CBSE Test Paper-04

Class - 12 Physics (Alternating Current)

1. A series circuit consists of an ac source of variable frequency, a 115.0Ω resistor, a 1.25 μF capacitor, and a 4.50-mH inductor. Impedance of this circuit when the angular frequency of the ac source is adjusted to half the resonant angular frequency is
   a. 156.0 Ω
   b. 166.0 Ω
   c. 176.0 Ω
   d. 146.0 Ω
2. For a parallel LC circuit the angular resonant frequency is
   a. $\frac{2}{\sqrt{LC}}$
   b. $\frac{1}{\sqrt{LC}}$
   c. $\frac{1}{LC}$
   d. $\frac{1}{\sqrt{2LC}}$
3. A pure inductor of 20.0 mH is connected to a source of 230 V. Inductive reactance and rms current in the circuit (if the frequency of the source is 50 Hz). are
   a. 6.28 Ω, 36.6 A
   b. 6.28 Ω, 46.6 A
   c. 8.28 Ω, 36.6 A
   d. 9.28 Ω, 36.6 A
4. Phase difference between voltage and current in a capacitor in ac circuit is:
   a. $\frac{\pi}{2}$
   b. 0
   c. π
   d. $\frac{\pi}{3}$
5. An electric hair dryer is rated at 1500 W (the average power) at 120 V (the rms voltage). Calculate (a) the resistance, (b) the rms current, and (c) the maximum instantaneous power. Assume that the dryer is a pure resistor.
   a. 8.6 Ω, 11.5 A, 3000 W
   b. 9.6 Ω, 12.5 A, 3000 W
   c. 10.6 Ω , 13.5 A, 3000 W

d. 7.6 Ω , 14.5 A, 3000 W

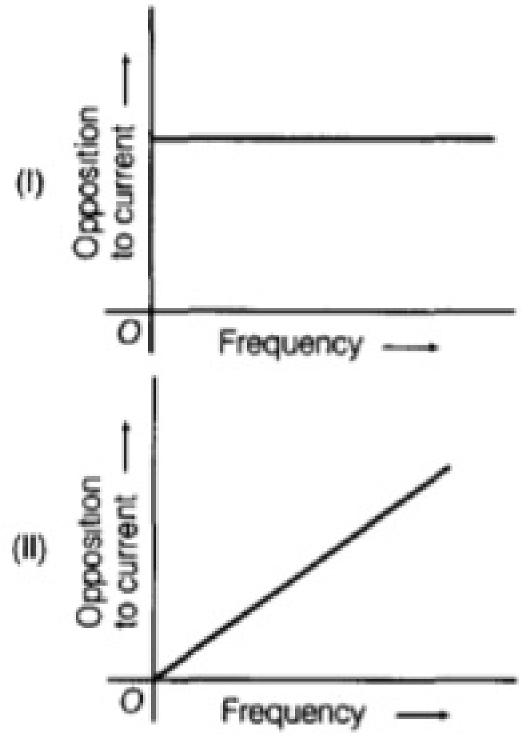
6. A $15.0 \mu F$ capacitor is connected to 220 V, 50 Hz source. Find the capacitive reactance and the rms current.

7. A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

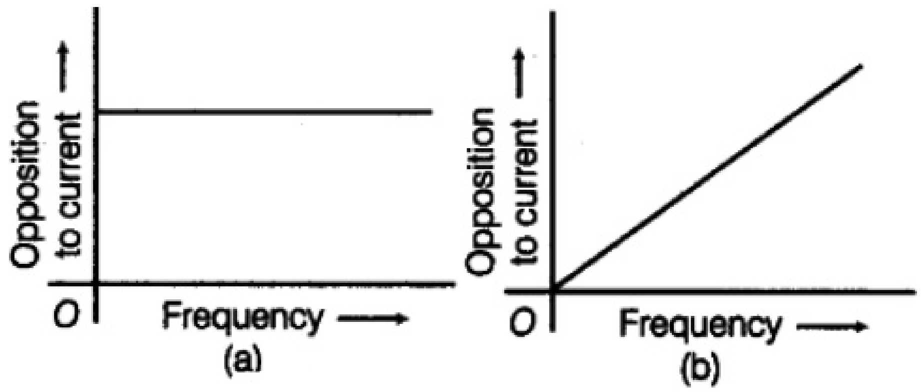
8. What do you mean by power factor? On what factors does it depend?

9. State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing variation of current with frequency of a.c. source in a series LCR circuit.

10. i. The graphs (I) and (II) represent the variation of the opposition offered by the circuit element to the flow of alternating current with frequency of the applied emf. Identify the circuit element corresponding to each graph.



ii. Write the expression for the impedance offered by the series combination of the above two elements connected across the AC sources. Which will be ahead in phase in this circuit, voltage or current?

