Long Answer Type Questions-II

Q. 1. Calculate the mean deviation about median for the following data : [NCT,2010]

Class	**0 - 10**	**10 - 20**	**20 - 30**	**30 - 40**	**40 - 50**	**50 - 60**
Frequency	**6**	**7**	**15**	**16**	**4**	**2**

Sol.

Class	**Frequency** f_i	**Cumulative Frequency** $c.f.$	**Mid – point** x_i	$\|x_i - M\|$	$f_i\|x_i - M\|$
0-10	6	6	5	23	138
10-20	7	13	15	13	91
20-30	15	28	25	3	45
30-40	16	44	35	7	112
40-50	4	48	45	17	68
50-60	2	50	55	27	54
Total	$\sum f_i = 50$				$\sum f_i\|x_i - M\| = 505$

The class interval containing $\left(\frac{N}{2}\right)^{th}$ or 25th term is 20-30.

∴ 20-30 is the median class.

∴ Median $= l + \frac{\frac{N}{2} - C_f}{f} \times h,$ where l = 20, c = 13, f=15, h = 10 and N = 50

$= 20 + \frac{25 - 13}{15} \times 10$

= 20 + 8

= 28

And mean deviation about median,

M.D. (M) $= \frac{1}{N}\sum f_i|x_i - M|$

$= \frac{1}{50} \times 508$

= 10.16.

Q. 2. Calculate the mean deviation about mean : **[DDE]**

Marks	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**
No. of Students	**6**	**7**	**15**	**16**	**4**

Sol.

Marks	f_i	x_i	$d_i = \frac{x_i - 25}{10}$	$f_i d_i$	$\lvert x_i - 27 \rvert$	$f_i \lvert x_i - 27 \rvert$
$0 - 10$	5	5	-2	-10	22	110
$10 - 20$	8	15	-1	-8	12	96
$20 - 30$	15	25	0	0	2	30
$30 - 40$	16	35	$+1$	$+16$	8	128
$40 - 50$	6	45	$+2$	$+12$	18	108
		$\sum f_i = 50$		$\sum f_i d_i = 10$		$\sum f_i \lvert x_i - 27 \rvert = 472$

Mean, $\bar{x}$ = assumed mean + $\frac{\Sigma f_i d_i}{\Sigma f_i} \times h$

= 25 + $\frac{10}{50} \times h$

= 25 + 2

= 27

Mean deviation about mean, M.D ($\bar{x}$) = $\frac{\Sigma f_i |x_i - 27|}{\Sigma f_i}$

$= \frac{472}{50}$

= 9. 44

Q. 3. Calculate the mean deviation about the mean of the set of first n natural numbers when 'n' is an odd number.

Sol. Let n = 2k + 1 ⇒ k = $\frac{n-1}{2}$

∴ $\bar{x} = \frac{1+2+3+\ldots+n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$

$= \frac{2k+1+1}{2} = k + 1.$

x_i	$x_i - \bar{x}$	$\|x_i - \bar{x}\|$
1 2 3 4	$1 - (k-1) = -k$ $-(k-1)$ $-(k-2)$ $-(k-3)$	k $k-1$ $k-2$ $k-3$
' ' '	' ' '	' ' '
$k-2$ $k-1$ k $k+1$ $k+2$ $k+3$	-3 -2 -1 0 1 2	3 2 1 0 1 2
' ' '	' ' '	' ' '
$2k$ $2k + 1$	$k-1$ k	$k-1$ k

$\sum|x_i - \bar{x}|$ = 2 (1 + 2 + 3 +…+ k) = 2 $\{\frac{k(k+1)}{2}\}$

$= k\,(k+1)$

$= (\frac{n-1}{2})(\frac{n-1}{2} + 1)$

$= \frac{n-1}{2} . \frac{n+1}{2}$

$= \frac{n^2-1}{4}$

∴ Read mean deviation about mean ($\bar{x}$)

$i.e.,$ M.D. ($\bar{x}$) = $\frac{1}{n}\sum|x_1 - \bar{x}|$

$= \frac{1}{n} . \frac{n^2-1}{4}$

$= \frac{n^2-1}{4n}$

Q. 4. Calculate the mean deviation about the mean of the set first n natured numbers when n is an even number.

Sol. Let $n = 2k \Rightarrow k = \frac{n}{2}$

$\therefore \quad \bar{x} = \dfrac{1 + 2 + 3 + \cdots + n}{n}$

$= \dfrac{n(n+1)}{2n}$

$= \dfrac{n+1}{2}$

$= \dfrac{2k+1}{2}$

x_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$
1	$1 - \left(\frac{2k+1}{2}\right) = -(\frac{2k-1}{2})$	$\frac{2k-1}{2}$
2	$-(\frac{2k-3}{2})$	$\frac{2k-3}{2}$
3	$-(\frac{2k-5}{2})$	$\frac{2k-5}{2}$
‘ ‘ ‘	‘ ‘ ‘	‘ ‘ ‘
$k-2$	$-\frac{5}{2}$	$\frac{5}{2}$
$k-1$	$-\frac{3}{2}$	$\frac{3}{2}$
k	$-\frac{1}{2}$	$\frac{1}{2}$
$k + 1$	$\frac{1}{2}$	$\frac{1}{2}$
$k + 2$	$\frac{3}{2}$	$\frac{3}{2}$
‘ ‘ ‘	‘ ‘ ‘	‘ ‘ ‘
$2k-1$	$\frac{2k-3}{2}$	$\frac{2k-3}{2}$

