Class XI Session 2023-24
Subject - Mathematics
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. The value of cos 1° cos 2° cos 3° ... cos 179° is **[1]**

a) $\frac{1}{\sqrt{2}}$ b) 1

c) -1 d) 0

2. Let f (x) = $\sqrt{9-x^2}$ then, dom f(x) =? **[1]**

a) (-∞, -3] b) [-3, 3]

c) (-∞, -3] ∪ (4, ∞) d) [3, ∞)

3. What is the standard deviation of 7, 9, 11, 13 and 15? **[1]**

a) 2.7 b) 2.8

c) 2.5 d) 2.4

4. $\lim\limits_{x\to 0} \frac{\sin x}{x(1+\cos x)}$ is equal to **[1]**

a) $\frac{1}{2}$ b) 0

c) 1 d) -1

5. The distance of the point P (1, – 3) from the line 2y – 3x = 4 is **[1]**

a) 13 b) None of these

c) $\sqrt{13}$ d) $\frac{7}{13}\sqrt{13}$

6. What is the locus of a point for which y = 0, z = 0? **[1]**

a) none of these b) equation of y-axis

c) equation of z-axis

d) equation of x-axis

7. If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number and $0 < \theta < 2\pi$, then θ = **[1]**

a) $\frac{\pi}{3}$

b) $\frac{\pi}{2}$

c) π

d) $\frac{\pi}{6}$

8. If $^nC_3$ = 220, then n = ? **[1]**

a) 11

b) 10

c) 12

d) 9

9. If f(x) = 1 - x + x^2 - x^3 + ... - x^{99} + x^{100}, then f'(1) is equal to: **[1]**

a) 150

b) -50

c) -150

d) 50

10. The value of $\sin^2 5°$ + $\sin^2 10°$ + $\sin^2 15°$ + ... + $\sin^2 85°$ + $\sin^2 90°$ is **[1]**

a) 10

b) 9.5

c) 8

d) 7

11. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively, **[1]**

a) 7, 7

b) 4, 4

c) 7, 4

d) 4, 7

12. In Pascal's triangle, each row begins with 1 and ends in **[1]**

a) -1

b) 0

c) 2

d) 1

13. In the expansion of $(x + a)^n$, if the sum of odd terms be P and the sum of even terms be Q, then 4PQ = ? **[1]**

a) $(x + a)^n - (x - a)^n$

b) $(x + a)^{2n} - (x - a)^{2n}$

c) $(x + a)^n + (x - a)^n$

d) None of these

14. Solve the system of inequalities $4x + 3 \geq 2x + 17$, $3x - 5 < -2$, for the values of x, then **[1]**

a) no solution

b) $\left(-\frac{3}{2}, \frac{2}{5}\right)$

c) $(-4, 12)$

d) (-2, 2)

15. If A = {x : x $\neq$ x} represents **[1]**

a) {1}

b) { }

c) {x}

d) {0}

16. If θ lies in quadrant II, then $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} - \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to **[1]**

a) cot θ

b) tan θ

c) 2cot θ

d) 2tan θ

17. $\lim\limits_{x \to 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$ is equal to **[1]**

a) 1
b) 0
c) 2
d) -1

18. How many numbers greater than 10 lacs be formed from 2, 3, 0, 3, 4, 2, 3? **[1]**

a) 360
b) 400
c) 420
d) 300

19. **Assertion (A):** Let A = {1, 2, 3} and B = {1, 2, 3, 4}. Then, $A \subset B$. **[1]**

Reason (R): If every element of X is also an element of Y, then X is a subset of Y.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If the sum of first two terms of an infinite GP is 5 and each term is three times the sum of the succeeding terms, then the common ratio is $\frac{1}{4}$. **[1]**

Reason (R): In an AP 3, 6, 9, 12 .... the 10th term is equal to 33.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the range of the relations: R = {(a, b) : a, b $\in$ N and 2a + b = 10} **[2]**

OR

Find the domain and the range of the real function: f(x) = $\frac{|x-4|}{x-4}$

22. Find the value of $\lim_{x \to 0} \frac{e^{3x}-1}{x}$. **[2]**

23. Four persons are to be chosen at random from a group of 3 men, 2 women and 4 children. Find the probability of selecting 2 women. **[2]**

OR

A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that one is red and two are white.

