Playing with Numbers

06

Guessing the Numbers

Often when we do operations like addition, multiplication on numbers, we know about the values of number and its digits. What happened if we don't know the value of numbers and its digits.

Fig. 1

If we have numbers like A, M, P, N, PQ, MN, ABC each of whose letter represents any one digit then how do we think about the possibilities to get to know the value of these digits?

If we think about number with one digit A, M, P, N as given above then we know that these digits can be any one from 0 to 9.

If we think about two digit numbers PQ and MN then P, Q, M and N are also between 0 to 9. In PQ, Q is the digit in the units place and P is the digit in the tens place. Therefore this number is actually 10P + Q. Similarly, number MN is 10M + N.

Similarly a three digit number like ABC is actually 100A + 10B + C.

Now, try to representing such number like ML, XY, AB, PQM, XYZ according to their place values. Think also of some four digit numbers.

Try This

1. If A = 3, B = 4, C = 5, D = 0 and each digit has to be used only one time then using these digits:-
   (i) What will be the largest number.
   (ii) Which number is bigger in ABCD and ADBC.
   (iii) What is the smallest number? Is it a three digit or four digit number?
   (iv) What is the value of DBAC? How many digits will be there in this number?

2. If A = 1, B = 2, C = 3, D = 4 then
   (i) Find the value of AB × CD. (ii) What will be the value of AB + CD.

Guessing the Numbers with Operation

Add these and see

Can you find value of P and Q here?

We know that the value of P and Q can be between 0 to 9.

$$\begin{array}{r} P \\ P \\ +\,P \\ \hline QP \end{array}$$

Now P + P + P = QP

3P = 10Q + P (According to the expanded form of the number)

2 P = 10 Q

$\frac{P}{5} = Q$

It means that P is such number which is divisible by 5.

i.e. P can only be 5, so P = 5

If P = 5 then $\frac{5}{5}$ = Q or Q = 1

Now on checking P + P + P = QP then 5 + 5 + 5 = 15

EXAMPLE-1. PQ – QP = 27 then what will be P and Q.

SOLUTION : (10P + Q) – (10Q + P) = 27

10P + Q – 10Q – P = 27

9P – 9Q = 27

9 (P – Q) = 27

P – Q = 3

Therefore, the possible answers of P and Q are:-

If (i) P = 9 then Q = 6 (ii) P = 8 then Q = 5

(iii) P = 7 then Q = 4 (iv) P = 6 then Q = 3 .....etc.

Thus we get 7 different values of P and Q.

Try This

1. $$\begin{array}{r} AB \\ +\,BA \\ \hline 77 \end{array}$$ than what is the value of A, B?
2. Similarly make same more questions which can have more than one answer.

Guessing the Three Digit Number

What will be the value of 'Y' in $5Y1 - 23Y = 325$.

Here if we compare the digit in units place then $1 - Y = 5$

Therefore $\quad 11 - Y = 5 \quad$ or $\quad Y = 6$

Another method:- $\quad (500 + 10Y + 1) - (200 + 30 + Y) = 325$

$$501 - 230 + 10Y - Y = 325$$

$$271 + 9Y = 325$$

$$9Y = 54 \qquad \text{Therefore } Y = 6$$

Guessing the Numbers when Multiplying and Dividing

EXAMPLE-2.

$$\begin{array}{r} AB \\ \times\, AB \\ \hline ACC \end{array}$$

SOLUTION : $(10A + B)(10A+B) = 100A^2 + 20\,AB + B^2$

It's obvious that $A = 1$

Since units digit is equal to tens digit in the answer,

$2\,AB = B^2$

Therefore $2\,B = B^2$ (Putting $A = 1$)

That is, $B = 2, \; C = 4$

$$\begin{array}{r} 12 \\ \times\, 12 \\ \hline 144 \end{array}$$

EXAMPLE-3.

$$\begin{array}{r} MN \\ \times \quad 3 \\ \hline LMN \end{array}$$

(Where LMN is a three digit number)

SOLUTION : We get N in the units place by multiplying N by 3. This is only possible when $N = 0$ or 5. The expanded form of MN will be $(10M + N)$.

Multiplying $(10M + N)$ by 3 is $(10M + N) \times 3$

$30M + 3N \quad = 100\,L + 10M + N \quad$ .....(i)

If $N = 5$ then

$20M + 15 \quad = 100L + 5$

$20M \quad = 100L - 10$

$$2M = 10L - 1$$

$$M = \frac{10L - 1}{2}$$

We know that L, M and N digits can be any whole number from 0 to 9.

A valid whole number M is not possible for any value of L from 0 to 9. Therefore N = 5 is not possible which implies it will be N = 0.

Come, let us now find out the value of M:-

Now similarly multiplying 3 by digit M in the tens place, we should get M in the tens place of product LMN which means either M = 5 or M = 0.

