CBSE Sample Paper -03 (solved)
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

SECTION – A

1. Explain why 7 × 11 × 13 + 13 is a composite number.

2. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ΔABC is a right triangle.

3. If tanA = cotB, prove that A + B = 90°.

4. Evaluate cos60°cos30° + sin60°sin30°.

SECTION – B

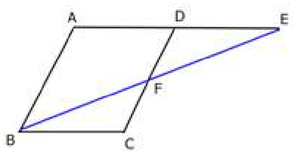
5. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$, then find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$.

6. Following table shows the weight of the bags of 12 students:

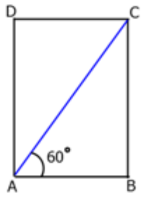
Weight (Kg)	67	70	72	73	75
Number of students	4	3	2	2	1

Find the mean weight.

7. E is a point on side AD produced of a parallelogram ABCD and BE intersects CD at F. Prove that ΔABE~ΔCFB.



8. In a rectangle ABCD, AB = 20 cm, ∠BAC = 60°. Calculate side BC.



9. There is a circular path around a sports field. Prenu takes 18 minutes to drive 1 round of the field, while Raj takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

10. Find a cubic polynomial with the sum, sum of the products of its zeros taken two at a time, and product of its zeros as 2, –7 and –14, respectively.

SECTION – C

11. Prove that $\sqrt{2}+\sqrt{5}$ is irrational.

12. Prove that $\frac{\sec\theta+\tan\theta-1}{\tan\theta-\sec\theta+1}=\frac{\cos\theta}{1-\sin\theta}$

13. If $a\cos\theta - b\sin\theta = c$, prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$.

14. Points A and B are 70 km apart on a highway. A car starts from a and another car starts from B at the same time. If they travel in same direction, they meet in 7 hours but if they travel in opposite direction, they meet in one hour. What are their speeds?

15. If the zeros of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a and$a + b$. Find a and b.

16. If the diagonals of a quadrilateral divide each other proportionally, the quadrilateral is a trapezium.

17. If the areas of two similar triangles are equal, prove that they are congruent.

18. Evaluate $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$.

19. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (kg/ha)	50-55	55-60	60-65	65-70	70-75	75-80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution and draw its ogive.

20. A 2-digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number.

SECTION – D

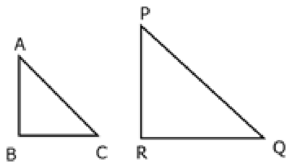
21. Prove that every positive integer different from 1 can be expressed as a product of non-negative power of 2 and an odd number.

22. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

23. State and prove Pythagoras theorem.

24. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row, there would be 3 rows more. Find the number of students in the class.

25. If $\angle$B and $\angle$Q are acute angles such that sinB = sinQ, then prove that $\angle$B = $\angle$Q.



26. Solve the following system of equations in x and y

$$(a-b)x+(a+b)y=a^2-2ab-b^2$$

$$(a+b)(x+y)=a^2+b^2$$

27. During the medical checkup of 35 studetns of a class, their weights were recorded as follows:

Weight (kg)	Less than 38	Less than 40	Less than 42	Less than 44	Less than 46	Less than 48	Less than 50	Less than 52
Number of students	0	3	5	9	14	28	32	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.

28. If cotθ= $\frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

29. Prove that the line segments joining the mid-points of the sides of a triangle form four triangles each of which is similar to the original triangle.

30. Let a, b, c and d be positive rationals such that $a+\sqrt{b}=c+\sqrt{d}$, then either $a = c$ and $b = d$ or b and d are squares of rationals.

31. A man hires a taxi to cover a certain distance. The fare is Rs 50 for first kilometre and Rs 25 for subsequent kilometers. Taking total distance covered as x km and total fare as y:

a. Write a linear equation for this.

b. The man covers a distance of 10 km and gave Rs 300 to the driver. Driver said “It is not the correct amount” and returned him the balance. Find the correct fare and the amount paid back by the driver.

c. Which values are depicted by the driver in the question?

