CBSE Test Paper 03
Chapter 7 Integrals

1. $\frac{d}{dx}\left(\int f(x)dx\right)$ is equal to
   a. $\frac{[(f(x)]^2}{2}$
   b. $\frac{f(x)}{2}$
   c. f(x)
   d. x
2. $\int_0^1 \frac{1-x}{1+x}$ dx is equal to
   a. $\sqrt{2}\log 2-1$
   b. 2 log 2 + 1
   c. 2 log 2 – 1
   d. 1 – 2 log 2
3. $\int_{-\pi/2}^{\pi/2} \sin^9 x\ dx$ is equal to
   a. 0
   b. $\frac{8.4.2}{9.7.5.3}$
   c. 1
   d. $\frac{2.8.4.2}{9.7.5.3}$
4. $\int_0^{\pi/2}$ log (tan x) dx is equal to
   a. $\int_0^{\pi/2}$ log (cot x) dx
   b. 1
   c. $-\frac{\pi}{2}\log 2$
   d. $\frac{\pi}{2}\log 2$
5. $\int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)dx$ is equal to

a. $-\cos\left(\frac{1}{x^2}\right)+C$
b. $\cos\left(\frac{1}{x^2}\right)+C$
c. $-\sin\left(\frac{1}{x}\right)+C$
d. $\cos\left(\frac{1}{x}\right)+C$

6. The indefinite integral of x^2 + 7 is _______.
7. The definite integral of $\int_1^2 \left(x^{-2}+2x^{-3}\right) dx$.
8. The definite indefinite integral of $\frac{2}{x^2}$ is _______.
9. Evaluate $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha} dx$.
10. Write the value of $\int \sec x(\sec x+\tan x)dx$.
11. $\int \frac{(x+1)(x+\log x)^2}{x} dx$.
12. Evaluate $\int \sqrt{\frac{1+x}{1-x}} dx, x\neq 1$.
13. Evaluate $\int \sqrt{2ax-x^2} dx$.
14. Evaluate $\int \frac{dt}{\sqrt{3t-2t^2}}$.
15. $\int e^{2x}\cdot\sin(3x+1)$dx.
16. $\int e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right)dx$.
17. Evaluate $\int_0^{\pi} \frac{x}{1+\sin x}$.
18. Evaluate $\int_1^4 \left(x^2-x\right) dx$ as a limit of a sum.

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Solution

1. c. f(x)
   Explanation: $\frac{d}{dx}(\int f(x)dx) = f(x)$
2. c. 2 log 2 – 1
   Explanation: $= \int\limits_0^1 (-1 + \frac{2}{1+x})dx = [-x + 2\log(1+x)]_0^1$
3. a. 0
   Explanation: Note that $\sin^9$x is an odd function, therefore, $\int\limits_{-\pi/2}^{\pi/2} \sin^9 x dx = 0$
4. a. $\int\limits_0^{\pi/2}$ log (cot x) dx
   Explanation:
   $\int\limits_0^{\pi/2} \log(\tan x)dx = \int\limits_0^{\pi/2} \log(\tan(\frac{\pi}{2} - x))dx. \implies \int\limits_0^{\pi/2} \log(\cot x)\ dx$
5. d. $\cos\left(\frac{1}{x}\right) + C$
   Explanation: $\frac{1}{x} = t,$
   $-\frac{1}{x^2}dx = dt = \int \sin t(-dt) = \int(-\sin t)dt = \cos t + C$
6. $\frac{1}{3}x^3 + 7x + c$
7. $\frac{5}{4}$
8. $\frac{6}{x^3} + c$
9. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx$
   $= \int \frac{(2\cos^2 x - 1) - (2\cos^2\alpha - 1)}{(\cos x - \cos\alpha)}dx$
   $= \int \frac{(2\cos^2 x) - (2\cos^2\alpha)}{(\cos x - \cos\alpha)}dx$
   $= \int \frac{2(\cos^2 x - \cos^2\alpha)}{(\cos x - \cos\alpha)}dx$
   $= \int \frac{2(\cos x + \cos\alpha)(\cos x - \cos\alpha)}{(\cos x - \cos\alpha)}dx$
   $= \int 2(cosx + cos\alpha)\ dx$
   $= 2(\sin x + \cos\alpha . x) + c$

