
SAMPLE PAPER-05 (solved)

PHYSICS (Theory)

Class - XI

Time allowed: 3 hours

Maximum Marks: 70

General Instructions:

- a) All the questions are compulsory.
- b) There are **26** questions in total.
- c) Questions **1** to **5** are very short answer type questions and carry **one** mark each.
- d) Questions **6** to **10** carry **two** marks each.
- e) Questions **11** to **22** carry **three** marks each.
- f) Questions **23** is value based questions carry **four** marks.
- g) Questions **24** to **26** carry **five** marks each.
- h) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions in five marks each. You have to attempt only one of the choices in such questions.
- i) Use of calculators is **not** permitted. However, you may use log tables if necessary.
- j) You may use the following values of physical constants wherever necessary:

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

- 1. What happens to surface tension when impurity is mixed in liquid?
 - 2. Is it possible for the relative velocity of two bodies moving in opposite direction to be greater than the absolute velocity of either?
 - 3. Comment: "A body at higher temperature contains more heat".
 - 4. What is apparent weight of the floating block, when a block of wood is floating in a lake?
 - 5. Fill in the blanks:
 - a) $6 \text{ ms}^{-2} = \text{----- kmh}^{-2}$
 - b) $1 \text{ m} = \text{----- light year}$
 - 6. If a bullet of mass 20g is moving with a speed of 150 m s^{-1} strikes a target and is brought to rest after piercing 10 cm into it, then calculate the average force of resistance offered by the target.
 - 7. Find the time period of motion if a mass m is dropped in a tunnel along the diameter of earth from height h above the surface of earth. Is the motion simple harmonic?
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8. Calculate the percentage of heat used for doing work, if a steam engine intakes steam at 200°C and after doing work exhausts it directly in air at 100°C . Assume the engine to be ideal engine.

Or

Calculate the temperature at which the root mean square velocity of nitrogen molecules will be equal to 8 km s^{-1}

9. Find an expression for the weight of a body at the centre of the earth.
10. Give reasons: "The springs made of steel and not of copper".
11. Explain the following w.r.t. associating it with vector.
- a) The length of a wire bent into a loop
 - b) A plane area
 - c) A sphere
12. State whether the following statement is true or false with proper reason.
- (i) The net acceleration of particle in circular motion is always along the radius of the circle towards the centre.
 - (ii) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.
 - (iii) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.
- 13.
- a) What is the coefficient of performance (β) of a Carnot refrigerator working between 30°C and 0°C ?
 - b) What would be the kinetic energy of 1 g of nitrogen gas at 77°C . Given $R = 8.31\text{ J mol}^{-1}\text{ K}^{-1}$
14. A block with a mass of 3.0 kg is suspended from an ideal spring having negligible mass and stretches the spring by 0.2 m
- (i) What is the force constant of the spring
 - (ii) What is the period of oscillation of the block if it is pulled down and released?
15. A wire of length l and area of cross section A is stretched by the application of a force. If the Young's modulus is Y what is the work done per unit volume?

Or

A spring balance reads 10 kg when a bucket of water is suspended from it. Explain what will be the reading of spring balance when

- i) An ice cube of mass 1.5 kg is put into bucket
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- ii) An iron piece of mass 7.8 kg suspended by another string is immersed with half of its volume inside the water in the bucket. [Relative density of iron = 7.8]
16. The acceleration due to gravity on the surface of moon is 1.7 ms^{-2} . What is the time period of a simple pendulum on the moon if its time period is 3.5 s on earth?
17. Discuss the effect of temperature on the velocity of sound in gases. State Newton's second law of Motion. On its basis derive the relation between force and acceleration.
18. A particle starts SHM from the mean position. Its amplitude is A and its time period T. at one time its speed is half that of the maximum speed. What is this displacement.
19. What is a thermostat? Briefly explain its principle?
20. A liquid drop of diameter 4 mm breaks into 1000 droplets of equal size. Calculate the resultant change in surface energy, the surface tension of the liquid is 0.07 Nm^{-1}
- 21.
- i) When is the work done said to be zero?
- ii) Give two examples from daily life where according to physics, work done is zero.
22. A wood ball of density ρ is immersed in water of density σ to depth h and then released. Find the height H above the surface of water upto which the ball jumps out of water.
23. Sara was afraid on going anywhere by air. But, she couldn't avoid going by an air. Her friend Hema who knew her problem was with her. Inside the plane Hema saw that Sara was very quiet and feeling uncomfortable. She tried to talk to Sara but she didn't answer. As the plane was about to take off, Hema started fighting with Sara without any cause for diverting her mind. While fighting Sara didn't realize that plane had taken off and now she was in air. She felt very happy to overcome her fear.
- i) What values do you associate with Hema?
- ii) An aeroplane takes off at an angle of 30° to the horizontal. If the component of its velocity along the horizontal is 250 km/h . what is the actual velocity? Find also the vertical component of velocity.
- iii) The blades of an aeroplane propeller are rotating at the rate of 600 revolutions per minute. Calculate its angular velocity?
24. If a body executing linear SHM has a velocity of 3 cm s^{-1} when its displacement is 4 cm and a velocity of 4 cm s^{-1} when its displacement is 3 cm, then
- (a) Find the amplitude and period of the oscillation
- (b) Calculate the total energy of oscillation if the mass of the body is 50 g,