CBSE Sample Paper -03 (solved)
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours **ANSWERS** Maximum Marks: 90

SECTION – A

1. **Solution:**

$7 \times 11 \times 13 + 13 = 1001 + 13 = 1014$

$1014 = 2 \times 3 \times 13 \times 13$

i.e., it is the product of prime factors.

$\therefore$ $7 \times 11 \times 13 + 13$ is a composite number.

2. **Solution:**

We have AC = BC and $AB^2 = 2AC^2$

Now, $AB^2 = 2AC^2$

$\Rightarrow$ $AB^2 = AC^2 + AC^2$

$\Rightarrow$ $AB^2 = AC^2 + BC^2$ [$\because$ AC = BC (Given)]

$\Rightarrow$ ΔABC is a right triangle right angled at C.

3. **Solution:**

We have

$\tan A = \cot B$

$\Rightarrow$ $\tan A = \tan(90° - B)$ $\Rightarrow$ $A = 90° - B$ $\Rightarrow$ $A + B = 90°$

4. **Solution:**

$\sec 67° + \text{cosec} 58° = \sec(90° - 23°) + \text{cosec}(90° - 32°)$

$= \text{cosec} 23° + \sec 32°$

SECTION – B

5. **Solution:**

Since α and β are the zeros of the polynomial $f(x) = x^2 - px + q$,

$\therefore$ $\alpha + \beta = -\left(\frac{-p}{1}\right) = p$ and $\alpha\beta = \frac{q}{1} = q$

Thus, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{p}{q}$

6. **Solution:**

Calculation of arithmetic mean

Weight (Kg) x_i	Frequency f_i	$f_i x_i$
67	4	268
70	3	210
72	2	144
73	2	146
75	1	75
	$N = \sum f_i = 12$	$\sum f_i x_i = 843$

∴ Mean = $\overline{X} = \frac{\sum f_i x_i}{N} = \frac{843}{12} = 70.25$

7. **Solution:**

In triangles ABE and CFB, we have

$\angle AEB = \angle CBF$ [Alternate angles]

$\angle A = \angle C$ [Opposite angles of a parallelogram]

Thus, by AA-criterion of similarity, we have

$\Delta ABE \sim \Delta CFB$

8. **Solution:**

In ΔABC, we have

$AB = 20, \angle BAC = 60°$

∴ $\tan\angle BAC = \frac{BC}{AB}$

⇒ $\tan 60° = \frac{BC}{20}$

⇒ $\sqrt{3} = \frac{BC}{20}$ ⇒ $BC = 20\sqrt{3}$ cm

9. **Solution:**

Required number of minutes is the LCM of 18 and 12.

We have,

$18 = 2 \times 3^2$ and $12 = 2^2 \times 3$

∴ LCM of 18 and 12 = $2^2 \times 3^2 = 36$

Thus, Prenu and Raj will meet again at the starting point after 36 minutes.

10. **Solution:**

If α, β and γ are the zeros of a cubic polynomial $f(x)$, then

$$f(x) = k\left\{x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma\right\},$$

where k is any non-zero real number.

Here, $\alpha+\beta+\gamma = 2$, $\alpha\beta+\beta\gamma+\gamma\alpha = -7$ and $\alpha\beta\gamma = -14$

$\therefore \quad f(x) = k\left(x^3 - 2x^2 - 7x + 14\right)$, where k is any non-zero real number.

