2.10 Roots of quadratic and cubic equations

Quadratic equations

Equation	$ax^2+bx+c=0 \quad (a\neq 0)$ (2.454)	x	variable
		a,b,c	real constants
Solutions	$x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ (2.455)	x_1,x_2	quadratic roots
	$=\dfrac{-2c}{b\pm\sqrt{b^2-4ac}}$ (2.456)		
Solution combinations	$x_1+x_2=-b/a$ (2.457)		
	$x_1x_2=c/a$ (2.458)		

Cubic equations

Equation	$ax^3+bx^2+cx+d=0 \quad (a\neq 0)$ (2.459)	x	variable
		a,b,c,d	real constants
Intermediate definitions	$p=\dfrac{1}{3}\left(\dfrac{3c}{a}-\dfrac{b^2}{a^2}\right)$ (2.460)		
	$q=\dfrac{1}{27}\left(\dfrac{2b^3}{a^3}-\dfrac{9bc}{a^2}+\dfrac{27d}{a}\right)$ (2.461)	D	discriminant
	$D=\left(\dfrac{p}{3}\right)^3+\left(\dfrac{q}{2}\right)^2$ (2.462)		

If $D\geq 0$, also define:	If $D<0$, also define:
$u=\left(\dfrac{-q}{2}+D^{1/2}\right)^{1/3}$ (2.463)	$\phi=\arccos\left[\dfrac{-q}{2}\left(\dfrac{\|p\|}{3}\right)^{-3/2}\right]$ (2.467)
$v=\left(\dfrac{-q}{2}-D^{1/2}\right)^{1/3}$ (2.464)	$y_1=2\left(\dfrac{\|p\|}{3}\right)^{1/2}\cos\dfrac{\phi}{3}$ (2.468)
$y_1=u+v$ (2.465)	$y_{2,3}=-2\left(\dfrac{\|p\|}{3}\right)^{1/2}\cos\dfrac{\phi\pm\pi}{3}$ (2.469)
$y_{2,3}=\dfrac{-(u+v)}{2}\pm\mathbf{i}\dfrac{u-v}{2}3^{1/2}$ (2.466)	
1 real, 2 complex roots (if $D=0$: 3 real roots, at least 2 equal)	3 distinct real roots

Solutions[a]	$x_n=y_n-\dfrac{b}{3a}$ (2.470)	x_n	cubic roots ($n=1,2,3$)
Solution combinations	$x_1+x_2+x_3=-b/a$ (2.471)		
	$x_1x_2+x_1x_3+x_2x_3=c/a$ (2.472)		
	$x_1x_2x_3=-d/a$ (2.473)		

[a] y_n are solutions to the reduced equation $y^3+py+q=0$.