CBSE Test Paper 01

Chapter 3 Matrices

1. If $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$. Then |A| is
   a. none of these
   b. Idempotent
   c. Nilpotent
   d. Symmetric
2. $\begin{vmatrix} 1 & 1 & 1 \\ e & 0 & \sqrt{2} \\ 2 & 2 & 2 \end{vmatrix}$ is equal to
   a. 0
   b. 3e
   c. none of these
   d. 2
3. A square matrix A is called idempotent if
   a. $A^2 = I$
   b. $A^2 = O$
   c. 2A=I
   d. $A^2 = A$
4. Let for any matrix M ,M^{-1} exist. Which of the following is not true.
   a. none of these
   b. $(M^{-1})^{-1} = M$
   c. $(M^{-1})^2 = (M^2)^{-1}$
   d. $(M^{-1})^{-1} = (M^{-1})^1$
5. The system of equations x + 2y = 11, -2 x – 4y = 22 has
   a. only one solution
   b. infinitely many solutions
   c. finitely many solutions
   d. no solution

6. Sum of two skew-symmetric matrices is always ________ matrix.
7. ________ matrix is both symmetric and skew-symmetric matrix.
8. If A and B are square matrices of the same order, then (kA)' = ________ where k is any scalar.
9. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, Prove that A – A^t is a skew – symmetric matrix.
10. $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$, Prove that A + A' is a symmetric matrix.
11. The no. of all possible matrics of order 3×3 with each entry as 0 or 1 is-
12. Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.
13. If A is a square matrix, such that A^2 = A, then $(I + A)^3$ – 7A is equal to
14. If $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$, then Prove that A + A' is a symmetric matrix.
15. If f(x) = x^2 – 5x + 7 and $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then find f(A).
16. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$.
17. Show that:
    i. $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$
    ii. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$
18. $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$,

    Prove $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$.

Solution

1. c. Nilpotent

   Explanation: The given matrix A is nilpotent, because |A| = 0,as determinant of a nilpotent matrix is zero.

2. a. 0

   Explanation: $\begin{vmatrix} 1 & 1 & 1 \\ e & 0 & \sqrt{2} \\ 2 & 2 & 2 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 & 1 \\ e & 0 & \sqrt{2} \\ 1 & 1 & 1 \end{vmatrix} = 0$, because, row 1 and row 3 are identical.

3. d. $A^2 = A$

   Explanation: If the product of any square matrix with itself is the matrix itself, then the matrix is called Idempotent.

4. d. $(M^{-1})^{-1} = (M^{-1})^1$

   Explanation: Clearly , $(M^{-1})^{-1} = (M^{-1})^1$ is not true.

5. d. no solution

   Explanation: For no solution , we have: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, for given system of equations we have: $\frac{1}{-2} = \frac{2}{-4} \neq \frac{11}{22}$.

6. skew symmetric
7. Null
8. kA'
9. $P = A - A^t$

$$= \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$P' = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

P' = - P

Proved.

10. $P = A + A' = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$

$$P = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

$$P' = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

Thus, P' = P.

11. $2^9 = 512$

12. Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Hence, a = 1, b = -2, c = 2, d = 0

$$X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

13. $(I + A)^3 - 7A = I^3 + A^3 + 3IA(I + A) - 7A$

$= I + A^3 + 3I^2A + 3IA^2 - 7A$

$= I + A^3 + 3A + 3A^2 - 7A$

$= I + A^3 + 3A + 3A - 7A$ $\{A^2 = A\}$

$= I + A^3 - A$

$= I + A^2 - A$ $[A^2 = A, A^3 = A^2]$

$= I + A - A$ $\{A^2 = A\}$

$= I$

14. $P = A + A' = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$

$$P = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

$$P' = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

P' = P

Therefore P = P'

Hence A+A' is symmetric.

15. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = AA = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

Now, f(A) = A^2 - 5A + 7I

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$f(A) = \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 + 5 + 0 & 3 - 10 + 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Therefore, $f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

16. We have, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

From the given equation, it is clear that order of A should be 2×3

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$\therefore \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a - d & 2b - e & 2c - f \\ a + 0d & b + 0.e & c + 0.f \\ -3a + 4d & -3b + 4e & -3c + 4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a - d & 2b - e & 2c - f \\ a & b & c \\ -3a + 4d & -3b + 4e & -3c + 4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

By equality of matrices, we get

a = 1, b = -2, c = -5

and 2a - d = -1 $\Rightarrow$ d = 2a + 1 = 3;

$\Rightarrow$ 2b - e = -8 $\Rightarrow$ e = 2(-2) + 8 =4

2c - f = -10 $\Rightarrow$ f = 2c + 10 = 0

$$\therefore A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

17. i. L.H.S. $= \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5(2)+(-1)3 & 5(1)+(-1)4 \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

R.H.S. $= \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

$\therefore$ L.H.S. $\neq$ R.H.S.

ii. L.H.S. $= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

R.H.S. $= \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} -1(1)+1(0)+0(1) & (-1)2+1(1)+0(1) & (-1)3+1(0)+0(0) \\ 0(1)+(-1)0+1(1) & (0)2+1(-1)+1(1) & (0)3+0(-1)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$\therefore$ L.H.S. $\neq$ R.H.S.

18. Put $\tan \frac{\alpha}{2} = t$

$$A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ -t & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$
$$\text{L.H.S.} = (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$
$$= (I - A) \begin{bmatrix} \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{-2\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \\ \frac{2\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{-2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1-t^2}{1+t^2} + \frac{t.2t}{1+t^2} & \frac{-2t}{1+t^2} + t\left(\frac{1-t^2}{1+t^2}\right) \\ -t\left(\frac{1-t^2}{1+t^2}\right) + \frac{2t}{1+t^2} & -t\left(\frac{-2t}{1+t^2}\right) + \left(\frac{1-t^2}{1+t^2}\right) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t^3-t}{1+t^2} \\ \frac{t^3+t}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

L.H.S = R.H.S

Hence proved