SECTION – C

11. **Solution:**

Let us assume on the contrary that $\sqrt{2}+\sqrt{5}$ is a rational number. Then, there exist co-prime positive integers a and b such that

$$\sqrt{2}+\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{a}{b} - \sqrt{2} = \sqrt{5}$$

$$\Rightarrow \quad \left(\frac{a}{b} - \sqrt{2}\right)^2 = \left(\sqrt{5}\right)^2$$

$$\Rightarrow \quad \frac{a^2}{b^2} - \frac{2a}{b}\sqrt{2} + 2 = 5$$

$$\Rightarrow \quad \frac{a^2}{b^2} - 3 = \frac{2a}{b}\sqrt{2}$$

$$\Rightarrow \quad \frac{a^2 - 3b^2}{2ab} = \sqrt{2}$$

$\Rightarrow \quad \sqrt{2}$ is rational number $\quad \left[\because a \text{ and } b \text{ are integers}, \therefore \frac{a^2-3b^2}{2ab} \text{ is rational.}\right]$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is wrong.

12. **Solution:**

$$\text{LHS} = \frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1} \qquad [\because \sec^2\theta - \tan^2\theta = 1]$$

$$= \frac{(\sec\theta + \tan\theta)\{1 - (\sec\theta - \tan\theta)\}}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)(\tan\theta - \sec\theta + 1)}{\tan\theta - \sec\theta + 1}$$

$$= \sec\theta + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1 + \sin\theta}{\cos\theta}$$

$$= \frac{1 + \sin\theta}{\cos\theta} \times \frac{1 - \sin\theta}{1 - \sin\theta}$$

$$= \frac{1 - \sin^2\theta}{\cos\theta(1 - \sin\theta)} = \frac{\cos^2\theta}{\cos\theta(1 - \sin\theta)} = \frac{\cos\theta}{1 - \sin\theta}$$

13. **Solution:**

We have,

$(a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$

$= a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$= a^2 + b^2$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$\Rightarrow$ $c^2 + (a\sin\theta + b\cos\theta)^2 = a^2 + b^2$

$\Rightarrow$ $(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

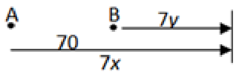
$\Rightarrow$ $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$

Thus, $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$

14. **Solution:**

Let the speed of faster car at A = x km/hr and the speed of slower car at B = y km.hr.

Case 1: When they travel in same direction



Distance covered by faster car in 7 hours = $7x$ km

Distance covered by slower car in 7 hours = $7y$ km

$\Rightarrow$ $7x = 7y + 70$

$\Rightarrow$ $7(x - y) = 70$

⇒ $x - y = 10$ ...(i)

Case 2: When they travel in opposite direction

Distance travelled by faster car in 1 hour = x km

Distance travelled by slower car in 1 hour = y km

⇒ $x + y = 70$ ...(ii)

Adding (i) and (ii), we get

$2x = 80$ ⇒ $x = 40$

Substituting $x = 40$ in (i), we get

$40 - y = 10$ ⇒ $y = 40 - 10 = 30$

∴ Speeds of cars would be 40 km/hr and 30 km/hr.

15. **Solution:**

$x^3 - 3x^2 + x + 1$ is a cubic polynomial.

∴ Sum of its zeros = $\frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3} = \frac{-(-3)}{1} = 3$

⇒ $a - b + a + a + b = 3$

⇒ $3a = 3$

⇒ $a = 3$

Also, product of its zeros = $\frac{-(\text{constant term})}{\text{coefficient of } x^3} = \frac{-1}{1} = -1$

⇒ $(a - b) \times a \times (a + b) = -1$

⇒ $a(a^2 - b^2) = -1$

⇒ $1(1 - b^2) = -1$ [∵ $a = 1$]

⇒ $1 - b^2 = -1$

⇒ $b^2 = 2$

⇒ $b = \pm\sqrt{2}$

Thus, $a = 1$ and $b = \pm\sqrt{2}$.

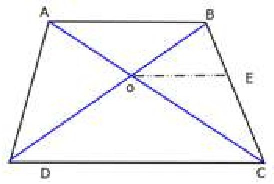
16. **Solution:**

Given: A quadrilateral ABCD whose diagonals AC and BC intersect at O such that

$$\frac{AO}{OC}=\frac{BO}{OD}$$

To prove: ABCD is a trapezium.

