CUET Mathematics Solved Paper-2023

Held on 21 May 2023, (Shift-III)

SECTION : COMMON

1. $\int \frac{x^3}{x+1} dx$ is equal to

(a) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1+x| + C$

(b) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1-x| + C$

(c) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1+x| + C$

(d) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1-x| + C$

2. If $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 2 & -4 \\ 0 & 2 & 9 \end{bmatrix}$ then A^{-1} is equal to

(a) $\begin{bmatrix} 2 & -2 & 4 \\ 0 & 2 & 9 \\ -1 & 1 & -2 \end{bmatrix}$ (b) does not exist

(c) $\begin{bmatrix} 0 & 2 & 9 \\ 1 & -1 & 2 \\ -2 & 2 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -2 & 0 \\ -1 & 2 & 2 \\ 2 & -4 & 9 \end{bmatrix}$

3. The derivative of $f(x) = 2|x+3|$ at $x = -2$ is

(a) -2 (b) 2

(c) -1 (d) 1

4. A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$, then $P(B \cap A')$ is equal to :

(a) 0.2 (b) 0.1

(c) 0.3 (d) 0.4

5. The feasible region represented by the system of inequalities $x + y \le 3, y \le 6, x \ge 0, y \ge 0$ is:

(a) Unbounded in first quadrant

(b) Bounded in first quadrant

(c) Unbounded in first and second quadrant

(d) Bounded in first and second quadrant

6. If A and B are two matrices of orders $3 \times p$ and $3 \times q$ respectively and $p = q$, then the order of $(2A - 5B)$ is:

(a) $3 \times q$ (b) $p \times q$

(c) 3×3 (d) $p \times 3$

7. The area (in sq. units) bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is:

(a) 8π (b) 20π

(c) 16π (d) 256π

8. The function $y = (x-9)^2$ is strictly increasing in the intervals

(a) $(-\infty, 3)$ (b) $(9, \infty)$

(c) $(-\infty, 3) \cup (9, \infty)$ (d) $(3, 9)$

9. Trials of a random experiment are called Bernoulli trials, if they satisfy certain conditions. Which is the correct condition?

A. There should be infinite number of trials.

B. The trials should be independent.

C. Each trial has exactly two outcomes: success and failure.

D. The probability of success changes in each trial.

Choose the correct answer from the options given below:

(a) A and B only (b) B and D only

(c) A and C only (d) B and C only

10. Solution of the differential equation $\frac{dy}{dx} + ay = e^{mx}$ is:

(a) $(a+m)y = e^{mx} + Ce^{-ax}$

(b) $y = e^{mx} + Ce^{-ax}$

(c) $(a+m)y = me^{mx} + C$

(d) $y.e^{ax} = me^{mx} + C$

11. If A is a square matrix of order 3×3 such that $|A| = 4$, then $|-3A|$ is equal to:

(a) 12 (b) -12

(c) -108 (d) 108

12. If $A = \begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then k is equal to:

(a) $\frac{1}{19}$ (b) -19

(c) 19 (d) $\frac{1}{19}$

13. The order and degree of the differential equation of the family of curves $y = a(x + b)$ where, a, b are arbitrary constants, are respectively.

(a) 1,2 (b) 2,1
(c) 1,1 (d) 2,2

14. The set of values of decision variables that satisfy the linear constraints and non-negative conditions of a linear programming problem is called its:
(a) Unbounded solution
(b) Bounded solution
(c) Feasible solution
(d) Optimum solution

15. The point on the curve $y = -x^2 + 12x + 5$ where the tanget is parallel to the x-axis is:
(a) $(-6, -103)$ (b) $(6, 41)$
(c) $(0, 5)$ (d) $(6, 103)$

SECTION : CORE MATHEMATICS

1. The function $f(x) = 3x^2$ is:
(a) increasing in $(-\infty, -1) \cup (1, \infty)$ and decreasing in $(-1, 1)$
(b) decreasing in $(-\infty, -1) \cup (1, \infty)$ and increasing in $(-1, 1)$
(c) increaing in $(0, \infty)$ and decreasing in $(-\infty, 0)$
(d) decreaing in $(0, \infty)$ and increasing in $(-\infty, 0)$

2. If $\cos^{-1} \alpha + \cos^{-1} \beta + \cos^{-1} \gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ equals:
(a) 0 (b) 1
(c) 6 (d) 12

3. If sin x is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$ then P is:
(a) tan x (b) cos x
(c) cot x (d) log sin x

4. The corner points of the feasible region for an LPP are (0, 10), (5, 5), (15, 15) and (0, 20). If the objective function is $Z = px + qy$; $p, q > 0$, then the condition on p and q so that the maximum of Z occurs at (15,15) and (0, 20) is:
(a) $p = q$ (b) $p = 2q$
(c) $q = 2p$ (d) $q = 3p$

5. If $A = \begin{bmatrix} 1 & 3 & 9 \\ 1 & x & x^2 \\ 4 & 6 & 9 \end{bmatrix}$ is singular, then x is equal to
(a) 3 (b) 3 or 6
(c) 3 or $\frac{3}{2}$ (d) -3 or $\frac{3}{2}$

6. In a Linear Programming Problem:
(a) The constraints are linear, in any objective function.
(b) The constraints and objective function both are linear.
(c) Constraints are linear or quadratic and objective function is linear.
(d) Only one of the constraints or the objective function is linear.

