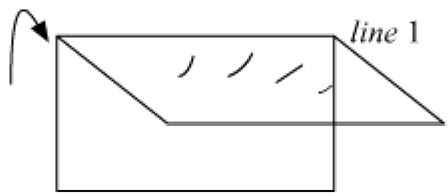
Practical Geometry

Construction of Parallel Lines Using Paper Folding Method

There are many methods to construct parallel lines on a paper. Let us start with the most basic method to draw parallel lines.

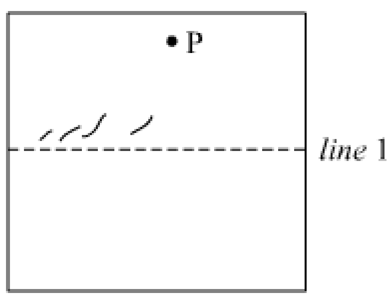
The steps we use to draw parallel lines are as follows.

First of all, we take a paper and fold it to make a straight line. Let this line be *line*1.



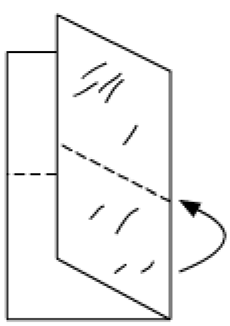
(i)

Now let us take any point outside this line and mark it as P. This is the point from which we will draw the line parallel to *line*1.

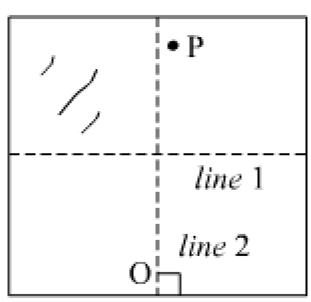


(ii)

From this point, we draw a line which is perpendicular to *line*1. Let this line be *line*2.

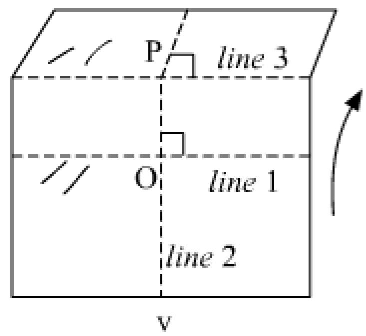


(iii)



(iv)

Now we draw a line that is perpendicular to *line*2 and that passes through point P. Let this line be *line*3.



What do you see?

You can easily figure out that *line*1 and *line*3 are parallel to each other.

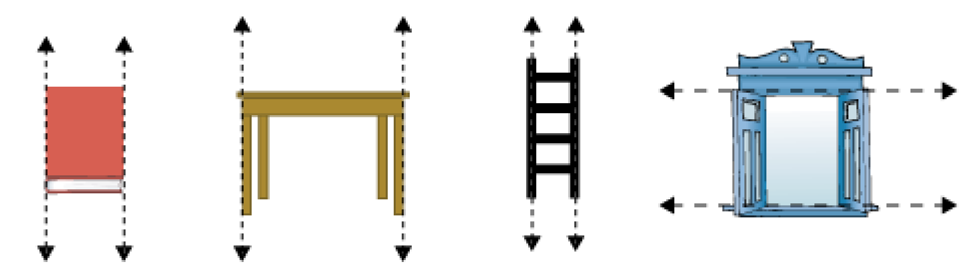
Now which property of parallel lines can we extract from this method of drawing parallel lines?

We can easily say, "If we draw two lines which are perpendicular to the same line, then the two lines will be parallel to each other."

We can also say, "A line which is perpendicular to one of the two parallel lines will also be perpendicular to the second one".

Construction of Parallel Lines

Look at the following figures.



Opposite edges of a book

Opposite edges of a table

Rungs on a ladder

Crossbars of a window

What do you notice? Is there something common in the given figures?

We can see that some lines have been shown in these figures such as the opposite edges of a book, the opposite edges of a table, the rungs on a ladder, the crossbars of a window, etc.

What is special about these lines?

These lines are parallel to each other and are called parallel lines.

When are two lines called parallel lines?

