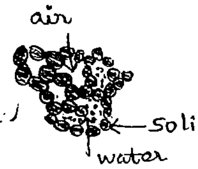
11. CONSOLIDATION

→ Compression

Compressibility is the property of soil due to which compression occurs. Clay has relatively more compressibility compared to gravel, sand and silt.

Compression of soil is due to :-

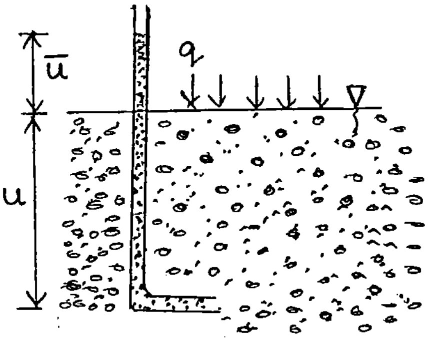


- compression and escape of air from voids → compaction
- escape of pore water; whereas compression of solid grains and water is negligible. → consolidation

∴ compression depends upon volume of voids. More the vol. of voids, more will be the compression.

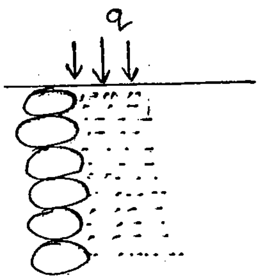
→ Consolidation

- It is the compression of soil due to expulsion of water under static long term loading.
- It is a slow process.
- It occurs in low permeable soil.



u = pore water pressure (or) Hydrostatic pressure.

$\bar{u}$ = excess pore water pressure (or) hydrodynamic pressure.



Hydro static → due to self wt.

Hydrodynamic → due to external load.

k_w → stiffness of water under confined condition

k_s → stiffness of soil grain structure.

$$k_w >> k_s.$$

At beginning, $\bar{u} = q$

At end of consolidation, $\bar{u} = 0$

Immediately after Loading	During Consolidation	At the end of consolidation
$\bar{u} = q$	$\bar{u} < q$	$\bar{u} = 0$
$\sigma' = 0$	$\sigma' > 0$	$\sigma' = q$
$\bar{u} + \sigma' = q$.	$\bar{u} + \sigma' = q$	$\bar{u} + \sigma' = q$

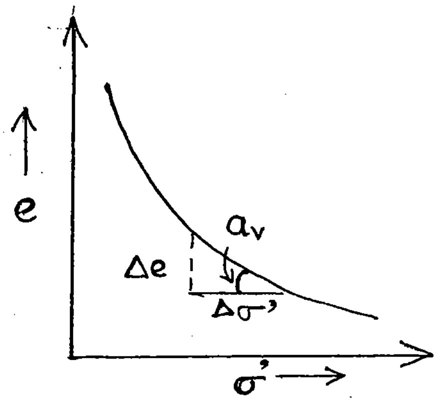
During consolidation, excess pore pressure ($\bar{u}$) decreases, effective stress (σ') increases, but total stress (σ) remains constant.

th Sept,
TURDAY

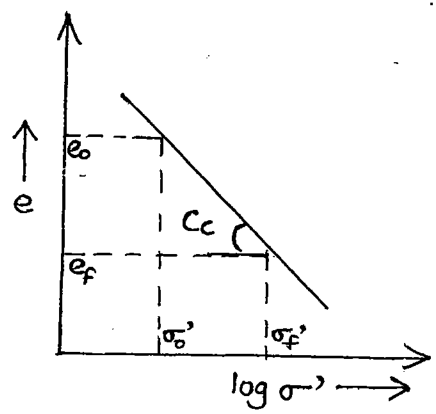
During Consolidation,

Properties which decrease.	Properties which increase	Properties which remains same
$\bar{u}$, Permeability, Compressibility, water content, void ratio.	σ', γ_d, settlement, shear strength.	σ, $S_r (= 100\%)$

→ $e - \sigma'$ curve.



→ $e - \log \sigma'$ curve



* Coefficient of Compressibility, a_v

$$a_v = \frac{\Delta e}{\Delta \sigma'}$$

* Compression Index, C_c

Coefficient of compression index, $C_c = \dfrac{e_0 - e_f}{\log\left(\dfrac{\sigma_f'}{\sigma_0'}\right)}$

$$C_c = \frac{\Delta e}{\log_{10}\left(\dfrac{\sigma_f'}{\sigma_0'}\right)}$$

$$\Rightarrow \Delta e = C_c \log_{10}\left(\frac{\sigma_f'}{\sigma_0'}\right)$$

$C_c = 0.007\,(w_L - 10)$; for remoulded clay

$C_c = 0.009\,(w_L - 10)$; for field consolidation (OR) normally consolidated clay

