CBSE Test Paper 03

Chapter 7 Coordinate Geometry

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1. Three given points will be collinear, if the area of the triangle formed by these points is **(1)**
   a. 0 sq. units
   b. 1 sq. units
   c. -1 sq. units
   d. 2 sq. units
2. The area of the triangle with vertices (a, b+c), (b, c+a) and (c, a+b) is **(1)**
   a. a + b + c
   b. $a^2 + b^2 + c^2$
   c. 0
   d. $(a + b + c)^2$
3. The points A(4, – 1), B(6, 0), C(7, 2) and D(5, 1) are the vertices of a **(1)**
   a. Square
   b. Parallelogram
   c. Rhombus
   d. Rectangle
4. If the co – ordinates of a point are (– 5, 11), then its abscissa is **(1)**
   a. -5
   b. 11
   c. 5
   d. -11
5. The vertices of a quadrilateral are (1, 7), (4, 2), (– 1, – 1) and (– 4, 4). The quadrilateral is a **(1)**
   a. rectangle
   b. parallelogram
   c. square
   d. Rhombus

6. If the points (0, 0), (1, 2) and (x, y) are collinear,then find x. **(1)**

7. Find the point on the X-axis which is equidistant from the points (-1,0) and (5,0) **(1)**

8. Find the distance of the point (α, β) from y-axis. **(1)**

9. Find the area of the triangle with vertices (0 ,0) (6 ,0) and (0 ,5). **(1)**

10. If the centre and radius of circle is (3, 4) and 7 units respectively, then what is the position of the point A(5,8) with respect to circle? **(1)**

11. Find the distance between the points P (-4, 7) and Q(2, -5). **(2)**

12. Prove that the coordinates of the centroid of a triangle ABC, with vertices A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$. **(2)**

13. Find the area of the rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order. **(2)**

14. Find the coordinates of the centre of the circle passing through the points (0, 0), (-2, 1) and (-3, 2). Also, find its radius. **(3)**

15. If the points P (-3, 9), Q (a, b) and R (4, -5) are collinear and a + b = 1, find the values of a and b. **(3)**

16. The centre of a circle is (2a, a -7). Find the values of a, if the circle passes through the point (11, -9) and has diameter $10\sqrt{2}$ units. **(3)**

17. Find the area of a triangle ABC with A(1, - 4) and mid-points of sides through A being (2, -1) and (0, -1). **(3)**

18. Find the lengths of the medians of a $\triangle$ABC having vertices at A (0, -1), B (2, 1) and C (0, 3). **(4)**

19. If the centre of circle is (2a, a – 7) then find the values of a if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units. **(4)**

20. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, -3) and (3,4), find the vertices of the triangle. **(4)**

CBSE Test Paper 03
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Solution

1. a. 0 sq. units

   Explanation: Three points that lie on a same straight line are colinear, and the area of the triangle formed by these collinear points is zero.
   Hence Three given points will be collinear, if the area of the triangle formed by these points is 0 sq. units.

2. c. 0

   Explanation: Given: Vertices of a triangle ABC, A(a, b+c), B(b, c+a) and C(c, a+b)
   $\therefore$ar $(\Delta \mathrm{ABC}) = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$
   = $\frac{1}{2}|a(c + a - a - b) + b(a + b - b - c) + c(b + c - c - a)|$
   = $\frac{1}{2}|a(c - b) + b(a - c) + c(b - a)|$
   = $\frac{1}{2}|ac - ab + ab - bc + bc - ac|$
   = $\frac{1}{2} \times 0$ = 0 sq. units
   Also, therefore, the three given points are collinear.