Parallel lines can be defined as follows:

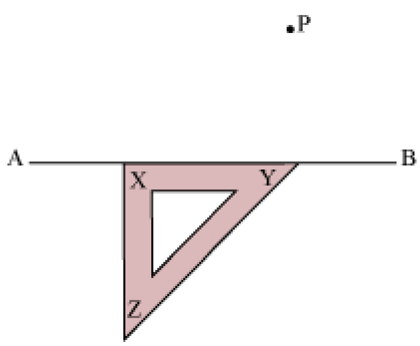
"Two lines are called parallel lines if they do not intersect anywhere and they are at the same distance from each other along their entire length".

In the given figure, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel lines.

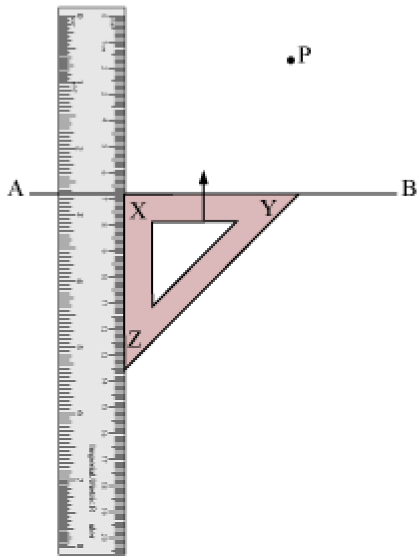
Now, we will learn how to construct a line parallel to a given line through a point which does not lie on the line, with the help of a ruler and compass.

The steps involved in the construction of a line parallel to $\overleftrightarrow{AB}$ through point P (using a ruler and a set square) are as follows:

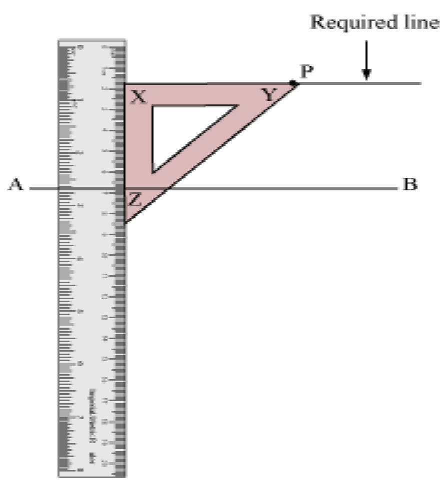
1. Place your set square such that one of its shorter edges i.e., XY lies just along line AB.



2. Place your ruler such that one of its edges lies just along the shorter edge i.e., XZ of the set square. Hold the ruler firmly and slide the set square along the ruler until the edge XY of the set square passes through P.



3. Draw a line along the edge XY of the set square. This is the required line through point P. Note that it is parallel to line AB.



Let us now see another example to understand the concept of construction of parallel lines better.

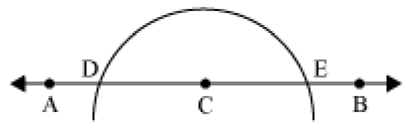
Example 1:

Draw a line perpendicular to a given line $\overleftrightarrow{AB}$. Then draw a line parallel to $\overleftrightarrow{AB}$ through any point on that perpendicular line using only ruler and compass.

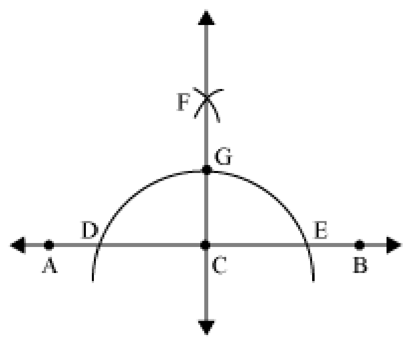
Solution:

First of all, draw a line $\overleftrightarrow{AB}$. Now, we are required to draw a perpendicular to $\overleftrightarrow{AB}$.

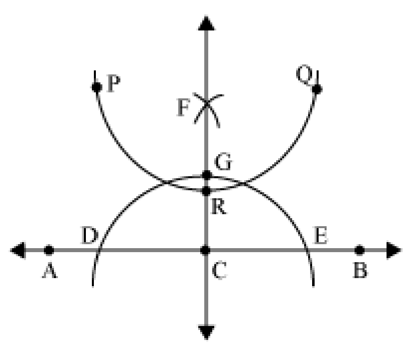
Let us take a point C on $\overleftrightarrow{AB}$. Then, taking C as the centre, we draw an arc using the compass that cuts $\overleftrightarrow{AB}$ at the points D and E respectively.