Construction: Through O, draw OE || AB.



∴ By Basic Proportionality Theorem, we have

$$\frac{AO}{OC}=\frac{BE}{EC}$$

But, $\frac{AO}{OC}=\frac{BO}{OD}$ [Given]

∴ $\frac{BE}{EC}=\frac{BO}{OD}$

Now, in ΔBCD, we have $\frac{BE}{EC}=\frac{BO}{OD}$

∴ By Basic Proportionality Theorem, we have

OE || DC

Now, OE || AB [By construction]

and, OE || DC

∴ AB || DC

Thus, ABCD is a trapezium.

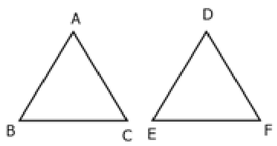
17. **Solution:**

Given that ABC and DEF are two similar triangles and their areas are equal.

∵ ΔABC~ΔDEF

∴ $\frac{ar\Delta ABC}{ar\Delta DEF}=\frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{DF^2}$

[Ratios of the areas of similar triangles is equal to the ratio of the squares of corresponding sides]



But, arΔABC = arΔDEF

$\therefore \quad \frac{ar\Delta ABC}{ar\Delta DEF} = 1$

$\Rightarrow \quad \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1$

Or, $AB^2 = DE^2$, $BC^2 = EF^2$ and $AC^2 = DF^2$

$\therefore \quad AB = DE$

$BC = EF$

and, $AC = DF$

Thus, $\Delta ABC \cong \Delta DEF$ [By SSS criterion]

18. **Solution:**

$$\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + \frac{2}{1}}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2\left(1+\sqrt{3}\right)}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}\left(1+\sqrt{3}\right)}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}\left(1+\sqrt{3}\right)} \times \frac{\left(1-\sqrt{3}\right)}{\left(1-\sqrt{3}\right)} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6}\left(1-\sqrt{3}\right)}{4\left(1-3\right)}$$

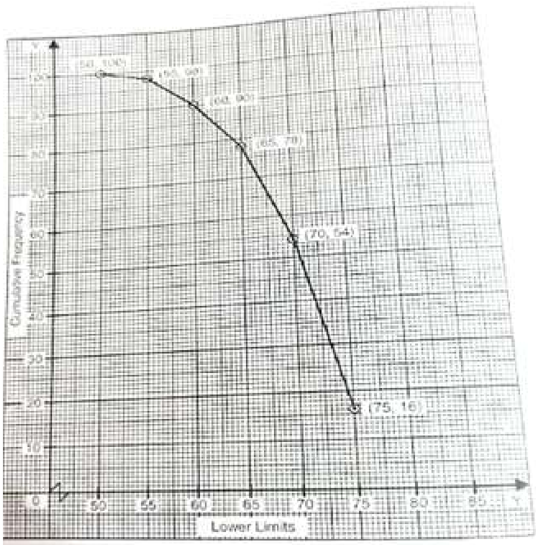
$$= \frac{\sqrt{6}\left(\sqrt{3}-1\right)}{8}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

19. **Solution:**

Production yield (kg/ha)	Number of farms	Production yield more than or equal to (kg.ha)	Cumulative frequency (*cf*)
50-55	2	50	100
55-60	8	55	98
60-65	12	60	90
65-70	24	65	78
70-75	38	70	54
75-80	16	75	16

To draw the required ogive, we plot the points corresponding to the ordered pairs given by (lower limit, corresponding frequency), i.e., (50, 100), (55, 98), (60, 90), (65, 78), (70, 54) and (75, 16) on a graph paper and join them by freehand curve.



20. **Solution:**

Let the tens and units digits of the required number be x and y, respectively. Then $xy = 14$.

