y$ | 0 | 5 | 7 | 9 | 10 |

Plot the points and join them using a smooth curve.




The radicand of square root should be nonnegative. Thus, the domain is all real numbers greater than or equal to 0, or $x \geq 0$. The range is all real numbers greater than or equal to zero, or $y \geq 0$.

## Answer 7gp.

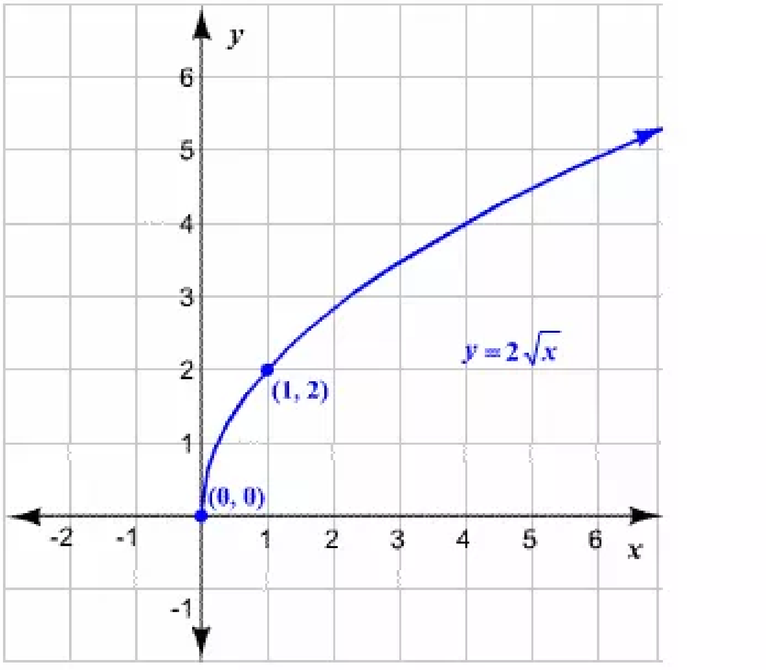
The given function is of the form $y = a\sqrt{x-h} + k$, where $a$ is 2, $h$ is $-1$, and $k$ is 0.

***STEP 1*** **Sketch** the graph of $y = a\sqrt{x}$. In the given case, we have to sketch $y = 2\sqrt{x}$.

Take a set of random values for $x$, and then find the corresponding $y$-values.

| | | |
|---|---|---|
| $x$ | 0 | 1 |
| $y$ | 0 | 2 |

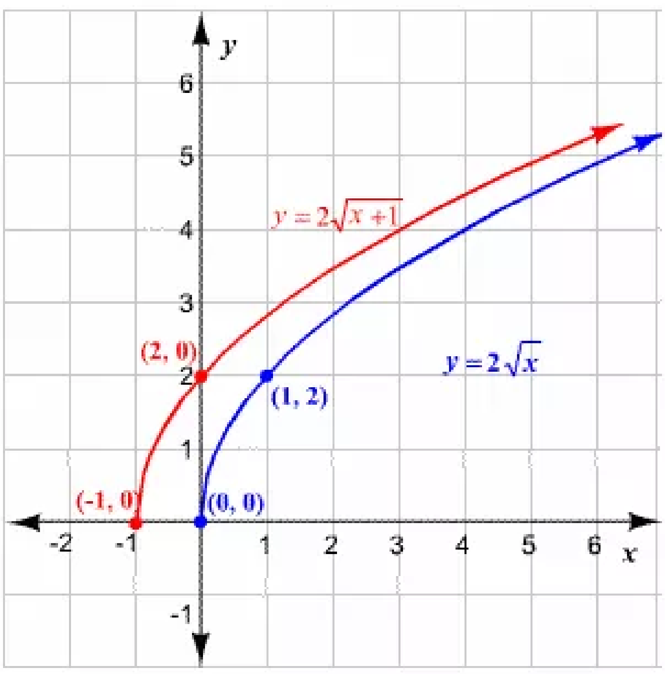
Plot the points and join them using a smooth curve.




***STEP 2*** **Translate** the graph.

We have to translate the graph of $y = a\sqrt{x}$, $h$ units horizontally and $k$ units vertically. Since $k$ is 0 for the given function, there is no vertical shift.

Shift the graph of $y = 2\sqrt{x}$ to the left by 1 unit to get the graph of $y = 2\sqrt{x+1}$. The resulting graph will begin from the point (−1, 0) and pass through the point (2, 0).




In the graph, we can see that the domain is $x \geq -1$ and the range is $y \geq 0$.

## Answer 8e.

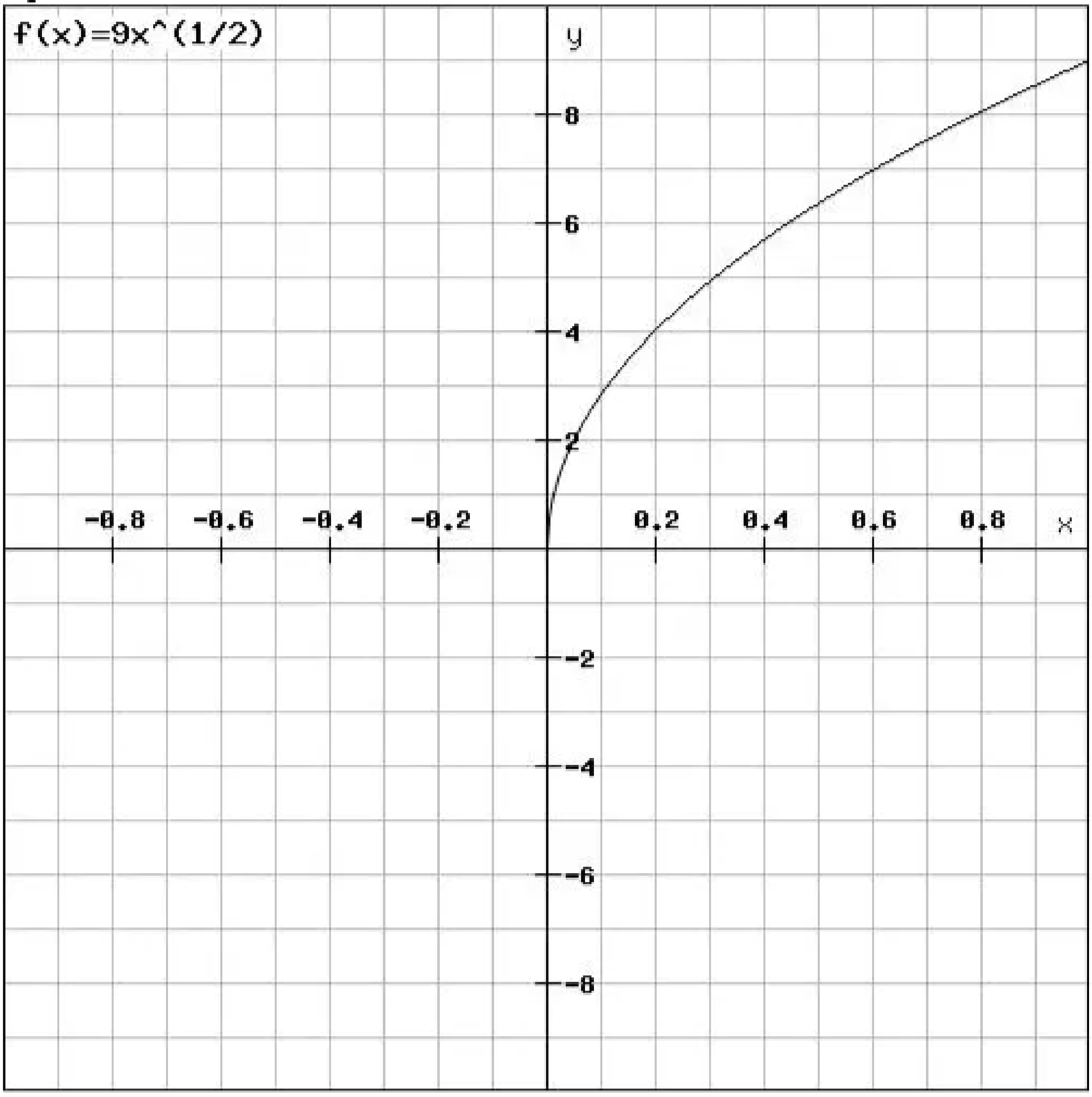
Given function $g(x)=9\sqrt{x}$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|
| $g(x)$ | 0 | 4.025 | 5.692 | 6.971 | 8.05 | 9 |

The graph is obtained as follows




The square root function exists if and only if the radicand of the square root must be nonnegative, so x should be nonnegative real number.
Therefore domain of the function is $x \geq 0$
From the graph, range of the function is $g(x) \geq 0$ for all $x \geq 0$ values.

Domain: $\boxed{x \geq 0}$

Range: $\boxed{g(x) \geq 0}$

**Answer 8gp.**

Given function $f(x)=\frac{1}{2}\sqrt{x-3}-1$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | 3 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|
| $f(x)$ | -1 | -0.5 | -0.134 | 0.118 | 0.323 | 0.5 |

The graph is obtained as follows


The function $\frac{1}{2}\sqrt{x-3}-1$ exists if and only if the radicand of the square root must be nonnegative, so $x-3$ should be nonnegative real number.
Therefore domain of the function is $x \geq 3$
From the graph, range of the function is $f(x) \geq -1$ for all $x \geq 3$ values.

Domain: $\boxed{x \geq 3}$

Range: $\boxed{f(x) \geq -1}$

**Answer 9e.**

The given graph starts from the origin and passes through the point (4, −3).

Assign 0 and 4 for $x$ in the given equations and check whether which equation gives a graph similar to the given graph.

Consider the equation $y = \frac{3}{4}\sqrt{x}$.

Find the values for $y$ when $x$ is 0 and $x$ is 4.

| $x$ | 0 | 4 |
|---|---|---|
| $y$ | 0 | $\frac{3}{2}$ |

The equation does not represent the graph because the graph does not go through these points.

Now, take the equation $y = -\frac{3}{4}\sqrt{x}$.

Make a table of values.

| | | |
|---|---|---|
| $x$ | 0 | 4 |
| $y$ | 0 | $-\frac{3}{2}$ |

Since the graph does not go through these points, ignore the equation.

Take the equation $y = \frac{3}{2}\sqrt{x}$.

Make a table of values.

| | | |
|---|---|---|
| $x$ | 0 | 4 |
| $y$ | 0 | 3 |

Reject this equation because the graph passes through −3 when $x$ is 4.

Consider the equation $y = \frac{-3}{2}\sqrt{x}$.

Make a table of values.

| | | |
|---|---|---|
| $x$ | 0 | 4 |
| $y$ | 0 | −3 |

We can see that the given graph passes through the same points. Thus, the given graph is of the equation $y = \frac{-3}{2}\sqrt{x}$, which matches with choice **D**.

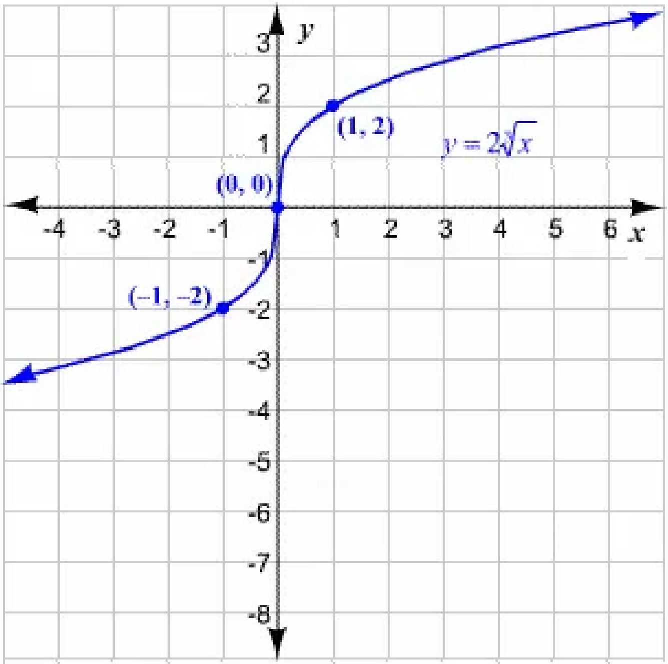
## Answer 9gp.

The given function is of the form $y = a\sqrt[3]{x-h}+k$, where $a$ is 2, $h$ is 4, and $k$ is 0.

***STEP 1*** **Sketch** the graph of $y = a\sqrt[3]{x}$. In the given case, we have to sketch $y = 2\sqrt[3]{x}$. Take a set of random values for $x$, and then find the corresponding $y$-values. List the values in a table.

| | | | |
|---|---|---|---|
| $x$ | −1 | 0 | 1 |
| $y$ | −2 | 0 | 2 |

Plot the points and join them using a curve.

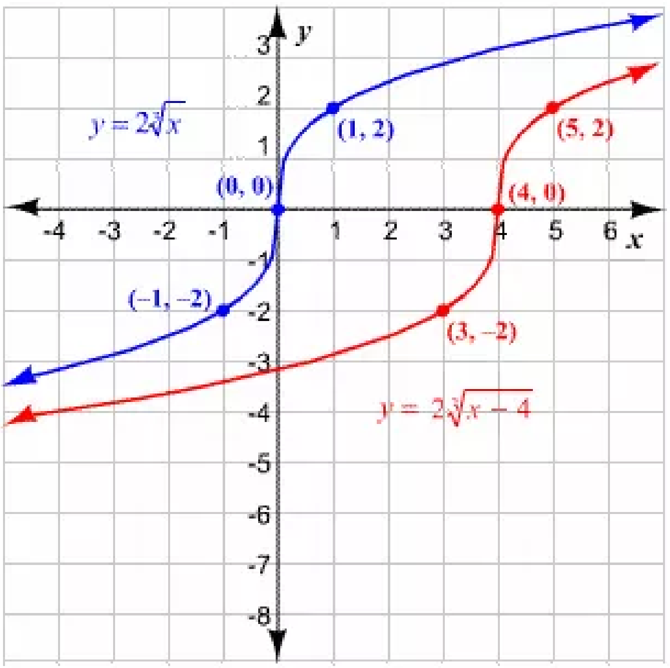



*STEP 2* **Translate** the graph.

We have to translate the graph of $y = a\sqrt[3]{x}$, $h$ units horizontally and $k$ units vertically. Since $k$ is 0 for the given function, there is no vertical shift.