24. Let A = {r : r = 6n ,n $\in$ N} and B = {x : r = 9n ,n $\in$ N}, find $A \cap B$. **[2]**

25. Find a point on the y-axis which is equidistant from A (-4, 3) and B(5, 2). **[2]**

Section C

26. In how many ways can 4 prizes be distributed among 5 students, when no student gets more than one prize? **[3]**

27. Find the locus of the point which is equidistant from A(3, 4, 0) and B(5, 2, -3). **[3]**

28. Simplify $(x + 2y)^8 + (x - 2y)^8$ **[3]**

OR

Using binomial theorem, expand: $(\sqrt{x} + \sqrt{y})^8$

29. Differentiate $\sqrt{cosec\ x}$ from the first principle. **[3]**

OR

Differentiate the function with respect to x using first principle: $\frac{ax+b}{cx+d}$

30. If the p^{th} and q^{th} terms of a GP are q and p respectively, then show that $(p + q)^{th}$ term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$. **[3]**

OR

If S_1, S_2 and S_3 be respectively the sum of n, 2n and 3n terms of a GP then prove that $S_1(S_3 - S_2) = (S_2 - S_1)^2$.

31. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that B = C. **[3]**

Section D

32. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test: **[5]**

Marks	0	1	2	3	4	5
Frequency	x – 2	x	x^2	$(x + 1)^2$	2x	x + 1

where x is a positive integer. Determine the mean and standard deviation of the marks.

33. Find the (i) lengths of major and minor axes, (ii) coordinate of the vertice, (iii) coordinate of the foci, (iv) eccentricity, and (v) length of the latus rectum of ellipe: $x^2 + 4y^2 = 100$. **[5]**

OR

A visitor with sign board 'DO NOT LITTER' is moving on a circular path in an exhibition. During the movement he stops at points represented by (3, - 2) and (-2, 0). Also, centre of the circular path is on the line 2x - y = 3. What is the equation of the path? What message he wants to give to the public?

34. Solve the following system of linear inequalities. **[5]**

2(2x + 3) - 10 < 6(x - 2)

and $\frac{2x-3}{4} + 6 \geq 2 + \frac{4x}{3}$

35. If 2 $\tan\frac{\alpha}{2}$ = $\tan\frac{\beta}{2}$, prove that $\cos\alpha = \frac{3+5\cos\beta}{5+3\cos\beta}$. **[5]**

OR

Prove that $\cos\frac{2\pi}{15} \cdot \cos\frac{4\pi}{15} \cdot \cos\frac{8\pi}{15} \cdot \cos\frac{16\pi}{15} = \frac{1}{16}$

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Ordered Pairs The ordered pair of two elements a and b is denoted by (a, b): a is first element (or first component) and b is second element (or second component).

Two ordered pairs are equal if their corresponding elements are equal.

i.e. (a, b) = (c, d) $\Rightarrow$ a = c and b = d

Cartesian Product of Two Sets For two non-empty sets A and B, the cartesian product A $\times$ B is the set of all ordered pairs of elements from sets A and B.

In symbolic form, it can be written as

A $\times$ B = {(a, b): a $\in$ A, b $\in$ B}

(i) Let A and B be two sets such that A $\times$ B consists of 6 elements. If three elements of A $\times$ B are (1, 4), (2, 6) and (3, 6), then find A$\times$B and B$\times$A.

(ii) If (x + 2, 4) = (5, 2x + y), then find the value of x and y.

(iii) If (x + 6, y - 2) = (0, 6), then find the value of x and y.