Required number = $(10x + y)$

Number obtained on reversing its digits = $(10y + x)$

$\therefore \quad (10x + y) + 45 = (10y + x)$

$\Rightarrow \quad 9(y - x) = 45$

$\Rightarrow \quad y - x = 45 \qquad$ ...(i)

Now, $(y+x)^2 - (y-x)^2 = 4xy$

$\Rightarrow \quad (y+x) = \sqrt{(y-x)^2 + 4xy}$

$= \sqrt{25+4\times14} = \sqrt{81}$

$\Rightarrow \quad y + x = 9 \qquad$ ...(ii)

On adding (i) and (ii), we get

$2y = 14 \quad \Rightarrow \quad y = 7$

Putting $y = 7$ in (ii), we get

$7 + x = 9 \quad \Rightarrow \quad x = 9 - 7 = 2$

$\therefore \quad x = 2$ and $y = 9$

SECTION – D

21. **Solution:**

Let n be a positive integer other than 1. By the fundamental theorem of Arithmetic, n can be uniquely expressed as powers of primes in ascending order. So, let

$n = p_1^{a_1} p_2^{a_2} ... p_k^{a_k}$ be the unique factorisation of n into primes with $p_1 < p_2 < p_3 < ... < p_k$.

Clearly, either $p_1 = 2$ and $p_2, p_3, ..., p_k$ are odd positive integers or each of $p_1, p_2, ..., p_k$ is an odd positive integer.

Therefore, we have the following cases:

Case I: When $p_1 = 2$ and $p_2, p_3, ..., p_k$ are odd positive integers.

In this case, we have

$n = 2^{a_1} p_2^{a_2} p_3^{a_3} ... p_k^{a_k}$

$\Rightarrow \quad n = 2^{a_1} \times \left(p_2^{a_2} p_3^{a_3} ... p_k^{a_k} \right)$

$\Rightarrow \quad n = 2^{a_1} \times$ An odd positive integer

$\Rightarrow \quad n$ = (A non-negative power of 2) × (An odd positive integer)

Case II: When each of $p_1, p_2, p_3, ..., p_k$ is an odd positive integer.

In this case, we have

$n = p_1^{a_1} p_2^{a_2} p_3^{a_3} ... p_k^{a_k}$

$\Rightarrow \quad n = 2^2 \times \left(p_1^{a_1} p_2^{a_2} p_3^{a_3} ... p_k^{a_k} \right)$

$\Rightarrow \quad n$ = (A non-negative power of 2) × (An odd positive integer)

Thus, in either case, n is expressible as the product of a non-negative power of 2 and an odd positive integer.

22. **Solution:**

Let $\quad p(x) = x^3 - 3x^2 + x + 2$

$q(x) = x - 2$

$r(x) = -2x + 4$

by division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$\Rightarrow \quad g(x) \times q(x) = p(x) - r(x)$

$\Rightarrow \quad g(x)(x - 2) = x^3 - 3x^2 + x + 2 - (-2x + 4)$

$= x^3 - 3x^2 + x + 2 + 2x - 4$

$= x^3 - 3x^2 + 3x - 2$

$\Rightarrow \quad g(x)$ is a factor of $x^3 - 3x^2 + 3x - 2$ other than $(x - 2)$.

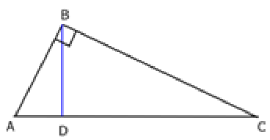
Dividing $x^3 - 3x^2 + 3x - 2$ by $(x - 2)$, we obtain $g(x)$ as follows:

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & + \quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

$\therefore \quad g(x) = x^2 - x + 1$

23. **Solution:**

Theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given: A right-angled triangle ABC in which $\angle B = 90°$.

To prove: $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $AC^2 = AB^2 + BC^2$

Construction: From B, draw $BD \perp AC$.