Shift the graph of $y = 2\sqrt[3]{x}$ to the right by 4 units to get the graph of $y = 2\sqrt[3]{x-4}$ .

The resulting graph will pass through the points (−1, −2), (4, 0), and (5, 2).




In the graph, we can see that both the domain and the range are all real numbers.

**Answer 10e.**

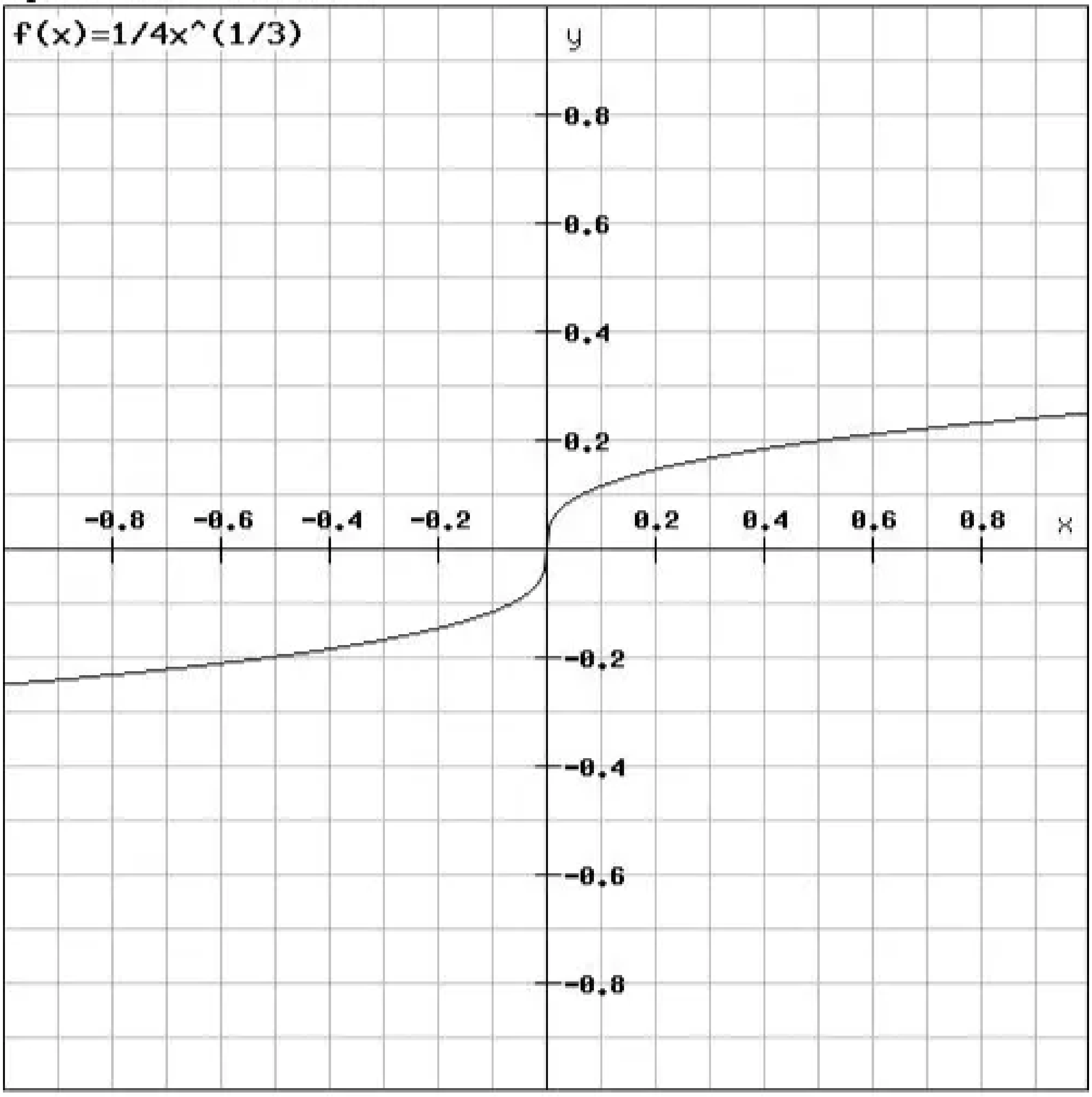
Given function $y=\frac{1}{4}\sqrt[3]{x}$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | -1 | -0.8 | -0.6 | -0.4 | -0.2 | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | -0.25 | -0.23 | -0.21 | -0.18 | -0.14 | 0 | 0.14 | 0.18 | 0.21 | 0.23 | 0.25 |

The graph is obtained as follows




The function $\frac{1}{4}\sqrt[3]{x}$ exists for all real numbers
Therefore domain of the function is all real numbers
Therefore range of the function is all real numbers

**Answer 10gp.**

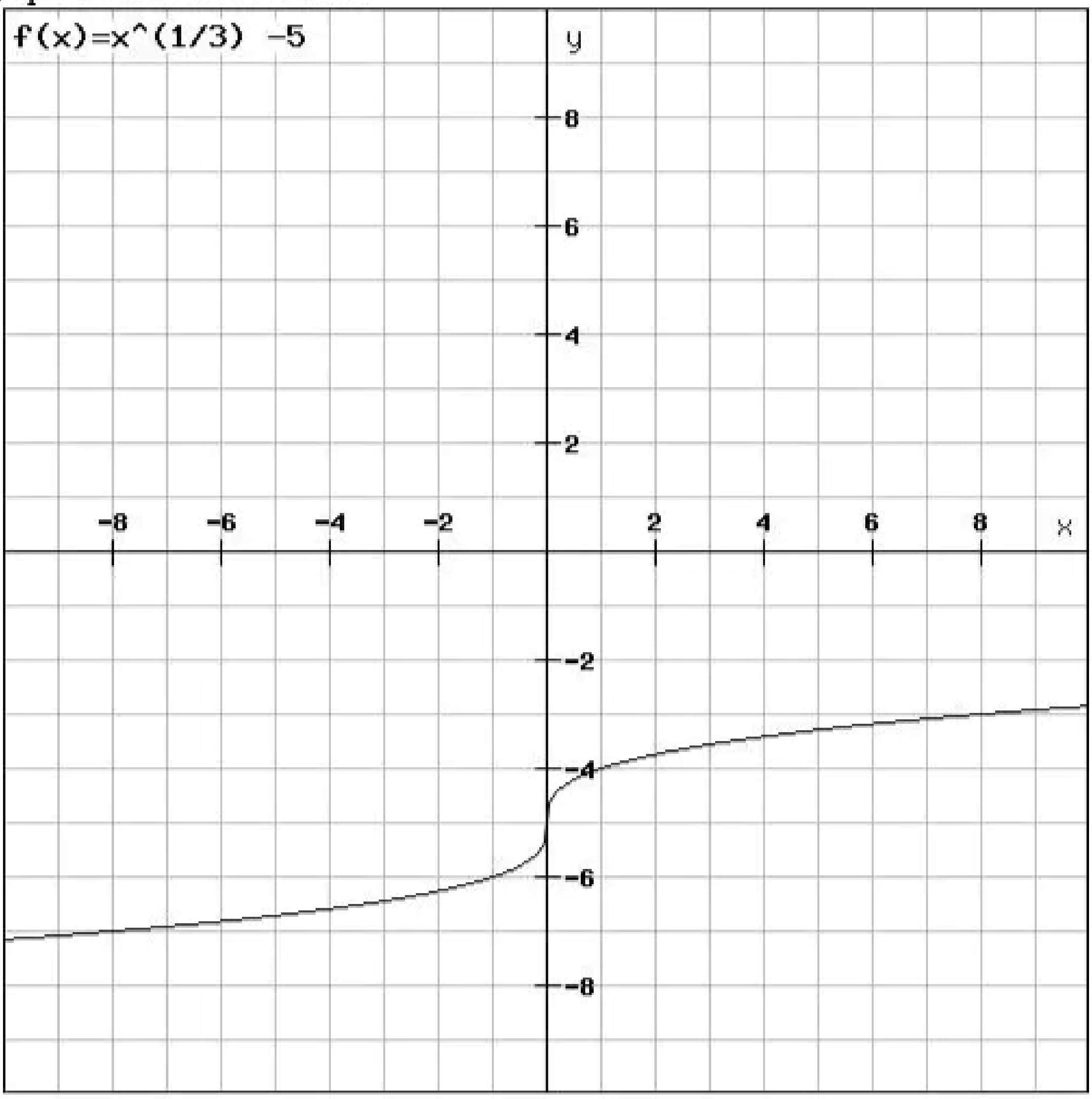
Given function $g(x)=\sqrt[3]{x}-5$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | -10 | -8 | -6 | -4 | -2 | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $g(x)$ | -7.15 | -7 | -6.81 | -6.58 | -6.26 | -5 | -3.74 | -3.41 | -3.18 | -3 | -2.84 |

The graph is obtained as follows




The function $\sqrt[3]{x}-5$ exists for all real numbers
Therefore domain of the function is all real numbers
Therefore range of the function is all real numbers

**Answer 11e.**

First, take a set of random values for $x$, and then find the corresponding $y$-values.
Substitute 0 for $x$ in the given equation.

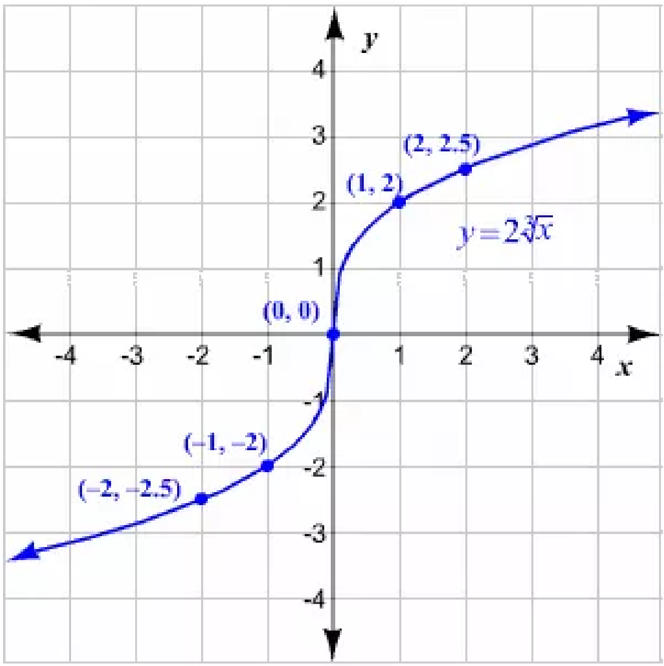
$$y = 2\sqrt[3]{0}$$

$$y = 0$$

Similarly, find some other values for $y$.
List the values in a table.

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $-2.5$ | $-2$ | $0$ | $2$ | $2.5$ |

Plot the points and join them using a smooth curve.

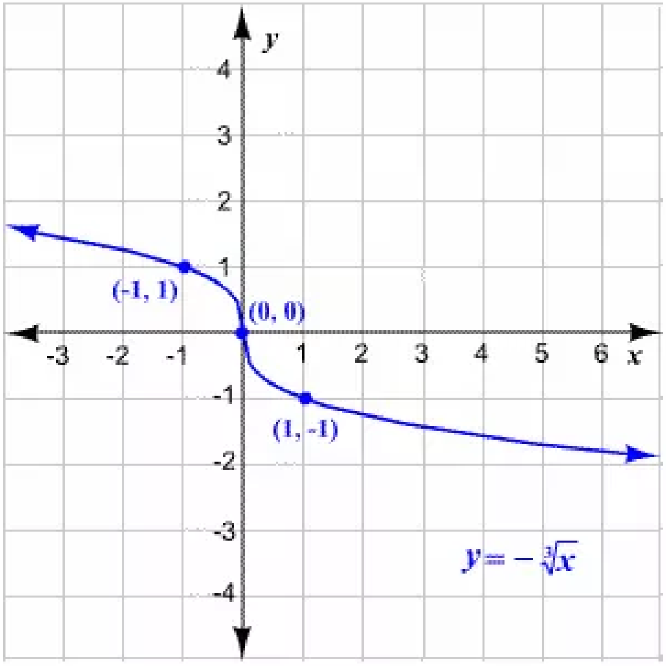



In the graph, we can see that the domain and the range are all real numbers.

## Answer 11gp.

The given function is of the form $y = a\sqrt[3]{x - h} + k$, where $a$ is $-1$, $h$ is $-2$, and $k$ is $-3$.

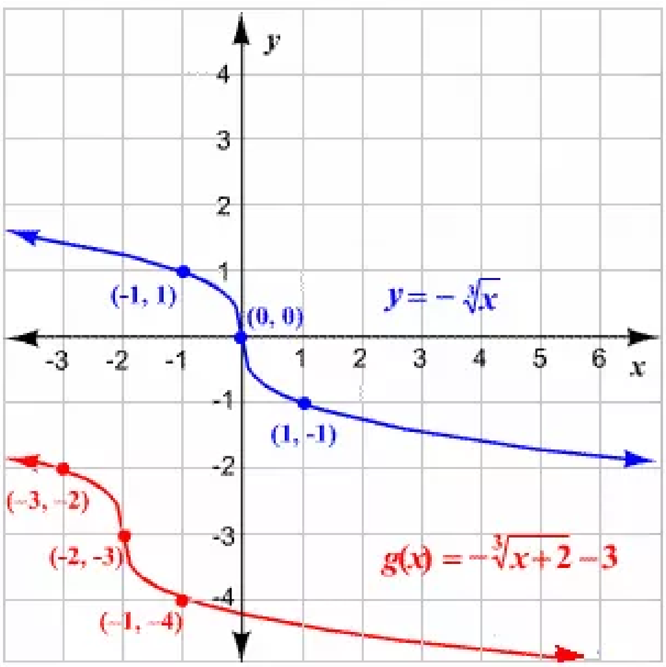
***STEP 1*** **Sketch** the graph of $y = a\sqrt[3]{x}$. In the given case, we have to sketch $y = -\sqrt[3]{x}$. The graph will pass through the point $(1, -1)$, $(0, 0)$, and $(-1, 1)$.




***STEP 2*** **Translate** the graph.

We have to translate the graph of $y = a\sqrt[3]{x}$, $h$ units horizontally and $k$ units to vertically.

Shift the graph of $y = -\sqrt[3]{x}$ to the left by 2 units, and 3 units down to get the graph of $y = -\sqrt[3]{x + 2} - 3$. The resulting graph passes through the points $(-2, -3)$, $(-3, -2)$, and $(-1, -4)$.




In the graph, we can see that the domain and the range are all real numbers.