$2k-1$	$\frac{2k-3}{2}$	$\frac{2k-3}{2}$

$\therefore \quad \sum |x-\bar{x}| =$

$\{\frac{2k-1}{2}+\frac{2k-3}{2}+\frac{2k-5}{2}+\cdots+\frac{5}{2}+\frac{3}{2}+1\}$

$= 1 + 3 + 5 + \cdots + (2k-1)$

$=\frac{k}{2}\{1+(2k-1)\} = k^2 = \left(\frac{n}{2}\right)^2 = \frac{n^2}{4}$

$\therefore$ Mean deviation about mean, M.D ($\bar{x}$)

$=\frac{1}{n}\sum|x-\bar{x}| = \frac{1}{n}.\frac{n^2}{4} = \frac{n}{4}$

Q. 5. Find the Mean deviation about the mean:

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	2	3	8	14	8	3	2

Sol. Solve as Q. no. 2.

Q. 6. Find the mean deviation about the median

Weight (in KG)	30-40	40-50	50-60	40-50	60-70	70-80	80-90
Frequency	2	3	8	14	8	3	2

Sol. Solve as Q. no. 1.

Q. 7. Calculate the Variance and Standard deviation for the following data :

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Sol.

Class	Freq. (f_1)	Mid-points (x_i)	$f_i x_i$	$\lvert x-x_1\rvert$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$

$30 - 40$	3	35	105	27	729	2187
$40 - 50$	7	45	315	17	289	2023
$50 - 60$	12	55	660	7	49	588
$60 - 70$	15	65	975	3	9	135
$70 - 80$	8	75	600	13	169	1352
$80 - 90$	3	85	255	23	529	1587
$90 - 100$	2	95	190	33	1089	2178
Total	**50**		**3100**			**10050**

Here, N = 50 and $\sum f_i x_i$ = 3100

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{7} f_i x_i = \frac{3100}{50} = 62$$

$$\sigma = \frac{1}{N}\sum_{i=1}^{7} f_i (x_i - \bar{x})^2 = \frac{1}{50} \times 10050$$

Standard deviation, σ = $\sqrt{201} = 14.18$

Q. 8. Calculate the mean and Standard deviation of the data given below:

Class Interval	**1-5**	**6-10**	**11-15**	**16-20**	**20-25**
Frequency	**3**	**8**	**13**	**18**	**23**

Sol.

Continuous Class Interval	**Mid-point x_i**	f_i	$f_i x_i$	$f_i x_i^2$
$0.5 - 5.5$	3	3	9	27
$5.5 - 10.5$	8	8	64	512
$10.5 - 15.5$	13	13	169	2197
$15.5 - 20.5$	18	18	324	5832
$20.5 - 25.5$	23	23	539	12167
		$\sum fi = 65$	$\sum fixi = 1095$	$\sum fixi^2 = 20735$

Here, Mean, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1095}{95} = 16.85$

and Standard deviation, $\sigma = \frac{1}{N}\sqrt{\sum f_i x_i^2 - (\sum f_i x_i)^2}$

$$= \frac{1}{65} = \sqrt{65 \times 20{,}735 - (1095)^2}$$

$$\sigma = \frac{1}{65}\sqrt{1347775 - 1199025}$$

$$= \frac{1}{65}\sqrt{1{,}48{,}750}$$

$$= \frac{1}{65} \times 385.7$$

$$= 5.93$$

∴ Mean = 16.85 and Standard deviation = 5.93

Q. 9. Calculate the mean and Standard deviation of the following cumulative data.

Wages upto (in Rs.)	15	30	45	60	75	90	105	120
Number of workers	12	30	65	107	157	202	222	230

Sol. Let us prepare the frequency distribution table.