10. Let I = $\int \sec x(\sec x+\tan x)dx$

$= \int \left(\sec^2 x+\sec x\tan x\right) dx$

$= \int \sec^2 x dx + \int \sec x \tan x dx$

= tan x + secx + C

11. $I = \int \frac{(x+1)(x+\log x)^2}{x} dx$

Put $x+\log x = t$

$\left(1+\frac{1}{x}\right) dx = dt$

$\left(\frac{x+1}{x}\right) dx = dt$

$\therefore I = \int t^2\, dt$

$= \frac{t^3}{3} + c$

$= \frac{(x+\log x)^3}{3} + c$

12. $I = \int \sqrt{\frac{1+x}{1-x}} dx$

$= \int \sqrt{\frac{(1+x)(1+x)}{(1-x)(1+x)}} dx$

$= \int \frac{1+x}{\sqrt{1-x^2}} dx$

$= \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{xdx}{\sqrt{1-x^2}} = \sin^{-1} x + I_1,$

where $I_1 = \int \frac{xdx}{\sqrt{1-x^2}}$

Put $1-x^2 = t^2 \Rightarrow -2xdx = 2tdt.$Therefore

$I_1 = -\int dt = -t + C = -\sqrt{1-x^2} + C$

Hence $I = \sin^{-1} x - \sqrt{1-x^2} + C.$

13. Let $I = \int \sqrt{2ax-x^2} dx = \int \sqrt{-\left(x^2-2ax\right) dx}$

$= \int \sqrt{-\left(x^2-2ax+a^2-a^2\right)} dx = \int \sqrt{-\left[(x-a)^2-a^2\right]} dx$

$= \int \sqrt{a^2-(x-a)^2} dx$

$= \frac{x-a}{2}\sqrt{a^2-(x-a)^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$

$= \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$

14. Let $I = \int \frac{dt}{\sqrt{3t-2t^2}} = \frac{1}{\sqrt{2}} \frac{dt}{\sqrt{-\left(t^2-\frac{3}{2}t\right)}}$

$= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{-\left[\left(t^2-2.\frac{1}{2}.\frac{3}{2}t\right)+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2\right]}}$

$= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{-\left[\left(t-\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2\right]}}$

$= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2-\left(t-\frac{3}{4}\right)^2}}$

$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{t-\frac{3}{4}}{\frac{3}{4}}\right) + C = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4t-3}{3}\right) + C$

15. According to the question, $I = \int \underset{II}{e^{2x}} \sin(3\underset{I}{x} + 1)dx$ ...(i)

$= \sin(3x+1) \int e^{2x}dx - \int \left\{ \frac{d}{dx} \sin(3x+1) \int e^{2x}dx \right] dx$

[Using integration by parts]

$= \frac{\sin(3x+1) \cdot e^{2x}}{2} - \int 3\cos(3x+1) \cdot \frac{e^{2x}}{2} dx$

$= \frac{e^{2x}\sin(3x+1)}{2} - \frac{3}{2} \int \underset{II}{e^{2x}} \underset{I}{\cos}(3x+1)dx$

$= \frac{e^{2x}\sin(3x+1)}{2} - \frac{3}{2} [\cos(3x+1) \int e^{2x}dx$

$- \int \left\{ \frac{d}{dx} \cos(3x+1) \int e^{2x}dx \right\} dx$

[using integration by parts]

$= \frac{e^{2x}\sin(3x+1)}{2} - \frac{3}{2} \left[\cos(3x+1) \cdot \frac{e^{2x}}{2} \right.$

$\left. - \int -3\sin(3x+1) \frac{e^{2x}}{2} dx \right] + C_1$

$\Rightarrow I = \frac{e^{2x}\sin(3x+1)}{2} - \frac{3}{4} e^{2x} \cos(3x+1)$

$- \frac{9}{4} \int e^{2x} \sin(3x+1)dx + C_1$

$\Rightarrow I = \frac{e^{2x}\sin(3x+1)}{2} - \frac{3}{4} e^{2x} \cos(3x+1) - \frac{9}{4} I + C_1$

[from Equation (i)]

$\Rightarrow \frac{13}{4} I = \frac{e^{2x}\sin(3x+1)}{2} - \frac{3e^{2x}\cos(3x+1)}{4} + C_1$

$\therefore \quad I = \frac{2e^{2x}\sin(3x+1)}{13} - \frac{3e^{2x}\cos(3x+1)}{13} + C$

where, $C = \frac{4C_1}{13}$.