7. Area bounded by the curve $y = x^3$, the x-axis and the abscissa $x = -2$ and $x = 3$ is:
(a) $\frac{97}{4}$ (b) $-\frac{65}{4}$
(c) $\frac{65}{4}$ (d) -97

8. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$, and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals
(a) -4 (b) -2
(c) 0 (d) 1

9. If $A = R - \{+1\}$ and funcion $f : A \to A$ is defined by $f(x) = \frac{x+1}{x-1}$, then $f^{-1}(x)$ is given by:
(a) $\frac{1}{x-1}$ (b) $\frac{1}{1-x}$
(c) $\frac{x+1}{1-x}$ (d) $\frac{x+1}{x-1}$

10. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}\alpha$ with x- axis. The value of α is equal to:
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{7}$
(c) $\frac{3}{7}$ (d) $\frac{\sqrt{2}}{3}$

11. Let A and B be two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$. Which of the following is correct?
A. $P(A|B)$ is $\frac{2}{5}$
B. $P(A'|B)$ is $\frac{4}{5}$
C. $P(A|B).P(A'|B)$ is $\frac{8}{25}$
D. $P(B'|A)$ is $\frac{1}{3}$

Choose the correct answer from the options given below:
(a) B and C only (b) B and D only
(c) A and D only (d) A and C only

12. Area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ is:
(a) $\frac{16}{3}$ (b) 16
(c) $\frac{8}{3}$ (d) $\frac{32}{3}$

13. The derivative of $\sin^3(\cos x^2)$ with respect to x is:
(a) $-6x\sin^2(\cos x^2)\sin x^2$
(b) $-6x\sin^2(\cos x^2)\sin x$
(c) $-6x\sin^3(\cos x^2)\sin x^2$
(d) $-6x\sin^3(\cos x^2)$

14. $\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt{\cot x}}dx$ is equal to:
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{12}$ (d) $\frac{\pi}{2}$

15. The order and degree of the differential equation $\sqrt{\sin x}\,(dx+dy)=\sqrt{\cos x}\,(dx-dy)$ is:
(a) 2, 1 (b) 2, 2
(c) 1, 2 (d) 1, 1

16. If $x=a\cos\theta$, $y=b\sin\theta$, then $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$ is:
(a) $-\frac{b}{a}$ (b) $-\frac{b}{a^2}$
(c) $\frac{a}{b}$ (d) $\frac{a}{b^2}$

17. Match List-I with List-II

List-I	List-II
A. $f: R\to R$, $f(x)=[x]$	I. Signum function
B. $f: R\to R$, $f(x)=\|x\|$	II. Identity function
C. $f: R\to R$, $f(x)=x$	III. Greatest Integer function
D. $f: R\to R$, $f(x)=\begin{cases}-1 & \text{if } x<0\\ 0 & \text{if } x=0\\ 1 & \text{if } x>0\end{cases}$	IV. Modulus function

Choose the correct answer from the options given below:
(a) A-I, B-III, C-II, D-IV
(b) A-II, B-I, C-III, D-IV
(c) A-II, B-IV, C-I, D-III
(d) A-III, B-IV, C-II, D-I

18. If $A=\begin{bmatrix}2+x & y\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}3+y & x-1\\ 2 & 10\end{bmatrix}$ such that $B=2A$, then values of x and y are respectively:
(a) 1, 1 (b) 1, –1
(c) –1, –1 (d) –1, 1

19. If the matrix $A=\begin{bmatrix}0 & 2 & c\\ a & b & 1\\ -3 & -1 & 0\end{bmatrix}$ is a skew - symmetric matrix, then a + b + c is equal to:
(a) –5 (b) 5
(c) 1 (d) –1

20. Match the following elements of $\begin{bmatrix}1 & -1 & 0\\ 0 & 4 & 2\\ 3 & -4 & 6\end{bmatrix}$ with their co-factors.

List-I	List-II
A. 6	I. 6
B. 3	II. 1
C. 2	III. 4
D. –1	IV. –2

Choose the correct answer from the options given below:
(a) A-III, B-IV, C-II, D-I
(b) A-I, B-IV, C-II, D-III
(c) A-II, B-I, C-III, D-IV
(d) A-III, B-II, C-I, D-IV

21. Rolle's theorem holds for the function $f(x)=x^3+\alpha x^2+\beta x$, $1\le x\le 2$ at the point $\frac{4}{3}$, the values of α and β are:
(a) $\alpha=5, \beta=-8$ (b) $\alpha=-5, \beta=-8$
(c) $\alpha=8, \beta=-5$ (d) $\alpha=-5, \beta=8$