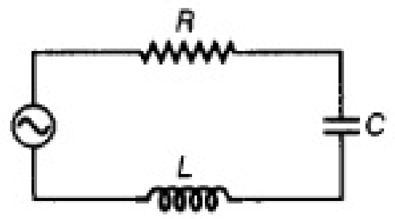


11. Write the expression for the impedance offered by the series combination of resistor, inductor and capacitor connected to an AC source of voltage V = V_0 sin wt. Show on a graph the variation of the voltage and the current with 'wt in the circuit.

12. What is iron loss in a transformer?

13. A LCR circuit has L = 10 mH, R = 3 ohm and C = $1\mu F$ connected in series to an a.c. source of the voltage 15 V. Calculate current amplitude and the average power dissipated per cycle at a frequency that is 10% lower than the resonant frequency.

14. The figure shows a series L-C-R circuit with L = 10 H, C= 40μF, R=600 connected to a variable frequency 240 V source. Calculate



i. the angular frequency of the source which derives the circuit at resonance.
ii. the current at the resonating frequency.
iii. the rms potential drop across the inductor at resonance.

15. An AC source of voltage V = V_0 sin ωt is connected to a series combination of L, C and R. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in the condition called?

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Answers

1. d. 146.0 Ω

   Explanation: $R = 115\Omega$

   $C = 1.25\mu F = 1.25 \times 10^{-6} F$

   $L = 4.5mH = 4.5 \times 10^{-3} H$

   resonant angular frequency

   $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.5 \times 10^{-3} \times 1.25 \times 10^{-6}}} = \frac{1}{7.5 \times 10^{-5}}$

   given that the angular frequency of the ac source

   $\omega = \frac{\omega_0}{2} = \frac{1}{15 \times 10^{-5}} = 6666.6 rad/s$

   impedance

   $Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$

   $Z = \sqrt{115^2 + \left[\left(\frac{1}{6666.6 \times 1.25 \times 10^{-6}}\right) - \left(6666.6 \times 4.5 \times 10^{-3}\right)\right]^2}$

   $Z = 146\Omega$

2. b. $\frac{1}{\sqrt{LC}}$

   Explanation: in case of resonance $X_L = X_C$

   $\omega_0 L = \frac{1}{\omega_0 C}$

   ${\omega_0}^2 = \frac{1}{LC}$

   $\omega_0 = \frac{1}{\sqrt{LC}}$

3. a. 6.28 Ω, 36.6 A

   Explanation: $L = 20mH = 20 \times 10^{-3} H$

   V = 230 volt

   f = 50Hz

   inductive reactance

   $X_L = \omega L = 2\pi f \times L = 2 \times 3.14 \times 50 \times 20 \times 10^{-3} = 6.28\Omega$

   rms current

   $i = \frac{V}{X_L} = \frac{230}{6.28} = 36.6A$

4. a. $\frac{\pi}{2}$

Explanation: An ac voltage v = v_m sinωt applied to a capacitor drives a current in the capacitor: i = i_m sin (ωt + π/2)$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$.

Here, $i_m = \frac{v_m}{X_C}$, $X_C = \frac{v_m}{i_m}$ is called capacitive reactance. The current through the capacitor is $\frac{\pi}{2}$ ahead of the applied voltage. As in the case of inductor, the average power supplied to a capacitor over one complete cycle is zero.

5. b. 9.6 Ω, 12.5 A, 3000 W

Explanation: P = 1500W

$V_{rms} = 120V$

$P = \frac{(V_{rms})^2}{R}$

$1500 = \frac{120 \times 120}{R}$

$R = \frac{120 \times 120}{R}$

$i_{rms} = \frac{V_{rms}}{R} = \frac{120}{9.6} = 12.5A$

maximum voltage $V_0 = V_{rms} \times \sqrt{2} = 120\sqrt{2}$

maximum current $i_0 = I_{rms} \times \sqrt{2} = 12.5\sqrt{2}$

maximum power $P_0 = V_0 \times i_0 = 120\sqrt{2} \times 12.5\sqrt{2} = 1500 \times 2 = 3000W$

6. Given $C = 15.0\mu F = 15 \times 10^{-6}F$

V = 220 V, f = 50 Hz

Capacitive reactance, $X_C = \frac{1}{2\pi f\ c}$

or $X_C = \frac{1}{2 \times 3.14 \times 50 \times 15 \times 10^{-6}}$

$= \frac{10^6}{314 \times 15} = \frac{1000000}{4710} = 212.31\Omega$

$\therefore I_{rms} = \frac{V}{X_C} = \frac{220}{212.31} = 1.03A$

7. Given L = 44 mH$= 44 \times 10^{-3}H$

f = 50 Hz, E_{rms} = 220V

$X_L = L \cdot \omega = L \cdot 2\pi f$

$= 44 \times 10^{-3} \times 2 \times 3.14 \times 50$

Now, $I_{rms} = \frac{E_{rms}}{X_L}$

$= \frac{220}{44 \times 10^{-3} \times 2 \times 3.14 \times 50}$

= 15.9 A

8. The power factor is defined as the cosine of the phase angle between alternating

e.m.f. and current in an a.c. circuit.

