Long Answer Type Questions

[5 Marks]

Que 1. Two chords AB and CD of length 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Sol.

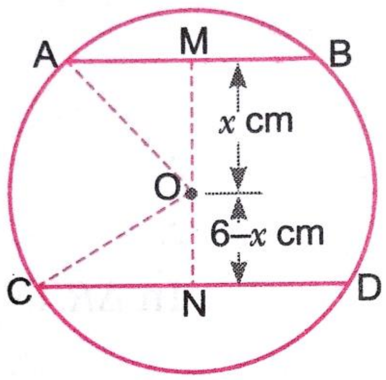

Fig. 10.48

Let, r be the radius of given circle and its centre be O. Draw OM ⊥ AB and ON ⊥ CD
Since, OM ⊥ AB, ON ⊥ CD and AB || CD
Therefore, points M, O and N are collinear. So, MN = 6 cm
Let, OM = x cm. Then, ON = (6 – x) cm.
Join OA and OC. Then OA = OC = r.
As the perpendicular from the centre to a chord of the circle bisects the chord.

∴ AM = BM = $\frac{1}{2}AB = \frac{1}{2} \times 5 = 2.5\ cm.$

CN = DN = $\frac{1}{2}CD = \frac{1}{2} \times 11 = 5.5\ cm.$

In right triangles OAM and OCN, we have

$OA^2 = OM^2 + AM^2$ and $OC^2 = ON^2 + CN^2$

$r^2 = x^2 + \left(\frac{5}{2}\right)^2$ ……(i)

$r^2 = (6 - x)^2 + \left(\frac{11}{2}\right)^2$ …….(ii)

From (i) and (ii), we have

$$X^2 + \left(\frac{5}{5}\right)^2 = (6 - x)^2 + \left(\frac{11}{2}\right)^2$$

$$X^2 + \frac{25}{4} = 36 + x^2 - 12x + \frac{121}{4}$$

$\Rightarrow \quad 4x^2 + 25 = 144 + 4x^2 - 48x + 121$

$\Rightarrow \quad 48x = 240$

$\Rightarrow \quad x = \frac{240}{48} \quad \Rightarrow x = 5$

Putting the value of x in equation (i), we get

$$r^2 = 5^2 + \left(\frac{5}{2}\right)^2 \Rightarrow r^2 = 25 + \frac{25}{4}$$

$\Rightarrow \quad r^2 = \frac{125}{4} \quad \Rightarrow r = \frac{5\sqrt{5}}{2} cm$

Que 2. Three girls Reshma, Salma and Mandeep are playing a game by standing on a circle of radius 5 cm drawn in a park. Reshma throws a ball to Salma, Salma to Mandeep to Reshma. If the distance between Reshma and Salma and between Salma and Mandeep is 6 cm each, what is the distance between Reshma and Mandeep?

Sol.

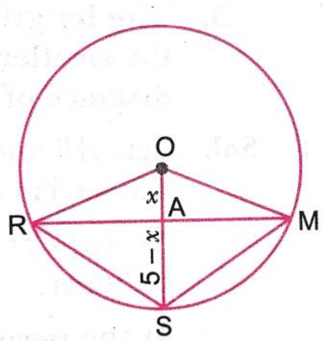

Fig. 10.49

Let R, S and M represent the position of Reshma, Salma and Mandeep respectively.
Clearly ΔRSM is an isosceles triangle as

RS = SM = 6 m

Join OS which intersects RM at A.