3. c. Rhombus

   Explanation: Given: The points A(4, – 1), B(6, 0), C(7, 2) and D(5, 1).
   $\therefore$ AB = $\sqrt{(6-4)^2 + (0+1)^2}$ = $\sqrt{4+1}$ = $\sqrt{5}$ units
   BC = $\sqrt{(7-6)^2 + (2-0)^2}$ = $\sqrt{1+4}$ = $\sqrt{5}$ units
   CD = $\sqrt{(5-7)^2 + (1-2)^2}$ = $\sqrt{4+1}$ = $\sqrt{5}$ units
   AD = $\sqrt{(5-4)^2 + (1+1)^2}$ = $\sqrt{1+4}$ = $\sqrt{5}$ units
   Therefore all 4 sides AB, BC, CD and DA are equal
   and diagonal AC = $\sqrt{(7-4)^2 + (2+1)^2}$ = $\sqrt{9+9}$ = $3\sqrt{2}$ units
   and BD = $\sqrt{(5-6)^2 + (1-0)^2}$ = $\sqrt{1+1}$ = $\sqrt{2}$ units
   Therefore diagonals AC and BD are not equal
   Since, all sides are equal and both diagonals are not equal.
   Therefore, the given quadrilateral is a rhombus.

4. a. -5

Explanation: Since $x-$coordinate of a point is called abscissa.

Therefore, abscissa is -5.

5. c. square

Explanation: Let A (1, 7), B (4, 2), C$(-1,-1)$ and D$(-4,4)$ are the vertices of a quadrilateral ABCD.

$\therefore$ AB = $\sqrt{(4-1)^2+(2-7)^2}$

= $\sqrt{9+25}$ = $\sqrt{34}$ units

and BC = $\sqrt{(-1-4)^2+(-1-2)^2}$

= $\sqrt{25+9}$ = $\sqrt{34}$ units

and CD = $\sqrt{(-4+1)^2+(4+1)^2}$

= $\sqrt{9+25}$ = $\sqrt{34}$ units

and AD = $\sqrt{(-4-1)^2+(4-7)^2}$ = $\sqrt{25+9}$ = $\sqrt{34}$ units

AC = $\sqrt{(-1-1)^2+(-1-7)^2}$

= $\sqrt{4+64}$ = $2\sqrt{17}$ units

BD = $\sqrt{(-4-4)^2+(4-2)^2}$

= $\sqrt{64+4}$ = $2\sqrt{17}$ units

Since, all sides are equal and both diagonals are also equal.

Therefore, the given quadrilateral is a square.

6. The points are collinear, then area of triangle = 0

$\therefore \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]=0$

or, $\frac{1}{2}[0(2-y)+1(y-0)+x(0-2)]=0$

or, $\frac{1}{2}[y-2x]=0$

or, 2x - y = 0

$\therefore \quad x=\frac{y}{2}$

7. Let A(x,o) be any point on the X-axis , which is equidistant from points (-1,0) and (5,0).

$\Rightarrow (x+1)^2=(x-5)^2$

$\Rightarrow x^2 + 2x + 1 = x^2 - 10x + 25$

⇒ 2x + 1 = -10x + 25

⇒ 2x + 10x = 25 - 1

⇒ 12x = 24

$\Rightarrow$ x = 24/12

$\Rightarrow x = 2$

Therefore , Required point is (2,0).

8. Distance of the point (α, β) from y-axis is the positive value of its x-coordinate.
   $\therefore$ Distance = $|\alpha|$

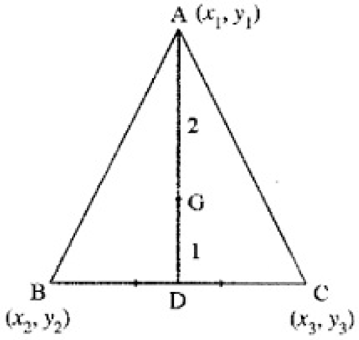
9. We have to find the area of the triangle with vertices (0 ,0) (6 ,0) and (0 ,5).
   Area of triangle
   $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
   $= \frac{1}{2}[0(0-5) + 6(5-0) + 0(0-0)]$
   = $\frac{1}{2}[6 \times 5]$
   =15 sq. units.

10. Distance of the point, from the centre
    a = $\sqrt{(5-3)^2 + (8-4)^2}$
    = $\sqrt{4+16} = \sqrt{20}$
    = $2\sqrt{5}$
    Since, $2\sqrt{5}$ is less than radius=7
    $\therefore$ The point lies inside the circle.