22. If A is square matrix of order 3 and B = –2A such that $|B|=k|A|$ then the value of k is
(a) 8 (b) 4
(c) 16 (d) –8

23. If A is a $m\times n$ matrix and B is $n\times p$ matrix and $m\neq p$ then,
(a) $(AB)'=B'A'$ (b) $(BA)'=A'B'$
(c) $(AB)'=A'B'$ (d) $(BA)'=B'A'$

24. If $3\tan^{-1}x+\cot^{-1}x=\pi$, then x equals to:
(a) –1 (b) 1
(c) $\frac{1}{2}$ (d) 0

25. Probability that A speaks truth is $\frac{3}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was a head is:
(a) $\frac{3}{5}$ (b) $\frac{4}{5}$
(c) $\frac{1}{5}$ (d) $\frac{1}{3}$

26. The function $f(x)=2x^3-3x^2-12x+4$ has:
(a) two points of local maximum
(b) two points of local minimum
(c) one maximum and one minimum
(d) no maximum and no minimum

27. If $f(x)=\begin{cases}\dfrac{\sqrt{x^2+5}-3}{x+2}, & x\neq -2\\ k, & x=-2\end{cases}$ is continuous at $x=-2$

then the value of k is :

(a) $-\frac{2}{3}$ (b) 0

(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

28. Given equation of line $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}$ and equation of a plane $2x-y+3z=1$. Which of the following is correct?

(A) Given line and plane intersect at (10,10,–3).
(B) The line passes through the point (–1, 2, –3).
(C) Direction ratio's of normal to the plane are –2, –1, –3.
(D) The line is parallel to the vector $3\hat{i}+4\hat{j}-2\hat{k}$.

Choose the correct answer from the options given below:
(a) A and C only (b) A and D only
(c) B and D only (d) C and D only

29. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors, then which of the following is correct?

A. $\vec{a}\times\vec{b}=\vec{0}$ if and only if $\vec{a}$ is perpendicular to $\vec{b}$.

B. $\vec{a}\times\vec{b}$ is vector.

C. $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\cos\theta\hat{n}$

D. If $\theta=\frac{\pi}{2}, |\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|$

Choose the correct answer from the options given below:
(a) A and B only (b) A and D only
(c) B and C only (d) B and D only

30. Match List I with List II

List I	List II
A. $\int_{-\pi/2}^{\pi/2}\sin^7 x\,dx$	I. π
B. $\int_{-\pi/2}^{\pi/2}(x\cos x+1)\,dx$	II. $\frac{\pi}{12}$
C. $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	III. 0
D. $\int_1^{\sqrt{3}}\frac{dx}{1+x^2}$	IV. $\frac{\pi}{4}$

Choose the correct answer from the options given below:
(a) A-II, B-III, C-IV, D-I
(b) A-III, B-I, C-IV, D-II
(c) A-I, B-II, C-III, D-IV
(d) A-IV, B-I, C-II, D-III

31. If $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x, (x>0)$, then value of x is:

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

32. The minimum value of $x\log_e x$ is equal to

(a) e (b) $\frac{1}{e}$

(c) $-\frac{1}{e}$ (d) 2e

33. The area of the region $\{(x,y): x^2+y^2\le 1\le x+y\}$ is:

(a) $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ (b) $\left(\frac{\pi}{4}+\frac{1}{2}\right)$

(c) $\left(\frac{\pi}{2}-\frac{1}{2}\right)$ (d) $\left(\frac{\pi}{2}+\frac{1}{2}\right)$

34. The area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$ is:

(a) $2\sqrt{5}$ (b) 15

(c) $15\sqrt{2}$ (d) 2

35. The equation of the plane passing through the point (–1, 3, 2) and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$ is:

(a) $7x+8y-3z+25=0$
(b) $7x-8y+3z-25=0$
(c) $7x+8y-3z-25=0$
(d) $7x-8y+3z+25=0$

Hints & Explanations

1. (a) Let $I = \int \frac{x^3}{x+1} dx.$

$= \int \frac{x^3+1-1}{x+1} dx = \int \frac{x^3+1}{x+1} dx - \int \frac{1}{x+1} dx.$

$= \int (x^2+1-x)\,dx - \ln|x+1| + C$

$= \frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1| + C.$

2. (b) $\because A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 2 & -4 \\ 0 & 2 & 9 \end{bmatrix}$

$\Rightarrow |A| = 1(18+8) + 1(-18+0) + 2(-4+0)$

$\Rightarrow |A| = 0$

$\therefore$ Inverse of matrix does not exist.

3. (a) Here, $f(x) = 2|x+3|$

Then, $f(x) = \begin{cases} 2(x+3) & x \geq -3 \\ -2(x+3) & x < -3 \end{cases}$

$\therefore \quad f'(x) = \begin{cases} 2 & x > -3 \\ -2 & x < -3 \end{cases}$

at, $x = -2, f'(x) = -2$

$\Rightarrow$ Derivative of $f(x)$ at $x = -2$ is -2.