**Answer 12e.**

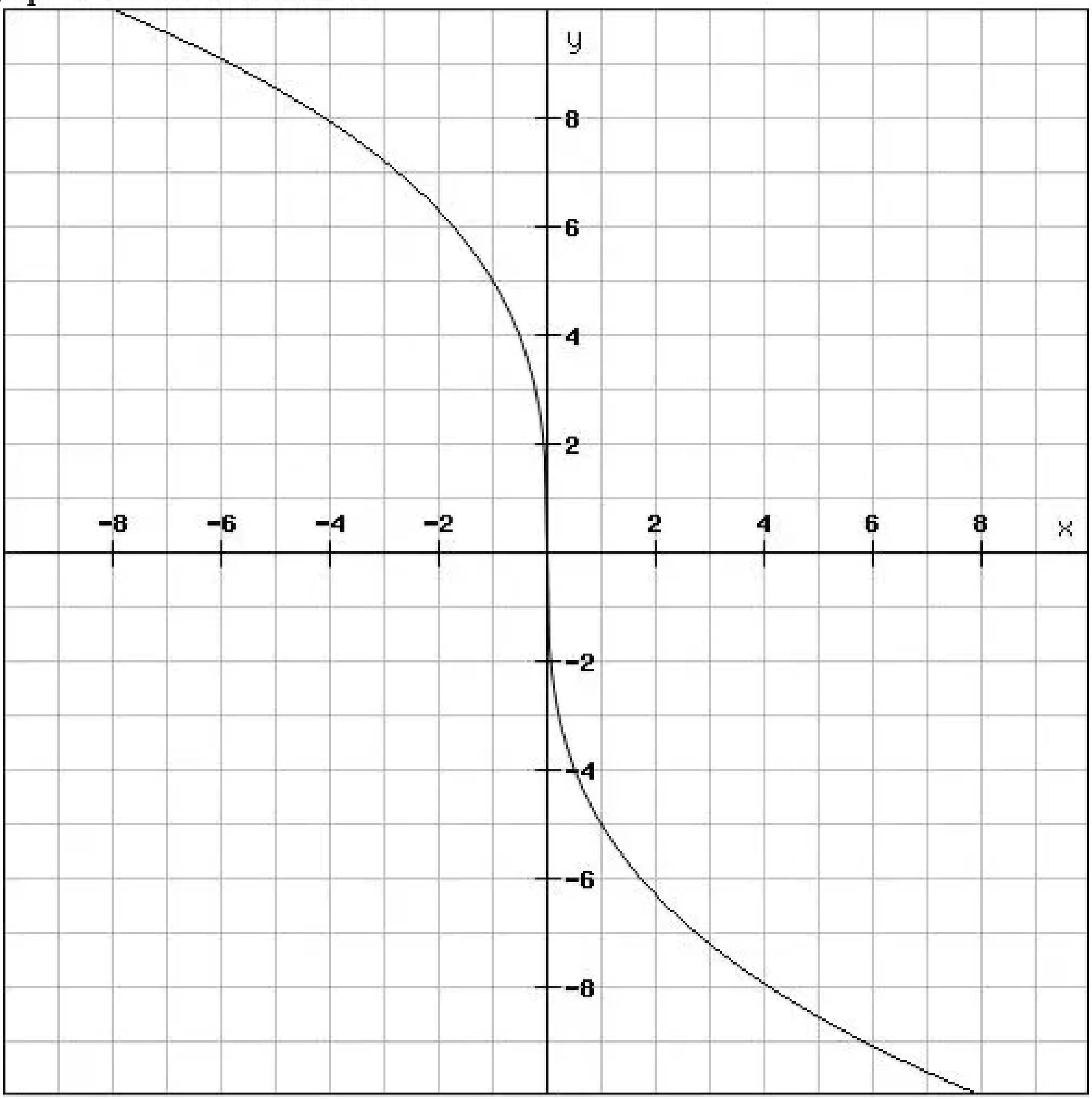
Given function $f(x) = -5\sqrt[3]{x}$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | -10 | -8 | -6 | -4 | -2 | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 10.77 | 10 | 9.08 | 7.93 | 6.3 | 0 | -6.3 | -7.93 | -9.08 | -10 | -10.77 |

The graph is obtained as follows




The function $-5\sqrt[3]{x}$ exists for all real numbers
Therefore domain of the function is all real numbers
Therefore range of the function is all real numbers

## Answer 13e.

First, take a set of random values for $x$, and then find the corresponding $y$-values.

Substitute 0 for $x$ in the given equation.

$$y = -\frac{1}{7}\sqrt[3]{0}$$

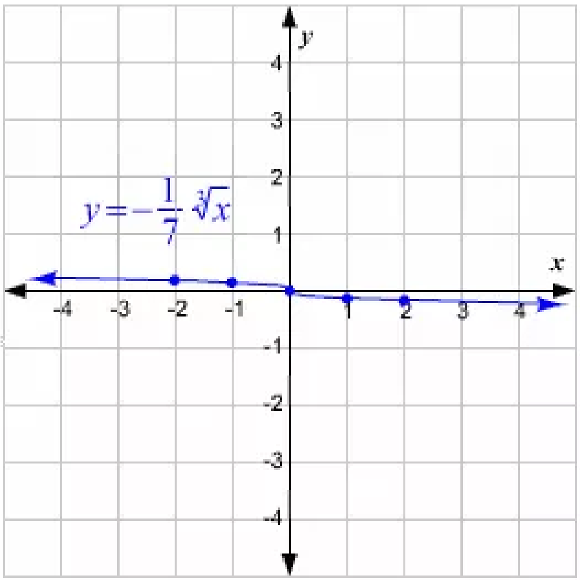
$$y = 0$$

Similarly, find some other values for $y$.
List the values in a table.

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 0.176 | 0.14 | 0 | −0.14 | −0.176 |

Now, sketch the graph.

Plot the points and join them using a smooth curve.




The domain and the range are all real numbers.

## Answer 14e.

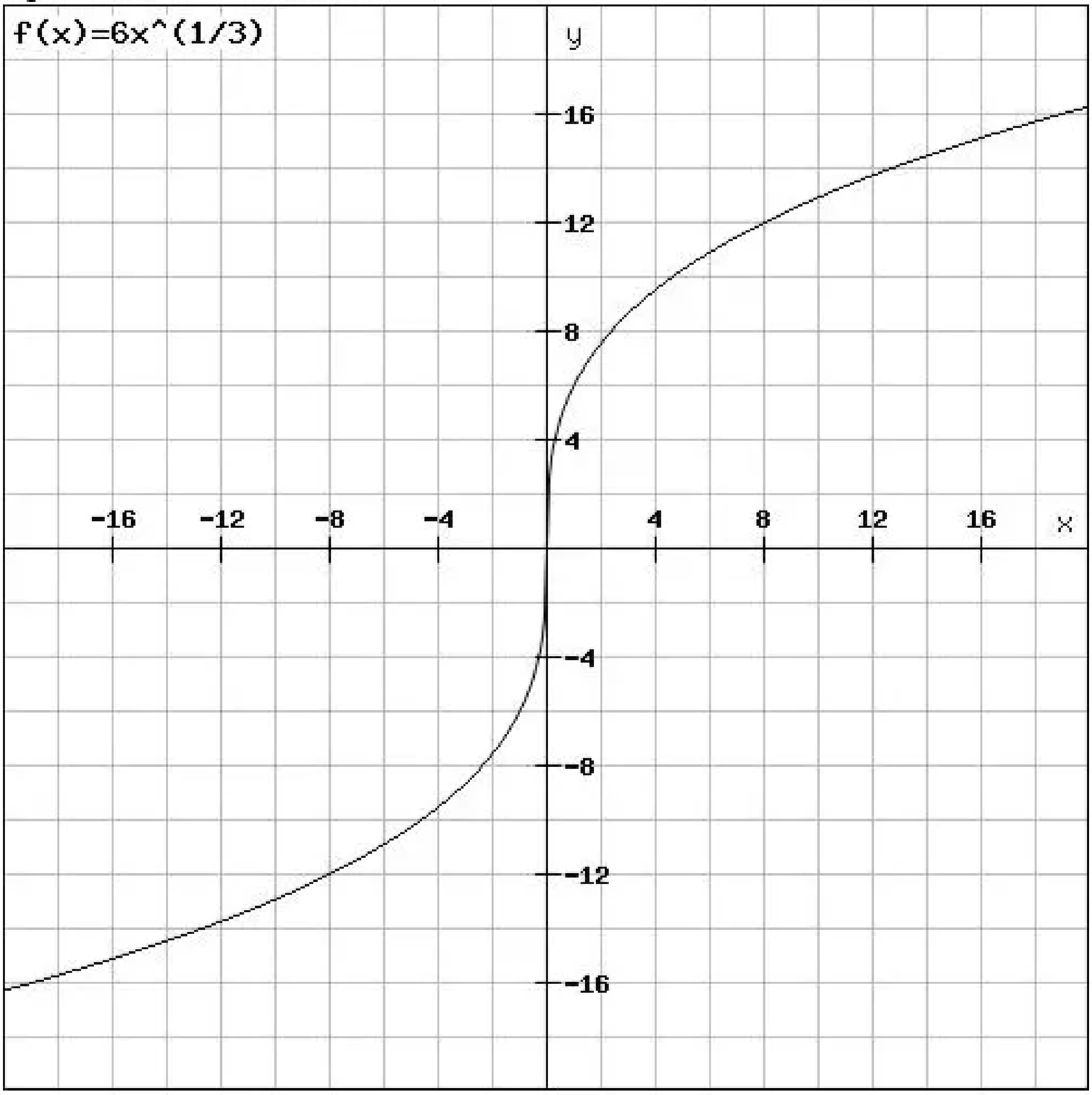
Given function $y = 6\sqrt[3]{x}$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | -16 | -12 | -8 | -6 | -4 | 0 | 4 | 6 | 8 | 12 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | -15.11 | -13.73 | -12 | -10.90 | -9.52 | 0 | 9.52 | 10.90 | 12 | 13.73 | 15.11 |

The graph is obtained as follows




The function $6\sqrt[3]{x}$ exists for all real numbers
Therefore domain of the function is all real numbers
Therefore range of the function is all real numbers

## Answer 15e.

First, take a set of random values for $x$, and then find the corresponding $y$-values.
Substitute 0 for $x$ in the given equation.

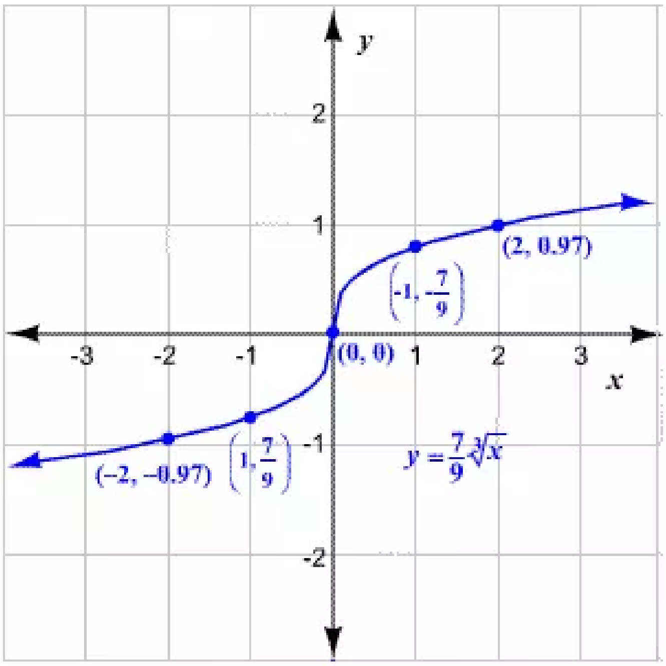
$$y = \frac{7}{9}\sqrt[3]{0}$$

$$y = 0$$

Similarly, find some other values for $y$.
List the values in a table.

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | $-0.97$ | $-\frac{7}{9}$ | 0 | $-\frac{7}{9}$ | 0.97 |

Plot the points and join them using a smooth curve.




In the graph, we can see that the domain and the range are all real numbers.

## Answer 16e.

Given function $f(x)=2\sqrt{x-1}+3$

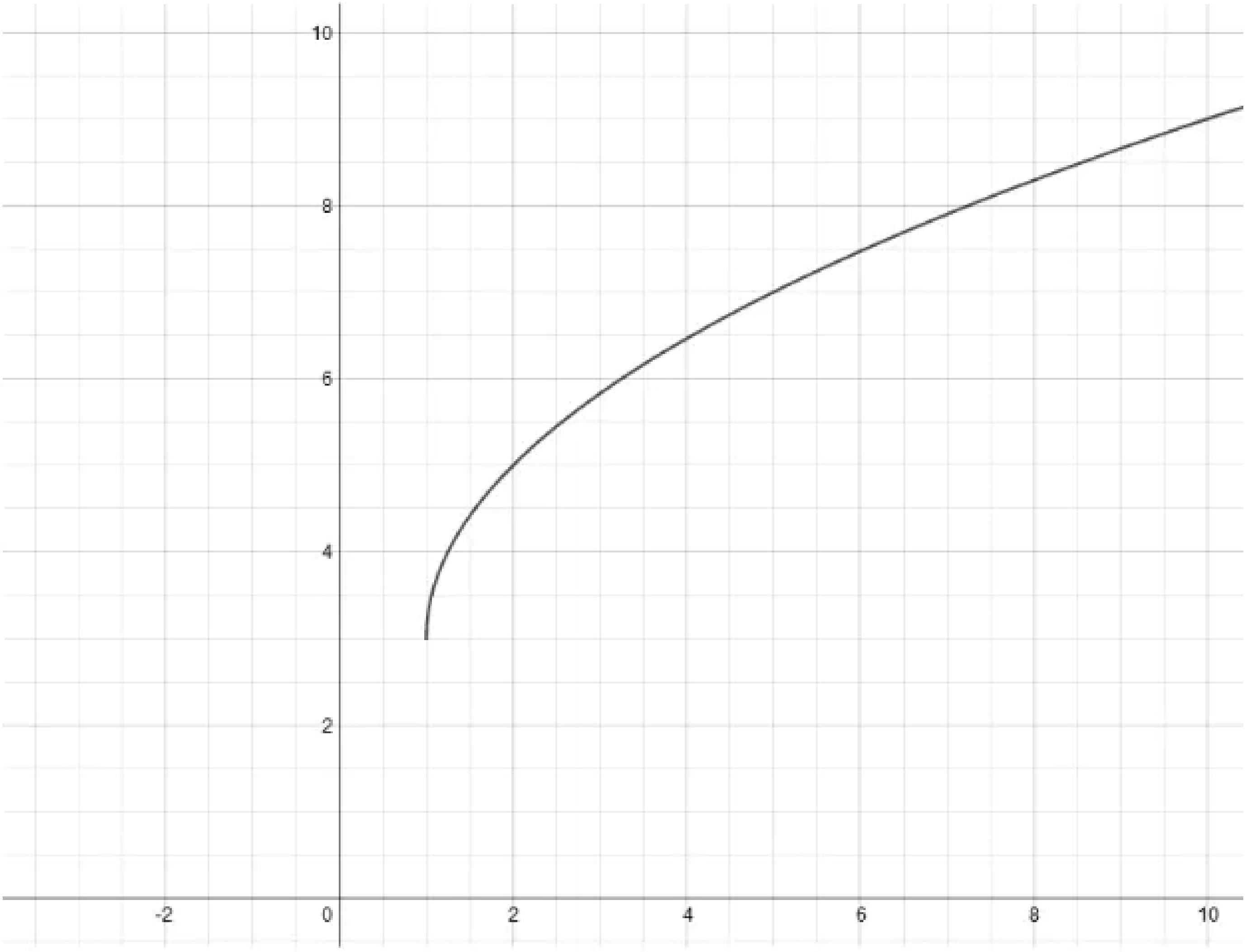
---

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | 1 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $f(x)$ | 3 | 5 | 6.464 | 7.472 | 8.291 | 9 |

The graph is obtained as follows




The function $2\sqrt{x-1}+3$ exists if and only if the radicand of the square root must be nonnegative, so $x-1$ should be nonnegative real number.
Therefore domain of the function is $x \geq 1$
From the graph ,range of the function is $f(x) \geq 3$ for all $x \geq 1$ values.