16. $\int e^x \left(\tan^{-1}x + \frac{1}{1+x^2}\right) dx$

$f(x) = \tan^{-1}x$

$f'(x) = \frac{1}{1+x^2}$

$\int e^x \left(\tan^{-1}x + \frac{1}{1+x^2}\right) dx = e^x \tan^{-1}x + c$

$[\because \int e^x [f(x) + f'(x)]dx = e^x f(x) + C]$

17. Let $I=\int\limits_0^\pi \frac{x}{1+\sin x}dx$ ...(i)

and $I=\int\limits_0^\pi \frac{(\pi-x)}{1+\sin(\pi-x)}dx=\int_0^\pi \frac{\pi-x}{1+\sin x}dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2I=\pi\int_0^\pi \frac{1}{1+\sin x}dx$

$=\pi\int_0^\pi \frac{(1-\sin x)dx}{(1+\sin x)(1-\sin x)}$

$=\pi\int_0^\pi \frac{(1-\sin x)dx}{\cos^2 x}$

$=\pi\int_0^\pi \left(\sec^2 x-\tan x.\sec x\right)dx$

$=\pi\int\limits_0^\pi \sec^2 xdx-\pi\int\limits_0^\pi \sec x\tan xdx$

$=\pi(\tan x)_0^\pi-\pi(\sec x)_0^\pi$

$=\pi(\tan\pi-\tan 0)-\pi(\sec\pi-\sec 0)$

$\Rightarrow 2I=\pi(0-0)-\pi(-1-1)=2\pi$

$2I=2\pi$

$\therefore I=\pi$

18. According to the question, $I=\int_1^4 \left(x^2-x\right)dx$

On comparing the given integral with $\int_a^b f(x)dx$, we get

$a=1, b=4, f(x)=x^2-x$

We know that,

$\int_a^b f(x)dx=\lim\limits_{h\to 0} h[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\ ]$ ...(i)

where, $h=\frac{b-a}{n}\Rightarrow nh=b-a.$

$nh=b-a=4-1=3$

Now,

f(a) = f(1) = $(1)^2$ - 1 = 0

f(a + h) = f(1 + h)= $(1 + h)^2$ - (1 + h) = 1 + h^2 + 2h - 1 - h = h^2 + h

f(a + 2h) = f(1 + 2h) = $(1 + 2h)^2$ - (1 + 2h) = 1 + $4h^2$ + 4h - 1 - 2h = $4h^2$ + 2h

so on

f[a + (n - 1)h] = f[1 + (n - 1)h] =$[1 + (n - 1)h]^2$ - [1 + (n - 1)h] = 1 + $(n - 1)^2h^2$ + 2(n - 1)h - 1 - (n - 1)h = $(n - 1)^2h^2$ + (n - 1)h

$\therefore \int_1^4 \left(x^2-x\right)dx=\lim\limits_{h\to 0}$

h[0 + (h^2 + h) + ($4h^2$ + 2h) +..........+ (n -1)$2h^2$ + (n - 1)h]

Now , by rearranging terms , we get

$= \lim_{h \to 0}$h[h^2{1 + 4 +......+$(n - 1)^2$ + h{1 + 2 +........(n - 1)}]

$= \lim_{h \to 0}$h[h^2{1^2 + 2^2 +.....+$(n - 1)^2$} + h{ 1 + 2.........+(n - 1)}]

$$= \lim_{h \to 0} h\left[h^2 \frac{n(n-1)(2n-1)}{6} + h\frac{n(n-1)}{2}\right] [\because \sum n = \frac{n(n+1)}{2}\ and\ \sum n^2 = \frac{n(n+1)(2n+1)}{6}]$$

$$= \lim_{h \to 0} h\left[\frac{n(nh-h)(2nh-h)}{6} + \frac{n(nh-h)}{2}\right]$$

$$= \lim_{h \to 0}\left[\frac{nh(nh-h)(2nh-h)}{6} + \frac{nh(nh-h)}{2}\right]$$

$$= \lim_{h \to 0}\left[\frac{3(3-h)(6-h)}{6} + \frac{3(3-h)}{2}\right]$$

$$= \frac{3\times3\times6}{6} + \frac{3\times3}{2}$$

$$= 9 + \frac{9}{2}$$

$= \frac{27}{2}$ sq units