4. (b) Given $P(A) = 0.4, P(B) = 0.3$ and $P(A \cup B) = 0.5$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow P(A \cap B) = 0.4 + 0.3 - 0.5 = 0.2$

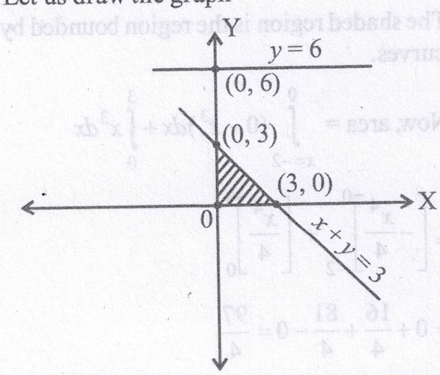
Now, $P(B \cap A') = P(B) - P(B \cap A)$

$= 0.3 - P(A \cap B) = 0.3 - 0.2 = 0.1$

5. (b) The given sysem of inequalities are

$x + y \leq 3, y \leq 6, x \geq 0, y \geq 0$

Let us draw the graph



The shaded region represent the feasible region, which is bounded in the first quadrant.

6. (a) $\therefore$ Order of matrix $A = 3 \times p$

Order of matrix $B = 3 \times q$

So, Order of $2A = 3 \times p = 3 \times q \quad \{\because p = q\}$

and Order of $5B = 3 \times q$

$\therefore$ Order of $(2A - 5B) = 3 \times q$

7. (a) Area bounded by the curve $y = \sqrt{16-x^2}$ and x-axis is

$A = \int_{x=-4}^{4} y\,dx = \int_{-4}^{4} \sqrt{16-x^2}\,dx.$

$= 2\int_0^4 \sqrt{16-x^2}\,dx.$

$= 2.\frac{1}{2}\left[x\sqrt{16-x^2} + 16\sin^{-1}\frac{x}{4}\right]_0^4$

$= \left[4\sqrt{16-16} + 16\sin^{-1}\frac{4}{4} - 0 - 16\sin^{-1} 0\right]$

$= 16 \times \frac{\pi}{2} = 8\pi$

8. (b) $\because y = (x-9)^2 \Rightarrow y' = 2(x-9)$

For strictly increasing function

$y' > 0 \Rightarrow 2(x-9) > 0$

$\Rightarrow x - 9 > 0 \Rightarrow x > 9$

$\Rightarrow x \in (9, \infty)$

9. (d) The trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :

(A) There should be a finite number of trials

(B) The trials should be independent.

(C) Each trial has exactly two outcomes: success or failure.

(D) The brobability of success remains the same in each trials.

10. (a) $\because \quad \frac{dy}{dx} + ay = e^{mx}.$

Which is a linear differntial equation; whose integrating factor is

$\text{I.F} = e^{\int a dx} = e^{ax}.35$

Now, the solution is

$y \cdot e^{ax} = \int e^{mx} \cdot e^{ax}\,dx + C_1$

$\Rightarrow y \cdot e^{ax} = \int e^{(m+a)x}\,dx + C_1.$

$\Rightarrow y \cdot e^{ax} = \dfrac{e^{(m+a)x}}{m+a} + C_1$

$\Rightarrow (m+a)y = e^{mx+ax-ax} + C_1(m+a)e^{-ax}.$

$\Rightarrow (m+a)y = e^{mx} + ce^{-ax}$ where $C = C_1(m+a)$

11. (c) $\because$ A is square matix of order 3×3 such that $|A| = 4$

Then $|-3A| = (-3)^3 |A| = -27 \times 4 = -108$

12. (d) $\because A = \begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix}$

So, $\text{Adj}(A) = \begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix} = -1\begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix} = -A.$

Now, $A^{-1} = \dfrac{1}{|A|} Adj(A) = \dfrac{1}{(-4-14)}(-A) = \dfrac{1}{19} A$ ...(1)

Given: $A^{-1} = kA$ ...(2)

From equations (1) and (2): $k = \dfrac{1}{19}$

13. (b) $\because$ $y = a(x+b) = ax + ab$

Then, $y^1 = a$

$\Rightarrow$ $y^{11} = 0$

which is required differential equation, whose degree = 1, order = 2.

14. (c) The set of values of decision variable that satisfy the linear constraints and non-negativity conditions of a linear programming problem is called its feasible solution.

15. (b) $\because$ $y = -x^2 + 12x + 5$

$\Rightarrow y' = -2x + 12$

$\because$ Tangent is parallel to x–axis

$\Rightarrow y' = 0$

$\Rightarrow -2x + 12 = 0 \Rightarrow x = 6$

when $x = 6$; $y = -6^2 + 12 \times 6 + S = 36 + S = 41$

So, the required point is (6,41).