OR

If (a - 3, b + 7) = (3, 7), then find the value of a and b.

37. **Read the text carefully and answer the questions:** **[4]**

On her vacation, Priyanka visits four cities. Delhi, Lucknow, Agra, Meerut in a random order.

Meerut

New Delhi

Agra

Lucknow

(i) What is the probability that she visits Delhi before Lucknow?

(ii) What is the probability she visit Delhi before Lucknow and Lucknow before Agra?

(iii) What is the probability she visits Delhi first and Lucknow last?

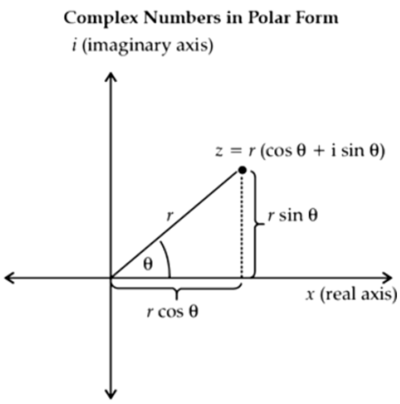
OR

What is the probability she visits Delhi either first or second?

38. **Read the text carefully and answer the questions:** **[4]**

Consider the complex number Z = 2 - 2i.

Complex Number in Polar Form



(i) Find the principal argument of Z.

(ii) Find the value of $\mathbf{z\bar{z}}$?

Solution

Section A

1.

(d) 0

Explanation: Since cos90° = 0

Thus cos 1° cos 2° cos 3°… cos90°… cos 179° = 0

2.

(b) [-3, 3]

Explanation: f(x) = $\sqrt{9-x^2}$

domain of the function can be defined for $\sqrt{9-x^2} \geq 0$

$\Rightarrow \sqrt{9-x^2} \geq 0$

$\Rightarrow 9 - x^2 \geq 0$

$\Rightarrow x^2 \leq 9$

$\Rightarrow -3 \leq x \leq 3$

Therefore, domain of f(x) is [-3, 3]

3.

(b) 2.8

Explanation: Mean of given observation $\bar{x} = \frac{7+9+11+13+15}{5} = \frac{55}{5}$ = 11

Standard deviation = $\sqrt{\frac{\sum|x-\bar{x}|^2}{n}}$

= $\sqrt{\frac{1}{5}\{(7-11)^2+(9-11)^2+(11-11)^2+(13-11)^2+(15-11)^2}\}$

= $\sqrt{\frac{1}{5} \times 40} = \sqrt{8}$ = 2.8

4. **(a)** $\frac{1}{2}$

Explanation: We have

$\lim\limits_{x\to 0} \frac{\sin x}{x(1+\cos x)} = \lim\limits_{x\to 0} \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{x \quad 2\cos^2\frac{x}{2}}$

$= \frac{1}{2} \lim\limits_{x\to 0} \frac{\tan\frac{x}{2}}{\frac{x}{2}} = \frac{1}{2}$

5.

(c) $\sqrt{13}$

Explanation: We know that the distance of the point P (1, – 3) from the line 2y – 3 x – 4 = 0 is the length of perpendicular from the point to the line which is given by

$\left|\frac{2(-3)-3-4}{\sqrt{13}}\right| = \sqrt{13}$

6.

(d) equation of x-axis

Explanation: Locus of the point y = 0, z = 0 is x-axis, since on x-axis both y-coordinate and z-coordinate = 0

7.

(c) π

Explanation: π

Given:

$\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number

On rationalising, we get,

$$\frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$= \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1)^2-(2i\sin\theta)^2}$
$= \frac{3+2i\sin\theta+6i\sin\theta+4i^2\sin^2\theta}{1+4\sin^2\theta}$
$= \frac{3-4\sin^2\theta+8i\sin\theta}{1+4\sin^2\theta}$ [$\because i^2 = -1$]
$= \frac{3-4\sin^2\theta}{1+4\sin^2\theta} + i\frac{8\sin\theta}{1+4\sin^2\theta}$ For the above term to be real, the imaginary part has to be zero.
$\therefore \frac{8\sin\theta}{1+4\sin^2\theta} = 0$
$\Rightarrow 8\sin\theta = 0$
For this to be zero,
$\sin\theta = 0$
$\Rightarrow \theta = 0$,
$\pi, 2\pi, 3\pi$ ...
But
$0 < \theta < 2\pi$
Hence,
$\theta = \pi$

8.