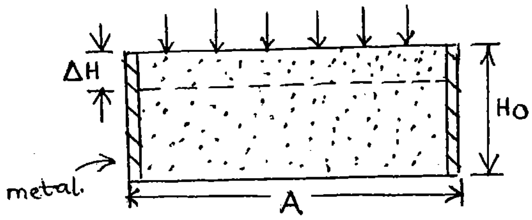
* Coeffecient of Volume Change (or) Coeffecient of Volume compressibility

$$m_v = \frac{\Delta V}{V_0 \cdot \Delta\sigma'}$$

It is the volumetric strain per unit change of effective str

If soil is laterally confined (area remains same),

$$\frac{\Delta V}{V_0} = \frac{\Delta H}{H_0}$$



We have, $V \propto 1 + e$.

$$\frac{\Delta V}{V_0} = \frac{\Delta e}{1 + e_0}$$

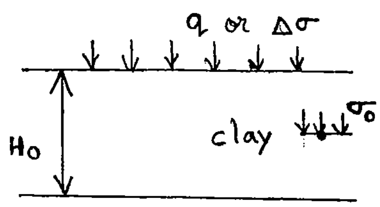
$$\Rightarrow \frac{\Delta H}{H_0} = \frac{\Delta e}{1 + e_0}$$

$$m_v = \frac{\Delta V}{V_0 \cdot \Delta\sigma'} = \frac{\Delta e}{(1+e_0)\,\Delta\sigma'}$$

$$= \frac{a_v}{1+e_0} = \frac{\Delta H}{H_0 \cdot \Delta\sigma'}$$

* To find ultimate or final consolidation settlement, S_f or ΔH

(i) $\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0}$



$\therefore$ ΔH or $S_f = H_0 \left(\frac{\Delta e}{1+e_0}\right)$

(ii) ΔH or $S_{f_1} = H_0 \cdot \frac{C_c}{(1+e_0)} \log_{10}\left(\frac{\sigma_f'}{\sigma_0'}\right)$

$\sigma_f' = \sigma_0' + \Delta\sigma'$

$\sigma_0' \rightarrow$ original or initial effective stress in the clay (due to self weight) at the centre of clay

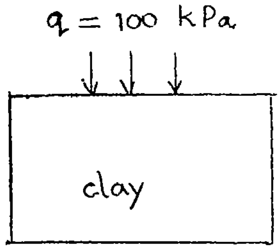
(iii) $m_v = \frac{\Delta H}{H_0 \cdot \Delta\sigma'}$

$\therefore$ ΔH or $S_f = m_v \cdot H_0 \cdot \Delta\sigma'$

(ii)nd equation is always preferred. because for a given soil, a_v & m_v are not constant; these decrease with increase in σ'. But C_c is always constant.

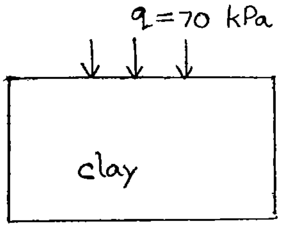
→ Depending upon Stress history, the States of Soil are:

- Normally Consolidated soil. (NC soil)
- Over Consolidated soil (OC soil)
- Under Consolidated soil (UC soil).



At the end of consolidation, $\sigma' = 100$ kPa

(Pre consolidated stress, $\sigma_c' = 100$ kPa)



Overconsolidated soil (or) preconsolidated soil.

Overconsolidated soil :- if the soil has ever been subjected to a pressure greater than existing pressure.

Normally consolidated soil :- if the soil has <u>never</u> been subjected to a pressure more than existing pressure.

Under consolidated soil :- when the soil is under consolidation.

→ Over Consolidation Ratio, OCR

$$OCR = \frac{\sigma_c'}{\sigma'}$$

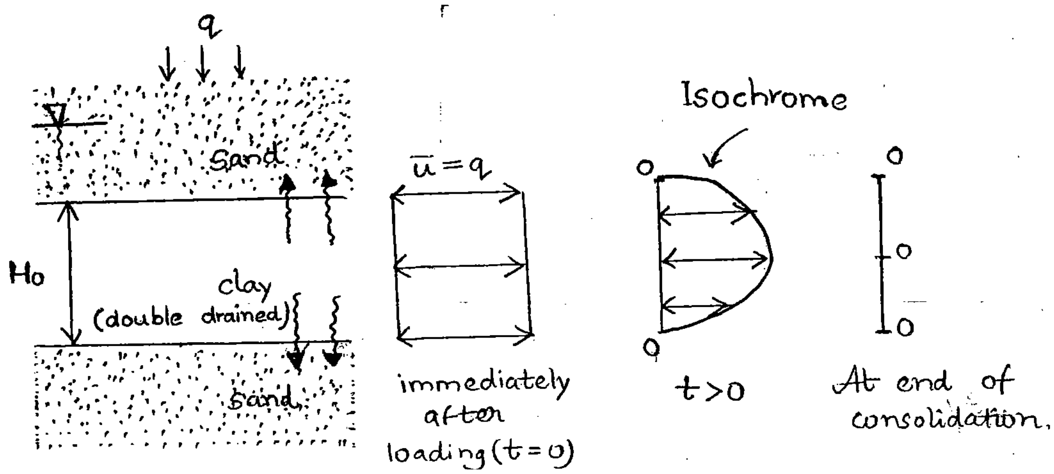
It is the ratio of preconsolidation stress to present effective stress in the soil.