Section : Core Mathematics

1. (c) $f(x) = 3x^2 \Rightarrow f'(x) = 6x$

For increasing function : $f'(x) > 0$

$\Rightarrow 6x > 0 \Rightarrow x > 0$

$\therefore$ $f(x)$ is increasing in $(0, \infty)$

For decreasing function : $f'(x) < 0$

$\Rightarrow 6x < 0 \Rightarrow x < 0$

$f(x)$ is decreasing in $(-\infty, 0)$

2. (c) Given: $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

Since, $0 \le \cos^{-1}x \le \pi$

$\therefore$ $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$ is possible if and only if

$\cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi$

$\Rightarrow \alpha = \beta = \gamma = -1.$

$\therefore$ $\alpha(\beta+\gamma) + \beta(\alpha+\gamma) + \gamma(\alpha+\beta)$

$= (-1)(-1-1) + (-1)(-1-1) + (-1)(-1-1)$

$= 2 + 2 + 2 = 6$

3. (d) $\because$ Integrating factor $= \sin x$

$e^{\int p dx} = \sin x$

$\int p dx \Rightarrow \log(\sin x)$

Differentiating both sides we get :

$p = \dfrac{\cos x}{\sin x} = \cot x$

4. (d) Since maximum of Z occurs at (15, 15) and (0, 20)

$\therefore$ $15p + 15q = 0p + 20q.$

$\Rightarrow 15p = 5q \Rightarrow q = 3p$

5. (c) Since, $A = \begin{bmatrix} 1 & 3 & 9 \\ 1 & x & x^2 \\ 4 & 6 & 9 \end{bmatrix}$

$\because$ A is singular $\Rightarrow \det(A) = 0$

$\Rightarrow 1(9x - 6x^2) - 1(27 - 54) + 4(3x^2 - 9x) = 0$

$\Rightarrow 9x - 6x^2 + 27 + 12x^2 - 36x = 0$

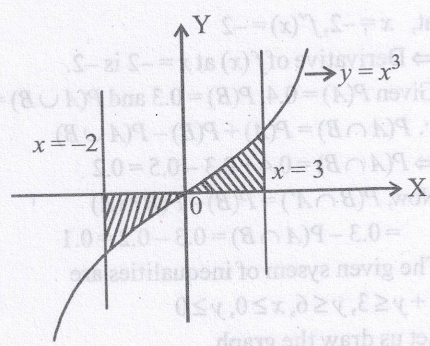
$\Rightarrow 6x^2 - 27x + 27 = 0 \Rightarrow 2x^2 - 9x + 9 = 0$

$\Rightarrow 2x^2 - 6x - 3x + 9 = 0 \Rightarrow (2x-3)(x-3) = 0$

$\Rightarrow x = \dfrac{3}{2}$ or 3

6. (b) In a linear programming problem, the constraints and objective function both are linear.

7. (a) Let us draw the given region



The shaded region is the region bounded by the given curves.

Now, area $= \int_{x=-2}^{0} (0 - x^3)dx + \int_0^3 x^3 dx$

$= \left[-\dfrac{x^4}{4}\right]_{-2}^{0} + \left[\dfrac{x^4}{4}\right]_0^3$

$= 0 + \dfrac{16}{4} + \dfrac{81}{4} - 0 = \dfrac{97}{4}$

8. (b) $\because$ $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$

$\Rightarrow$ $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1(1-2(x-2))-1(-1-2x)+1(x-2+x)=0$$
$$\Rightarrow 1-2x+4+1+2x+2x-2=0$$
$$\Rightarrow 2x+4=0 \Rightarrow x=-2$$

9. (d) $\because f(x)=\dfrac{x+1}{x-1}$

Let $y=f(x)$

$$\Rightarrow y=\frac{x+1}{x-1} \Rightarrow xy-y=x+1$$

$$\Rightarrow x(y-1)=y+1 \Rightarrow x=\frac{y+1}{y-1}$$

$$\Rightarrow f^{-1}(y)=\frac{y+1}{y-1} \qquad \{\because y=f(x)\Rightarrow x=f^{-1}(y)\}$$

$$\Rightarrow f^{-1}(x)=\frac{x+1}{x-1}$$

10. (b) Given plane is: $2x-3y+6z-11=0$ ...(1)

Equation of x-axis: $\dfrac{x-0}{1}=\dfrac{y}{0}=\dfrac{z}{0}$ ...(2)

Angle between line (2) and plane (1) $\sin^{-1}a$.

$$\Rightarrow \sin^{-1}\left(\frac{2\times 1-3\times 0+6\times 0}{\sqrt{2^2+3^2+6^2}\sqrt{1^2+0+0}}\right)=\sin^{-1}a$$

$$\Rightarrow a=\frac{2}{7}$$

11. (c) $P(A)=\dfrac{3}{8}, P(B)\dfrac{5}{8}=$ and $P(A\cup B)=\dfrac{3}{4}$

$$P(A\cap B)=P(A)+P(B)-P(A\cup B)$$

$$P(A\cap B)=\frac{3}{8}+\frac{5}{8}-\frac{3}{4}=\frac{1}{4}$$

$$\therefore P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{4}}{\frac{5}{8}}=\frac{2}{5}.$$

$$P(A'|B)=\frac{P(A'\cap B)}{P(B)}=\frac{P(B)-P(A\cap B)}{P(B)}=\frac{\frac{5}{8}-\frac{1}{4}}{\frac{5}{8}}$$