If we put N = 0 and M = 0 in equation (i) then we will get L = 0 which is not possible because LMN is a three digit number.

This implies M = 5.

If we put N = 0 and M = 5 in equation (i) then we will get L = 1.

Therefore L = 1, M = 5 and N = 0.

Try This

1. Find out the value of B if 1B × B = 96
2. Find out the value of M and N in 73 M ÷ 8 = 9N

Exercise - 6.1

Find out value of letters A, B, X, Y, Z, L, M, N as used in following questions.

(i) $\begin{array}{r} BA \\ +33 \\ \hline 12B \\ \hline \end{array}$ (ii) $\begin{array}{r} 3XY \\ +YY2 \\ \hline 1018 \\ \hline \end{array}$ (iii) $\begin{array}{r} MN \\ \times 6 \\ \hline MLN \\ \hline \end{array}$ (iv) $\begin{array}{r} 1Z \\ \times Z \\ \hline 7Z \\ \hline \end{array}$

(v) $\begin{array}{r} XX \\ 6 \\ +YYY \\ \hline 461 \\ \hline \end{array}$ (vi) $\begin{array}{r} 2PQ \\ +PQ1 \\ \hline Q18 \\ \hline \end{array}$ (vii) $\begin{array}{r} ML \\ \times 6 \\ \hline LLL \\ \hline \end{array}$

Number Riddle (Puzzle)

Alok and Neha are making number riddles and asking each other:

Fig. 2

Neha : Think of any 3 digits and write them.
Don't show me and don't take any zeros.

Alok : Jotted down (Writes on copy 3, 2 and 7)

Neha : Now write all two digit number which can be made by these 3 digits.

Alok : Alright. He writes, 32, 27, 23, 72, 37 and 73.

Neha : Now add all these two digit numbers.

Alok : 32 + 27 + 23 + 73 + 37 + 73 = ...
(He writes in the copy and adds them)

Neha : If you divide this sum by 22 then it will be completely divisible and the quoitient will be sum of digits that you selected.

Alok : 264 ÷ 22 = 12, Yes it is completely divisible.
Now the sum of 3, 2 and 7 (3 + 2 + 7) is 12.
Oh yes! How did you do it? I hope you didn't see what I wrote.

Neha : No, this will work for any three digits you take. I don't need to know your digits.

Try This

Select any 3 number of 3 digits and do Neha's activity.

Did you find sum of digit as Alok did?

Let us Understand Why this is So

If we select a, b and c as three digits then the two digit number which can be made from these numbers are ab, bc, ac, cb, ca and ba.

Expanded form of these are:-

$ab = 10a + b$ $\qquad$ $bc = 10b + c$ $\qquad$ $ac = 10a + c$

$cb = 10c + b$ $\qquad$ $ca = 10c + a$ $\qquad$ $ba = 10b + a$

By adding all these we get-

$$= 10a + b + 10b + c + 10a + c + 10c + b + 10c + a + 10b + a$$

$$= 22a + 22b + 22c = 22\,(a + b + c)$$

This means that the sum is a multiple of 22 so on dividing this sum by 22, the answer is sum of the digits $(a + b + c)$.

Think and Discuss

1. Take any two digit number. Now get a new number by inter changing the place of digits. Add both the numbers. Now their sum will be completely divisible by 11. Can you say how is happens?
2. Think of a three digit number. Now get a new number by placing the digits in reverse order. Subtract the smaller number that we thus obtained, from the bigger number. Is this a multiple of 99? Why?

Which is Divisible by What

Checking the Divisibility

Do you know that how to check which number can be divided by divisors like 10, 5, 2, 3, 9 etc. How does it work? Let us see:-

Divisibility Rule for 10

Checking the divisibility by 10 is easy in comparison to other numbers. See some multiples of 10-

10, 20, 30, 40, 50, ...

For comparing, see some non multiples of 10- 12, 25, 33, 46, 57, 64, 77, 89, ... we can see that numbers that have a zero in the units place are multiples of 10. Whereas the numbers in which units place is not zero are not the multiples of 10. We get the rule of checking the divisibility by 10 through this analysis.

Now we will see how does this rule work? For this we have to use the rules of place value.

Take any number..... *cba*. In expanded form this can be written as

$$\ldots\ldots\ldots\ldots + 100c + 10b + a$$

Here a is a digit in the units place, b is a digit in the tens place and c is a digit in the hundreds place. The..... dots shows that there can be more digits on left side of c.

Here 10, 100, 1000 etc. are divisible by 10 therefore $10b$, $100c$, ... will also be divisible by 10. About a we can say that if the given number is divisible by 10 then a also has to be divisible by 10, which is only possible if $a = 0$.

Obviously, any number will be divisible by 10 when the digit in the units place is 0.

Now give examples of some numbers which are divisible by 10.