Or

If a sonometer wire of length 110 cm is stretched with a tension T and fixed at its ends is divided into three segments by placing two bridges below, then where the bridges should be placed so that the fundamental frequencies of the segments are in the ratio 1:2:3?

25. If a ball of mass 100 g is projected vertically upwards from the ground with a velocity of 49 m/s and at the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as followed by the first ball, then after sometime the two balls collide and stick together and finally fall together. Find the time of flight of the masses.

Or

A gun kept on a straight horizontal road to hit a car traveling along the same road away from the gun with a uniform speed of 72 km/h. The car is at a distance of 500 m from the gun when the gun is fired at an angle of 45° to the horizontal. Find

- (i) The distance of the car from the gun when the shell hits it
- (ii) The speed of projection of the shell from the gun

26. An object starts from rest and covers total distance X in the below manner:

It first has uniform acceleration a_1 from some time t_1 moves with the speed acquired at the end of t_1 for some distance and is then given a uniform retardation a_2 so that it is again at rest at the end of the journey. Explain that the journey is covered in least time if this body is accelerated

$$\text{For a time } \left[\frac{2Xa_2}{a_1(a_1 + a_2)} \right]^{\frac{1}{2}} \text{ and this minimum time is } \left[2x \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \right]^{\frac{1}{2}}$$

Or

From the top of the tower 156.8 m high a projectile is thrown up with velocity of 39.2 ms^{-1} , making an angle 30° with horizontal direction. Find the distance from the foot of tower where it strikes the ground and the time taken by it.

SAMPLE PAPER-05 (solved)

PHYSICS (Theory)

Class - XI

ANSWER

1. What happens to surface tension when impurity is mixed in liquid?

Ans:

If impurity is mixed in liquid, the surface tension of the liquid decreases.

2. Is it possible for the relative velocity of two bodies moving in opposite direction to be greater than the absolute velocity of either?

Ans:

Yes. When two bodies move in opposite direction then the relative velocity of each is greater than the individual velocities.

3. Comment: "A body at higher temperature contains more heat".

Ans:

This is not always true because heat content of a body depends on mass of the body, its specific heat and temperature.

4. What is apparent weight of the floating block, when a block of wood is floating in a lake?

Ans:

The apparent weight of the floating block is equal to zero because the weight of the block acting vertically downwards is balanced by the buoyant force acting on the block upwards.

5. Fill in the blanks:

c) $6 \text{ ms}^{-2} = \text{----- kmh}^{-2}$

d) $1 \text{ m} = \text{----- light year}$

Ans:

a) 7.776×10^4

b) 1.06×10^{-16}

6. If a bullet of mass 20g is moving with a speed of 150 m s^{-1} strikes a target and is brought to rest after piercing 10 cm into it, then calculate the average force of resistance offered by the target.

Ans:

$$m = 20 \text{ g} = 0.02 \text{ kg}, u = 150 \text{ m s}^{-1}, v = 0 \text{ and } s = 10 \text{ cm} = 0.1 \text{ m}$$

According to work - K.E. theorem

$$k - K' = W = Fs$$

$$\frac{1}{2}mu^2 - 0 = Fs$$

$$F = \frac{mu^2}{2s} = \frac{0.02 \times (150)^2}{2 \times 0.1} = 2250N$$

7. Find the time period of motion if a mass m is dropped in a tunnel along the diameter of earth from height h above the surface of earth. Is the motion simple harmonic?

Ans:

When a ball is dropped from a height h , it gains velocity due to gravity pull. The body will enter the tunnel of earth with velocity $v = \sqrt{2gh}$ after a time $t = \sqrt{2h/g}$. The body will go out of the earth on the other side through the same distance before coming back towards the earth. When the body is outside the earth, the restoring force $F \propto (-1/r^2)$ and not $(-r)$ so the motion does not remain SHM.