$$\Rightarrow P(A'|B)=\frac{3}{5}$$

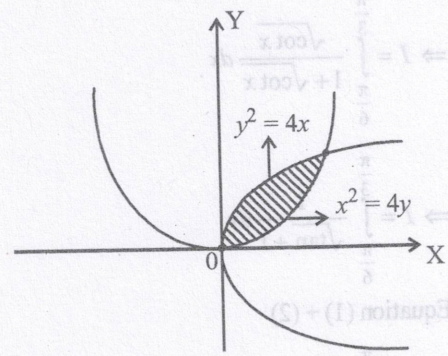
Now, $P(A|B).P(A'|B)=\dfrac{2}{5}\times\dfrac{3}{5}=\dfrac{6}{25}$

$$P(B'|A)=\frac{P(B'\cap A)}{P(A)}=\frac{P(A)-P(A\cap B)}{P(A)}$$

$$\Rightarrow P(B'|A)=\frac{\frac{3}{8}-\frac{1}{4}}{\frac{3}{8}}=\frac{1}{3}$$

$\therefore$ Only (a) and (d) are correct.

12. (a) For intersection point, let us solve $y^2=4x$ and $x^2=4y$



$$\therefore \left(\frac{x^2}{4}\right)^2=4x \Rightarrow x^4=16\times 4x.$$

$$\Rightarrow x=0,\ x^3-64=0$$
$$\Rightarrow x=4$$

$\therefore$ Point are (0,0), (4,4)

So, the required area is

$$A=\int_{x=0}^{4}(y_1-y_2)\,dx=\int_0^4\left(\sqrt{4x}-\frac{x^2}{4}\right)dx.$$

$$=\left[2\cdot\frac{2}{3}x^{3/2}-\frac{x^3}{12}\right]_0^4$$

$$=\frac{4}{3}\cdot(4)^{3/2}-\frac{64}{12}-0$$

$$=\frac{32}{3}-\frac{64}{12}=\frac{128-64}{12}=\frac{64}{12}=\frac{16}{3}$$

13. (Bonus) Let $y=\sin^3(\cos x^2)$

Then, $y'=3\sin^2(\cos x^2)\cdot\cos(\cos x^2)\cdot(-\sin x^2)\cdot(2x)$

$\Rightarrow y'=-6x\sin^2(\cos x^2)\cdot\cos(\cos x^2)\cdot\sin x^2$

14. (c) Let $I=\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{1+\sqrt{\cot x}}dx.$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx \quad \text{...(1)}$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}\,dx$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sqrt{\cot x}}{1+\sqrt{\cot x}}\,dx$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{\sqrt{\tan}+1}\,dx \quad \text{...(2)}$$

Equation (1) + (2)

$$\Rightarrow 2I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}1\cdot dx=[x]_{\pi/6}^{\pi/3}=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$$

$$\Rightarrow I=\frac{\pi}{12}$$

15. (d) Since, $\sqrt{\sin x}\,(dx+dy)=\sqrt{\cos x}\,(dx-dy)$

$$\Rightarrow \sqrt{\tan x}\left(1+\frac{dy}{dx}\right)=1-\frac{dy}{dx}.$$

$\therefore$ Order = 1, degree = 1

16. (b) $\because x=a\cos\theta \Rightarrow \dfrac{dx}{d\theta}=-a\sin\theta.$

and $y=b\sin\theta \Rightarrow \dfrac{dx}{d\theta}=b\cos\theta.$

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{b\cos\theta}{-a\sin\theta}=-\frac{b}{a}\cot\theta$$

Now, $\dfrac{d^2y}{dx^2}=\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=\dfrac{d}{d\theta}\left(\dfrac{dy}{dx}\right)\cdot\dfrac{d\theta}{dx}$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{-\frac{b}{a}(-\text{cosec}^2\theta)}{-a\sin\theta}=-\frac{b}{a^2}\text{cosec}^3\theta$$

At; $\theta=\dfrac{\pi}{2},\ \dfrac{d^2y}{dx^2}=-\dfrac{b}{a^2}\text{cosec}^3\dfrac{\pi}{2}=-\dfrac{b}{a^2}$

17. (d) (A) $f:R\to R, f(x)=[x]\to$ Greatest Integer function.

(B) $f:R\to R, f(x)=|x|\to$ Modulus function.

(C) $f:R\to R, f(x)=x\to$ Indentify function.

(D) $f:R\to R, f(x)=\begin{cases}-1 & \text{if } x<0\\ 0 & \text{if } x=0\\ 1 & \text{if } x>0\end{cases}$

$\to$ Signum function

18. (c) $\because A=\begin{bmatrix}2+x & y\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}3+y & x-1\\ 2 & 10\end{bmatrix}$

Also, $B=2A$.

$$\Rightarrow \begin{bmatrix}3+y & x-1\\ 2 & 10\end{bmatrix}=\begin{bmatrix}4+2x & 2y\\ 2 & 10\end{bmatrix}$$

Comparing both sides, we get.