(c) 12

Explanation: ${}^nC_3 = 220$
$\Rightarrow \frac{n(n-1)(n-2)}{6} = 220$
$\Rightarrow n(n - 1)(n - 2) = 1320$
$\Rightarrow n = 12$ [$\because 12 \times 11 \times 10 = 1320$]

9.

(d) 50

Explanation: Given that $f(x) = 1 - x + x^2 - x^3 + ... - x^{99} + x^{100}$
$f'(x) = -1 + 2x + 3x^2 + .. - 99x^{98} + 100x^{99}$
$\therefore f'(x) = -1 + 2 - 3 + .. - 99x^{98} + 100x^{99}$
$= (-1 -3 - 5 .. -99) + (2 + 4 + 6 + .. + 100)$
$= \frac{50}{2}[2 \times -1 + (50 - 1)(-2)] + \frac{50}{2}[2 \times 2 + (50 - 1)2]$
$= 25[-2 - 98] + 25[4 + 98] = 25 \times -100 + 25 \times 102$
$= 25[-100 - 102] = 25 \times 2 = 50$

10.

(b) 9.5

Explanation: We have $\sin^2 5° + \sin^2 10° + \sin^2 15° + ... + \sin^2 85° + \sin^2 90°$
$\sin^2 5° + \sin^2 10° + \sin^2 15° + ... + \sin^2(90° - 10°) + \sin^2(90° - 5°) + \sin^2 90°$
$= \sin^2 5° + \sin^2 10° + \sin^2 15° + ... + \cos^2 10° + \cos^2 5° + \sin^2 90°$
$= (\sin^2 5° + \cos^2 5°) + (\sin^2 10° + \cos^2 10°) + (\sin^2 15° + \cos^2 15°)$
$+ (\sin^2 20° + \cos^2 20°) + (\sin^2 25° + \cos^2 25°) + (\sin^2 30° + \cos^2 30°)$
$+ (\sin^2 35° + \cos^2 35°) + (\sin^2 40° + \cos^2 40°) + \sin^2 45° + \sin^2 90°)$
$= 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + \left(\frac{1}{\sqrt{2}}\right)^2 + (1)^2$ $[\because \sin^2\theta + \cos^2\theta = 1]$
$= 8 + \frac{1}{2} + 1$
$= 9.5$

11.

(c) 7, 4

Explanation: Now to find value of m and n

The number of subsets of a set containing x elements is given by 2^x

According to question: $2^m - 2^n = 112$

$\Rightarrow 2^n(2^{m-n} - 1) = 16 \times 7$

$\Rightarrow 2^n (2^{m-n} - 1) = 2^4 \times 7$

On comparing on both sides $2^n = 2^4$ and $2^{m-n} - 1 = 7$

$\Rightarrow$ n = 4 and $2^{m-n} = 8$

$\Rightarrow 2^{m-n} = 2^3$

$\Rightarrow$ m – n = 3

$\Rightarrow$ m – 4 = 3

$\Rightarrow$ m = 7

Therefore,the value of m and n is 7 and 4 respectively

12.

(d) 1

Explanation:

The pascal's triangle is given by

1 1

1 2 1

1 3 3 1

13.