Divisibility Rule for 5

See some multiples of 5:-

5, 10, 15, 20, 25, 30, 35, 40, 45, 50.......

we see that the digit of unit place are alternatively either 5 or 0.

This gives us the rule of divisibility for 5, that is if the digit in the units place of any number is either 5 or 0 then that number is divisible by 5. Now, for understanding this rule better, take any number .... cba.

Write it in expanded form .... $100c + 10b + a$

Here a is a digit in the units place, b is a digit in the tens place and c in a digit in the hundreds place and there can be more digits on left side of c. Since 10, 100, ... are divisible by 10. Therefore, $10b$, $100c$, ... will also be divisible by 10. These numbers are therefore also divisible by 5. $(10 = 5 \times 2)$.

Now, about a we can say that if a number is divisible by 5 then a has to be divisible by 5. It is clear that a must be either 0 or 5.

Try This

Are all numbers that are divisible by 5, also divisible by 10? Explain why.

Divisibility Rule for 2

Following are multiples of 2:-

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ... which are all even numbers.

Where the digits in the units place are 2, 4, 6, 8, 0. Therefore one can say that if the units digit of any number is 0, 2, 4, 6, 8 then that number will be divisible by 2.

Now we will check this rule. For this take any number..... cba

Its expanded form will be .... + $100c + 10b + a$

Here a is the digit in the units place, b is a digit in the tens place and c is a digit in the hundreds place. 10, 100, ... are divisible by 10 and so are also divisible by 2. Therefore $10b$, $100c$, ... will be divisible by 2. Let us talk of a now. If a given number is divisible by 2 then a has to be divisible by 2. But it is only possible when $a = 0, 2, 4, 6, 8$.

Divisibility Rule for 9 and 3

You have seen the rules to check for the divisibility of 10, 5 and 2. Have you seen anything special about these rules? In all these rules divisibility is decided by the digit of units place in the given number.

The divisibility rule of 9 and 3 are different. Take any number 3429. Its expanded form will be:-

$$3 \times 1000 + 4 \times 100 + 2 \times 10 + 9 \times 1$$

This can be written in the following way:-

$$3 \times (999 + 1) + 4 \times (99 + 1) + 2 \times (9+1) + 9 \times 1$$

$$= 3 \times 999 + 4 \times 99 + 2 \times 9 + (3 + 4 + 2 + 9)$$

$$= 9\,(3 \times 111 + 4 \times 11 + 2 \times 1) + (3 + 4 + 2 + 9)$$

We see that the given number will be divisible by 9 or 3 only when the number (3 + 4 + 2 + 9) is divisible by 9 and 3.

Here $3 + 4 + 2 + 9 = 18$ which is divisible by both 9 and 3.

Now take one more example:-

Say the number 3579. Its expanded form will be-

$$3 \times 1000 + 5 \times 100 + 7 \times 10 + 9 \times 1$$

This can be written in following way:-

$$3 \times (999 + 1) + 5 \times (99 + 1) + 7 \times (9 + 1) + 9 \times 1$$

$$= 3 \times 999 + 5 \times 99 + 7 \times 9 + (3 + 5 + 7 + 9)$$

Here $3 + 5 + 7 + 9 = 24$ which is not divisible by 9 but is divisible by 3.

If we take any number is *abcd*, then its expanded form will be:-

$$1000a + 100b + 10c + d = 999a + 99b + 9c + (a + b + c + d)$$
$$= 9(111a + 11b + c) + (a + b + c + d)$$

Therefore the divisibility of 9 and 3 is only possible when the sum of all digits of that four digit number is divisible by 9 or 3.

(i) A number is divisible by 9 if sum of its digits is divisible by 9.

(ii) A number is divisible by 3 if sum of its digits is divisible by 3.

Checking the Divisibility Rule for 6

We know that any number that is divisible by 6, will have to be divisible by the prime factors of 6 i.e. 2 and 3.

Hence in order to check the divisibility of the number by 6, we will have to check for its divisibility by both 2 and 3.

Let us recall the rule of divisibility by 2 and 3-

The digit in the units place has to be an even number to make the number divisible by 2.

The sum of all digits of a number has to be divisible by 3 to make the number divisible by 3.

For example- take the number 1248

1248 has 8 in the units place, therefore, 1248 is divisible by 2 and the sum of all four digits of 1248 is 1 + 2 + 4 + 8 = 15 which is divisible by 3.

Hence, 1248 is divisible by 6.

Divisibility Rule for 7 and 11

Rule for checking divisibility rule by 7

Divide 91 by 7. Is 91 completely divisible by 7? On dividing, is the remainder zero? If yes, then is 91 divisible by 7?