8. Calculate the percentage of heat used for doing work, if a steam engine intakes steam at 200°C and after doing work exhausts it directly in air at 100°C . Assume the engine to be ideal engine.

Ans:

$$T_1 = 200^\circ\text{C} = 473\text{ K and } T_2 = 100^\circ\text{C} = 373\text{ K}$$

$$\text{The efficiency of engine } \eta = \frac{W}{Q_1} = \left(\frac{T_1 - T_2}{T_1} \right) = \frac{473 - 373}{473} = \frac{100}{473} = 0.21$$

$$W = 0.21, Q_1 = 21\% \text{ o}$$

Hence the engine will convert 21% of heat used for doing work.

Or

Calculate the temperature at which the root mean square velocity of nitrogen molecules will be equal to 8 km s^{-1}

Ans:

$$\text{r.m.s velocity } C = 8\text{ km s}^{-1} = 8 \times 10^5\text{ cm s}^{-1}$$

$$\text{Molar gas constant } R = 8.31 \times 10^7\text{ erg mol}^{-1}\text{ K}^{-1}$$

$$\text{Molecular weight of nitrogen } M = 28$$

Let T be the required temperature

$$C = \sqrt{\frac{3RT}{M}} \text{ or } C^2 = \frac{3RT}{M}$$

$$T = \frac{MC^2}{3R} = \frac{28 \times (8 \times 10^5)^2}{3 \times 8.31 \times 10^7}\text{ K} = 71881\text{ K}$$

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9. Find an expression for the weight of a body at the centre of the earth.

Ans:

The value of acceleration due to gravity at a depth 'd' below the surface of earth is given by,

$$g_a = g \left(1 - \frac{d}{R} \right)$$

At the centre of earth $d = R$ and hence

$$g_{\text{centre}} = g \left(1 - \frac{R}{R} \right) = g(1 - 1) = 0$$

Weight of a body at the centre of earth $= mg_{\text{centre}} = m \times 0$ which means that at the centre of earth a body will be weightless.

10. Give reasons: "The springs made of steel and not of copper".

Ans:

A spring will be better one if a large restoring force is set up in it on being deformed, which in turn depends upon the elasticity of the material of the spring. Since the Young's modulus of elasticity of steel is more than that of copper, hence the steel is preferred in making the springs.

11. Explain the following w.r.t. associating it with vector.

- d) The length of a wire bent into a loop
- e) A plane area
- f) A sphere

Ans:

- a) We cannot associate a vector with the length of a wire bent into a loop. This is because the length of the loop does not have a definite direction.
- b) We can associate a vector with a plane area. Such vector is called area vector and its direction is represented by a normal drawn outward to the area.
- c) The area of a sphere does not point in any definite direction. However we can associate a null vector with the area of the sphere. We cannot associate a vector with the volume of a sphere.

12. State whether the following statement is true or false with proper reason.

- (iv) The net acceleration of particle in circular motion is always along the radius of the circle towards the centre.
 - (v) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.
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- (vi) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Ans:

- (i) False, the net acceleration of a particle in circular motion is along the radius of the circle towards the centre only in uniform circular motion.
- (ii) True, because while leaving the circular path, the particle moves tangentially to the circular path
- (iii) True, the direction of acceleration vector in a uniform circular motion is directed towards the centre of circular path. It is constantly changing with time. The resultant of all these vectors will be a zero vector.

13.

- c) What is the coefficient of performance (β) of a Carnot refrigerator working between 30°C and 0°C ?
- d) What would be the kinetic energy of 1 g of nitrogen gas at 77°C . Given $R = 8.31\text{ J mol}^{-1}\text{ K}^{-1}$

Ans:

a)

$$T_2 = 0^\circ\text{C} = 273\text{ K}$$

$$T_1 = 30^\circ\text{C} = 273 + 30 = 303\text{ K}$$

$$\begin{aligned}\beta &= \frac{T_2}{T_1 - T_2} \\ &= \frac{273}{303 - 273} = \frac{273}{30} = 9.1\end{aligned}$$

b)

Mass of Nitrogen $M = 28$

Temperature $T = 77 + 273 = 350\text{ K}$

Gas constant $R = 8.31\text{ J mol}^{-1}\text{ K}^{-1}$

$$\begin{aligned}\text{K.E. of 1g of nitrogen} &= \frac{3}{2} \frac{RT}{M} \\ &= \frac{3 \times 8.31 \times 350}{2 \times 28} = 155.8\text{ J}\end{aligned}$$

14. A block with a mass of 3.0 kg is suspended from an ideal spring having negligible mass and stretches the spring by 0.2 m
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-
- (iii) What is the force constant of the spring
- (iv) What is the period of oscillation of the block if it is pulled down and released?