Class Interval	cf	Mid-value (x_i)	f_i	$u_i = \frac{x_i - 67.5}{15}$	$f_i u_i$	$f_i u_i^2$
$0-15$	12	7.5	12	-4	-48	192
$15-30$	30	22.5	18	-3	-54	162
$30-45$	65	37.5	35	-2	-70	140
$45-60$	107	52.5	42	-1	-42	42
$60-75$	157	67.5	50	0	0	0
$75-90$	202	82.5	45	1	45	45
$90-105$	222	97.5	20	2	40	80
$105-120$	230	112.5	8	3	24	72
Total			**230**		**-105**	**733**

Here, a = 67.5, h = 15, N = $\sum f_i$ = 230

$$\sum f_i u_i = -105 \text{ and } \sum f_i u_i^2 = 733$$

∴ Mean, $\bar{x} = a + h\left(\frac{1}{N}\sum f_i u_i\right) = 67.5 + 15\,(-\frac{105}{230})$

= 67.5 – 6.85

= 60.65

and Standard deviation,

$$\sigma = \sqrt{h^2\left[\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2\right]}$$

$$= \sqrt{255\left[\frac{733}{230} - \left(-\frac{105}{230}\right)^2\right]} = \sqrt{225[3.187 - (0.46)^2]}$$

$$= \sqrt{669.465}$$

= 25.87

∴ Mean = 60.65, Standard deviation = 25.87

Q. 10. Following are the marks obtained out of 100, by two students Reeta and Seeta in 10 tests.

Reeta	**25**	**50**	**45**	**30**	**70**	**42**	**36**	**48**	**35**	**60**
Seeta	**10**	**70**	**50**	**20**	**95**	**55**	**42**	**60**	**48**	**80**

Who is more intelligent and who is more consistent?

Sol. For Reeta:

x_i	x_i^2
25	625
50	2500
45	2025
30	900
70	4900
42	1764
36	1296
48	2304
35	1225
60	3600
$\sum_{i=1}^{10} x_i = 441$	$\sum_{i=1}^{10} x_i^2 = 21139$

∴ Mean, $\bar{x} = \frac{\sum x_i}{n} = \frac{441}{10} = 44.1$

And, $\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$

$$= \sqrt{\frac{21{,}139}{10} - \left(\frac{441}{10}\right)^2}$$

$$= \sqrt{\frac{2,11,930 - 1,94,481}{100}} = \sqrt{169.09} = 13(\text{approx.})$$

For Seeta:

x_i	x_i^2
10	100
70	4900
50	2500
20	400
95	9025
55	3025
42	1764
60	3600
48	2304
80	6400
$\sum_{i=1}^{10} x_i = 530$	$\sum_{i=1}^{10} x_i^2 = 34018$

∴ Mean, $\bar{x} = \frac{\sum x_i}{n} = \frac{530}{10} = 53$

and $\sigma = \sqrt{\frac{\sum xi^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$

∴ $\sigma = \sqrt{\frac{34{,}081}{10} - (53)^2} = \sqrt{5{,}928} = 24.35$

For Reeta, C.V. $= \frac{6}{\bar{x}} \times 100 = \frac{13}{44.1} \times 100 = 29.48$

and For Geeta, C.V. $= \frac{6}{\bar{x}} \times 100 = \frac{24.35}{53} \times 100 = 45.94$

Since co-effecient for Seeta is more than that of Reeta, so Seeta is more variable and hence Reeta is more consistent. And Seeta is more intelligent.

Q. 11. In a series of $2n$ observation, half of them equal 'a' and remaining half equal $' - a'$. If the standard deviation of the observations is 2, then find the value of $|a|$. [DDE]

Sol. Given, N = $2a$,

$$\text{Mean, } \bar{x} = \frac{a + a + \ldots + a + (-a) + (-a) + \ldots (-a)}{2n}$$

$$= \frac{0}{2n}$$

$$= 0$$

Also, Standard deviation, $\sigma = 2$

$\Rightarrow \quad \sigma^2 = 2^2$

$\Rightarrow \quad \frac{\sum x_i^2}{\text{N}} - \left(\frac{\sum x_i}{\text{N}}\right)^2 = 4$

$\Rightarrow \quad \frac{\sum x_i^2}{2n} - x^{-2} = 4$

$\Rightarrow \quad \frac{\sum x_i^2}{2n} - 0 = 4$

$\Rightarrow \quad \sum x_i^2 = 8n$

$\Rightarrow \quad a^2 + a^2 + \cdots + a^2 (\text{to 2n terms}) 2a^2 = 8n = 8n$

$\Rightarrow \quad a^2 = 4$

$\Rightarrow \quad a = \pm 2$

$\therefore \quad |a| = |\pm 2|$

$= 2$

Q. 12. If each of the observation $x_1, x_2, \ldots, x_n$ is increased by 'a', where 'a' is a negative or position number, then show that the variance remains unchanged.

Sol. Let $\bar{x}$ be the mean of $x_1, x_2, \ldots, x_n$.

Then, the variance is given by

$$\sigma_1^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

If $'a'$ is added to each observation, then the new observation will be $y_i = x_i + a$

let the mean of the new observation be $\bar{y}$. Then :

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i + a)$$

$$= \frac{1}{n}[\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} a]$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i + na\right] = \bar{x} + a$$

$\Rightarrow \quad \bar{y} = \bar{x} + a$

New variance, $\sigma_1^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i + a - \bar{x} - a)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$= \sigma_1^2$

∴ Variance remains unchanged.