Yes 91 is divisible by 7. How did we come to this conclusion? If we have to test the divisibility by 7 then we divide the number by 7. If remainder in zero then that number is divisible by 7. Check of divisibility for 7 of two digit number is done directly by division. But do we check the divisibility of a three digit number also by the same process? Or is there any other method?

There are many methods to check the divisibility by 7. We will check the divisbility by some of these methods:-

Take a three digit number *abc*.

Its expanded form will be- $100a + 10b + c$

$$100a + 10b + c = 98a + 7b + (2a + 3b + c)$$

Let's write it such that it shows 7 as common factor:-

$$7\,(14a + b) + (2a + 3b + c)$$

Here $7\,(14a + b)$ divisible by 7. Now if $(2a + 3b + c)$ is divisible by 7 then number *abc* will also be divisible by 7.

Try This

Check the divisibility of following numbers by 7 using the above methods-

373, 644, 343, 861

Now think of some more three digit numbers and check the divisibility by 7.

There are some more methods for checking the divisibility of numbers with more than three digits by 7. Come, let us see one more method-

$$\begin{array}{l} \underline{36484} \mid 7 \\ 36484 \\ \underline{-\quad 14} \leftarrow (7\times 2) \\ 36470 \end{array}$$

$$\begin{array}{l} \underline{3647} \mid 0 \\ 3647 \\ \underline{-\quad 0} \leftarrow (0\times 2) \\ 3647 \end{array}$$

$$\begin{array}{l} \underline{364} \mid 7 \\ 364 \\ \underline{-14} \leftarrow (7\times 2) \\ 350 \end{array}$$

We can see from this whole process that we double the units digit of the number and subtract it from the number leaving out the units digit.

35 | 0

35

-0 ← (0×2)

35

$\because$ 35 is divisible by 7, therefore 364847 is also divisible by 7.

Check the Divisibility for 11

Take any number *abcd*. Its expanded form is $(1000\,a + 100\,b + 10c + d)$

This can also be written as:-

$(1001 - 1)\ a + (99 + 1)\ b + (11 - 1)\ c + d$

Let's write it such that it shows 11 as a factor-

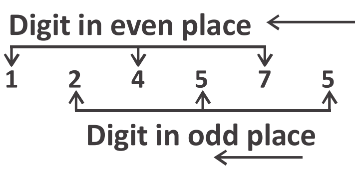
$11\ (91a + 9b + c\ ) - (a - b + c - d)$

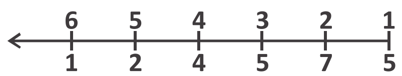
$11\ (91\ a + 9b + c)$ is obviously divisible by 11.

If $a - b + c - d$ is 0 or divisible by 11 then the whole number *abcd* is also divisible by 11.

Therefore, the rule is that if the difference between sum of digits in even places and sum of digits in odd places of the number is either 0 or divisible by 11 then the number is divisible by 11.

Check for number 124575-





Difference = (Sum of digits in odd places) – (Sum of digits in even places)

$= (5 + 5 + 2) - (7 + 4 + 1)$

$= \quad 12 - 12$

$= \quad 0$

Here the meaning of taking difference is to substract the smaller sum from bigger sum.

Difference is zero, therefore number is divisible by 11.

Try This

To check the divisibility of number 19151 by 11.

Similarly there are interesting rules of checking the divisibility by 13. Think and try to find by discussing with friends.

Think and Discuss

1. Are all numbers that are divisible by 3, also divisible by 9? Why or why not?
2. Choose any one number which are completely divisible by 6. Divide this number by 2 and 3 and see. Is this number is divisible by 2 and 3 both? What can you say about the divisibility rule of 6?
3. Using digits 2, 5, 4 and 7, make all the numbers that are completely divisible by 15. Without actually dividing by 15 how will you find out which of these numbers will be divisible by 15? (Use the divisibility rules)
4. Find the smallest number that will be divisible by 7 and 11 both.

Exercise - 6.2

1. Which of the following numbers are multiples of 5 and 10.
   316, 9560, 205, 311, 800, 7936
2. If a number that is divisible by 3, has 8 in its unit place, then what can be possible digits in tens place?
3. If number 35P is a multiple of 5 then find the value of P?
4. If 6A 3B is a number that is divisible by 9, find the value of A and B?
5. Check the divisibility by 7 for following numbers.
   (i) 672 (ii) 905 (iii) 2205 (iv) 9751
6. Check the divisibility by 11.
   (i) 913 (ii) 987 (iii) 3729 (iv) 198
7. Check the divisibility by 13.
   (i) 169 (ii) 2197 (iii) 3146 (iv) 5280

What Have We Learnt

1. We can write numbers in expanded form like a two digit number ab can be written as $10a + b$ and a three digit number abc can be written as $100a + 10b + c$.
2. We take help of expanded form of number in solving puzzle or number games.
3. We learnt about the divisibility rules of 2, 3, 5, 6, 7, 9 and 11.