Ans:

(i) Force constant $k = \frac{F}{l} = \frac{mg}{l}$

$M = 3.0 \text{ kg}$ and elongation in length of spring $l = 0.2 \text{ m}$

$$\text{Force constant } k = \frac{3.0 \times 9.8}{0.2} = 147 \text{ Nm}^{-1}$$

(ii) Period of oscillation $T = 2\pi\sqrt{\frac{m}{k}} = 2 \times 3.14 = \sqrt{\frac{3}{147}} = 0.9 \text{ s}$

15. A wire of length l and area of cross section A is stretched by the application of a force. If the Young's modulus is Y what is the work done per unit volume?

Or

A spring balance reads 10 kg when a bucket of water is suspended from it. Explain what will be the reading of spring balance when

- iii) An ice cube of mass 1.5 kg is put into bucket
- iv) An iron piece of mass 7.8 kg suspended by another string is immersed with half of its volume inside the water in the bucket. [Relative density of iron = 7.8]

Ans:

The wire has length l , area of the cross-section A made of material constant Y . Let force F be applied and at any instance, x be the extension associated ($x < L$) where L is the maximum

extension. At this instant $F = \frac{AY \cdot x}{l}$.

Since force is a variable with x , work done to stretch wire is

$$W = \int_0^L F dx$$

$$W = \frac{1}{2} \frac{AY}{l} \cdot L^2$$

$$W = \frac{1}{2} (Al) \left(\frac{YL}{l} \right) \left(\frac{L}{l} \right)$$

$$W = \frac{1}{2} \times \text{Volume} \times \text{Stress} \times \text{Strain}$$

$$\text{Therefore Work done per unit volume} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

Or

- i) Reading on balance = $10 + 1.5 = 1.5 \text{ Kg}$
- ii) Volume of Iron = $\frac{M}{P} = \frac{7.8}{7.8 \times 10^3} = 10^{-3} \text{ m}^3$
- Mass of water displaced = $\frac{10^{-3}}{2} \times 10^3 = \frac{1}{2} \text{ kg}$

Reading of balance = $10 + 0.5 = 10.5 \text{ kg}$

16. The acceleration due to gravity on the surface of moon is 1.7 ms^{-2} . What is the time period of a simple pendulum on the moon if it time period is 3.5 s on earth?

Ans:

On earth $T = 3.5 \text{ s}$, $g = 9.8 \text{ m/s}^2$

Using $T = 2\pi \sqrt{\frac{l}{g}}$

$$\Rightarrow 3.5 = 2\pi \sqrt{\frac{l}{9.8}}$$

On Moon $T' = ?$, $g' = 1.7 \text{ m/s}^2$

$$\Rightarrow T' = 2\pi \sqrt{\frac{l}{g'}} = 2\pi \sqrt{\frac{l}{1.7}}$$

$$\frac{T'}{3.5} = \sqrt{\frac{9.8}{1.7}}$$

$$T' = 240 \times 3.5 = 8.4 \text{ s}$$

17. Discuss the effect of temperature on the velocity of sound in gases. State Newton's second law of Motion. On its basis derive the relation between force and acceleration.

Ans:

Velocity of sound in gases depends directly on the square root of the absolute temperature. According to Newton's second law, force acting on an object is the rate of change of its momentum

$$F = \frac{dp}{dt} = \frac{m(v-u)}{t} = ma$$

18. A particle starts SHM from the mean position. Its amplitude is A and its time period T . at one time its speed is half that of the maximum speed. What is this displacement.

Ans:

In a SHM velocity is given by $v = \omega \sqrt{A^2 - x^2}$ where x is the displacement from mean position.

Velocity at $x = \frac{\sqrt{3}A}{2}$

$$v_1 = \omega \sqrt{A^2 - \frac{3}{4}A^2} = \omega A \sqrt{\frac{1}{4}} = \frac{\omega A}{2}$$

Velocity at central position = $\omega \sqrt{A^2 - 0^2} = \omega A$

Therefore Velocity at $\frac{\sqrt{3}A}{2} = \frac{1}{2}$

19. What is a thermostat? Briefly explain its principle?

Ans:

It is a device that helps in maintaining a constant temperature. It consists of a bi-metallic strip which comprises of two thin strips of different materials welded together along their lengths. On heating, this combination bends into an arc. It happens because brass has a higher coefficient of expansion than invar.

20. A liquid drop of diameter 4 mm breaks into 1000 droplets of equal size. Calculate the resultant change in surface energy, the surface tension of the liquid is 0.07 Nm^{-1} ?