$3+y=4+2x \Rightarrow y=1+2x$ ...(1)

and $x-1=2y$

$\Rightarrow x-1=2(1+2x)$ {from equation (1)}

$\Rightarrow 3x=-3 \Rightarrow x=-1.$

$\therefore\ y=1+2(-1)=-1$

So, we have, the values of x and y are respectively $-1, -1$.

19. (c) Given: $A=\begin{bmatrix}0 & 2 & c\\ a & b & 1\\ -3 & -1 & 0\end{bmatrix}$

$\because$ A is skew-symmetric matrix.

$\therefore\ b=0$

$a=-2$

$c=3$

Now, $a+b+c=-2+0+3=1.$

20. (a) Co-factor of $6=\begin{vmatrix}1 & -1\\ 0 & 4\end{vmatrix}=4$

Co-factor of $3=\begin{vmatrix}-1 & 0\\ 4 & 2\end{vmatrix}=-2$

Co-factor of $2=-\begin{vmatrix}1 & -1\\ 3 & -4\end{vmatrix}=-(-4+3)=1$

Co-factor corresponding to $-1=-\begin{vmatrix}0 & 2\\ 3 & 6\end{vmatrix}=6$

21. (d) $\because\ f(x)=x^3+\alpha x^2+\beta x$

$\Rightarrow f'(x)=3x^2+2\alpha x+\beta$

$\because$ Rolle's theorem holds for $f(x)$ at $x=\dfrac{4}{3}$

$$\therefore f'\left(\frac{4}{3}\right)=0$$

$$\Rightarrow 3\left(\frac{4}{3}\right)^2+2\alpha\cdot\frac{4}{3}+\beta=0$$

$\Rightarrow \frac{16}{3}+\frac{8\alpha}{3}+\beta=0$

$\Rightarrow 8\alpha+3\beta+16=0$ ...(1)

$\because$ Rolle's theorem hold in $1\le x\le 2$.

$\therefore f(1)=f(2)$

$\Rightarrow 1+\alpha+\beta=8+4\alpha+2\beta$

$\Rightarrow 3\alpha+\beta+7=0$ ...(2)

Solving equations (1) and (2), we get

$\alpha=-5, \beta=8$

22. (d) $\because B=-2A$

$\Rightarrow |B|=|-2A|$

$=(-2)^3|A|$

$\Rightarrow |B|=-8|A|$

$\therefore k=-8.$

23. (a) The transpose of product of two matrices is

$\Rightarrow (AB)'=B'A'$

24. (b) $3\tan^{-1}x+\cot^{-1}x=\pi$

$\Rightarrow 3\tan^{-1}x+\frac{\pi}{2}-\tan^{-1}x=\pi$

$\Rightarrow 2\tan^{-1}x\frac{\pi}{2}\Rightarrow \tan^{-1}x=\frac{\pi}{4}$

$\Rightarrow x=\tan\left(\frac{\pi}{4}\right)=1.$

25. (a) Let E_1 and E_2 be the events such that

E_1 : *A* Speak truth

E_2 : *A* Speak false.

Let *X* be the event that a head appears.

Given : $P(E_1)=\frac{3}{5}$

Then $P(E_2)=1-\frac{3}{5}=\frac{2}{5}$

Also, $P(X/E_1)=P(X/E_2)=\frac{1}{2}$

Now, the probability that there is actually a head is

$$=P(E_1|X)=\frac{P(E_1)\cdot P(X/E_1)}{P(E_1)\,P(X/E_1)+P(E_2)\,P(X/E_2)}$$

$$=\frac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2}+\frac{2}{5}\cdot\frac{1}{2}}=\frac{3}{5}$$

26. (c) $\because f(x)=2x^3-3x^2-4$

Then $f'(x)=6x^2-6x$

$f''(x)=12x-6$

For critical points :

$f'(x)=0\Rightarrow 6x^2-6x=0$

$\Rightarrow x(x-1)=0\Rightarrow x=0, 1$

At $x=0, f''(0)=-6<0$ (maximum)

At $x=1, f''(1)=12-6=6>0$ (minima)

So, $f(x)$ has one maxium and one minima.

27. (a) $\lim\limits_{x\to-2} f(x)=\lim\limits_{x\to-2}\frac{\sqrt{x^2+5}-3}{x+2}$

$$=\lim_{x\to-2}\frac{\frac{2x}{2\sqrt{x^2+5}}-0}{1}$$

$$=\frac{-2}{\sqrt{4+5}}=\frac{-2}{3}$$

$\because f(x)$ is continuous at $x=-2$

So, $\lim\limits_{x\to-2} f(x)=f(-2)\Rightarrow -\frac{2}{3}=k$

28. (b) The given equation of line is

$\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}=t$ (say) ...(1)

Then an arbitrary point on line is

$\Rightarrow (3t+1, 4t-2, -2t+3)$

The equation of plane is

$2x-y+3z=1$ ...(2)

(A) For point intersection of (1) and (2)

$2(3t+1)-(4t-2)+3(-2t+3)=1$

$\Rightarrow 6t+2-4t+2-6t+9=1$

$\Rightarrow 4t=12\Rightarrow t=3$

So, point of intersection is

$(3\times3+1, 4\times3-2, -2\times3+3)=(10, 10, -3)$

(B) Suppose if possible the line (1) passing through $(-1, 2, -3)$

$\therefore 3t+1=-1\Rightarrow t=-\frac{2}{3}$

$4t-2=2\Rightarrow 4\times\left(-\frac{2}{3}\right)-2=2$

$\Rightarrow \frac{-8}{3}-2=2\Rightarrow \frac{-14}{3}=2$ (not possible)

$\therefore (-1, 2, -3)$ is not passing through the given line.