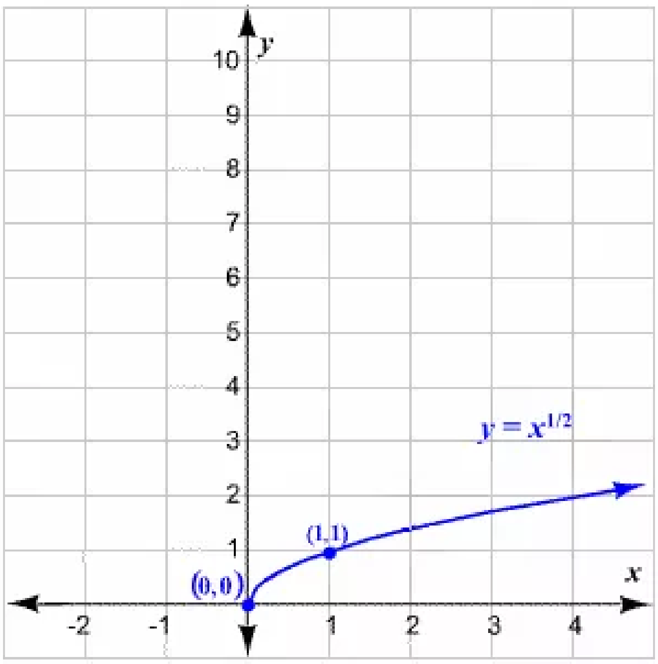
Domain: $\boxed{x \geq 1}$

Range: $\boxed{f(x) \geq 3}$

## Answer 17e.

We can rewrite the given function as $y = \sqrt{x+1} + 8$, which is of the form $y = a\sqrt{x-h} + k$, where $a$ is 1, $h$ is $-1$, and $k$ is 8.

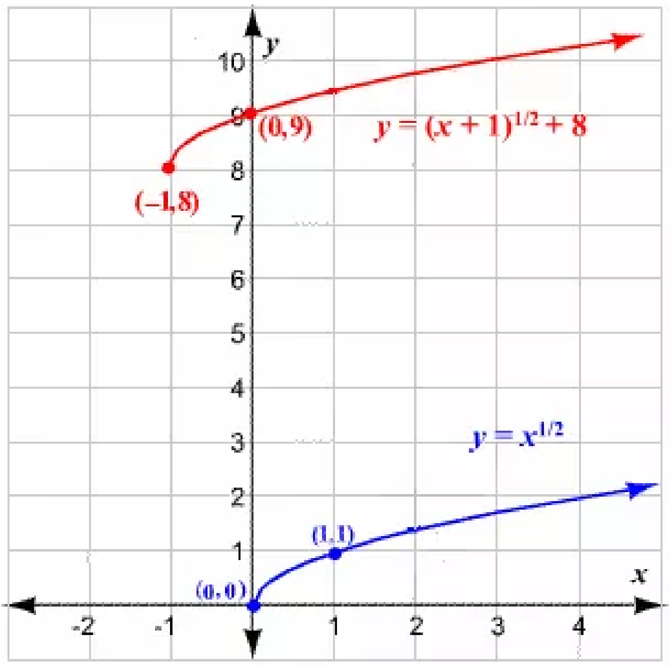
***STEP 1*** **Sketch** the graph of $y = a\sqrt{x}$. In the given case, we have to sketch $y = x^{1/2}$, or $y = \sqrt{x}$. The graph will begin at the origin and pass through the point (1, 1).




***STEP 2*** **Translate** the graph.

We have to translate the graph of $y = a\sqrt{x}$, $h$ units horizontally and $k$ units vertically.

We have to translate the graph of $y = x^{1/2}$, to the left by 1 unit and up by 8 units. The resulting graph will start at $(-1, 8)$ and pass through (0, 9).




In the graph, we can see that the domain of the function is $x \geq -1$ and the range is $y \geq 8$.

## Answer 18e.

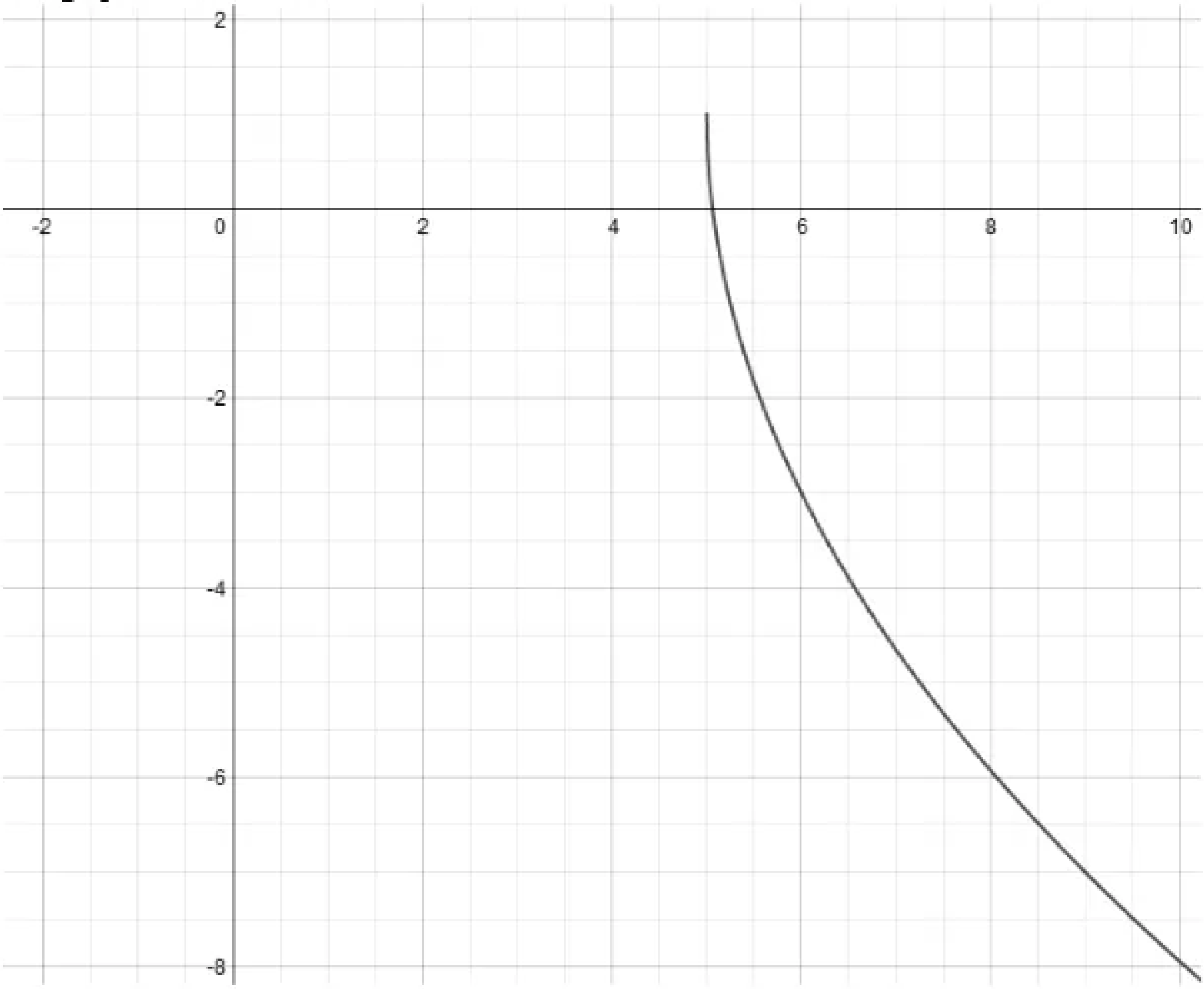
Given function $y = -4\sqrt{x-5} + 1$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|
| $y$ | 1 | -3 | -4.65 | -5.92 | -7 | -7.94 |

The graph is obtained as follows




The function $-4\sqrt{x-5}+1$ exists if and only if the radicand of the square root must be nonnegative, so $x-5$ should be nonnegative real number.
Therefore domain of the function is $x \geq 5$
From the graph, range of the function is $y \leq 1$ for all $x \geq 5$ values.

Domain: $\boxed{x \geq 5}$

Range: $\boxed{y \leq 1}$

## Answer 19e.

Rewrite the rational exponent in the expression as a radical.

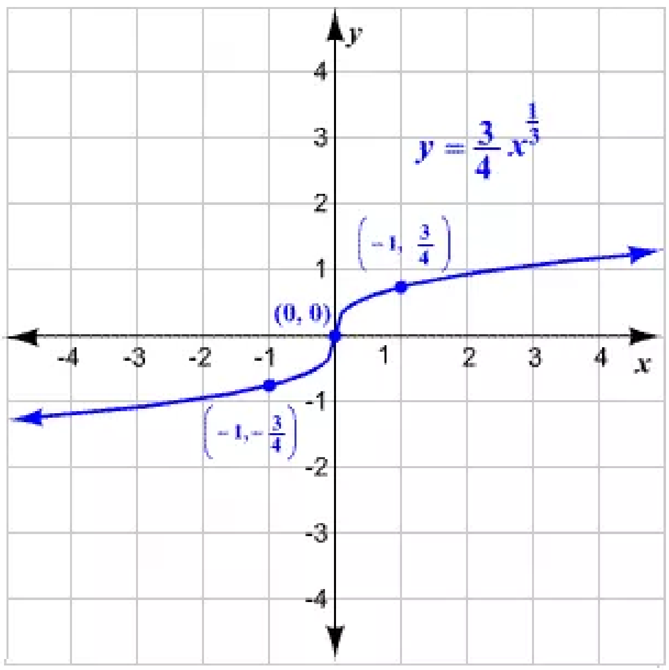
$y = \frac{3}{4}\sqrt[3]{x} - 1$

The function is of the form $y = a\sqrt[3]{x-h}+k$, where $a$ is $\frac{3}{4}$, $h$ is 0, and $k$ is $-1$.

***STEP 1*** **Sketch** the graph of $y = a\sqrt[3]{x}$. In the given case, we have to sketch $y = \frac{3}{4}\sqrt[3]{x}$. Take a set of random values for $x$, and then find the corresponding $y$-values. List the values in a table.

| $x$ | $-1$ | 0 | 1 |
|---|---|---|---|
| $y$ | $-\frac{3}{4}$ | 0 | $\frac{3}{4}$ |

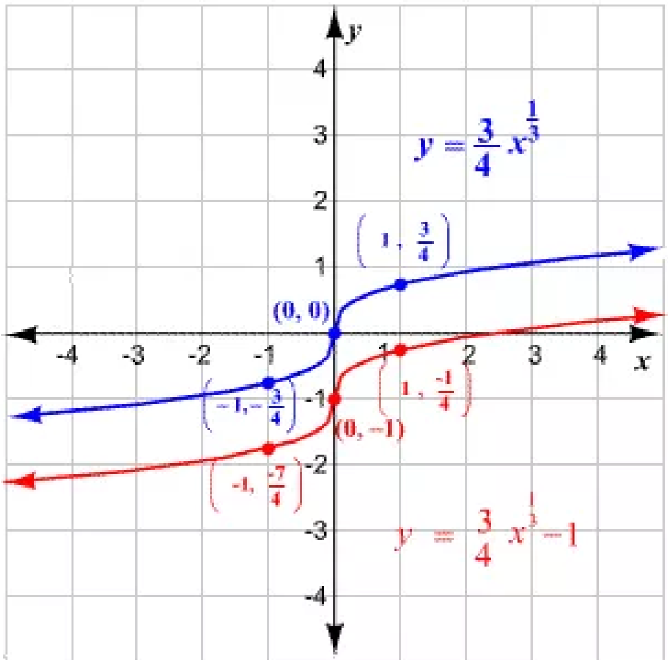
Plot the points and join them using a curve.



*STEP 2* **Translate** the graph.

We have to translate the graph of $y = a\sqrt[3]{x}$ , $h$ units horizontally and $k$ units vertically. Since $h$ is 0 for the given function, there is no horizontal shift.

Shift the graph of $y = \frac{3}{4}\sqrt[3]{x}$ downwards by 1 unit to get the graph of $y = \frac{3}{4}\sqrt[3]{x} - 1$. The resulting graph will pass through the points $(0, -1)$, $\left(-1, -\frac{7}{4}\right)$, and $\left(1, -\frac{1}{4}\right)$.




In the graph, we can see that the domain and the range are all real numbers.