In ΔROS and ΔMOS

OR = OM	(Radii of the same circle)
OS = OS	(Common)
RS = SM	(Each 6 cm)

∴ ΔROS ≅ ΔMOS (By SSS congruence criterion)

∴ ∠RSO = ∠MSO (CPCT)

In ΔRAS and ΔMAS

AS = AS (Common)

$\therefore \quad \angle RSA = \angle MSA \quad (\because \angle RSO = \angle MSO)$

$RS = MS \quad$ (Given)

$\therefore \quad \Delta RAS \cong \Delta MAS \quad$ (By SAS congruence criterion)

$\therefore \quad \angle RAS = \angle MAS \quad$ (CPCT)

$\therefore \quad \angle RAS + \angle MAS = 180^\circ$ (Linear pair)

$\Rightarrow \quad \angle RAS = \angle MAS = 90^\circ$

Let $\quad OA = x$ m $\Rightarrow \quad AS = (5 - x)$ m

In right triangle RAS,

$RS^2 = RA^2 + AS^2$

$\Rightarrow \quad 6^2 = RA^2 + (5 - x)^2 \quad$ ……(i)

$\Rightarrow \quad RA^2 = 6^2 - (5 - x)^2$

In right triangle RAO,

$RO^2 = RA^2 + OA^2$

$\Rightarrow \quad 5^2 = RA^2 + x^2$

$\Rightarrow \quad RA^2 = 5^2 - x^2 \quad$ ………(ii)

From equation (i) and (ii), we get

$6^2 - (5 - x)^2 = 5^2 - x^2$

$6^2 - 5^2 = (5 - x)^2$

$36 - 25 = 25 + x^2 - 10x - x^2$

$11 = 25 - 10x \Rightarrow \quad 10x = 14 \quad \Rightarrow = 1.4$ m

From equation (ii), we have

$RA^2 = 5^2 - (1.4)^2 = 25 - 1.96$

$RA^2 = 23.04 \quad \Rightarrow \quad RA = \sqrt{23.04}$

As the Perpendicular from the centre of a bisects the chord.

$\therefore \quad RM = 2RA$

$RM = 2 \times 4.8 = 9.6$ m

Hence, distance between Reshma and Mandeep is 9.6 m.

Que 3. The length of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of other chord from the centre?

Sol.

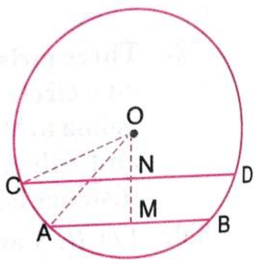

Fig. 10.50

Let, AB and CD be two parallel chords of a circle with centre O such that AB = 6 cm and CD = 8 cm. Draw OM ⊥ AB and ON ⊥ CD.
As AB || CD and OM ⊥ AB, ON ⊥ CD. Therefore, Points O, N and M are collinear.
As the perpendicular from the centre of a circle to the chord bisects the chord.
Therefore,

$$AM = \frac{1}{2}AB = \frac{1}{2} \times 6 = 3\ cm$$

$$CN = \frac{1}{2}CD = \frac{1}{2} \times 8 = 4\ cm$$

In right triangle OAM, we have

$OA^2 = OM^2 + AM^2$

$OA^2 = 4^2 + 3^2$ ⇒ $OA^2 = 25$ ⇒ OA = 5cm

Also, OA = OC (Radii of the same circle)

⇒ OC = 5 cm

In right triangle OCN, we have

$OC^2 = ON^2 + CN^2$

⇒ $5^2 = ON^2 + 4^2$ ⇒ $ON^2 = 5^2 - 4^2$

⇒ $ON^2 = 9$ ⇒ ON = 3 cm

Que 4. AC and BD are chords of a circle that bisect each other. Prove that AC and BD are diameter and ABCD is a rectangle.

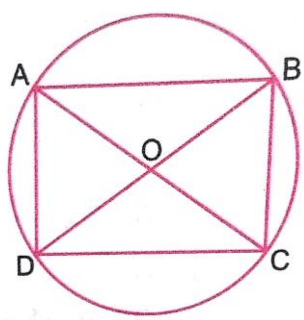

Fig. 10.51

Sol. Let AC and BD bisect each other at point O. Then,

OA = OC and OB = OD ….(i)

In triangles AOB and COD, we have

OA = OC

OB = OD

and ∠AOB = ∠COD (Vertically opposite angles)

∴ ΔAOB ≅ ΔCOD (SAS congruence criterion)

⇒ AB = CD (CPCT)

⇒ $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ ……(ii)

Similarly BC = DA

$\Rightarrow \quad \overset{\frown}{BC} \cong \overset{\frown}{DA}$ ……(iii)

From (ii) and (ii), we have

$$\overset{\frown}{AB} + \overset{\frown}{BC} \cong \overset{\frown}{CD} + \overset{\frown}{DA}$$

$\Rightarrow \quad \overset{\frown}{ABC} = \overset{\frown}{CDA}$

⇒ AC divides the circle into two equal parts.