(C) The direction ratios of normal to plane is 2, –1, 3.

(D) Since, direction ratio of the line is 3, 4, –2.

$\therefore$ The line is parallel to the vector $3\hat{i}+4\hat{j}-2\hat{k}$.

So, statement (A) and (D) are correct.

29. (d) (A) $\vec{a}\times\vec{b}=\vec{0}\Leftrightarrow\vec{a}$ is parallel to $\vec{b}$ as $\theta=0$

(B) $\vec{a}\times\vec{b}$ is a vector.

(C) $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$

(D) If $\theta = \frac{\pi}{2}, |\vec{a} \times \vec{b}| = \left| |\vec{a}| \cdot |\vec{b}| \sin\frac{\pi}{2} \hat{n} \right|$

$= |\vec{a}||\vec{b}| \cdot 1 = |\vec{a}||\vec{b}|$

$\therefore$ Statement (B) and (D) are correct.

30. (b) (A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx = 0$ {$\sin^7 x$ is an odd function}

(B) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x\cos x + 1)\, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x \, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \, dx.$

$= 0 + [x]_{-\pi/2}^{\pi/2}$ {x cos x is odd function}

$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

(C) $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} = \frac{\pi}{4}$

(D) $\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \left[\tan^{-1} x\right]_1^{\sqrt{3}} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

31. (c) $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x.$

$\Rightarrow 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1} x.$

$\Rightarrow \tan^{-1}\left(\frac{2\left(\frac{1-x}{1+x}\right)}{1 - \left(\frac{1-x}{1+x}\right)^2}\right) = \tan^{-1} x.$

$\Rightarrow \frac{1-x^2}{2x} = x \Rightarrow 3x^2 = 1 \Rightarrow x = \frac{1}{\sqrt{3}}$

32. (c) Let, $y = x \log_e x$

$y' = x \cdot \frac{1}{x} + \log_e x = 1 + \log_e x.$

$y'' = \frac{1}{x}$

For critical points : $y' = 0$

$\Rightarrow 1 + \log_e x = 0$

$\Rightarrow \log_e x = -1 \Rightarrow x = e^{-1}$

Now, at $x = e^{-1}, y'' = \frac{1}{e^{-1}} = e > 0$

So, $y = x \log_e x$ has minima at $x = e^{-1}$

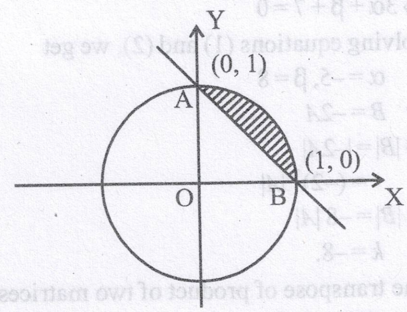
Now, minimum value is

$y(e^{-1}) = (e^{-1}) \log_e e^{-1}.$

$= -e^{-1} \times 1 = -\frac{1}{e}$

33. (a) Given, region is : $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$

We have to find the area of the shaded region.



Area of circle in I[st] quadrant $= \frac{\pi \times 1^2}{4} = \frac{\pi}{4}$

Area of triangle AOB is $= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

So, the required area $= \frac{\pi}{4} - \frac{1}{2}$

34. (c) $\because \vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i}(20) - \hat{j}(-5) + \hat{k}(-5)$

$= 20\hat{i} + 5\hat{j} - 5\hat{k}$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{400 + 25 + 25} = \sqrt{450} = 15\sqrt{2}$

35. (d) Equation of plane passing through (–1, 3, 2) is

$a(x+1) + b(y-3) + c(z-2) = 0$ ...(1)

$\because$ Plane (1) is perpendicular to the plane

$x + 2y + 3z = 5$

$\Rightarrow a + 2b + 3c = 0$ ...(2)

Also, plane (1) is perpendicular to the plane

$3x + 3y + z = 0$

$\Rightarrow 3a + 3b + c = 0$ ...(3)

From equations (2) and (3)

$\frac{a}{(2-9)} = \frac{-b}{(1-9)} = \frac{c}{(3-6)}$

$\Rightarrow \frac{9}{7} = \frac{b}{-8} = \frac{c}{3} = k$ (say)

$\Rightarrow a = 7k, b = -8k, c = 3k.$

Putting above values in equation (1) :

$7k(x+1) - 8k(y-3) + 3k(z-2) = 0$

$\Rightarrow 7x - 8y + 3z + 25 = 0$