Power factor of a.c. circuit is given by

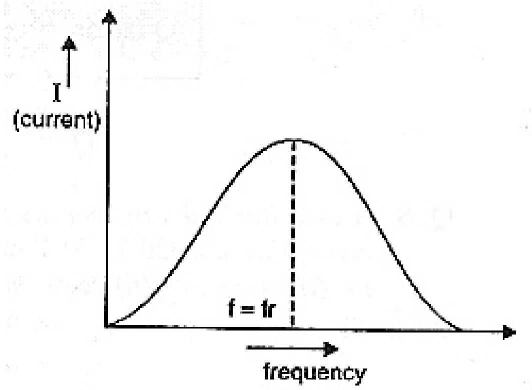
$$\cos\phi = \frac{R}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

It depends upon the frequency of the a.c. source.

9. Resonance occurs in a series of LCR circuit when

   $X_L = X_C$

   The graph showing the variations of current with frequency of a.c. source in a series LCR circuit is given below.



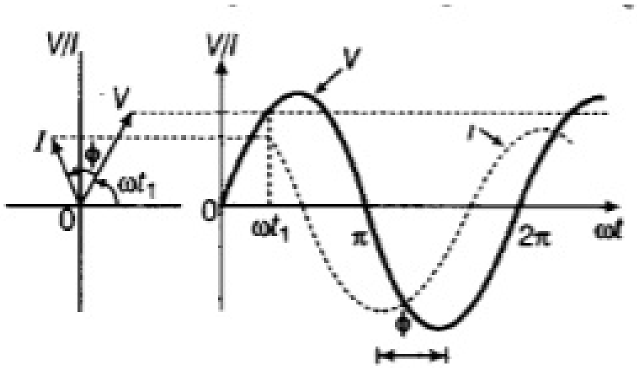
10. i. From graph (I), it is clear that resistance (opposition to current) is not changing with frequency, and we know that resistance of a resistor does not depend on frequency of applied source, so the circuit element here is a pure resistance (R). i.e. the circuit is a pure resistive circuit.
       From graph (II), it is clear that resistance increases linearly with frequency, so the circuit element here is an inductor.
       As we know that, inductive resistance, $X_L = 2\pi f L$
       $\Rightarrow \quad X_L \propto f$, f being frequency of the ac voltage or current.
    ii. Impedance offered by the series combination of resistance R and inductor L.
       $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi f L)^2}$, X_L being the inductive reactance of the circuit.
       In an L-R circuit, the applied ac voltage leads the ac current in phase by $\pi/2$

11. Impedance offered by series L-C-R circuit,

    $$z = \sqrt{R^2 + (X_c - X_L)^2}, V = \sqrt{V_R^2 + (V_c - V_L)^2}.$$

V_C or V_L may be greater than V.

The situation may be shown in figure, where $V_c > V_L$



12. It is loss of energy in the form of heat in iron core of a transformer.

There are two types of core or iron losses in a Transformer.

1. **Hysteresis losses (Transformer Losses) :** Each time the magnetic field is reversed, a small amount of energy is lost due to hysteresis within the core. For a given core material, the transformer losses are proportional to the frequency, and is a function of the peak flux density to which it is subjected.
2. **Eddy current Losses (Transformer Losses)** : Ferromagnetic materials are also good conductors, and a core made from such a material also constitutes a single short-circuited turn throughout its entire length. Eddy currents therefore circulate within the core in a plane normal to the flux, and are responsible for resistive heating of the core material. The eddy current loss is a complex function of the square of supply frequency and inverse square of the material thickness. Eddy current losses can be reduced by making the core of a stack of plates electrically insulated from each other, rather than a solid block; all transformers operating at low frequencies using laminated or similar cores.