⇒ AC is the diameter of the circle. Similarly, we can prove that BD is also a diameter of the circle.

Since AC and BD are diameter of the circle.

∴ ∠ABC = 90° = ∠ADC

Also, ∠BAD = 90° = ∠BCD

Also, AB = CD and BC = DA (Proved above)

Hence, ABCD is a rectangle.

Que 5. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

Sol.

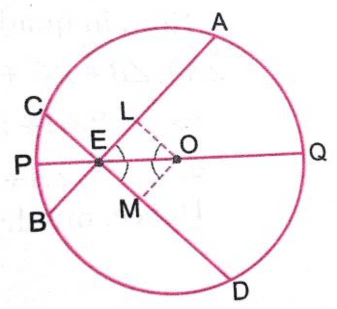

Fig. 10.52

Given: AB and CD are two chords of a circle with centre O, intersecting at point E. PQ is a diameter through E, such that ∠AEQ = ∠DEQ.

To prove: AB = CD

Construction: Draw OL ⊥ AB and OM ⊥ CD

Proof: ∠LOE + ∠LEO + ∠OLE = 180° (Angle sum property of a triangle)

⇒ ∠LOE + ∠LEO + 90° 180°

∠LOE + ∠LEO = 90° …..(i)

Similarly ∠MOE + ∠MEO + ∠OME = 180°

⇒ ∠MOE + ∠MEO + 90° = 180°

∠MOE + ∠MEO = 90° …..(ii)

From (i) and (ii) we get

∠LOE + ∠LEO = ∠MOE + ∠MEO ……(iii)

Also, ∠LEO = ∠MEO (Given) ….(iv)

From (iii) and (iv) we get

$$\angle LOE = \angle MOE$$

Now in triangle OLE and OME

	$\angle LEO = \angle MEO$	(Given)
∴	$\angle LOE = \angle MOE$	(Proved above)
	$EO = EO$	(Common)
∴	$\Delta OLE \cong \Delta OME$	(ASA congruence criterion)
∴	$OL = OM$	(CPCT)

Thus, chords AB and CD are equidistance from the centre are equal.

∴ $AB = CD$

Que 6. If the non-parallel sides of a trapezium are equal, prove that it is cyclin.

Sol.

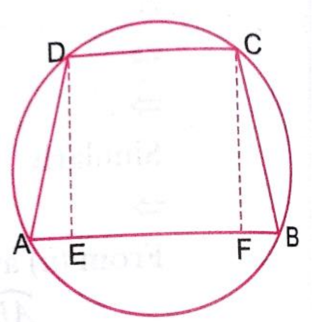

Fig. 10.53

Given: A trapezium ABCD in which AB || CD and AD = BC
To prove: ABCD is a cyclin trapezium.
Construction: Draw $DE \perp AB$ and $CF \perp AB$
In right triangle AED and BFC, We have

	$AD = BC$	(Given)	
	$\angle DEA = \angle CFB$	(Each equal to 90°	
and,	$DE = CF$	(Distance between two parallel lines)	
⇒	$\Delta DEA \cong \Delta CFB$	(RHS congruence criterion)	
⇒	$\angle A = \angle B$	(CPCT)	…….(i)
	$\angle ADE = \angle BCF$	(CPCT)	…….(ii)
⇒	$\angle C = \angle BCF + 90° = \angle ADE + 90° = \angle ADC$		…….(iii)
⇒	$\angle C = \angle D$		

Now, in quadrilateral ABCD, we have

$\angle A + \angle B + \angle C + \angle D = 360°$ (By Angle sum property)

⇒ $2\angle A + 2\angle C = 360°$ (From (i) and (iii))

⇒ $\angle A + \angle C = 180°$

Hence, quadrilateral ACBD is cyclin.