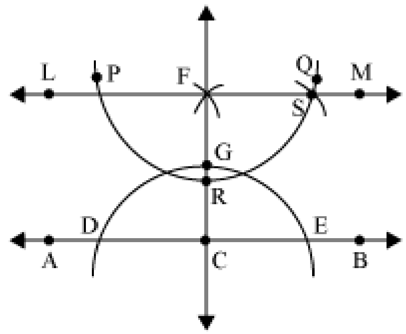
Now, with a radius greater than $\overline{CD}$, we draw two arcs with D and E as centres that intersect each other at point F. Then, we draw a line through the points C and F, which cuts the arc DE at point G. $\overleftrightarrow{CF}$ is a line perpendicular to $\overleftrightarrow{AB}$.



Taking the same radius and with F as centre, we draw an arc $\overset{\frown}{PQ}$, which cuts $\overleftrightarrow{CF}$ at a point R.



Now, let us measure the distance DG or EG with the help of compass. Then, taking that distance as the radius with R as the centre, we draw an arc that cuts the arc $\overset{\frown}{PQ}$ at a point S.



Now, we draw a line $\overleftrightarrow{LM}$ through the points F and S using a ruler. The line $\overleftrightarrow{LM}$ is perpendicular to $\overleftrightarrow{FC}$ through a point F.

Thus, $\overleftrightarrow{LM}$ is the required line, which is parallel to $\overleftrightarrow{AB}$ and passing through a point on a line, which is perpendicular to $\overleftrightarrow{AB}$.

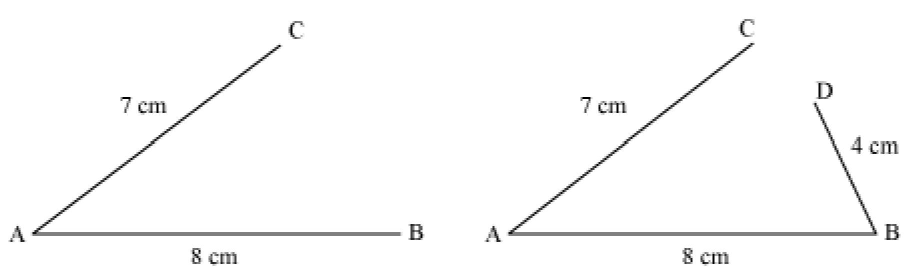
Construction of a Triangle when the Lengths of its Sides Are Given

Suppose if someone asks us to draw a triangle. The first question that strikes us is that what are the lengths of the sides of the triangle which is to be drawn?

Therefore, if the three sides of a triangle are given to us, then can we draw the triangle?

If we try to draw the triangle only with the help of a ruler, then it is not possible to draw it. With the help of a ruler, we can draw two sides of the triangle very easily. However, when we try to draw the third side, it may or may not intersect the third side.

Let us assume that we are asked to draw a triangle and the sides of the triangle are 8 cm, 7 cm, and 4 cm. Firstly, we draw the two sides of the triangle, which are 8 cm and 7 cm and then the third side of length 4 cm as shown in the following figure.



In these figures, we can see that a triangle is not formed.

Thus, we cannot draw a triangle only with the help of a ruler, but by using the ruler and compass.

Now, let us see the construction of a triangle using ruler and compass.

Before constructing a triangle, we should check whether the triangle is possible with the given sides or not.

In a triangle, the sum of any two sides must be greater than the third side.

For example: Can we draw a triangle with sides of length 6 cm, 9 cm, and 2 cm?

Here, 6 cm + 2 cm = 8 cm < 9 cm

i.e., the sum of the lengths of two sides is less than the length of the third side.

Therefore, we cannot draw a triangle with sides of given lengths.

Let us solve some examples based on the construction of triangles.

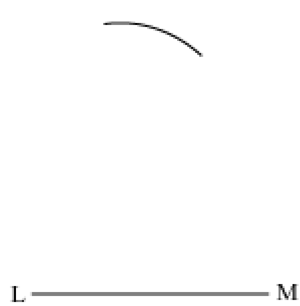
Example 1:

Construct an isosceles triangle such that the two equal sides are of lengths 9 cm each and the unequal side is of length 4 cm.