11. The given points are P (-4, 7) and Q(2, -5).
    Then, x_1 = -4, y_1 = 7 and x_2 = 2, y_2 = -5.
    $\therefore \quad PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
    $= \sqrt{[(2-(-4)]^2 + (-5-7)^2} = \sqrt{6^2 + (-12)^2}$
    $= \sqrt{36+144} = \sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5}\ units.$

12. Let the coordinates of vertices of $\triangle$ABC be A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) respectively. Let D be the midpoint of BC.
    Using section formula, coordinates of D will be
    $\left(\frac{x_3+x_2}{2}, \frac{y_3+y_2}{2}\right)$
    Now since centroid G will divide the line joining A and D in the ratio of 2 : 1, therefore again using section formula, coordinates of G will be

$$\left[\frac{\left(\frac{x_3+x_2}{2}\right)2+x_1\times 1}{2+1}, \frac{\left(\frac{y_3+y_2}{2}\right)2+y_1\times 1}{2+1}\right]$$
$$=\left(\frac{x_3+x_2+x_1}{3}, \frac{y_3+y_2+y_1}{3}\right)$$



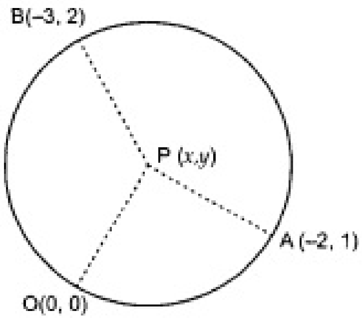
13. Let A (3, 0), B (4, 5), C (-1, 4) and D (-2, -1)

$$AC=\sqrt{(-1-3)^2+(4-0)^2}=4\sqrt{2}$$
$$BD=\sqrt{(-2-4)^2+(-1-5)^2}=\sqrt{36+36}=6\sqrt{2}$$

Area of rhombus = $\frac{1}{2}d_1\times d_2$

$=\frac{1}{2}AC\times BD$

$=\frac{1}{2}\times 4\sqrt{2}\times 6\sqrt{2}=24$ Sq. unit.

14. 

Let P (x, y) be the centre of the circle passing through the points O (0, 0), A (-2, 1) and B (-3, 2). Then,

$$OP=AP=BP$$

Now, OP = AP

$\Rightarrow OP^2 = AP^2$

$\Rightarrow x^2 + y^2 = (x + 2)^2 + (y - 1)^2$

$\Rightarrow x^2 + y^2 = x^2 + y^2 + 4x - 2y + 5$

$\Rightarrow$ 4x - 2y + 5 = 0 ...(i)

and, OP = BP

$\Rightarrow OP^2 = BP^2$

$\Rightarrow x^2 + y^2 = (x + 3)^2 + (y - 2)^2$

$\Rightarrow x^2 + y^2 = x^2 + y^2 + 6x - 4y + 13$

$\Rightarrow 6x - 4y + 13 = 0$.........(ii)

On solving equations (i) and (ii), we get $x = \frac{3}{2}$ and $y = \frac{11}{2}$

Thus, the coordinates of the centre are $\left(\frac{3}{2}, \frac{11}{2}\right)$

Therefore, Radius = OP = $\sqrt{x^2 + y^2} = \sqrt{\frac{9}{4} + \frac{121}{4}} = \frac{1}{2}\sqrt{130}$sq. units.

15. It is given that the points P (-3, 9), Q (a, b) and R(4,-5) are collinear.

$\therefore$ Area of $\triangle$ PQR = 0

$\Rightarrow \quad |\{-3b - 5a + 36\} - \{15 + 4b + 9a\}| = 0$

$\Rightarrow \quad |(-14a - 7b + 21)| = 0$

$\Rightarrow 14a + 7b - 21 = 0$

$\Rightarrow 2a + b - 3 = 0$.....(i)

It is given that a + b =1 ... (ii)

Solving (i) and (ii), we obtain a = 2 and b = - 1.