Ans:

$$\sigma = 0.07 \text{ Nm}^{-1}, R = \frac{D}{2} = 2 \times 10^{-3} \text{ m}$$

$$N = 1000$$

Change in surface energy $W = \sigma[N4\pi r^2 - 4\pi R^2]$ where $r = RN^{-1/3}$

$$\therefore W = \sigma[N^{1-3/2}4\pi R^2 - 4\pi R^2]$$

$$W = \sigma4\pi R^2[N^{1/3} - 1]$$

$$W = 0.07 \times 4 \times \frac{22}{7} \times (2 \times 10^{-3})^2 \times [(1000)^{1/3} - 1]$$

$$W = 0.07 \times 4 \times \frac{22}{7} \times 4 \times 10^{-6} \times 9$$

$$W = 31.68 \times 10^{-6} \text{ J}$$

21.

iii) When is the work done said to be zero?

iv) Give two examples from daily life where according to physics, work done is zero.

Ans:

i) When the force applied \vec{F} or the displacement \vec{s} or both are zero then work done $W = Fs \cos \theta$ is zero. Also when angle θ between \vec{F} and \vec{s} is 90° , $\cos \theta = \cos 90^\circ = 0$ therefore work done is zero.

ii) A person carrying a load on his head and standing at a place. A body moving in a circle.

22. A wood ball of density ρ is immersed in water of density σ to depth h and then released. Find the height H above the surface of water upto which the ball jumps out of water.

Ans

Let V be the volume of the ball. Weight of the ball = $V\rho g$

Upward thrust on the ball = $V\sigma g$

Effective upward thrust = $V\rho g - V\sigma g$

$$\text{Upward acceleration } a = \frac{V\rho g - V\sigma g}{V\rho} = \left(\frac{\sigma - \rho}{\rho} \right) g$$

Let v be the velocity of ball while reaching the surface of the water, after being released at depth h in water.

$$v^2 = 2as = 2x \left(\frac{\sigma - \rho}{\rho} \right) gh$$

For the motion of ball outside the water,

Kinetic Energy at the surface of water = Potential Energy at height H

$$\frac{1}{2}mv^2 = mgH$$

$$H = \frac{v^2}{2g} = \left(\frac{\sigma - \rho}{\rho} \right) h = \left(\frac{\sigma}{\rho} - 1 \right) h$$

23. Sara was afraid on going anywhere by air. But, she couldn't avoid going by an air. Her friend Hema who knew her problem was with her. Inside the plane Hema saw that Sara was very quiet and feeling uncomfortable. She tried to talk to Sara but she didn't answer. As the plane was about to take off, Hema started fighting with Sara without any cause for diverting her mind. While fighting Sara didn't realize that plane had taken off and now she was in air. She felt very happy to overcome her fear.

- iv) What values do you associate with Hema?
- v) An aeroplane takes off at an angle of 30° to the horizontal. If the component of its velocity along the horizontal is 250 km/h. what is the actual velocity? Find also the vertical component of velocity.
- vi) The blades of an aeroplane propeller are rotating at the rate of 600 revolutions per minute. Calculate its angular velocity?

Ans:

i) Hema is very understanding and helpful.

ii) $\theta = 30^\circ$

Horizontal component

$$A_x = 250 \text{ km/h}$$

Actual velocity $A = ?$

Vertical component $A_y = ?$

$$A_x = A \cos \theta$$

$$A = A_x / \cos \theta = 250 / \cos 30^\circ$$

$$A = 250 \times 2\sqrt{3}$$

$$A = 288.7 \text{ km/h}$$

$$A_y = A \sin \theta$$

$$A_y = 288.7 \times \sin 30^\circ$$

$$A_y = 288.7 \times \frac{1}{2}$$

$$A_y = 144.35 \text{ km/h}$$

iii) Here $v = 600$ revolution/ minute
 $= \frac{600}{60}$ revolution / minute
 $\omega = 2\pi v$
 $\omega = 2\pi \times 600 / 60$
 $\omega = 20\pi \text{ rad.s}^{-1}$

24. If a body executing linear SHM has a velocity of 3 cm s^{-1} when its displacement is 4 cm and a velocity of 4 cm s^{-1} when its displacement is 3 cm , then

(c) Find the amplitude and period of the oscillation

(d) Calculate the total energy of oscillation if the mass of the body is 50 g ,

Ans:

(a) In SHM the velocity V at a displacement x is given by

$$V = \omega(A^2 - x^2)^{1/2}$$

$$V^2 = \omega^2 (A^2 - x^2)$$

$$V = 3 \text{ cm s}^{-1} \text{ when } x = 4 \text{ cm.}$$

$$9 = \omega^2 (A^2 - 16) \text{ ----- (i)}$$

$$V = 4 \text{ cm s}^{-1} \text{ when } x = 3 \text{ cm}$$

$$16 = \omega^2 (A^2 - 9) \text{ ----- (ii)}$$

Simultaneous solution of equations (i) and (ii)

Amplitude $A = 5 \text{ m}$ and Angular frequency $\omega = 1 \text{ rad s}^{-1}$

$$\text{Hence time period } T = \frac{2\pi}{\omega} = 2\pi \text{ seconds} = 6.28 \text{ s}$$

(b) $m = 50 \text{ g} = 50 \times 10^{-3} \text{ kg}$, $A = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$, $\omega = 1 \text{ rad s}^{-1}$

$$\text{Total energy} = \frac{1}{2} mA^2\omega^2$$

$$= \frac{1}{2} \times (50 \times 10^{-3}) \times (5 \times 10^{-2})^2 (1)^2$$

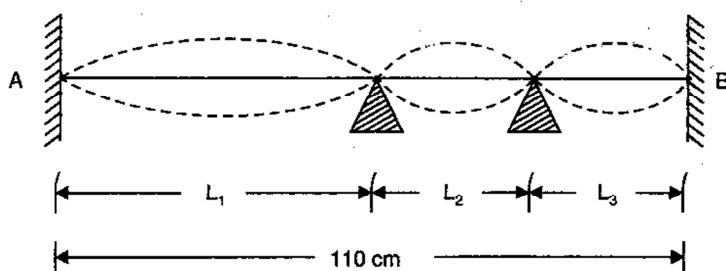
$$= 6.25 \times 10^{-5} \text{ J}$$

Or

If a sonometer wire of length 110 cm is stretched with a tension T and fixed at its ends is divided into three segments by placing two bridges below, then where the bridges should be placed so that the fundamental frequencies of the segments are in the ratio $1:2:3$?

Ans:

Let L_1 , L_2 and L_3 be the length of the segments of wire AB.



$$L_1 + L_2 + L_3 = 110 \text{ cm} \quad \text{----- (i)}$$

Let n_1 , n_2 and n_3 be their respective fundamental frequencies

$$n_1 = \frac{1}{2L_1} \sqrt{\frac{T}{m}}$$

$$n_2 = \frac{1}{2L_2} \sqrt{\frac{T}{m}}$$

$$n_3 = \frac{1}{2L_3} \sqrt{\frac{T}{m}}$$

$$\text{Hence } n_1 L_1 = n_2 L_2 = n_3 L_3 \quad \text{----- (ii)}$$

$$n_1 : n_2 : n_3 = 1 : 2 : 3$$

$$n_2 = 2n_1 \text{ and } n_3 = 3n_1 \quad \text{----- (iii)}$$

From (ii) and (iii) we get

$$L_1 = 2L_2 = 3L_3 \quad \text{----- (iv)}$$

Substituting (4) in (1) we get

$$L_1 + \frac{1}{2}L_1 + \frac{1}{3}L_1 = 110$$

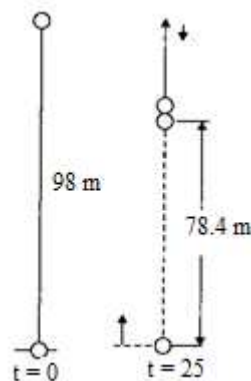
$$L_1 = 60 \text{ cm}$$

$$L_2 = 30 \text{ cm and } L_3 = 20 \text{ cm}$$

Thus the bridges should be placed at distance of 60 cm and 90 cm from A.

25. If a ball of mass 100 g is projected vertically upwards from the ground with a velocity of 49 m/s and at the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as followed by the first ball, then after sometime the two balls collide and stick together and finally fall together. Find the time of flight of the masses.

Ans:



Let the balls collide at an instant t seconds after they start their respective motion. Clearly the two balls are at the same height above the ground at that instant.