For NC soil, $OCR = 1$.

For OC soil, $OCR > 1$

For UC soil, $OCR < 1$

→ Terzaghi's Theory of 1-D Consolidation :



Water tends to escape in vertical direction (to sand hav higher permeability), one direction, ∴ called 1D consolidation.

Water near the sand escapes first (lesser drainage distance) and $\bar{u} = 0$. Since there is sand in both sides of clay, it is called double drained clay.

* Slope of Isochrome $= \frac{\bar{u}}{z} = \frac{h}{z} = i$ = hydraulic gradie.

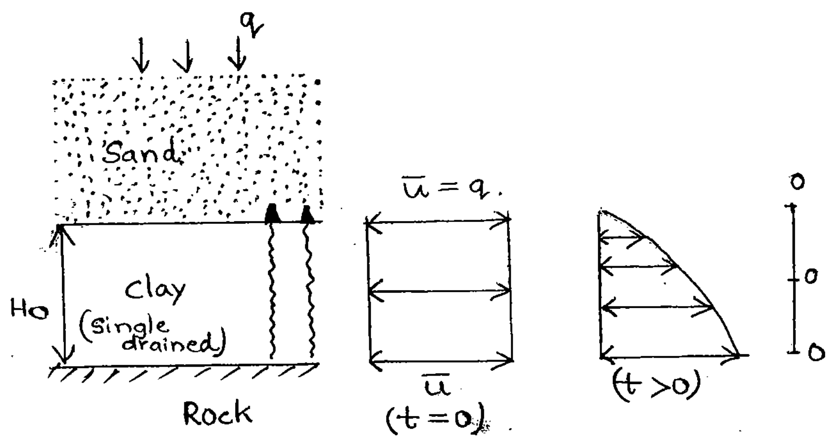
But $\bar{u}$ varies with time and depth. ∴ slope is not constant.

* Drainage path, d

$$d = \frac{H_o}{2} \quad ; \text{ for double drained condition}$$

$$d = H_o \quad ; \text{ for single drained condition}$$

Drainage path is the max distance that the water has to travel to escape.



$$\frac{\partial \bar{u}}{\partial t} = \frac{k}{m_v \gamma_w} \frac{\partial^2 u}{\partial z^2}$$

DIFFERENTIAL EQUATION OF 1D CONSOLIDATION

$$\boxed{\frac{\partial \bar{u}}{\partial t} = C_v \cdot \frac{\partial^2 \bar{u}}{\partial z^2}}$$

→ Coefficient of Consolidation, C_v

$$C_v = \frac{k}{m_v \gamma_w}$$

Units: m^2/sec , cm^2/sec

$$\frac{\partial \bar{u}}{\partial t} = C_v \frac{\partial^2 \bar{u}}{\partial z^2}$$

Solution is given in terms of (i) Time factor, T_v
(ii) Degree of consolidation, U

* Time factor, $T_v = \frac{C_v t}{d^2}$

$t \rightarrow$ time of consolidation.

* Degree of consolidation, $U = \frac{S}{S_f} \times 100$

$S \rightarrow$ settlement occured upto certain time, t
$S_f \rightarrow$ final settlement.

At beginning, $S = 0$; $U = 0$ $0 \leq U \leq 100\%$
At end, $S = S_f$; $U = 100\%$

Also, $U = \frac{\text{dissipated excess pore pressure}}{\text{initial excess pore pressure}} \times 100$

$$= \frac{\bar{u}_i - \bar{u}}{\bar{u}_i} \times 100$$

$\bar{u}_i \rightarrow$ Initial excess pore pressure
$\bar{u} \rightarrow$ excess pore pressure after certain time, t

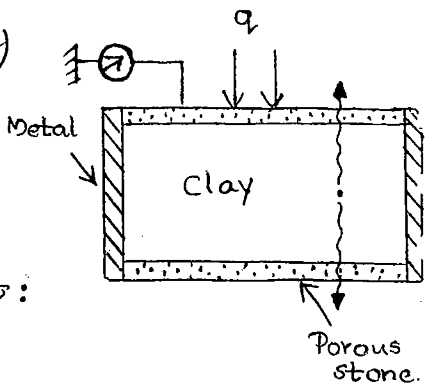
or $U = \frac{\sigma'}{\bar{u}_i} \times 100$; $\sigma' = \bar{u}_i - \bar{u}$

* Relation b/w U & T_v

(i) $T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$; $U \leq 60\%$

(ii) $T_v = 1.781 - 0.933 \log_{10}(100 - U\%)$; $U > 60\%$

→ Consolidation Test (Oedometer Test)



- undisturbed sample is used
- diameter of sample $\geq 3 \times$ thickness.