Proof: In triangles ADB and ABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90°]

and, $\angle A = \angle A$ [Common]

So, by AA-similarity criterion, we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow \quad \frac{AD}{AB} = \frac{AB}{AC}$ [∵ In similar triangles corresponding sides are proportional]

$\Rightarrow \quad AB^2 = AD \times AC$ ...(i)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ [Each equal to 90°]

and, $\angle C = \angle C$ [Common]

So, by AA-similarity criterion, we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow \quad \frac{DC}{BC} = \frac{BC}{AC}$ [∵ In similar triangles corresponding sides are proportional]

$\Rightarrow \quad BC^2 = AC \times DC$ ...(ii)

Adding equations (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$\Rightarrow \quad AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow \quad AB^2 + BC^2 = AC \times AC$

$\Rightarrow \quad AB^2 + BC^2 = AC^2$

Or, $AC^2 = AB^2 + BC^2$

24. **Solution:**

Let the number of students be x and the number of rows be y.

Then, number of students in each row = $\frac{x}{y}$

When one student is extra in each row, there are 2 rows less, i.e., when each row has $\left(\frac{x}{y}+1\right)$ students, the number of rows is $(y-2)$.

∴ Total number of students = No. of rows × No. of students in each row

$\Rightarrow \quad x = \left(\frac{x}{y}+1\right)(y-2)$

$\Rightarrow \quad x = x - \frac{2x}{y} + y - 2$

$\Rightarrow \quad -\frac{2x}{y} + y - 2 = 0$ ...(i)

If one student is less in each row, then there are 3 rows more, i.e., when each row has $\left(\frac{x}{y}-1\right)$ students, the number of rows is $(y+3)$.

∴ Total number of students = No. of rows × No. of students in each row

$\Rightarrow \quad x = \left(\frac{x}{y}-1\right)(y+3)$

$\Rightarrow \quad x = x+\frac{3x}{y}-y-3$

$\Rightarrow \quad \frac{3x}{y}-y-3=0 \qquad$ ...(ii)

Putting $\frac{x}{y} = u$ in (i) and (ii), we get

$-2u + y - 2 = 0 \qquad$ ...(iii)

and, $3u - y - 3 = 0 \qquad$ ...(iv)

Adding (iii) and (iv), we get

$u - 5 = 0 \quad \Rightarrow \quad u = 5$

Putting $u = 5$ in (iii), we get y = 12

Now, $u = 5 \quad \Rightarrow \quad \frac{x}{y} = 5 \quad \Rightarrow \quad \frac{x}{12} = 5 \quad \Rightarrow \quad x = 60$

Thus, the number of students in the class is 60.

25. **Solution:**

Consider two right triangles ABC and PQR such that sinB = sinQ.

We have,

$\sin B = \frac{AC}{AB}$ and $\sin Q = \frac{PR}{PQ}$

∴ sinB = sinQ

$\Rightarrow \quad \frac{AC}{AB}=\frac{PR}{PQ}$

$\Rightarrow \quad \frac{AC}{PR}=\frac{AB}{PQ} = k$ (say) $\qquad$ ...(i)

$\Rightarrow \quad AC = kPR$ and $AB = kPQ \qquad$ ...(ii)

Using Pythagoras theorem in triangles ABC and PQR, we have

$AB^2 = AC^2 + BC^2$ and $PQ^2 = PR^2 + QR^2$

$\Rightarrow$ BC = $\sqrt{AB^2 - AC^2}$ and QR = $\sqrt{PQ^2 - PR^2}$

$$\Rightarrow \quad \frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}}$$

$$= \frac{\sqrt{k^2 PQ^2 - k^2 PR^2}}{\sqrt{PQ^2 - PR^2}}$$

$$= \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k \qquad \text{...(iii)}$$