## Answer 20e.

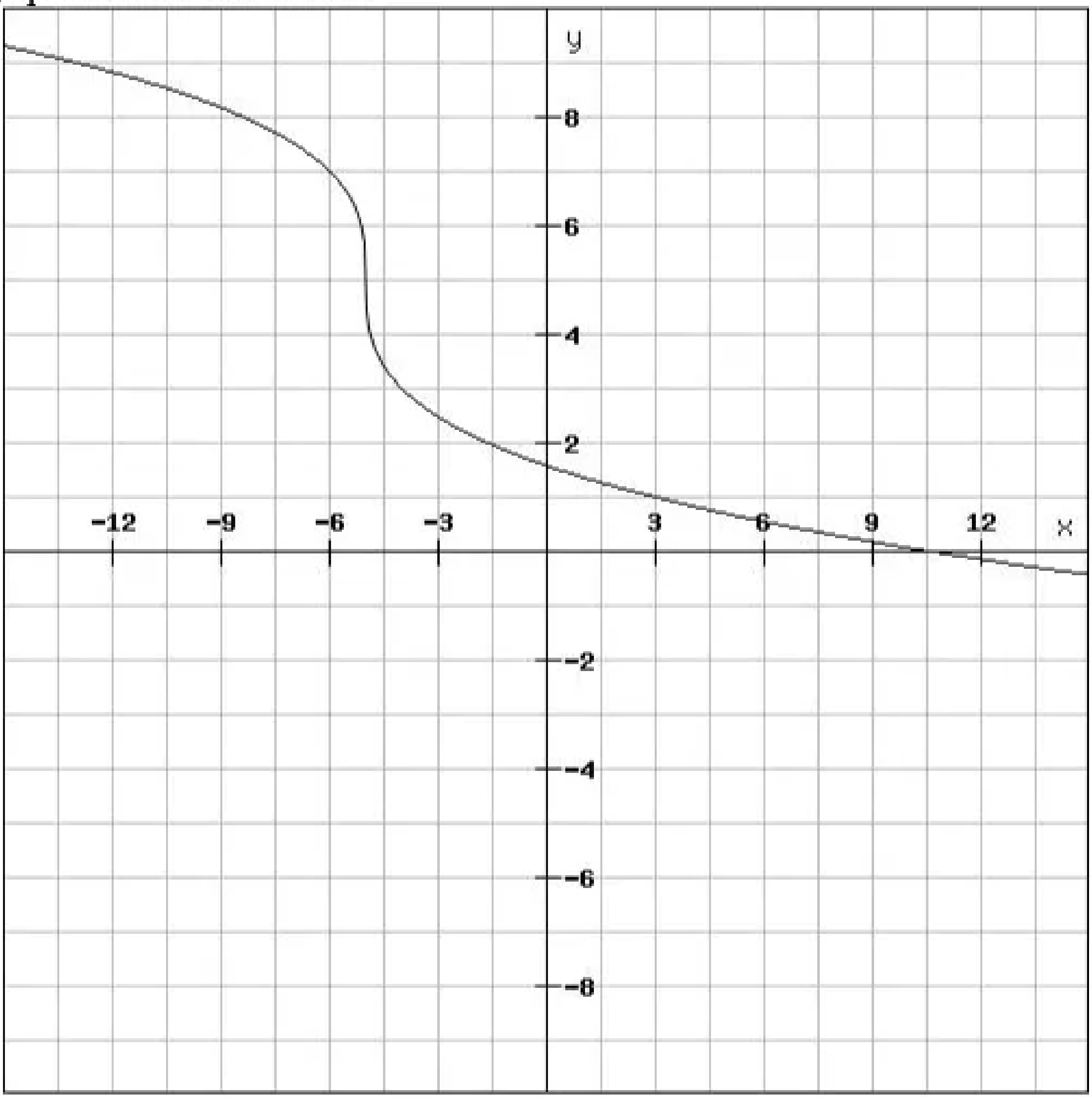
Given function $y = -2\sqrt[3]{x+5} + 5$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | -12 | -7 | -5 | -3 | -1 | 0 | 1 | 3 | 5 | 7 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 8.82 | 7.52 | 5 | 2.48 | 1.82 | 1.58 | 1.36 | 1 | 0.69 | 0.42 | -0.14 |

The graph is obtained as follows




The function $-2\sqrt[3]{x+5}+5$ exists for all real numbers
Therefore domain of the function is all real numbers
Therefore range of the function is all real numbers

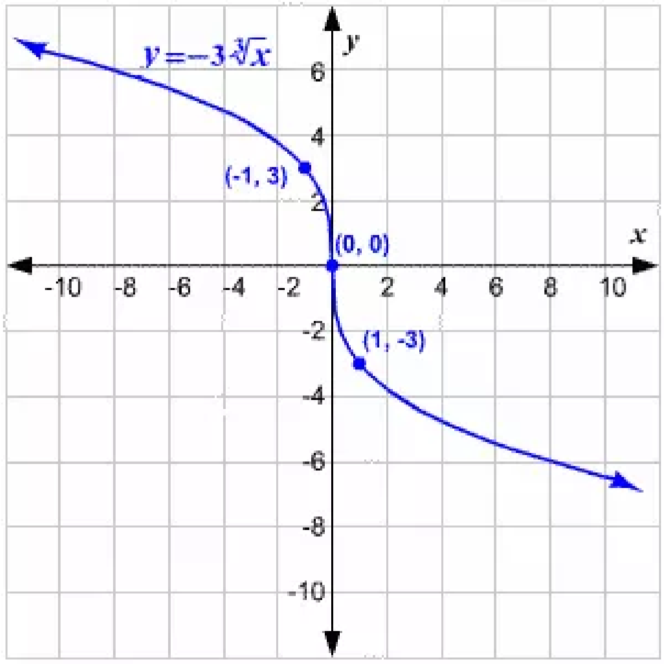
## Answer 21e.

The given function is of the form $y = a\sqrt[3]{x-h}+k$, where $a$ is $-3$, $h$ is $-7$, and $k$ is -6.

***STEP 1*** **Sketch** the graph of $h(x) = -3\sqrt[3]{x}$. Take a set of random values for $x$, and then find the corresponding $y$-values. List the values in a table.

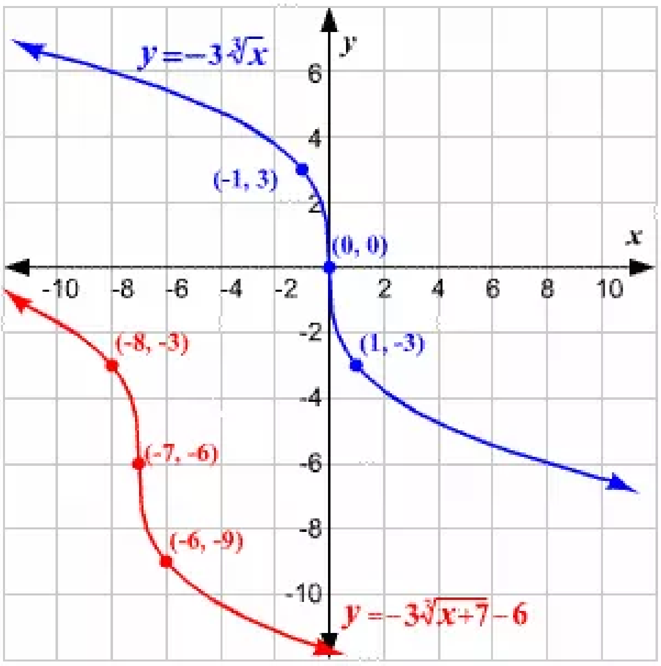
| $x$ | $-1$ | 0 | 1 |
|---|---|---|---|
| $h(x)$ | 3 | 0 | $-3$ |

Plot the points and join them using a smooth curve.




*STEP 2* **Translate** the graph.

Shift the graph of $h(x) = -3\sqrt[3]{x}$ to the left by 7 units and down by 6 units to get the graph of $h(x) = -3\sqrt[3]{x+7} - 6$. The resulting graph will pass through the points (−8, −3), (−7, −6), and (−6, −9).




The domain and the range are all real numbers.

## Answer 22e.

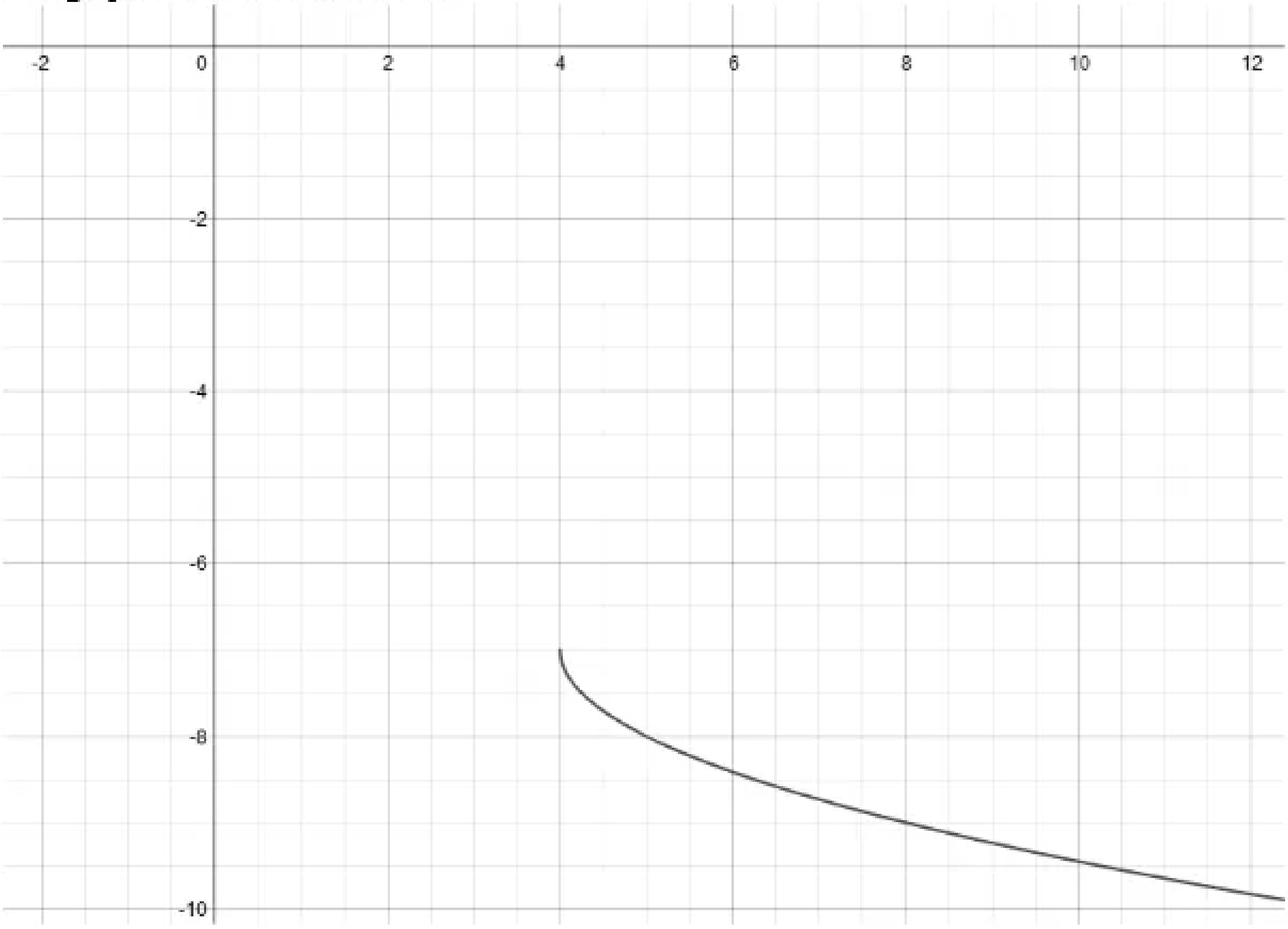
Given function $y = -\sqrt{x-4} - 7$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | 4 | 5 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|
| $y$ | -7 | -8 | -8.41 | -9 | -9.44 | -9.82 |

The graph is obtained as follows




The function $-\sqrt{x-4} - 7$ exists if and only if the radicand of the square root must be nonnegative, so $x-4$ should be nonnegative real number.
Therefore domain of the function is $x \geq 4$
From the graph, range of the function is $y \leq -7$ for all $x \geq 4$ values.

Domain: $\boxed{x \geq 4}$

Range: $\boxed{y \leq -7}$

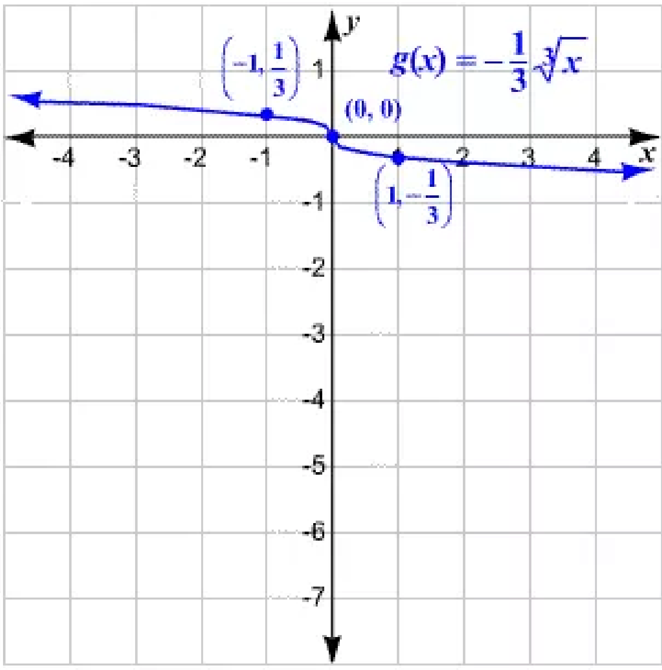
## Answer 23e.

In order to graph $y = a\sqrt[3]{x-h}+k$, first sketch the graph of $y = a\sqrt[3]{x}$. Then translate the graph horizontally $h$ units and vertically $k$ units.

***STEP 1*** We have to sketch $g(x) = -\frac{1}{3}\sqrt[3]{x}$. Take a set of random values for $x$, and then find the corresponding $y$-values.
List the values in a table.

| $x$ | $-1$ | 0 | 1 |
|---|---|---|---|
| $y$ | $\frac{1}{3}$ | 0 | $-\frac{1}{3}$ |

Plot the points and join them using a smooth curve.