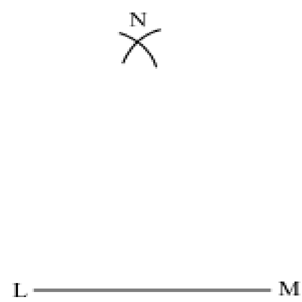
Solution:

Firstly, we draw a line-segment $\overline{LM}$ of length 4 cm using a ruler.

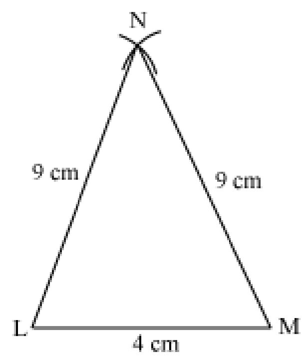
Then using compass, we draw an arc of radius 9 cm taking L as the centre.



Again, taking M as the centre, we draw another arc of radius 9 cm. Now, both the arcs intersect each other at a point N.



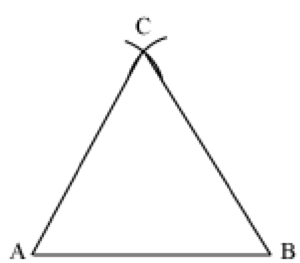
Now, we join the line segments $\overline{LN}$ and $\overline{MN}$.



Hence, ΔLMN is the required isosceles triangle with sides of given lengths.

Example 2:

A line segment $\overline{AB}$ is drawn. Then, two arcs with radius equal to the length of $\overline{AB}$ are drawn taking A and B as centres. The arcs intersect at C as shown in the given figure.



What type of a triangle is ΔABC?

Solution:

ΔABC is an equilateral triangle. As the two arcs have been drawn with radius equal to the length of $\overline{AB}$, therefore, $\overline{AC}$ and $\overline{BC}$ both are equal to $\overline{AB}$ i.e., all the three sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ of ΔABC are equal.

Example 3:

Construct an equilateral triangle of side 4.9 cm.

Solution:

We know that to construct a triangle, we require the measure of the lengths of all its three sides.

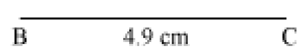
Now, here, we are required to construct an equilateral triangle of side 4.9 cm.

To construct the required triangle, we will use a property of an equilateral triangle.

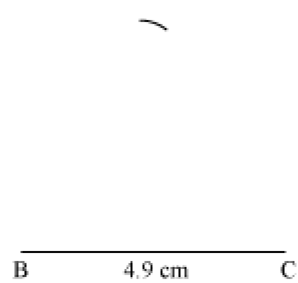
We know that all sides of an equilateral triangle are of equal length. So, we have to construct a triangle ABC with AB = BC = CA = 4.9 cm.

The steps of construction are as follows:

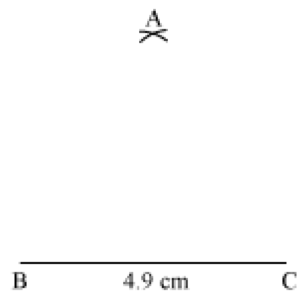
1. Draw a line segment BC of length 4.9 cm.



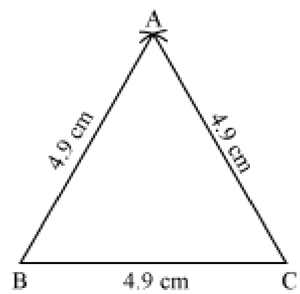
2. Taking point B as centre draw an arc of 4.9 cm radius.



2. Taking point C as centre draw an arc of 4.9 cm radius to meet the previous arc at the point A.



2. Join A to B and A to C.

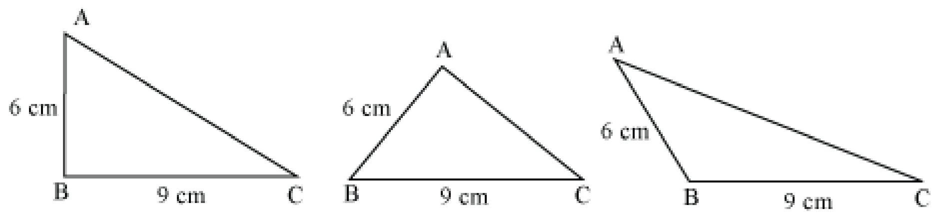


ABC is the required equilateral triangle.