The height of the first ball after t seconds = $49t - \frac{1}{2} \times 9.8t^2 = 4.9t(10 - t)$

The height of the second ball after t seconds = $98 - \text{downward moved in } t \text{ seconds}$

$$= 98 - \frac{1}{2} \times 9.8t^2$$

$$= 4.9(20 - t^2)$$

$$4.9t(10 - t) = 4.9(20 - t^2)$$

$$10t - t^2 = 20 - t^2$$

$$t = 2\text{s}$$

The balls are colliding two seconds after the start of their motion. Their velocities at that instant are

$$\begin{aligned} \text{First ball: } v_1 &= (49 - 9.8 \times 2) \text{ m/s} \\ &= 29.4 \text{ m/s directed upwards} \end{aligned}$$

$$\begin{aligned} \text{Second ball: } v_2 &= (0 + 9.8 \times 2) \text{ m/s} \\ &= 19.6 \text{ m/s directed downwards} \end{aligned}$$

If v is the velocity of the combined mass of the two balls after they stick together following their collision, we have by principle of conservation of momentum

$$200 \times v = 100 \times 29.4 - 100 \times 19.6$$

$$v = 4.9 \text{ m/s}$$

The combined mass moves forward after collision with the velocity of 439 m/s. its height above the ground at this instant

$$(98 - \frac{1}{2} \times 9.8 \times 2^2)m = (98 - 19.6) m = 78.4 m$$

Now, to find the time t' taken by the combined mass of the two balls to fall to ground,

$$\text{Combined mass } u = 4.9 \text{ m/s}$$

$$s = -78.4 \text{ m}$$

$$a = -g = -9.8 \text{ ms}^{-2}$$

$$-78.4 = 4.9 t' + \frac{1}{2} (-9.8) t'^2$$

$$t'^2 - t' - 16 = 0$$

$$t' = \frac{1 \pm \sqrt{1+64}}{2} = \frac{1 \pm 8.06}{2}$$

$$t' = 4.532 \text{ s}$$

The combined mass thus takes 4.53 s to fall to the ground. Since the balls collided 2s after they started their motion, their total time of flight is $(2 + 4.53) \text{ s} = 6.53 \text{ s}$

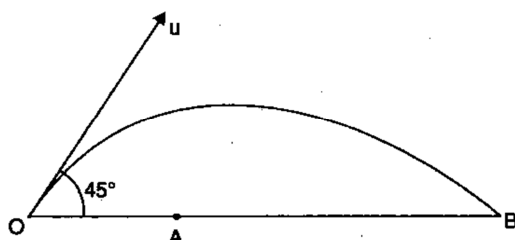
Or

A gun kept on a straight horizontal road to hit a car traveling along the same road away from the gun with a uniform speed of 72 km/h. The car is at a distance of 500 m from the gun when the gun is fired at an angle of 45° to the horizontal. Find

- (iii) The distance of the car from the gun when the shell hits it
- (iv) The speed of projection of the shell from the gun

Ans:

The gun and the card are at O and A at $t=0$



Let at $t=t$, the shell and the car reach B simultaneously so that the shell hits the car when it is at a distance OB from the gun.

Let u be the speed of projection of the shell from the gun. Then the initial horizontal component of the velocity of the shell $= u \cos 45^\circ = \frac{u}{\sqrt{2}}$ and the initial vertical component of the velocity of the shell $= u \sin 45^\circ = \frac{u}{\sqrt{2}}$

$$\text{Time of flight of the shell} = \frac{2(u / \sqrt{2})}{g} = \sqrt{2}(u / g)$$

The car takes this time to cover the distance AB while the shell covers the distance OB in this time

$$OB = OA + AB = 500 + AB$$

$$OB = \frac{u}{\sqrt{2}} \cdot \frac{\sqrt{2}u}{g} = \frac{u^2}{g}$$

$$AB = 20 \times \sqrt{2} \left(\frac{u}{g} \right) = 20\sqrt{2} \frac{u}{g}$$

$$\frac{u^2}{g} = 500 + 20\sqrt{2} \frac{u}{g}$$

$$u^2 - 20\sqrt{2}u - 4900 = 0$$

$$u = \frac{20\sqrt{2} \pm \sqrt{400 \times 4 + 4 \times 4900}}{2} \text{ ms}^{-1}$$

$$u = (10\sqrt{2} \pm \sqrt{5300}) \text{ ms}^{-1}$$

$$u = 10 \left[\sqrt{2} \pm \sqrt{53} \right] \text{ ms}^{-1}$$

$$u = 10 \left[1.414 + 7.280 \right] \text{ ms}^{-1} = 86.94 \text{ ms}^{-1}$$

This is the speed of projection of the shell from the gun. The distance of the car from the gun when the shell hits it is OB where

$$OB = \frac{u^2}{g} = \frac{(86.94)^2}{9.8} \text{ m} = 771.3 \text{ m}$$

26. An object starts from rest and covers total distance X in the below manner:

It first has uniform acceleration a_1 from some time t_1 moves with the speed acquired at the end of t_1 for some distance and is then given a uniform retardation a_2 so that it is again at rest at the end of the journey. Explain that the journey is covered in least time if this body is accelerated