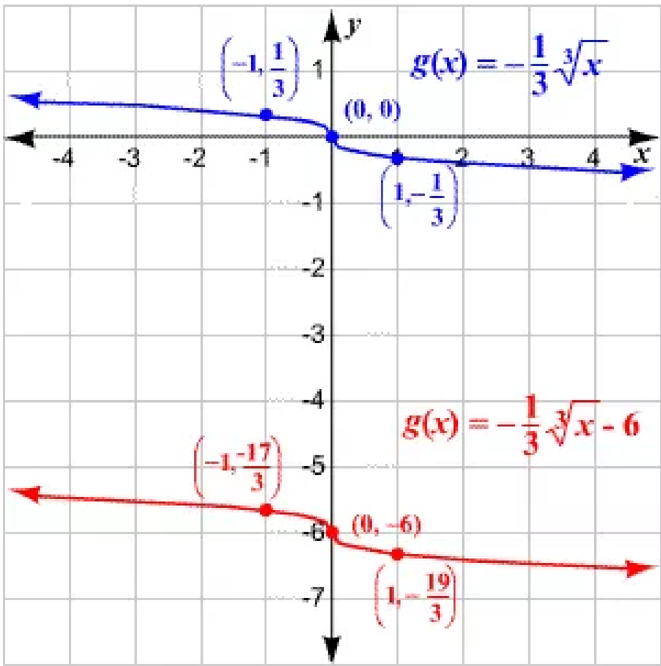


***STEP 2*** **Translate** the graph.

We have to translate the graph of $y = a\sqrt[3]{x}$, $h$ units horizontally and $k$ units vertically. Since $h$ is 0 for the given function, there is no horizontal shift.

Shift the graph of $g(x) = -\frac{1}{3}\sqrt[3]{x}$ downwards by 6 units to get the graph of $g(x) = -\frac{1}{3}\sqrt[3]{x} - 6$.

The resulting graph will pass through the point $(0, -6)$, $\left(-1, -\frac{17}{3}\right)$, and $\left(1, -\frac{19}{3}\right)$.




In the graph, we can see that the domain and the range are all real numbers.

## Answer 24e.

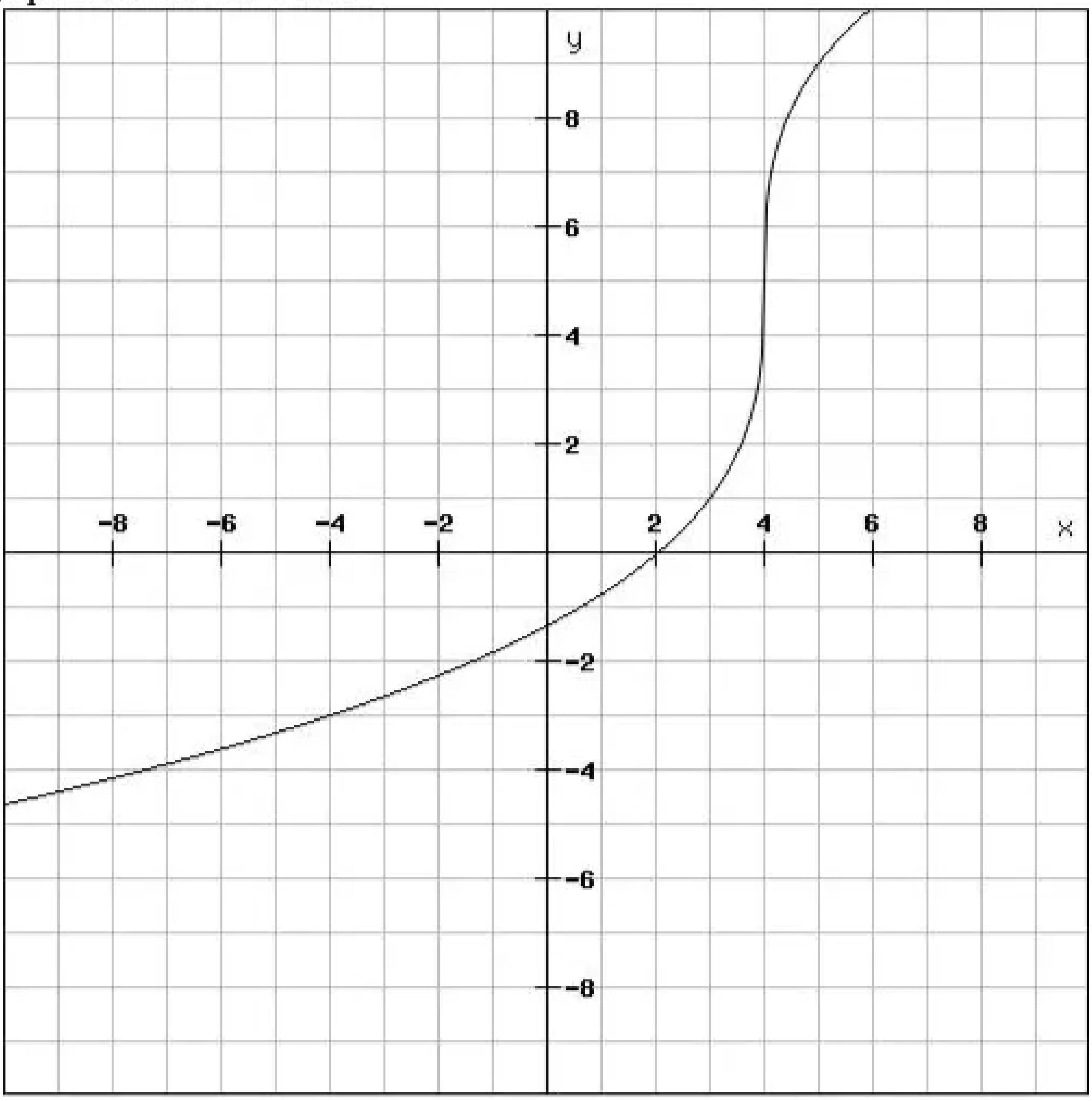
Given function $y = 4\sqrt[3]{x-4} + 5$

We need to sketch the graph and state the domain and range of the function.
For graphing the function, we need to make a table of values. Use a graphing calculator to make a table and sketch the graph.

The table is obtained as follows

| $x$ | -6 | -5 | -4 | -3 | -2 | 0 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | -3.61 | -3.32 | 3 | -2.65 | -2.26 | -1.35 | -0.04 | 1 | 5 | 9 | 10.04 |

The graph is obtained as follows




The function $4\sqrt[3]{x-4}+5$ exists for all real numbers
Therefore domain of the function is all real numbers
Therefore range of the function is all real numbers

## Answer 25e.

The given function is a radical function.
We know that the radicand of a square root should be nonnegative. For this condition to be satisfied, $x - 5$ should be greater than or equal to 0. In other words, $x$ should be greater than or equal to 5.

The domain of a function is the set of all $x$-values. So, the domain of the given function is $x \ge 5$. Since the range depends upon the domain, there will be limitation for the range also.

## Answer 26e.

Given parent function $y = -2\sqrt[3]{x}$

Given function $y = -2\sqrt[3]{x+1} - 3$

Student explained the relation in a way as 'The graph of $y = -2\sqrt[3]{x+1} - 3$ is the graph of $y = -2\sqrt[3]{x}$ translated right 1 unit and down 3 units'

In generalized form, the graph of $y = a\sqrt[3]{x-h} + k$ will be obtained from the graph of $y = a\sqrt[3]{x}$ with translation of $h$ units horizontally right and $k$ units vertically up

Here for the given function, $h = -1$ and $k = -3$
Hence, the graph of given function $y = -2\sqrt[3]{x+1} - 3$ will be obtained from the graph of parent function $y = -2\sqrt[3]{x}$ with translation of -1 units horizontally right and -3 units vertically up

In other words, the graph of given function $y = -2\sqrt[3]{x+1} - 3$ is the graph of $y = -2\sqrt[3]{x}$ with translation of 1 units horizontally left and 3 units vertically down

The error in the student explanation is in the translation stated in horizontal direction. The correct direction is left instead of right as explained by student.

## Answer 27e.

The given equation is of the form $y = a\sqrt[3]{x}$, where $a$ is 3.

In order to graph $y = a\sqrt[3]{x - h} + k$, we will translate the graph of $y = a\sqrt[3]{x}$ horizontally $h$ units and vertically $k$ units.

It is given that the graph is shifted to the left by 2 units. The variable $h$ will be $-2$.
Since there is no vertical shift, $k$ value is zero.

Thus, the equation of the translated graph is $y = 3\sqrt[3]{x + 2}$.

Therefore, choice C represents the equation of the translated graph.

## Answer 28e.

Given function $y = \sqrt{x+5}$

The function $\sqrt{x+5}$ exists if and only if the radicand of the square root must be nonnegative, so $x+5$ should be nonnegative real number.
Therefore domain of the function is $x \geq -5$
Substitute $x \geq -5$ in the function, we get range of the function is all non negative real numbers.
Therefore range is $y \geq 0$ for all $x \geq -5$ values.

Domain: $\boxed{x \geq -5}$

Range: $\boxed{y \geq 0}$

## Answer 29e.

The given function is a square root function.
We know that the radicand of a square root should be nonnegative. For this condition to be satisfied, $x - 12$ should be greater than or equal to 0. In other words, $x$ should be greater than or equal to 12.

The domain of a function is the set of all $x$-values. So, the domain of the given function is $x \geq 12$.

The range is the set of all $y$-values.
Substitute the least value, 12, for $x$ in the given function and find $y$.

$$\begin{aligned} y &= \sqrt{12 - 12} \\ &= 0 \end{aligned}$$

The range for the given function is $y \geq 0$.

## Answer 30e.

Given function $y = \frac{1}{3}\sqrt{x} - 4$

The function $\frac{1}{3}\sqrt{x} - 4$ exists if and only if the radicand of the square root must be nonnegative, so $x$ should be nonnegative real number.
Therefore domain of the function is $x \geq 0$
Substitute $x \geq 0$ in the function, we get range is $y \geq -4$ for all $x \geq 0$ values.

Domain: $\boxed{x \geq 0}$

Range: $\boxed{y \geq -4}$

## Answer 31e.

The given function is a cube root function.
We know that the radicand of a cube root can be any real number.

The domain of a function is the set of all $x$-values and the range is the set of all $y$-values. Since there is no restriction on the selection of $x$-values, the domain and the range of the given function is the set of all real numbers.

## Answer 32e.

Given function $g(x)=\sqrt[3]{x+7}$

The cube root function $\sqrt[3]{x+7}$ exists for all real numbers
Therefore domain of the function is all real numbers
Therefore range of the function is all real numbers

## Answer 33e.

The given function is a square root function.
We know that the radicand of a square root should be nonnegative. For this condition to be satisfied, $x-3$ should be greater than or equal to 0. In other words, $x$ should be greater than or equal to 3.

The domain of a function is the set of all $x$-values. So, the domain of the given function is $x \geq 3$.

The range is the set of all $y$-values.
Substitute the least value, 3, for $x$ in the given function and find $y$.

$$y = \frac{1}{4}\sqrt{3-3}+6$$
$$= 6$$

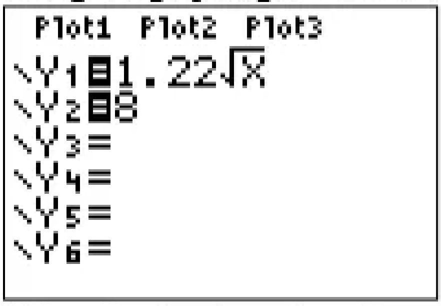
The range for the given function is $y \geq 6$.

## Answer 35e.

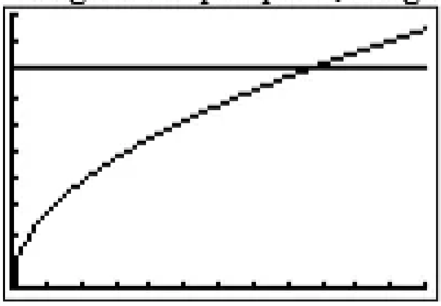
For the pilot, the distance he can see to horizon is given by $d=1.22\sqrt{a}$ where $d$ is distance (in miles) and $a$ is altitude (in feet above sea level)
Distance $d=8$ miles

To solve the given equation $d = 1.22\sqrt{a}$ for altitude, using the graphing calculator and intersect feature will ease the process

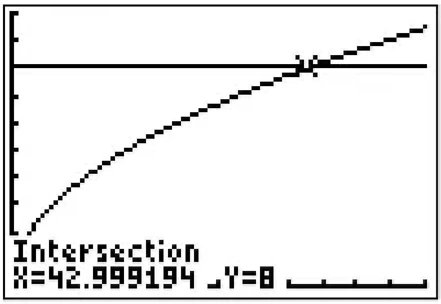
Using the graphing calculator and enter the functions as shown below




Using the Graph option, the graphs for the entered functions are obtained as shown below



Using the Intersect option, it is showed that the both graphs intersect at $x = 42.999$ and $y = 8$




Hence, it is shown that the solution from the graph is $x = 42.999$

Hence, the altitude where the pilot can view the horizon at distance of 8 miles is

$$\boxed{a = 42.999 \text{ feet}}$$

## Answer 36e.

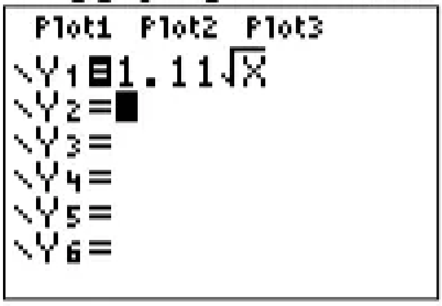
In a pendulum, the relation between period of pendulum $T$ (in seconds) and length of pendulum $l$ (in feet) is given by $T = 1.11\sqrt{l}$

(a) Length of pendulum $l = 2$ feet

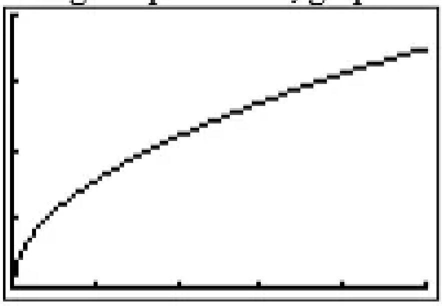
Given function $T = 1.11\sqrt{l}$

In generalized form, it can be written as $y = 1.11\sqrt{x}$

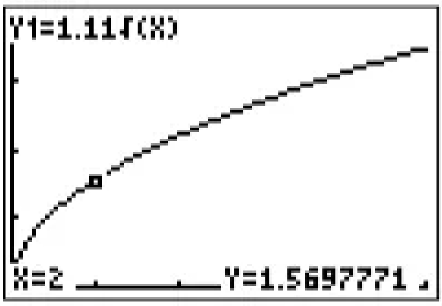
Using graphing calculator, enter the function as shown below