Construction of a Triangle when the Lengths of Two Sides and Angle Between Them Are Given

Abhijit constructed ΔABC. He told Ravi that the lengths of two sides of ΔABC are $\overline{AB}$ = 6 cm and $\overline{BC}$ = 9 cm. Then, he asked Ravi to construct a triangle with the same dimensions.

Ravi started constructing the triangle and observed that more than one triangle could be constructed using the same dimensions as shown in the following figures.



He asked Abhijit to tell something else about the triangle also. Therefore, Abhijit told him the measure of the angle between the two known sides.

Now, can Ravi construct a unique triangle based on the information given by Abhijit?

Yes, Ravi can construct a unique triangle because with the given information, one and only one triangle can be constructed.

The point to remember here is that

'If the lengths of any two sides and the measure of the angle between them are given, then a unique triangle can be constructed".

Let us take an example. Assume that we have to construct a triangle whose lengths of any two sides are 7 cm and 5 cm and the measure of the angle between them is 45°.

Let us look at some more examples.

Example 1:

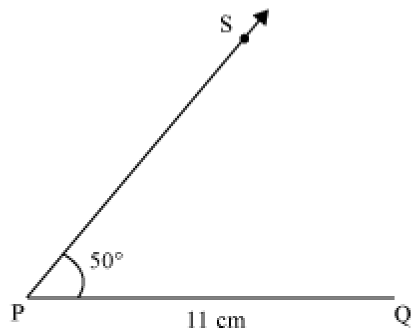
Construct a triangle PQR in which $\overline{PQ}$ = 11 cm, $\overline{PR}$ = 9 cm, and ∠QPR = 50°.

Solution:

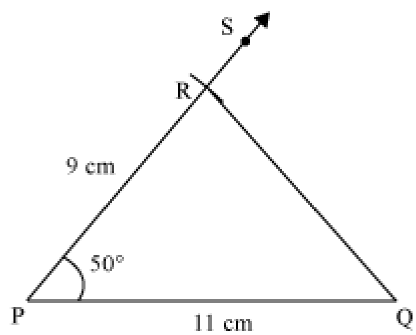
First, we draw a line segment $\overline{PQ}$ of length 11 cm using a ruler.

This is one of the sides of the triangle.

Now, we draw a ray $\overrightarrow{PS}$ from point P making an angle of measure 50° with the line segment $\overline{PQ}$.



Next, we draw an arc of radius 9 cm taking point P as the centre, which cuts $\overrightarrow{PS}$ at a point R. Then, we join the points Q and R to obtain the line segment $\overline{QR}$.



Thus, ΔPQR is the required triangle.

Example 2:

Construct a triangle ABC, where $\overline{AB}$ = 6 cm, $\overline{AC}$ = 2 $\overline{AB}$, and ∠BAC = 110°.

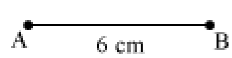
Solution:

Given, ∠BAC = 110°

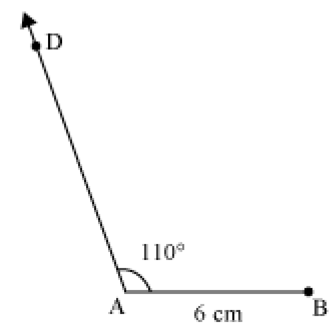
$\overline{AB}$ = 6 cm

$\overline{AC}$ = 2 $\overline{AB}$ = 2 × 6 cm = 12 cm

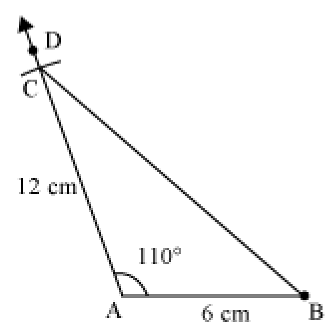
First, we draw a line segment $\overline{AB}$ of length 6 cm.



Then, we draw a line segment $\overrightarrow{AD}$ making an angle of measure 110° with $\overline{AB}$ such that ∠BAD = 110°.



Now, using compass, we draw an arc of radius 12 cm taking A as the centre, which cuts $\overrightarrow{AD}$ at a point C. Then, we join points B and C.



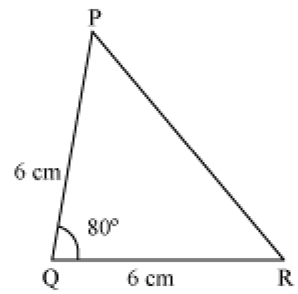
Thus, ΔABC so obtained is the required triangle.