* To find 'e' in Consolidation test:

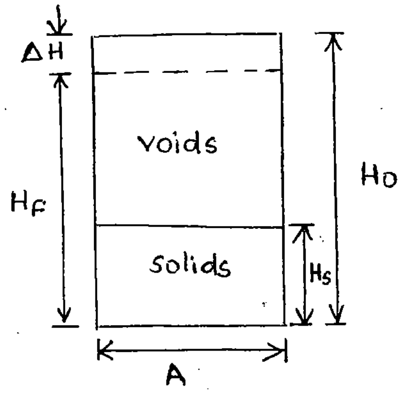
(i) Change in void ratio method:-

Final most void ratio, $e_f = w_f G$

$$\frac{\Delta e}{1+e_f} = \frac{\Delta H}{H_f}$$

Initial void ratio, $e_0 = e_f \pm \Delta e$

(ii) Height of Solids method:-



$H_s \rightarrow$ Height of solids

$$H_s = \frac{W_d}{G \gamma_w \cdot A}$$

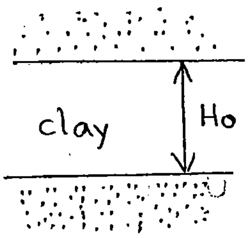
$$e_0 = \frac{H_0 - H_s}{H_s}$$

$$H_f = H_0 - \Delta H.$$

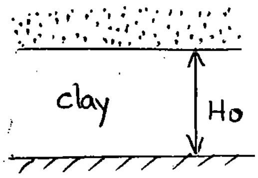
$$e_f = \frac{H_f - H_s}{H_s}$$

NOTE :

- For same loading and same clay properties, the ultimate consolidation settlement remains the same for both double drain clay condition and single drained clay condition.



$d = \frac{H_0}{2}$; S_{f1}



S_{f2} ; $d = H_0$

$S_{f1} = S_{f2}$

$$\Delta H = H_0 \left(\frac{\Delta e}{1+e_0}\right) \Rightarrow \Delta H \propto H_0$$

- For a given clay, C_v is constant (assumed).

For a given clay, to undergo same degree of consolidation the time required for a double drained condition is $\frac{1}{4}$th time required for a single drained condition

$$T_v = \frac{C_v t}{d^2}$$

If C_v & U (or T_v) are same, $t \propto d^2 \Rightarrow \frac{t_1}{t_2} = \left(\frac{d_1}{d_2}\right)^2$

$$\boxed{t \propto \frac{d^2 m_v}{K}}$$

$$t = \frac{T_v d^2}{C_v} = \frac{T_v d^2 \cdot m_v \gamma_w}{k}$$

∴ For same degree of consolidation, $t \propto \frac{d^2 m_v}{K}$.

$$\frac{t_2}{t_1} = \left(\frac{d_2}{d_1}\right)^2 \left(\frac{m_{v2}}{m_{v1}}\right) \frac{K_1}{K_2}$$

15th Sept, MONDAY

P-58

01 From 1 to 2 kg/cm², $\Delta H = 1$ cm $\Rightarrow \Delta\sigma' = 2-1 = 1$; $\frac{\sigma_f'}{\sigma_0'} = \frac{2}{1} = 2$

From 2 to 4 kg/cm², $\Delta H = ?$ $\Rightarrow \Delta\sigma' = 4-2 = 2$; $\frac{\sigma_f'}{\sigma_0'} = \frac{4}{2} = 2$

$$\Delta H = H_0 \cdot \frac{C_c}{1+e_0} \log_{10}\left(\frac{\sigma_f'}{\sigma_0'}\right)$$

Since $\frac{\sigma_f'}{\sigma_0'}$ ratio is same, ΔH is also same.