Que 7. Prove that quadrilateral formed by angle bisectors of a cyclin quadrilateral is also cyclin.

Sol.

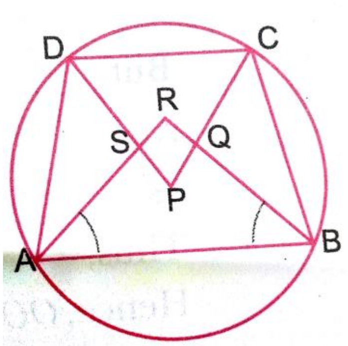

Fig. 10.54

Given: A cyclin quadrilateral ABCD in which the angle bisectors AR, CP and DP of internal angles A, B, C and D respectively form a quadrilateral PQRS.

To prove: PQRS is a cyclin quadrilateral.
Proof: In ΔARB, we have

$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle R = 180°$ ….(i) (∵ AR, BR are bisectors of ∠A ∠B)

In ΔDPC, We have

$\frac{1}{2}\angle D + \frac{1}{2}\angle C + \angle P = 180°$ ...(ii)

(∵ DP, CP are bisectors of ∠D and ∠C respectively)

Adding (i) and (ii), we get

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle R + \frac{1}{2}\angle D + \frac{1}{2}\angle C + \angle P = 180° + 180°$$

$$\angle P + \angle R = 360° - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$$

$$\angle P + \angle R = 360° - \frac{1}{2} \times 360° = 360° - 180°$$

⇒ ∠P + ∠R = 180°

As the sum of a pair of opposite angles of quadrilateral PQRS is 180°. Therefore, quadrilateral PQRS is cyclin.

Que 8. If two circles intersects at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Sol.

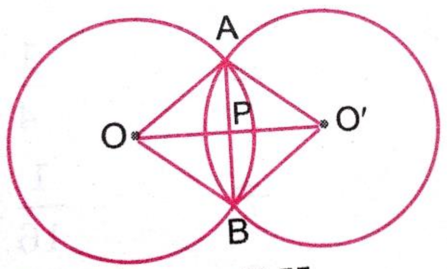

Fig. 10.55

Given: Two circles, with centres O and O' intersect at two points A and B. AB is the common chord of the two circles and OO' is the line segment joining the centres of the two circles. Let OO' intersect AB at P.

To prove: OO' is the perpendicular bisector of AB.
Construction: Join OA, OB, O' A and O' B
Proof: In triangles OAO' and OBO', we have

$OO' = OO'$ (Common)
$OA = OB$ (Radii of the same circle)
$O'A = O'B$ (Radii of the same circle)
$\Rightarrow$ $\Delta OAO' \cong \Delta OBO'$ (SSS congruence criterion)
$\Rightarrow$ $\angle AOO' = \angle BOO'$ (CPCT)
I.e., $\angle AOP = \angle BOP$

In triangle AOP and BOP, we have

$OP = OP$ (Common)
$\angle AOP = \angle BOP$ (Proved above)
$OA = OB$ (Radio of the same circle)
$\therefore$ $\Delta AOR \cong \Delta BOP$ (By SAS congruence criterion)
$\Rightarrow$ $AP = BP$ (CPCT)

And $\angle APO = \angle BPO$ (CPCT)

But $\angle APO + \angle BPO = 180°$ (Linear)
$\therefore$ $\angle APO + \angle APO = 180°$ $\Rightarrow$ $2\angle APO = 180°$
$\Rightarrow$ $\angle APO = 90°$
Thus, $AP = BP$ and $\angle APO = \angle BPO = 90°$
Hence, OO' is the perpendicular bisectors of AB.