(b) $(x + a)^{2n} - (x - a)^{2n}$

Explanation: P + Q = $(x + a)^n$ and P - Q = $(x - a)^n$

$\Rightarrow$ 4PQ = $(P + Q)^2 - (P - Q)^2 = (x + a)^{2n} - (x - a)^{2n}$

14. **(a)** no solution

Explanation: We have given: $4x + 3 \geq 2x + 17$

$\Rightarrow 4x - 2x \geq 17 - 3 \Rightarrow 2x \geq 1$

$\Rightarrow x \geq \frac{14}{2}$ [Dividing by 2 on both sides]

$\Rightarrow x \geq 7$ ..... (i)

Also we have 3x - 5 < -2

$\Rightarrow \quad 3x < -2 + 5 \Rightarrow 3x < 3$

$\Rightarrow \quad x < 1$

On combining (i) and (ii), we see that solution is not possible because nothing is common between these two solutions.(i.e., x < 1, x $\geq$ 7)

15.

(b) { }

Explanation: Here value of x is not possible so A is a null set.

16.

(d) 2tan θ

Explanation: Given exp. $= \frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}} - \frac{\sqrt{1+\sin\theta}}{\sqrt{1-\sin\theta}} = \frac{(1-\sin\theta)-(1+\sin\theta)}{\sqrt{1-\sin^2\theta}} = \frac{-2\sin\theta}{|\cos\theta|}$

$= \frac{-2\sin\theta}{-\cos\theta} = 2\tan\theta \qquad \begin{bmatrix} \because \text{ in quadrant II, } \cos\theta < 0 \\ \Rightarrow |\cos\theta| = -\cos\theta \end{bmatrix}$

17. **(a)** 1

Explanation: Given, $\lim\limits_{x\to 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$

$= \lim\limits_{x\to 0} \frac{\sin x[\sqrt{x+1}+\sqrt{1-x}]}{(\sqrt{x+1}-\sqrt{1-x})(\sqrt{x+1}+\sqrt{1-x})}$

$= \lim\limits_{x\to 0} \frac{\sin x[\sqrt{x+1}+\sqrt{1-x}]}{x+1-1+x}$

$= \lim\limits_{x\to 0} \frac{\sin x[\sqrt{x+1}+\sqrt{1-x}]}{2x} = \frac{1}{2} \cdot \lim\limits_{x\to 0} \frac{\sin x}{x}[\sqrt{x+1} + \sqrt{1-x}]$

Taking limits, we get

$= \frac{1}{2} \times 1 \times [\sqrt{0+1} + \sqrt{1-0}] = \frac{1}{2} \times 1 \times 2 = 1$

18. **(a)** 360

Explanation: We know, 10 lacs = 10,00,000 and it has 7 places.

The given numbers are 2, 3, 0, 3, 4, 2, 3.

As the required number is greater than 10 lacs, so the 7^{th} place can be filled with all the digits except 0.

So, the place can be filled in 6 ways.

The other 6 places can be filled with other 6 digits, so the total number of ways to fill the remaining 6 places is = 6!

But 2 is repeated twice and 3 is repeated thrice.

So, total numbers greater than 10 lacs be formed from 2, 3, 0, 3, 4, 2, 3 is

$= \frac{6 \times 6!}{2! \times 3!}$

$= \frac{6 \times 720}{2 \times 6}$

= 360

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion Since, every element of A is in B, so A $\subset$ B.

20.

(c) A is true but R is false.

Explanation: Assertion Let a be the first term and r(|r| < 1) be the common ratio of the GP.

$\therefore$ The GP is a, ar, ar^2,...

According to the question,

$T_1 + T_2 = 5 \Rightarrow a + ar = 5 \Rightarrow a(1 + r) = 5$

and $T_n = 3(T_{n+1} + T_{n+2} + T_{n+3} + ...)$

$\Rightarrow ar^{n-1} = 3(ar^n + ar^{n+1} + ar^{n+2} + ...)$

$\Rightarrow ar^{n-1} = 3ar^n(1 + r + r^2 + ...)$

$\Rightarrow 1 = 3r(\frac{1}{1-r})$

$\Rightarrow 1 - r = 3r$

$\Rightarrow r = \frac{1}{4}$

Reason: Given, 3, 6, 9, 12 ...