$$\therefore \Delta H = 1 \text{ cm}$$

02. In $t_1 = 4$ years ; $S_1 = 80$ mm

$t_2 = 9$ years ; $S_2 = ?$

$$T_v = \frac{C_v t}{d^2}$$

$$\frac{\pi}{4} u^2 = \frac{C_v t}{d^2} \Rightarrow t \propto u^2$$

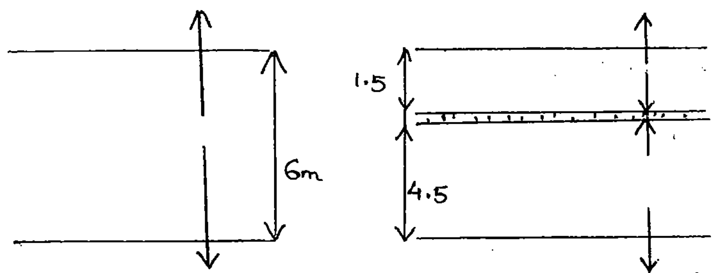
$$u = \frac{S}{S_f} \Rightarrow u \propto S.$$

$$\therefore \boxed{t \propto S^2}$$

$$\frac{t_1}{t_2} = \left(\frac{S_1}{S_2}\right)^2 \quad \text{(valid only when } u < 60\%\text{)}$$

$$\frac{4}{9} = \left(\frac{80}{S_2}\right)^2 \Rightarrow S_2 = \frac{80 \times 3}{2} = \underline{\underline{120 \text{ mm}}}$$

3.



Two drainage paths. 4 drainage paths

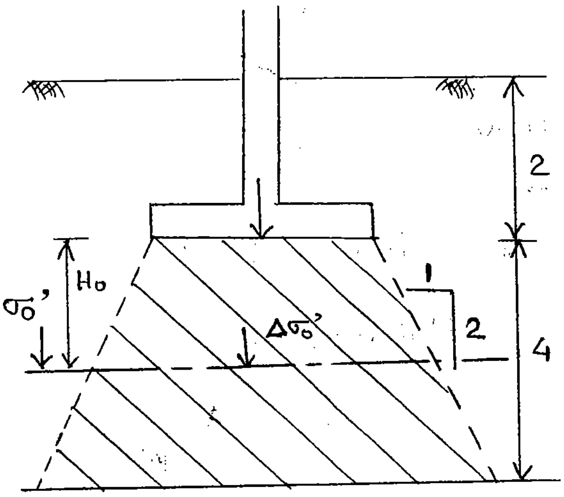
But ultimate settlement remains the same

4.

$H_o = 4\,m$, $C_c = 0.36$

$e_o = 0.92$

$$\sigma_0' = \left(2 + \frac{4}{2}\right)\gamma' = \underline{\underline{37.2 \text{ kPa}}}$$



$$\Delta\sigma' = \frac{Q}{(B+z)^2}$$

z : distance b/w pt. where load is acting & pt. where $\Delta\sigma$ load is required.

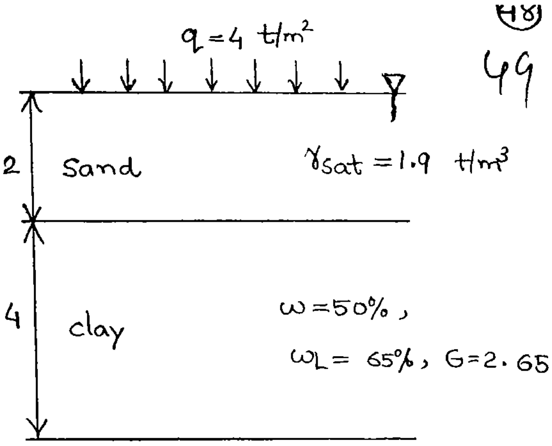
$$\Delta\sigma' = \frac{500}{(B+2)^2}$$

$$S_f = H_o \cdot \frac{C_c}{1+e_o} \log_{10}\left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}\right)$$

$$0.12 = 4 \times \frac{0.36}{1+0.92} \log_{10}\left(\frac{37.2 + \frac{500}{(B+2)^2}}{37.2}\right)$$

$$\Rightarrow B = \underline{\underline{3.5 m}}$$

05. $H_0 = 4m$;



$C_c = 0.009\,(\omega_L - 10) = 0.495$

$e_0 = \frac{\omega G}{S_r} = 0.5 \times 2.65 = 1.325$

$\sigma_0' = 2\gamma' + 2\gamma'$

$= 2(1.9-1) + 2(1.71-1)$

$= 3.2\ t/m^2$

For clay, $\gamma_{sat} = \gamma_w \frac{(G+e_0)}{1+e_0} = 1.71\ t/m^3$.