Example 3:

Construct an isosceles triangle in which the length of each of its equal sides is 6 cm and the angle between them is 80°.

Solution:

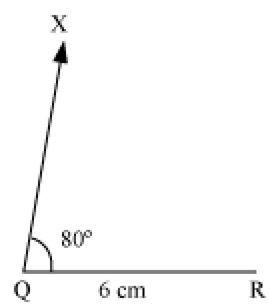
We have to construct an isosceles triangle PQR with PQ = QR = 6 cm. A rough sketch of the required triangle may be drawn as follows:



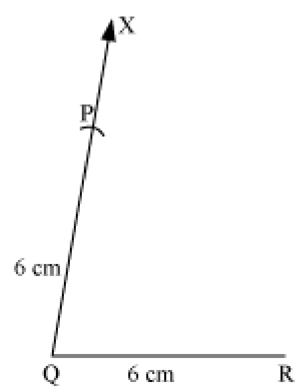
The steps of construction are as follows:

1. Draw the line segment QR of length 6 cm.

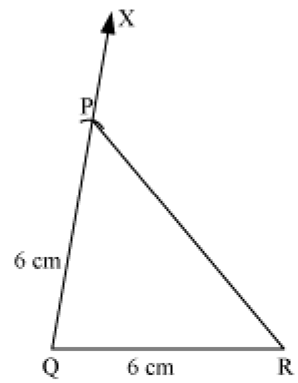
2. At point Q, draw a ray QX making an angle 80° with QR.



3. Taking Q as centre, draw an arc of 6 cm radius. It intersects QX at the point P.



iv. Join P to R to obtain the required triangle PQR.



Example 4:

Construct a triangle ABC, given that AB = 5 cm, BC = 6.5 cm and $\angle B = 60^\circ$.

Solution:

The steps of construction are as follows:

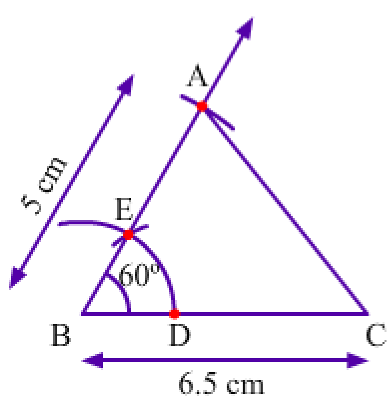
1. Draw a line segment BC of length 6.5 cm.

2. At B, using a compass draw an angle ∠EBC = 60 º.

3. With B as centre and radius 5 cm, draw an arc intersecting BE at A.

4. Join AC.

Thus, we get the required triangle ABC as shown below.



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Construction of a Triangle when Two Angles and the Length of Side Between Them Are Given

Riya had studied in a book that if the measure of two angles of a triangle and the length of included side are given, then a unique triangle can be constructed based on this information.

She wants to try and see if this is really true or not? So, she tries to construct a triangle ABC such that two angles ∠C = 80°, ∠B = 40° and one side $\overline{BC} = 6$ cm are given.

Now, let us see how she draws the triangle.

Thus, to construct a triangle when the measure of two angles and the length of the included side are given, first we draw the side whose length is given and then we draw two rays making given angles with this side.

Let us solve some examples using this method.

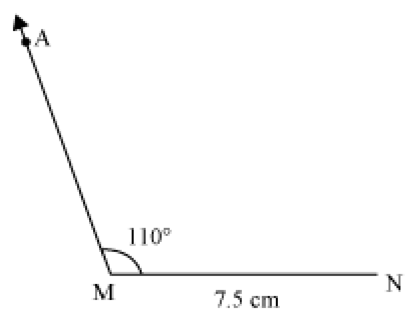
Example 1:

Construct a triangle such that the measures of two of its angles are 30° and 110° and the length of the side included between these two angles is 7.5 cm.

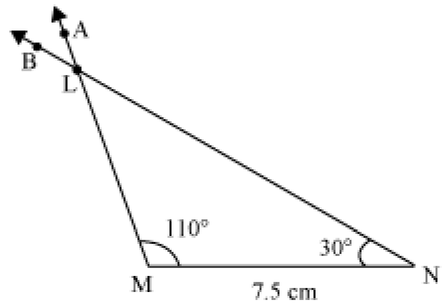
Solution:

First we draw a line segment $\overline{MN}$ of length 7.5 cm using a ruler.