From (i) and (iii), we have

$$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$$

$\Rightarrow$ $\Delta ACB \sim \Delta PRQ$

$\Rightarrow$ $\angle B = \angle Q$

26. **Solution:**

The given system of equations may be written as

$$(a-b)x + (a+b)y - (a^2 - 2ab - b^2) = 0$$

$$(a+b)x + (a+b)y - (a^2 + b^2) = 0$$

By cross-multiplication, we have

$$\frac{x}{(a+b)\times(a^2+b^2)-(a+b)\times-(a^2-2ab-b^2)} = \frac{-y}{(a-b)\times-(a^2+b^2)-(a+b)\times-(a^2-2ab-b^2)} = \frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\Rightarrow \quad \frac{x}{-(a+b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)} = \frac{-y}{-(a-b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)} = \frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\Rightarrow \quad \frac{x}{(a+b)\{(a^2+b^2)+(a^2-2ab-b^2)\}} = \frac{-y}{(a+b)(a^2-2ab-b^2)-(a-b)(a^2+b^2)} = \frac{1}{(a+b)(a-b-a-b)}$$

$$\Rightarrow \quad \frac{x}{(a+b)(-2ab-2b^2)} = \frac{-y}{a^3 - a^2b - 3ab^2 - b^3 - a^3 - ab^2 + a^2b + b^3} = \frac{1}{-(a+b)2b}$$

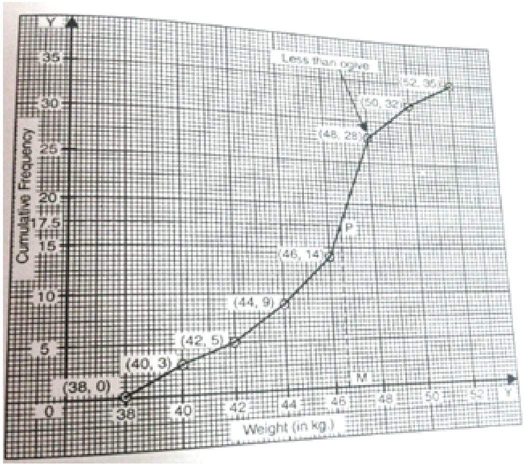
$$\Rightarrow \quad \frac{x}{-2b(a+b)^2} = \frac{-y}{-4ab^2} = \frac{1}{-2b(a+b)}$$

$$\Rightarrow \quad x = \frac{-2b(a+b)^2}{-2b(a+b)} = a+b \text{ and } y = \frac{4ab^2}{-2b(a+b)} = \frac{-2ab}{a+b}$$

Hence, the solution of the given system of equations is $x = a + b$, $y = \frac{-2ab}{a+b}$.

27. **Solution:**

To draw the required ogive, we plot the points (38, 0), (40, 3), (44, 9), (46, 14), (48, 28), (50, 32) and (52, 35) and join them by a freehand curve.



To obtain the value of the median, we locate the point $\frac{n}{2} = \frac{35}{2}$ = 17.5 on the y-axis. From this point, we draw a line parallel to the x-axis, meeting the ogive at the point P. From P, we draw a perpendicular PM on the x-axis. The x-coordinate of the point where this perpendicular meets the x-axis, i.e., M gives the value of the median.

∴ The required value of the median is 46.5 kg.

Verification:

Weight (kg)	Number of students (f_i)	Cumulative frequency (cf)
38-40	3	3
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35

Here, $n = 35$, ∴ $\frac{n}{2} = 17.5$

Median class is 46-48

∴ $l = 46, f = 14, cf = 14, h = 2$

Median $= l+\left(\dfrac{\frac{n}{2}-cf}{f}\right)\times h$

$= 46+\left(\dfrac{17.5-14}{14}\right)\times 2$

$= 46+\dfrac{3.5}{14}\times 2$

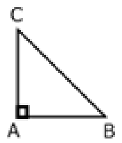
= 46.5 kg

The value of the median in both the cases is same, i.e., 46.5 kg.

Hence verified.