$\Delta\sigma' = q = 4\ t/m^2$.

$$S_f = 4 \times \frac{0.495}{1+1.325} \log_{10}\left(\frac{3.2+4}{3.2}\right) = 29.8\ cm$$

06. $u_1 = 25\%$, $t_1 = 10$ min

$u_2 = 50\%$, $t_2 = ?$

$$\frac{t_1}{t_2} = \left(\frac{25}{50}\right)^2 \qquad (t \propto u^2)$$

$$\frac{10}{t_2} = \frac{1}{4} \Rightarrow t_2 = 40\ min$$

07. $$U = \frac{\bar{u_i} - \bar{u}}{\bar{u_i}} \times 100 = \frac{2-0.6}{2} \times 100 = 70\%$$

$$U = \frac{S}{S_f} \times 100.$$

$$70 = \frac{S}{20} \times 100 \Rightarrow S = 14\ mm$$

08. $$t \propto \frac{d^2 m_v}{k}$$

$$\frac{t_2}{t_1} = \left(\frac{d_2}{d_1}\right)^2 \left(\frac{m_{v2}}{m_{v1}}\right) \cdot \left(\frac{k_1}{k_2}\right)$$

$$= \left(\frac{2d_1}{d_1}\right)^2 \left(\frac{4m_{v1}}{m_{v1}}\right)\left(\frac{k_1}{3k_1}\right) = \frac{16}{3} \Rightarrow t_2 = \frac{16}{3} \times 15 = 80\ ye$$

9. **Lab specimen** | **Field Clay**

Lab specimen:

$H_1 = 20$ mm

$d_1 = \frac{H_1}{2} = 10$ mm.

$U_1 = 50\%$

$t_1 = 45$ min.

Field Clay:

$H_2 = 10$ m

$d_2 = \frac{H_2}{2} = 5000$ mm

$U_2 = 50\%$

$t_2 = ?$

For same C_v & U, $t \propto d^2$

$$\frac{t_2}{t_1} = \left(\frac{d_2}{d_1}\right)^2$$

$$\frac{t_2}{45} = \left(\frac{5000}{10}\right)^2$$

$t_2 = 21.4$ years

10. If field clay is single drained = 4 times of 21.4 years
= 85.6 years

Q. 11. In the laboratory ..... Find the time required to undergo 70% consolidation of a field clay of 3m thick and double drained?

Lab Specimen:

$H_1 = 25$ mm

$d_1 = H_1 = 25$ mm (single drained)

$U_1 = 50\%$

$t_1 = 11$ min.

Field Clay:

$H_1 = 3$ m

$d_1 = \frac{H_1}{2} = 1.5$ m $= 1500$ mm (double drai

$U_2 = 70\%$

$t_2 = ?$

For same C_v, $t \propto d^2 T_v$.
(same soil)

$$\frac{t_2}{11} = \frac{0.405}{0.197} \times \left(\frac{1500}{25}\right)^2$$

$t_2 = 56.5$ days

12. Double drainage | Single drainage

Double drainage	Single drainage
H	H
$d_1 = \frac{H}{2}$	$d_1 = H.$
$t_1 = 5$ years.	$t_2 = 5$ years.
$S_1 = 9$ cm.	$S_2 = ?$
$S_f = 45$ cm	$S_f = 45$ cm.

$$T_v = \frac{\pi}{4} U^2$$

$$\frac{C_v t}{d^2} = \frac{\pi}{4} U^2 \Rightarrow U \propto \frac{1}{d}$$

$$U \propto S \Rightarrow \boxed{S \propto \frac{1}{d.}} \quad (\text{when } u < 60\%).$$

$$\frac{S_1}{S_2} = \frac{d_2}{d_1}$$

$$\frac{9}{S_2} = \frac{2H}{H} \Rightarrow S_2 = \underline{\underline{4.5}} \text{ cm}$$

13. For $t_1 = 4$ years. ; $S_1 = 80$ mm.

For $t_2 = 9$ years ; $S_2 = ?$

$$U_1 = \frac{S_1}{S_f} \times 100 = 26.67\%$$

$$T_{v1} = \frac{\pi}{4}\left(\frac{U_1}{100}\right)^2 = \underline{\underline{0.0558}}$$

$$T_v = \frac{C_v t}{d^2} \Rightarrow t \propto T_v$$

$$\frac{t_2}{t_1} = \frac{T_{v2}}{T_{v1}}$$

$$\frac{9}{4} = \frac{T_{v2}}{0.0558} \Rightarrow \underline{\underline{T_{v2}}} = 0.1255$$

From the table given, $T_{v2} = \frac{\pi}{4}\left(\frac{U_2}{100}\right)^2$

$$0.125 = \frac{\pi}{4} \times \left(\frac{U_2}{100}\right)^2 \Rightarrow U_2 = 40\%$$

$$U_2 = \frac{S_2}{S_f}$$

$$0.4 = \frac{S_2}{300} \Rightarrow S_2 = \underline{\underline{120}} \text{ mm}$$