Next we draw a ray $\overrightarrow{MA}$ from point M making an angle of measure 110° with the line segment $\overline{MN}$.



Now, we draw another ray $\overrightarrow{NB}$ from the point N, making an angle of 30° with $\overline{MN}$. Let it intersect the previously drawn ray $\overrightarrow{MA}$ at point L.



Thus, ΔLMN is the required triangle with the given measures.

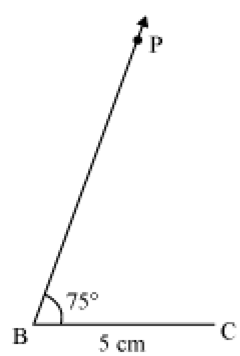
Example 2:

Construct an isosceles triangle such that the length of its unequal side is 5 cm and each of the two angles opposite to the equal sides is of measure 75°.

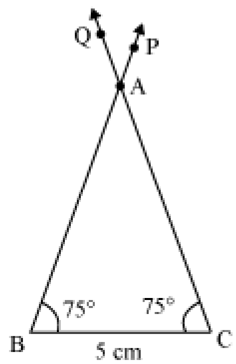
Solution:

First we draw a line segment $\overline{BC}$ of length 5 cm using a ruler.

Now, we draw a ray $\overrightarrow{BP}$ from point B making an angle of measure 75° with $\overline{BC}$.



Again, we draw another ray $\overrightarrow{CQ}$ from point C making an angle of measure 75° with $\overline{BC}$. Let it intersect the previously drawn ray $\overrightarrow{BP}$ at point A.



Thus, ΔABC is the required isosceles triangle with the given measures.

Construction of a Right-angled Triangle when the Length of One Leg and Hypotenuse Are Given

We know what a right-angled triangles is. Also, we know that it has two perpendicular sides and a longest side which is known as the hypotenuse.

Let us try to construct a right-angled triangle using the least information.

Now, we can say that:

"A right-angled triangle can be constructed if the length of one of its sides or arms and the length of its hypotenuse are known".

Note: We can also construct a triangle if the lengths of its two arms are given.

The stepwise method to construct a right-angled triangle, when the length of one of the perpendicular sides and the length of hypotenuse is given, is as follows.

1. Firstly we draw one of the perpendicular sides of the triangle.
2. Then we draw the perpendicular on one of its end points.
3. Then we draw an arc from its other end point taking radius as the length of the hypotenuse to intersect the perpendicular. This point of intersection gives the third vertex of the right triangle.

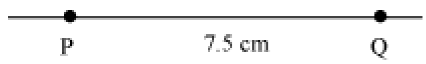
Now let us see one more example to understand the method of construction better.

Example1:

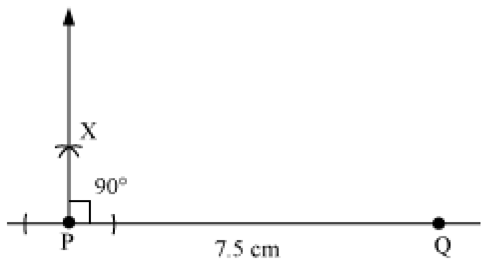
Construct a right-angled triangle such that the lengths of its hypotenuse and one of its sides are 10 cm and 7.5 cm respectively.

Solution:

Firstly, we draw a line segment PQ of length 7.5 cm using a ruler.

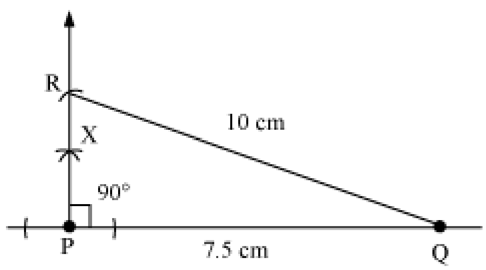


Now, we draw the perpendicular PX to the line segment $\overline{PQ}$ at point P using compasses. Therefore, ∠QPX so formed is a right angle.



Now, we draw an arc of radius 10cm taking Q as the centre, which cuts $\overline{PX}$ at a point R.

Q and R are joined to make the hypotenuse.



Now, ΔPQR so obtained is the required right-angled triangle.