13. Resonant frequency, $\omega_r = \frac{1}{\sqrt{LC}}$

Here, L = 10 mH $= 10 \times 10^{-3} H$

and $C = 1 \mu F = 1 \times 10^{-6} F$

$\therefore \omega_r = \sqrt{\left[\frac{1}{(10 \times 10^{-3})(1 \times 10^{-6})}\right]}$ = 10^4/second

Now, 10% less frequency will be

$\omega = 10^4 - 10^4 \times \frac{10}{100} = 9 \times 10^3/\text{second}$

At this frequency, $X_L = \omega L = 9 \times 10^3 \times (10 \times 10^{-3})$ = 90 ohm

$X_C = \frac{1}{\omega C} = \frac{1}{(9\times10^3)(1\times10^{-6})}$ = 111.11 ohm

$\therefore Z = \sqrt{\left[R^2 + (X_L - X_C)^2\right]}$

$= \sqrt{\left[(3)^2 + (90 - 111.11)^2\right]}$ = 21.32 ohm

Current amplitude,

$I_0 = \frac{E_0}{Z} = \frac{15}{21.32}$ = 0.704 amp.

Average power, $P = \frac{1}{2} E_0 I_0 \cos\phi$

Where $\cos\phi = \frac{R}{Z} = \frac{3}{21.32}$ = 0.141

$P = \frac{1}{2} \times 15 \times 0.704 \times 0.141$ = 0.744 watt.

14. Given, $L = 10\mathrm{H}, C = 40\mu\mathrm{F}$

    $R = 60\Omega V_{ms} = 240\mathrm{V}$

    i. $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\times40\times10^{-6}}}$

       $\omega_r = \frac{1}{20\times10^{-3}} = 50\mathrm{rad/s}$

    ii. $I_{\mathrm{ms}} = \frac{V_{\mathrm{ms}}}{Z} = \frac{V_{\mathrm{ms}}}{R}$

       = 240/60 = 4A

    iii. $\because$inductive reactance, $X_L = \omega L$

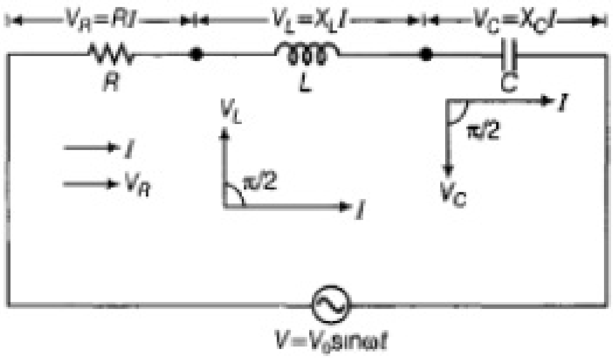
       At resonance, $X_L = \omega L = 50 \times 10 = 500\Omega$

       Potential drop across to inductor,

       $V_{ms} = I_{ms} \times X_L = 4 \times 500 = 2000\mathrm{V}$

15. If I is the current in the circuit containing L,C and R in series, then the voltage drop across the inductor is

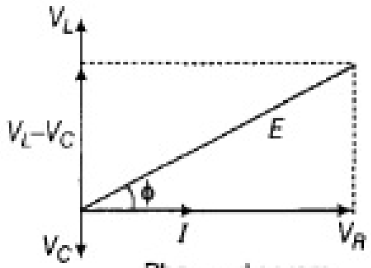
    V_i=1 x X_L



which leads current I by phase angle of $\pi/2$ and voltage drop across the capacitor is

$V_2 = I \times X_c$.which lags behind current I by phase angle of $\pi/2$ and voltage E across the circuit is

V_R = IR which is in phase with current I. So the net voltage E across the circuit is (using phasor diagram)



Phasor diagram

$$V = \sqrt{V_k^2 + (V_L - V_C)^2}$$

$$\Rightarrow \quad V = I\sqrt{R^2 + (X_L - X_c)^2} \Rightarrow V = IZ$$

where, $z = \sqrt{R^2 + (X_L - X_C)^2}$ is known as impedance. Phase angle between voltage and current is given by tan $\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$.

Above mentioned condition is known as the condition of resonance. In this condition

i. Inductive and capacitive reactances are equal
$X_L = X_c \Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$
$[\omega = 2\pi v]$
$\Rightarrow \quad v = \frac{1}{2\tau\sqrt{LC}}$

ii. Potential drop across inductor and capacitor are equal, V_L = VC

iii. The series resonant circuit is also called an acceptor circuit because when a number of different frequency currents are into the circuit, the circuit offers minimum impedance to natural frequency current.
For L-R circuit, $X_L = R$
Power factor, $P_1 = \cos\phi$
$= \frac{R}{\sqrt{R^2 + x_L^2}} = \frac{R}{R^2 + R^2} = \frac{1}{\sqrt{2}}$
For L-C-R circuit, as C is put in series with L-R circuit and $X_L = X_C$
Power factor, $P_2 = \cos\phi$
$= \frac{R}{\sqrt{R^2 + (X_L - X_c)^2}} = \frac{R}{\sqrt{R^2 + (x_L - X_L)^2}} = \frac{R}{R} = 1$
Required ratio = $P_1/P_2 = 1/\sqrt{2}$