14. $$\frac{T_{v2}}{T_{v1}} = \frac{t_2}{t_1}$$

$$\frac{T_{v2}}{0.0558} = \frac{25}{4} \Rightarrow T_{v2} = 0.348$$

$\therefore U_2 = 65\%$ (from table)

$$U_2 = \frac{S}{300} \Rightarrow S = 300 \times 0.65 = \underline{195\ mm}$$

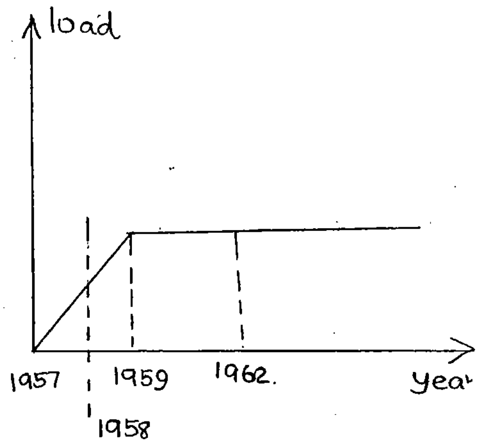
15. $$U_2 = \frac{S_2}{S_f} \times 100 = 70\%$$

$\therefore T_{v2} = 0.403$ (from table)

$$\frac{t_2}{t_1} = \frac{T_{v2}}{T_{v1}} \Rightarrow \frac{t_2}{4} = \frac{0.403}{0.0558}$$

$\therefore t_2 = \underline{28.8}$ years

16. Consolidation time is measured from the middle of construction period. (1958)



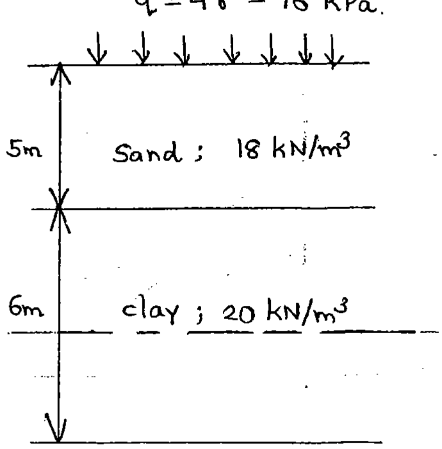
$t = 1962 - 1958 = 4$ years ; $S_1 = 90$ mm.

$t = 1967 - 1958 = 9$ years ; $S_2 = ?$

$$\frac{t_2}{t_1} = \left(\frac{S_2}{S_1}\right)^2$$

$$\frac{9}{4} = \left(\frac{S_2}{90}\right)^2$$

$S_2 = \underline{135\ mm}$



17. **Immediately after loading**

$\sigma = q + 5\gamma_{sat} + 3\gamma_{sat}$

$= 76 + 5 \times 18 + 3 \times 20$

$= 226$ kPa.

Total $U = U_{st} + \bar{U}$

$= (5+3)10 + 76$

$= 156$ kPa

After many years (at end of consolidation)

$\sigma = 226$ kPa.

Total $\bar{U} = U_{static}$

$= 80$ kPa.

$\sigma' = 226 - 80 = \underline{146\ kPa}$

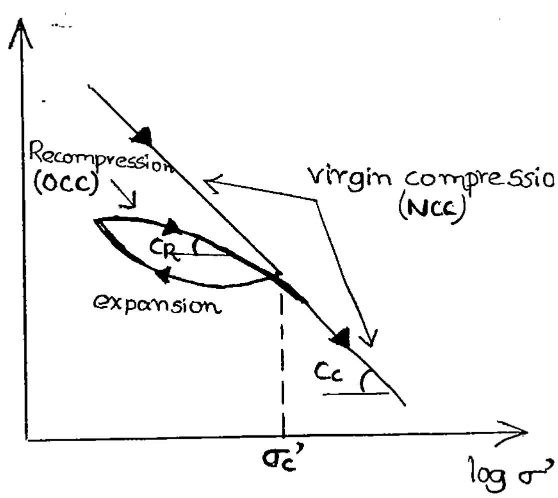
σ' (immediately after loading) $= 5\gamma' + 3\gamma'$

$= 5 \times (18-10) + 3(20-10).$

$= \underline{\underline{70 \text{ kPa}}}$

σ' (after many years) $= \sigma_0' + \Delta\sigma = 70 + 76 = \underline{\underline{146 \text{ kPa}}}$

→ Recompression Index, C_R



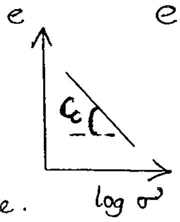
Slope of recompression curve (OCC) is called C_R.

$$C_R \approx \frac{1}{5} C_c$$

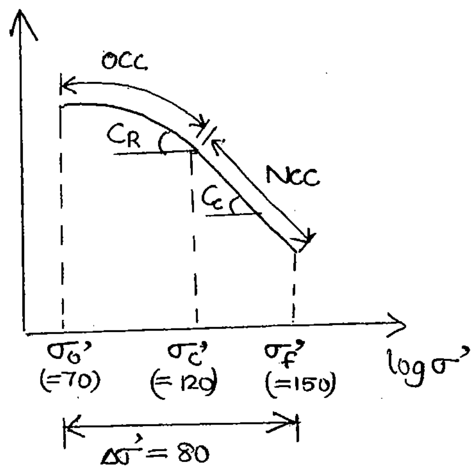
Straight line portion is called Virgin compression or NCC.