For a time $\left[\frac{2Xa_2}{a_1(a_1 + a_2)} \right]^{\frac{1}{2}}$ and this minimum time is $\left[2x \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \right]^{\frac{1}{2}}$

Ans:

Let x_1 be the distance travelled by the object in t_1 second then v , the speed acquired after travelling distance x_1 . $v = a_1 t_1$ ----- (i)

$$2a_1 t_1 = v^2 - 0^2 = v^2$$

$$x_1 = v^2 / 2a_1 \text{ ----- (ii)}$$

let x_2 and x_3 are distance travelled in the second and third leg of the journey of the particle extending over the time t_2 and t_3 .

$$x_2 = v t_2 \text{ ----- (iii)}$$

$$-2a_2 x_3 = 0^2 - v^2 = -v^2 \text{ ----- (iv)}$$

$$0 = v - a_2 t_3$$

$$V = a_2 t_3 \text{ ----- (v)}$$

The total time t of the journey,

$$t = t_1 + t_2 + t_3 = \frac{v}{a_1} + \frac{x_2}{v} + \frac{v}{a_2} \text{ ----- (vi)}$$

$$X = x_1 + x_2 + x_3 = \frac{v^2}{2a_1} + x_2 + \frac{v^2}{2a_2}$$

$$x_2 = X - \frac{v^2}{2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \text{ ----- (vii)}$$

From the equation (vi) and (vii)

$$t = \frac{v}{a_1} + \frac{X}{v} - \frac{v}{2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) + \frac{v}{a_2}$$

$$t = \frac{X}{v} + \frac{v}{2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \text{ ----- (viii)}$$

Using equation (viii) and (i)

$$t = \frac{X}{a_1 t_1} + \frac{a_1 t_1}{2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \text{ ----- (ix)}$$

For a particular value of X , t is least

$$\frac{dt}{dt_1} = 0 \quad \text{----- (x)}$$

Differentiating equation (ix) we get

$$\frac{dt}{dt_1} = -\frac{X}{a_1 t_1^2} + \frac{a_1}{2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) = 0$$

$$\frac{X}{a_1 t_1^2} = \frac{a_1 (a_1 + a_2)}{2a_1 a_2}$$

$$t_1 = \left[\frac{X \cdot 2a_2}{a_1 (a_1 + a_2)} \right]^{\frac{1}{2}}$$

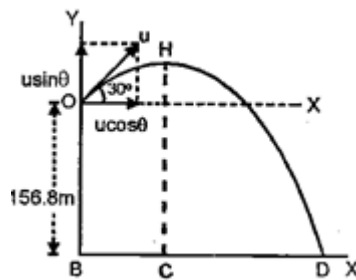
Corresponding to these values of t_1 we get

$$t = \left[2X \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \right]^{\frac{1}{2}}$$

Or

From the top of the tower 156.8 m high a projectile is thrown up with velocity of 39.2 ms⁻¹, making an angle 30° with horizontal direction. Find the distance from the foot of tower where it strikes the ground and the time taken by it .

Ans:



The height of the tower OB = 156.8 m; $u = 39.2 \text{ m/s}$; $\theta = 30^\circ$

Component of the velocity along OX = $u \cos \theta = 39.2 \cos 30^\circ = 33.947 \text{ ms}^{-1}$

Component of the velocity along OY = $u \sin \theta = 39.2 \sin 30^\circ = 19.6 \text{ ms}^{-1}$

Let 't' be the total time of flight. Since the vertical downward direction OB is the positive direction of y axis. Taking motion of a projectile from O to D along Y axis

$$y_0 = 0, y = 156.8 \text{ m}$$

$$u_y = -u \sin 30^\circ = -19.6 \text{ m/s}$$

$$a_y = 9.8 \text{ m/s}^2, t = t$$

$$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

$$156.8 = 0 + (-19.6) t + \frac{1}{2} \times 9.8 \times t^2$$

$$156.8 = -19.6t + 4.9 t^2$$

$$4.9t^2 - 19.6t - 156.8 = 0$$

$$t^2 - 4t - 32 = 0$$

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t-8) + 4(t-8) = 0$$

$$(t+4)(t-8) = 0$$

$$t = -4 \text{ or } 8$$

$$t = -4 \text{ s is not possible}$$

$$t = 8 \text{ s}$$

Distance from the foot of tower where it strikes the ground,

$$BD = u \cos 30^\circ \times t = 30.947 \times 8$$

$$= 271.57 \text{ m}$$