16. Diameter of a circle = $10\sqrt{2}$ units

$\Rightarrow$ Radius of a circle = $5\sqrt{2}$ units

Let the centre of a circle be O(2a, a - 7) which passes through the point P(11, -9).

$\Rightarrow$OP is the radius of the circle.

$\Rightarrow OP = 5\sqrt{2}$ units

$\Rightarrow OP^2 = (5\sqrt{2})^2$

$\Rightarrow (11 - 2a)^2 + (-9 - a + 7)^2 = 50$

$\Rightarrow 121 + 4a^2 - 44a + (-2 - a)^2 = 50$

$\Rightarrow 121 + 4a^2 - 44a + 4 + a^2 + 4a = 50$

$\Rightarrow 5a^2 - 40a + 125 = 50$

$\Rightarrow 5a^2 - 40a + 75 = 0$

$\Rightarrow a^2 - 8a + 15 = 0$

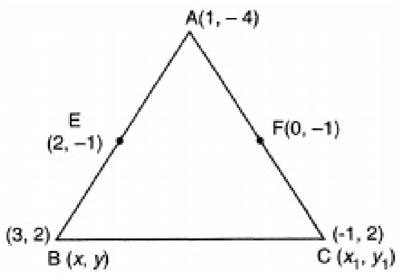
$\Rightarrow$ a^2 - 5a - 3a + 15 = 0

$\Rightarrow$ a(a - 5) -3(a - 5) =0

$\Rightarrow$ (a - 5)(a - 3) = 0

$\Rightarrow$ a - 5 = 0 or a - 3 = 0

$\Rightarrow$ a = 5 or a = 3

17. 

Let E be the midpoint of AB.

$\therefore \quad \frac{x+1}{2} = 2$ or x = 3

and $\frac{y+(-4)}{2} = -1$ or, y = 2

or, B(3, 2)

Let F be the mid-point of AC.Then,

$0 = \frac{x_1+1}{2}$ or $x_1 = -1$

and $\frac{y_1+(-4)}{2}$ = -1 or, y_1 = 2

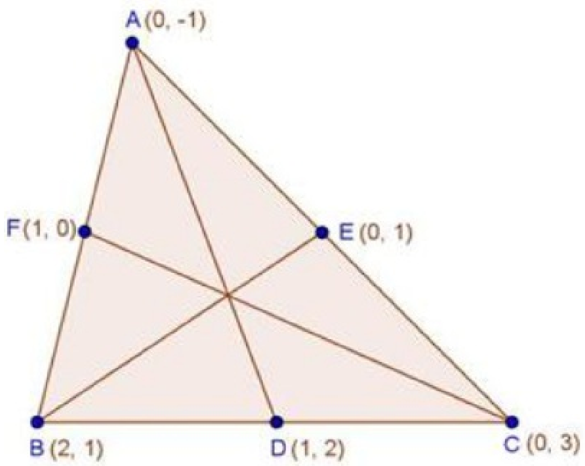
or, C= (-1, 2)

Now the co-ordinates are A(1, -4), B(3, 2), C (-1, 2)

Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[1(2-2) + 3(2+4) - 1(-4-2)]$

$= \frac{1}{2}[0 + 18 + 6]$ = 12 sq units.

18. 

Let A(0, -1), B(2, 1) and C(0, 3) be the given points.

Let AD, BE and CF be the medians

Coordinates of D are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$

Coordinates of E are $\left(\frac{0}{2}, \frac{3-1}{2}\right) = (0, 1)$

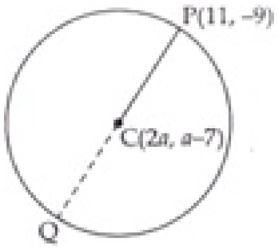
Coordinates of F are $\left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1, 0)$

Length of median $AD = \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{10}\ units$