OCC cannot be marked in 'e−log σ'' as eqbm void ratios are used (ie void ratio at the end of consolidation)

20. $S_f = H_0 \times \frac{C_c}{1+e_0} \times \log_{10}\left(\frac{\sigma_f'}{\sigma_0'}\right)$



Above eqn is used only when 'e−log σ'' curve is a straight line.



$$\therefore S_f = H_0 \cdot \frac{C_R}{1+e_0} \log_{10} \frac{\sigma_c'}{\sigma_0'} +$$

$$H_0 \cdot \frac{C_c}{1+e_0} \log_{10} \frac{\sigma_f'}{\sigma_c'}$$

$$= 5 \times \frac{0.03}{(1+0.9)} \log_{10}\left(\frac{120}{70}\right) + 5 \times \frac{0.27}{(1+0.9)} \times \log_{10}\left(\frac{150}{120}\right)$$

$= 0.087 \text{ m} = \underline{\underline{87.34}} \text{ mm}$

→ Stages of Consolidation

(i) Initial Consolidation.

Settlement that occurs immediately after loading due to elastic nature and escape of air.

(ii) Primary Consolidation

Due to expulsion of pore water.

(iii) Secondary Consolidation (Creep)

due to plastic readjustment of particles and escape of some double layer water or highly viscous water.

→ Elastic (or Immediate) Settlement :

$$S_i = \frac{q_n}{E_s} \cdot B \cdot (1 - \mu^2) \cdot I$$

where q_n → net pressure intensity.

E_s → Young's Modulus of Soil.

μ → Poisson's Ratio of Soil.

B → Characteristic linear dimension (usually width of footing or diameter of footing)

I → Influence factor

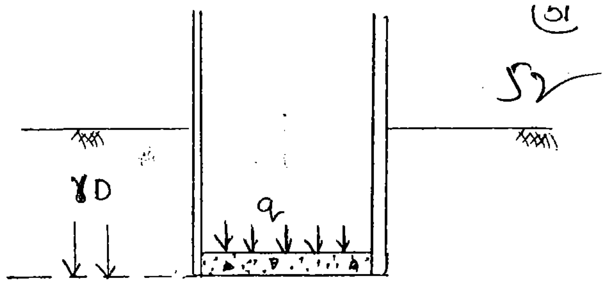
Influence factor, (I) depends on stiffness, shape, $\frac{L}{B}$ ratio of footing and location of point.

NOTE : The immediate settlement of a rigid footing is about 0.8 times the maximum settlement of an equal flexible footing (at the centre).

For a circular footing,

	Centre	Corner	Average
– Flexible	1.0	0.64	C
– Rigid	0.8		

$$\boxed{q_n = q - \gamma D}$$



Q21. Oil tank foundation → flexible footing.

$I_{(centre)}$ for circular flexible footing $= 1$.

$$q_n = q - \gamma D = 250 - 22 \times 3 = 184 \text{ kN/m}^2$$

$$B = \text{diameter} = 20 \text{ m}.$$

$$\mu = 0.45$$

$$\Rightarrow \quad S_i = \frac{q_n B (1-\mu^2) I}{E_s} = \frac{184 \times 20 (1 - 0.45^2) \times 1}{6 \times 10^4}$$

$$= 0.04891 \text{ m} = \underline{\underline{48.9 \text{ mm}}}$$

Q. The average effective overburden pressure on a 10m thick saturate clay layer is 150 kPa. Consolidation test on an undisturbed soil sample taken from clay layer showed that void ratio decreased from 0.6 to 0.5, By increasing the stress intensity from 100 kPa to 300 kPa, Determine the initial void ratio of clay layer. Also determine the total consolidation settlement of the clay layer due to construction of a structure imposing additional stress intensity of 200 kPa.

0.209

e_0

$\frac{\Delta e}{1+e} = \frac{\Delta H}{H_0}$

$$C_c = \frac{\Delta e}{\log \frac{\sigma_f'}{\sigma_0'}} = \frac{0.6 - 0.5}{\log_{10}\left(\frac{300}{100}\right)} = \underline{\underline{0.209}}.$$

$$C_c = \frac{0.6 - e}{\log_{10} \frac{150}{100}} \quad \Rightarrow e = \underline{\underline{0.563}}$$

$$\Delta H = H_0 \cdot \frac{C_c}{1+e_0} \log_{10}\left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}\right) = \frac{10 \times 0.209}{1 + 0.563} \log\left(\frac{150 + 200}{150}\right)$$

$$= \underline{\underline{0.492 \text{ m}}}$$