Using Graph feature, graph the function as shown below



Using the trace feature to find the period of pendulum for the length of 2 feet as shown below




Hence, it is shown that the solution from the graph is $y = 1.5697$

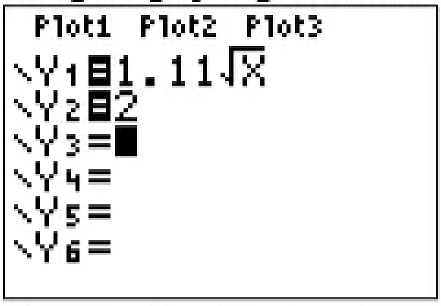
The period of pendulum for the given length of 2 feet is

$$\boxed{T = 1.5697 \text{ sec}}$$

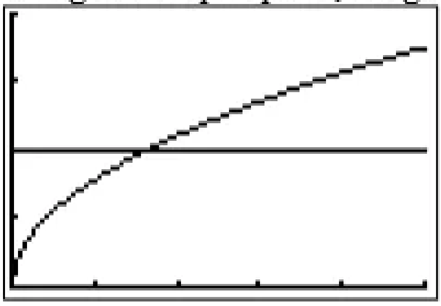
(b) Period of pendulum $T = 2\,\text{sec}$

To solve the above equation for length for the given period, using the graphing calculator and intersect feature will ease the process

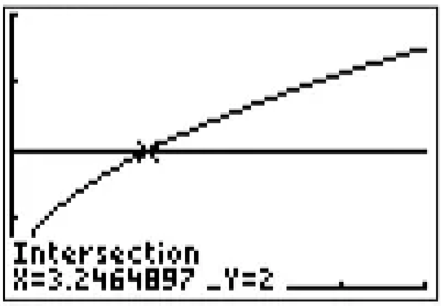
Using the graphing calculator and enter the functions as shown below




Using the Graph option, the graphs for the entered functions are obtained as shown below



Using the Intersect option, it is showed that the both graphs intersect at $x = 3.2465$ and $y = 2$




Hence, it is shown that the solution from the graph is $x = 3.2465$

The length of pendulum for the given period of 2 seconds is

$$\boxed{l = 3.2465 \text{ feet}}$$

## Answer 37e.

**a.** Substitute 273.15 + C for $K$ in the equation for $v$.

$$v = 331.5\sqrt{\frac{273.15 + C}{273.15}}$$

Simplify the equation.

$$v = 331.5\sqrt{1 + \frac{C}{273.15}}$$

**b.** We know that the radicand of a square root should be nonnegative. For this condition to be satisfied, $1+\dfrac{C}{273.15}$ should be greater than or equal to 0.

$$1+\frac{C}{273.15} \geq 0$$

On solving the inequality, we get $C \geq -273.15$.

Substitute the least value, -273.15, for $C$ in the function and find $v$.

$$\begin{aligned} v &= 331.5\sqrt{1+\frac{-273.15}{273.15}} \\ &= 331.5\sqrt{1-1} \\ &= 0 \end{aligned}$$

The domain of the function is the set of all $C$-values and the range is the set of all $v$-values.
Thus, the domain for the function will be $C \geq -273.15$, and the range is $v \geq 0$.

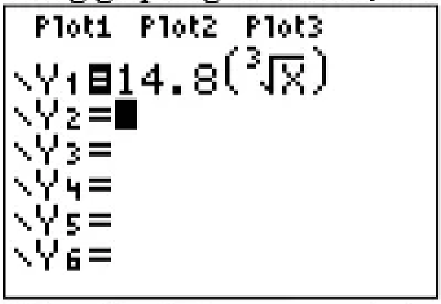
## Answer 38e.

In a drag racing, for a given weight, the relation between speed of car at end of race $s$ (in miles per hour) and car's power $p$ (in horsepower) is given by $s = 14.8\sqrt[3]{p}$
Weight of car is 3500 pounds
Speed of car $s = 200$ miles per hour

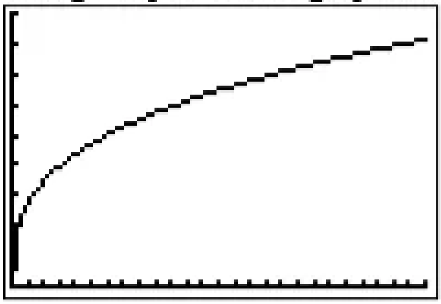
Given function $s = 14.8\sqrt[3]{p}$

In generalized form, it can be written as $y = 14.8\sqrt[3]{x}$

Using graphing calculator, enter the function as shown below

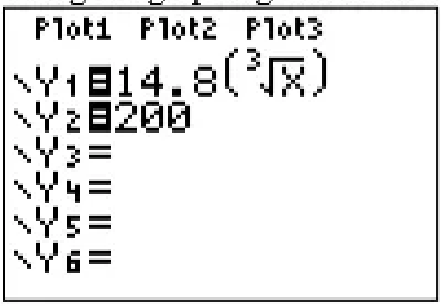



Using Graph feature, graph the function as shown below

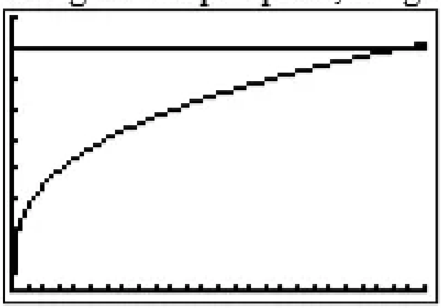


To solve the above equation for power, using the graphing calculator and intersect feature will ease the process

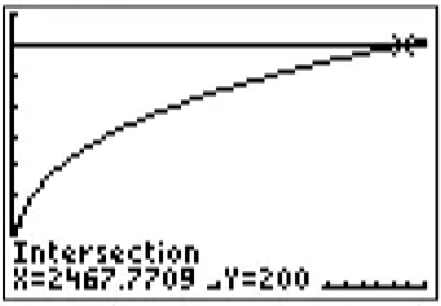
Using the graphing calculator and enter the obtained functions as shown below




Using the Graph option, the graphs for the entered functions are obtained as shown below



Using the Intersect option, it is showed that the both graphs intersect at $x = 2467.77$ and $y = 200$




Hence, it is shown that the solution from the graph is $x = 2467.77$

The power of the given car is

$p = 2467.77$ horse power

## Answer 39e.

**(a)** Substitute 165 for $W$ in the given equation.

$$v_t = 33.7\sqrt{\frac{165}{A}}$$

Thus, the equation for $v_t$ in terms of $A$ is $v_t = 33.7\sqrt{\frac{165}{A}}$.

**(b)** Take a set of random values for $A$, and then find the corresponding values of $v_t$.

Substitute 5 for $A$ in the equation and simplify.

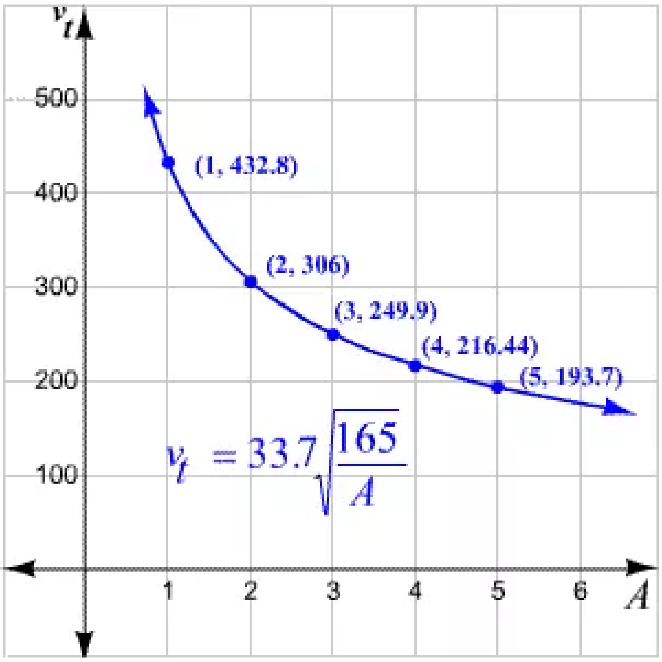
$$v_t = 33.7\sqrt{\frac{165}{5}}$$
$$\approx 193.6$$

Similarly, find some other values for $v_t$.

List the values in a table.

| $A$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $v_t$ | 432.8 | 306 | 249.9 | 216.44 | 193.6 |

**(c)** Plot the points and join them using a smooth curve.




## Answer 40e.

Given a right circular cone with surface area $S$, radius $r$ and slant height of 1 unit

The relation is given by $S = \pi r + \pi r^2$

(a) Solve for $r$ using the given relation

Rewriting the given relation as follows

$$S = \pi r + \pi r^2$$
$$= \pi\left(r + r^2\right)$$
$$= \pi\left(2\times\frac{1}{2}\times r + r^2\right)$$

Adding $\pi/4$ on both sides and rewrite the equation as follows

$$S = \pi\left(2\times\frac{1}{2}\times r + r^2\right)$$
$$S+\frac{\pi}{4} = \frac{\pi}{4} + \pi\left(2\times\frac{1}{2}\times r + r^2\right)$$
$$S+\frac{\pi}{4} = \pi\left(\frac{1}{4} + 2\times\frac{1}{2}\times r + r^2\right)$$
$$S+\frac{\pi}{4} = \pi\left(\left(\frac{1}{2}\right)^2 + 2\times\frac{1}{2}\times r + (r)^2\right)$$
$$S+\frac{\pi}{4} = \pi\left(r+\frac{1}{2}\right)^2$$

Square root on both sides and rewrite the equation as follows

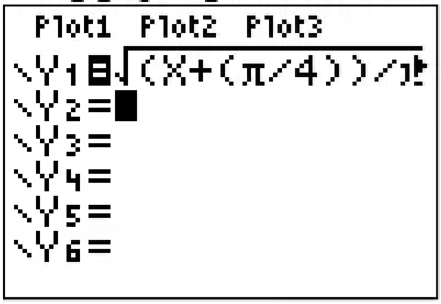
$$S+\frac{\pi}{4} = \pi\left(r+\frac{1}{2}\right)^2$$
$$\sqrt{S+\frac{\pi}{4}} = \sqrt{\pi\left(r+\frac{1}{2}\right)^2}$$
$$\sqrt{S+\frac{\pi}{4}} = \sqrt{\pi}\sqrt{\left(r+\frac{1}{2}\right)^2}$$
$$\sqrt{S+\frac{\pi}{4}} = \sqrt{\pi}\left(r+\frac{1}{2}\right)$$
$$\frac{1}{\sqrt{\pi}}\sqrt{S+\frac{\pi}{4}} = \left(r+\frac{1}{2}\right)$$
$$\frac{1}{\sqrt{\pi}}\sqrt{S+\frac{\pi}{4}} - \frac{1}{2} = \left(r+\frac{1}{2}\right) - \frac{1}{2}$$
$$\frac{1}{\sqrt{\pi}}\sqrt{S+\frac{\pi}{4}} - \frac{1}{2} = r$$

Finally, the relation of $r$ is showed as

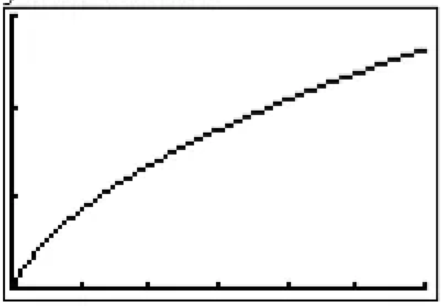
$$\boxed{r = \frac{1}{\sqrt{\pi}}\sqrt{S+\frac{\pi}{4}} - \frac{1}{2}}$$

(b) In generalized form, it can be written as $y=\frac{1}{\sqrt{\pi}}\sqrt{x+\frac{\pi}{4}}-\frac{1}{2}$

Using graphing calculator, enter the function as shown below



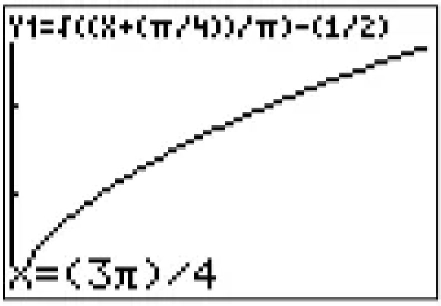

Using Graph feature, graph the function as shown below where x-axis is surface area and y-axis is radius



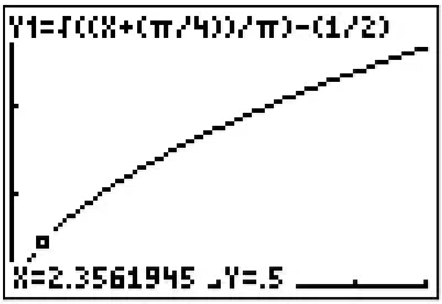
(c) Given surface area is $3\pi/4$ square units

To solve for radius for the given surface area, using the graphing calculator and trace feature will ease the process

Using the graphing calculator after graph is obtained; enter the surface area using trace feature as shown below




Using trace feature, the radius obtained as shown below




Hence, it is shown that the solution from the graph is $y=0.5$

The radius of the given cone is

$$\boxed{r=0.5\text{ units}}$$

## Answer 41e.

Multiply each side of the equation by 5.

$$\frac{1}{5}(x+8)^2(5) = 3(5)$$

$$(x+8)^2 = 15$$

Take the square root of each side.

$$x+8 = \pm\sqrt{15}$$

Subtract 8 from each side and simplify.

$$x+8-8 = \pm\sqrt{15}-8$$

$$x = -8\pm\sqrt{15}$$

Thus, the solution are $-8+\sqrt{15}$, and $-8-\sqrt{15}$.