28. **Solution:**

Draw a right triangle ABC in which ∠ABC = θ



Since, $\cot\theta = \dfrac{AB}{AC}=\dfrac{7}{8}$

∴ Let AB = 7 units and AC = 8 units

∴ $BC = \sqrt{AB^2+AC^2}$ (By Pythagoras Theorem)

$= \sqrt{7^2+8^2}$

$= \sqrt{49+64}$

$= \sqrt{113}$ units

∴ $\sin\theta = \dfrac{AC}{BC}=\dfrac{8}{\sqrt{113}}$ and $\cos\theta = \dfrac{AB}{BC}=\dfrac{7}{\sqrt{113}}$

Now, $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}=\dfrac{1-\sin^2\theta}{1-\cos^2\theta}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

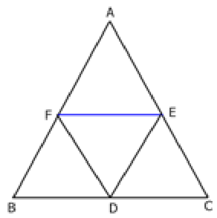
$$= \frac{113-64}{113-49} = \frac{49}{64}$$

29. **Solution:**

Given: ΔABC in which D, E and F are the mid-points of sides BC, CA and AB, respectively.

To prove: Each of the triangles AFE, FBD, EDC and DEF is similar to ΔABC.

Proof: Consider triangles AFE and ABC.



Since F and E are the mid-points of AB and AC, respectively

∴ FE || BC

⇒ ∠AFE = ∠B [Corresponding angles]

Thus, in ΔAFE and ΔABC, we have

∠AFE = ∠B

and, ∠A = ∠A [Common]

Similarly, we have

ΔFBD~ΔABC and ΔEDC~ΔABC.

Now, we shall show that ΔDEF~ΔABC.

Clearly, ED || AF and DF || EA.

∴ AFDE is a parallelogram.

⇒ ∠EDF = ∠A [∵ Opposite angles of a parallelogram are equal.]

Similarly, BDEF is a parallelogram.

∴ ∠DEF = ∠B [∵ Opposite angles of a parallelogram are equal.]

Thus, in triangles DEF and ABC, we have

∠EDF = ∠A and ∠DEF = ∠B

So, by AA-criterion of similarity, we have

ΔDEF~ΔABC.

Thus, in each one of the triangles AFE, FBD, EDC and DEF is similar to ΔABC.

30. **Solution:**

If $a = c$, then

$$a+\sqrt{b}=c+\sqrt{d} \quad \Rightarrow \quad \sqrt{b}=\sqrt{d} \quad \Rightarrow \quad b = d$$

So, let $a \neq c$. Then, there exists a positive rational number x such that $a = c + x$.

Now, $a+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow \quad c+x+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow \quad x+\sqrt{b}=\sqrt{d}$ ...(i)

$\Rightarrow \quad \left(x+\sqrt{b}\right)^2=\left(\sqrt{d}\right)^2$

$\Rightarrow \quad x^2+2\sqrt{b}x+b=d$

$\Rightarrow \quad d-x^2-b=2x\sqrt{b}$

$\Rightarrow \quad \sqrt{b}=\dfrac{d-x^2-b}{2x}$

$\Rightarrow \quad \sqrt{b}$ is rational. $\left[\because d, x \text{ and } b \text{ are rationals}, \therefore \dfrac{d-x^2-b}{2x} \text{ is rational}\right]$

From (i), we have

$\sqrt{d}=x+\sqrt{b}$

$\Rightarrow \quad \sqrt{d}$ is rational

$\Rightarrow \quad d$ is the square of a rational number.

Thus, either $a = c$ and $b = d$ or b and d are the squares of rationals.

31. **Solution:**

a. According to the given condition:

$y = 50 + 25(x - 1)$

$= 50 + 25x - 25$

$\Rightarrow \quad y = 25x + 25$

b. Correct fare = 25 × 10 + 25

= 250 + 25

= Rs 275

Amount paid back by the driver = 300 – 275 = Rs 25

c. The values depicted by the driver in the question are honesty and truthfulness.