Length of median $BE = \sqrt{(2-0)^2 + (1-1)^2} = 2 units$

Length of median $CF = \sqrt{(1-0)^2 + (0-3)^2} = \sqrt{10}\ units$

19. Let C(2a, a – 7) be the centre of the circle and it passes through the point P(11, –9).



∴ PQ = $10\sqrt{2}$

⇒ CP = $5\sqrt{2}$

⇒ $CP^2 = (5\sqrt{2})^2 = 50$

⇒ $(2a – 11)^2 + (a – 7 + 9)^2 = 50$

⇒ $(2a)^2 + (11)^2 – 2(2a)(11) + (a + 2)^2 = 50$

⇒ $4a^2 + 121 – 44a + (a)^2 + (2)^2 + 2(a)(2) = 50$

⇒ $5a^2 – 40a + 125 = 50$

⇒ $a^2 – 8a + 25 = 10$

⇒ $a^2 – 8a + 25 – 10 = 0$

⇒ $a^2 – 8a + 15 = 0$

⇒ $a^2 – 5a – 3a + 15 = 0$

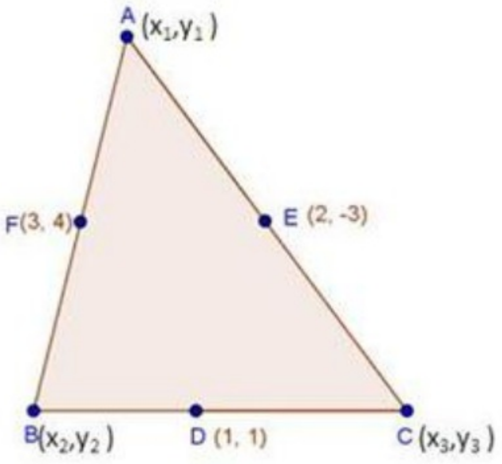
⇒ a(a – 5) –3(a – 5) = 0

⇒ (a – 5) (a – 3) = 0

⇒ a – 5 = 0 or a – 3 = 0

⇒ a = 5 or a = 3

Hence, the required values of a are 5 and 3.

20. 

Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of $\Delta ABC.$

Let D(1, 1), E(2, -3) and F(3, 4) be the mid-points of sides BC, CA and AB respectively.

Since, D is the mid-point of BC

$\therefore \quad \frac{x_2+x_3}{2} = 1$ and $\frac{y_2+y_3}{2} = 1$

$\Rightarrow x_2 + x_3 = 2$ and $y_2 + y_3 = 2$ ...(i)

Similarly E and F are the mid-points of CA and AB respectively.

$\therefore \quad \frac{x_1+x_3}{2} = 2$ and $\frac{y_1+y_3}{2} = -3$

$\Rightarrow x_1 + x_3 = 4$ and $y_1 + y_3 = -6$ ...(ii)

and, $\frac{x_1+x_2}{2} = 3$ and $\frac{y_1+y_2}{2} = 4$

$\Rightarrow x_1 + x_2 = 6$ and $y_1 + y_2 = 8$ ...(iii)

From (i), (ii) and (iii) we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 2 + 4 + 6$

and, $y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 2 +(-6) + 8$

$\Rightarrow 2(x_1 + x_2 + x_3) = 12$ and $2(y_1 + y_2 + y_3) = 4$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 2$ ...(iv)

From (i) and (iv) we get

$x_1 + 2 = 6$ and $y_1 + 2 = 2$

$\Rightarrow x_1 = 6 - 2 \Rightarrow y_1 = 2 - 2$

$\Rightarrow x_1 = 4 \Rightarrow y_1 = 0$

So the coordinates of A are (4, 0)

From (ii) and (iv) we get

$x_2 + 4 = 6$ and $y_2 + (-6) = 2$

$\Rightarrow x_2 = 2 \Rightarrow y_2 - 6 = 2$

$\Rightarrow y_2 = 8$

So the coordinates of B are (2, 8)

From (iii) and (iv) we get

$6 + x_3 = 6$ and $8 + y_3 = 2$

$\Rightarrow x_3 = 6 - 6 \Rightarrow y_3 = 2 - 8$

$\Rightarrow x_3 = 0 \Rightarrow y_3 = -6$

So the coordinates of c are (0, -6)

Hence, the vertices of triangle ABC are:

A(4, 0), B(2, 8) and C(0, -6)