## Answer 42e.

Given equation is $9(x-3)^2+22=130$

The equation can be solved as follows

Subtract 22 from both the sides of the equation

$$9(x-3)^2+22-22=130-22$$

$$9(x-3)^2=108$$

Divide both the sides of the equation by 9

$$\frac{9(x-3)^2}{9}=\frac{108}{9}$$

$$(x-3)^2=11$$

Square root on both the sides to eliminate the radical

$$\sqrt{(x-3)^2}=\sqrt{11}$$

$$x-3=\pm\sqrt{11}$$

Add 3 on both the sides

$$x-3+3=\pm\sqrt{11}+3$$

$$x=3\pm\sqrt{11}$$

The solution to the given equation is

$$\boxed{x=3\pm\sqrt{11}}$$

## Answer 43e.

Add 20 to both sides of the equation.

$$7x^2-20+20 = 36-20$$

$$7x^2 = 16$$

Divide both the sides by 7.

$$\frac{7x^2}{7} = \frac{16}{7}$$

$$x^2 = \frac{16}{7}$$

Take the square root of each side.

$$x = \pm\sqrt{\frac{16}{7}}$$

Use the product property and simplify.

$$x = \pm\frac{\sqrt{16}}{\sqrt{7}}$$

$$= \pm\frac{4}{\sqrt{7}}$$

Therefore, the solution for the given equation is $\pm\frac{4}{\sqrt{7}}$.

## Answer 44e.

Given equation is $x^2 - 14x + 37 = 0$

The equation can be solved as follows

Write the given polynomial using the formula $(a-b)^2 = a^2 - 2ab + b^2$

$$x^2 - 14x + 37 = 0$$

$$(x^2 - 2\cdot 7x + 49) - 12 = 0$$

$$(x-7)^2 - 12 = 0$$

Add both the sides of the equation with 12

$$(x-7)^2 - 12 = 0$$

$$(x-7)^2 - 12 + 12 = 12$$

$$(x-7)^2 = 12$$

Square root on both the sides to eliminate the square

$$\sqrt{(x-7)^2} = \pm\sqrt{12}$$

$$x - 7 = \pm\sqrt{12}$$

Add 7 on both the sides to obtain the value of $x$

$$x - 7 + 7 = \pm\sqrt{12} + 7$$

$$x = 7 \pm \sqrt{12}$$

$$x = 7 \pm 2\sqrt{3}$$

The solution for the given equation is

$$\boxed{x = 7 \pm 2\sqrt{3}}$$

## Answer 45e.

First, write the left side in the form $x^2 + bx$. For this, subtract 383 from each side.

$$x^2 - 40x + 383 - 383 = 0 - 383$$
$$x^2 - 40x = -383$$

Square half the coefficient of $x$.

$$\left(\frac{-40}{2}\right)^2 = (-20)^2 = 400$$

Add 400 to each side of the equation.

$$x^2 - 40x + 400 = -383 + 400$$

Write the left side as a binomial squared and simplify.

$$(x - 20)^2 = -383 + 400$$
$$(x - 20)^2 = -17$$

Now, take the square roots of each side.

$$x - 20 = \pm\sqrt{-17}$$

Add 20 to each side to solve for $x$.

$$x - 20 + 20 = \pm\sqrt{-17} + 20$$
$$x = 20 \pm \sqrt{-17}$$

Write in terms of the imaginary unit $i$.

$$x = 20 \pm \sqrt{-1 \cdot 17}$$
$$x = 20 \pm i\sqrt{17}$$

Thus, the solutions are $20 + i\sqrt{17}$ and $20 - i\sqrt{17}$.

## Answer 46e.

Given equation is $x^2 - 22x + 97 = 0$

The equation can be solved as follows

Write the given polynomial using the formula $(a-b)^2 = a^2 - 2ab + b^2$

$$x^2 - 22x + 97 = 0$$
$$(x^2 - 2\cdot 11x + 121) - 24 = 0$$
$$(x-11)^2 - 24 = 0$$

Add both the sides of the equation with 24

$$(x-11)^2 - 24 = 0$$
$$(x-11)^2 - 24 + 24 = 24$$
$$(x-11)^2 = 24$$

Square root on both the sides to eliminate the square

$$\sqrt{(x-11)^2} = \sqrt{24}$$
$$x - 11 = \pm\sqrt{24}$$

Add 11 on both the sides to obtain the value of $x$

$$x - 11 + 11 = \pm\sqrt{24} + 11$$
$$x = 11 \pm \sqrt{24}$$
$$x = 11 \pm 2\sqrt{6}$$

The solution for the given equation is

$$\boxed{x = 11 \pm 2\sqrt{6}}$$

## Answer 47e.

Take out the common factor $x$.

$f(x) = x(x^3 + 5x^2 - x - 5)$

Now, let us factor the trinomial $x^3 + 5x^2 - x - 5$.

Find all the possible rational zeros.

The leading coefficient of the given function is 1 and the constant term is 5. Divide the factors of the constant term by the factors of leading coefficient to get the list of possible rational zeros.

$$x = \pm\frac{1}{1}, \pm\frac{5}{1}$$

Test these zeros using synthetic division.

**Test $x = 1$**

$$\begin{array}{r|rrrr} 1 & 1 & 5 & -1 & -5 \\ & & 1 & 6 & 5 \\ \hline & 1 & 6 & 5 & 0 \end{array}$$

Now, we have 1 as a zero of $x^3 + 5x^2 - x - 5$.
Express $f(x)$ as a product of factors.
$f(x) = x(x-1)(x^2 + 6x + 5)$

Now, factor $x^2 + 6x + 5$.
$x^2 + 6x + 5 = (x+5)(x+1)$

Write $f(x)$ as a product of factors.
$f(x) = x(x-1)\,(x+5)(x+1)$

Therefore, the zeros of $f$ are $-1$, 0, 1, and $-5$.

## Answer 49e.

***STEP 1*** List the possible rational zeros. The leading coefficient is 1 and the constant term is $-8$. So, the possible rational zeros are

$$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1} \text{ and } \pm\frac{8}{1}.$$

***STEP 2*** Test these zeros using synthetic division.

Test $x = 1$

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 2 & 4 & -8 \\ & & 1 & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 8 & 0 \end{array}$$

Test $x = -1$

$$\begin{array}{r|rrrrr} -1 & 1 & 1 & 2 & 4 & -8 \\ & & -1 & 0 & -2 & -2 \\ \hline & 1 & 0 & 2 & 2 & -6 \end{array}$$

Test $x = 2$

$$\begin{array}{r|rrrrr} 2 & 1 & 1 & 2 & 4 & -8 \\ & & 2 & 6 & 16 & 40 \\ \hline & 1 & 3 & 8 & 20 & 32 \end{array}$$

Test $x = -2$

$$\begin{array}{r|rrrrr} -2 & 1 & 1 & 2 & 4 & -8 \\ & & -2 & 2 & -8 & 8 \\ \hline & 1 & -1 & 4 & -4 & 0 \end{array}$$

We can see that 1 and $-2$ are zeros of $f$. It means that $x - 1$ and $x + 2$ are factors of $f$. The product of $x - 1$ and $x + 2$ is $x^2 + x - 2$.

Divide $x^4 + x^3 + 2x^2 + 4x - 8$ by $x^2 + x - 2$.
The quotient is $x^2 + 4$. Thus, the third factor is $x^2 + 4$.

**STEP 3** Factor $x^2 + 4 = 0$.

It is not possible to factor this binomial. Solve and find the value of $x$.
Subtract 4 from both the sides of the equation and simplify.

$$x^2 + 4 - 4 = 0 - 4$$
$$x^2 = -4$$

Take the square roots of both sides.

$$\sqrt{x^2} = \pm\sqrt{-4}$$

Write in terms of the imaginary unit $i$.

$$x = \pm\sqrt{-1 \cdot 4}$$
$$x = \pm 2i$$

Thus, the zeros of $f$ are 1, −2, $2i$, and $-2i$.

## Answer 51e.

Let $h$ be a new function defined by $h(x) = \frac{f(x)}{g(x)}$.

Substitute $5x^{2/3}$ for $f(x)$, and $3x^{1/2}$ for $g(x)$ in $h(x) = \frac{f(x)}{g(x)}$.

$$h(x) = \frac{5x^{2/3}}{3x^{1/2}}$$

Use the quotient of powers property and simplify.

$$\frac{5x^{2/3}}{3x^{1/2}} = \frac{5}{3}x^{2/3 - 1/2}$$
$$= \frac{5}{3}x^{1/6}$$

Therefore, $\frac{f(x)}{g(x)}$ evaluates to $\frac{5}{3}x^{1/6}$.

## Answer 52e.

Given functions are $f(x) = 5x^{2/3}$ and $g(x) = 3x^{1/2}$

Required function $f(x)\cdot f(x)$ is obtained as follows

$$f(x)\cdot f(x)=5x^{2/3}\times 5x^{2/3}$$

Using the property of radicals $a^m a^n = a^{m+n}$

$$\begin{aligned}5x^{2/3}\times 5x^{2/3} &= 25x^{2/3+2/3}\\ &= 25x^{4/3}\\ &= 25\sqrt[3]{(x\times x\times x)\times x}\\ &= 25\sqrt[3]{(x)^3\times x}\\ &= 25x\sqrt[3]{x}\end{aligned}$$

The multiplication of $f(x)=5x^{2/3}$ with itself gives

$$\boxed{f(x)f(x)=25x\sqrt[3]{x}}$$

## Answer 53e.

Substitute $3x^{1/2}$ for $g(x)$.

$$g(x) \cdot g(x) = 3x^{1/2} \cdot 3x^{1/2}$$

Rewrite such that the exponential expression are groped together.

$$3x^{1/2} \cdot 3x^{1/2} = (3 \cdot 3)(x^{1/2} \cdot x^{1/2})$$
$$= 9(x^{1/2} \cdot x^{1/2})$$

Use the product of powers property and simplify.

$$9(x^{1/2} \cdot x^{1/2}) = 9 \cdot x^{1/2+1/2}$$
$$= 9x$$

Thus, the product is $9x$.

## Answer 54e.

Given functions are $f(x)=5x^{2/3}$ and $g(x)=3x^{1/2}$

Required function $f(x)\cdot g(x)$ is obtained as follows

$$f(x)\cdot g(x)=5x^{2/3}\times 3x^{1/2}$$

Using the property of radicals $a^m a^n = a^{m+n}$

$$5x^{2/3}\times 3x^{1/2} = 15x^{2/3+1/2}$$
$$=15x^{4+3/6}$$
$$=15x^{7/6}$$
$$=15x^{6/6}x^{1/6}$$
$$=15x\sqrt[6]{x}$$

The multiplication of the functions $f(x)=5x^{2/3}$ and $g(x)=3x^{1/2}$ gives

$$\boxed{f(x)g(x)=15x\sqrt[6]{x}}$$

## Answer 55e.

Substitute the expression of $g(x)$ for $x$ in the expression of $f(x)$.

$$f(g(x)) = f(3x^{1/2})$$

Now, replace $x$ with $3x^{1/2}$ in $5x^{2/3}$.

$$f\left(3x^{1/2}\right) = 5\left(3x^{1/2}\right)^{2/3}$$

Use the power of a product property and rewrite the expression.

$$5\left(3x^{1/2}\right)^{2/3} = 5\left(3^{2/3}\right)\left(x^{1/2}\right)^{2/3}$$

Apply the power of a power property and simplify.

$$5\left(3^{2/3}\right)\left(x^{1/2}\right)^{2/3} = 5\left(3^{2/3}\right)x^{(1/2)(2/3)}$$

$$= 5\left(3^{2/3}\right)x^{1/3}$$

Now, convert the rational exponents to radical form.

$$5\left(3^{2/3}\right)x^{1/3} = 5\left(\sqrt[3]{3^2}\right)\left(\sqrt[3]{x}\right)$$

$$= 5\sqrt[3]{3^2 \cdot x}$$

$$= 5\sqrt[3]{9x}$$

Thus, $f\left(g(x)\right) = 5\sqrt[3]{9x}$.

**Answer 56e.**

Given functions are $f(x)=5x^{2/3}$ and $g(x)=3x^{1/2}$

Required function $g\left(f(x)\right)$ is obtained as follows

In the function $g(x)=3x^{1/2}$ substitute the function $f(x)=5x^{2/3}$ in the place of $x$

$$g\left(f(x)\right)=3\left(5x^{2/3}\right)^{1/2}$$

$$=3\times 5^{1/2}\times\left(x^{2/3}\right)^{1/2}$$

Using the property of radicals $\left(a^m\right)^n=a^{mn}$

$$g\left(f(x)\right)=3\left(5x^{2/3}\right)^{1/2}$$

$$=3\times\sqrt{5}\times\left(x^{2/3\times 1/2}\right)$$

$$=3\times\sqrt{5}\times\left(x^{1/3}\right)$$

$$=3\sqrt{5}\sqrt[3]{x}$$

The function $g\left(f(x)\right)$ where $f(x)=5x^{2/3}$ and $g(x)=3x^{1/2}$ gives

$$\boxed{g\left(f(x)\right)=3\sqrt{5}\sqrt[3]{x}}$$

## Answer 57e.

Substitute the expression of $f(x)$ for $x$ in the expression of $f(x)$.

$$f(f(x)) = f(5x^{2/3})$$

Now, replace $x$ with $5x^{2/3}$ in the expression $5x^{2/3}$.

$$f(5x^{2/3}) = 5(5x^{2/3})^{2/3}$$

Use the power of a product property and rewrite the expression.

$$5(5x^{2/3})^{2/3} = 5(5^{2/3})(x^{2/3})^{2/3}$$

Apply the power of a power property and simplify.

$$5(5^{2/3})(x^{2/3})^{2/3} = 5(5^{2/3})x^{2/3 \cdot 2/3}$$

$$= 5(5^{2/3})x^{4/9}$$

Use the product of powers property.

$$5(5^{2/3})x^{4/9} = (5^{1+2/3})x^{4/9}$$

$$= 5^{5/3}x^{4/9}$$

Thus, $f(f(x)) = 5^{5/3}x^{4/9}$.

## Answer 58e.

Given functions are $f(x)=5x^{2/3}$ and $g(x)=3x^{1/2}$

Required function $g(g(x))$ is obtained as follows

In the function $g(x)=3x^{1/2}$ substitute the function $g(x)=3x^{1/2}$ in the place of $x$

$$g(g(x))=3\left(3x^{1/2}\right)^{1/2}$$

$$=3\times 3^{1/2}\times\left(x^{1/2}\right)^{1/2}$$

Using the property of radicals $(a^m)^n = a^{mn}$

$$g(g(x))=3\times 3^{1/2}\times\left(x^{1/2}\right)^{1/2}$$

$$3\times\sqrt{3}\times\left(x^{1/2\times 1/2}\right)$$

$$=3\times\sqrt{3}\times\left(x^{1/4}\right)$$

$$=3\sqrt{3}\sqrt[4]{x}$$

The function $g(g(x))$ where $f(x)=5x^{2/3}$ and $g(x)=3x^{1/2}$ gives

$$\boxed{g(g(x))=3\sqrt{3}\sqrt[4]{x}}$$