Here, a = 3, d = 6 - 3 = 3

$\therefore T_{10} = a + (10 - 1)d$

= 3 + 9 $\times$ 3

= 3 + 27 = 30

Section B

21. Given, R = {(a, b) : a, b $\in$ N and 2a + b = 10}

$\because$ 2a + b = 10

$\Rightarrow$ b = 10 - 2a

a	**b**
1	8
2	6
3	4
4	2

Thus, R = {(1, 8), (2, 6), (3, 4), (4, 2)}

$\therefore$ Range of R = (8, 6, 4, 2} or {2, 4, 6, 8}

OR

Here we have, f(x) = $\frac{|x-4|}{x-4}$

We need to find where the function is defined.

To find the domain of the function f(x) we need to equate the denominator of the function to 0

Therefore,

x - 4 = 0 or x = 4

It means that the denominator is zero when x = 4

So, the domain of the function is the set of all the real numbers except 4

The domain of the function, $D_{\{f(x)\}} = (-\infty, 4) \cup (4, \infty)$

The numerator is an absolute function of the denominator.

So, for any value of x from the domain set, we always get either +1 or -1 as the output.

So, the range of the function is a set containing -1 and +1

Therefore, the range of the function, $R_{f(x)} = \{-1, 1\}$

22. We have, $\lim_{x\to 0} \frac{e^{3x}-1}{x} = \lim_{x\to 0} \frac{e^{3x}-1}{x} \times \frac{3}{3}$ [multiplying numerator and denominator by 3]

$= 3 \lim_{x\to 0} \frac{e^{3x}-1}{3x}$ ..... (i)

Let h = 3x, as $x \to 0$, then $h \to 0$

Then, from Eq. (i), we get

$\lim_{x\to 0} \frac{e^{3x}-1}{x} = 3 \lim_{h\to 0} \frac{e^{h}-1}{h} = 3(1)\ \left[\because \lim_{x\to 0} \frac{e^{x}-1}{x} = 1\right]$

= 3

23. We have to find the probability of selecting 2 women

We have to select 4 persons of which 2 are women and the remaining 2 are chosen from 7 persons consisting of 3 men and 4 children.

This can be done in ${}^2C_2 \times {}^7C_2$ ways

$\therefore$ Favourable number of elementary events ${}^2C_2 \times {}^7C_2$ = 21

So, required probability $= \frac{21}{126} = \frac{1}{6}$

OR

We have to find the probability that one is red and two are white.

Given: bag which contains 6 red, 8 blue and 4 white balls

Formula: $\mathrm{P(E)} = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

two balls are drawn at random, therefore

total possible outcomes are ${}^{18}C_3$

therefore n(S) = 816

let E be the event of getting one red and two white balls

n(E) = ${}^6C_1 {}^4C_2$ = 36

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{36}{816} = \frac{3}{68}$

24. A = {x : x = 6n $\forall$ n $\in$ N)

As x = 6n hence for n = 1, 2, 3, 4, 5, 6… x = 6, 12, 18, 24, 30, 36…

Therefore ,A = {6, 12, 18, 24, 30, 36…}

B = {x : x = 9n $\forall$ n $\in$ N)

As x = 9n Therefore, for n = 1, 2, 3, 4… x = 9, 18, 27, 36…

Therefore, B = {9, 18, 27, 36…}

A $\cap$ B means common elements to both sets

The common elements are 18, 36, 54, …

Therefore, A $\cap$ B = {18, 36, 54, …}

All the elements are multiple of 18

Therefore, A $\cap$ B = {x: x = 18n $\forall$ n $\in$ N}

25. Let the point on the y-axis be P(0, y)

Here, it is given that P is equidistant from A(-4, 3) and B(5, 2).

i.e., PA = PB

$\Rightarrow \sqrt{(-4-0)^2 + (3-y)^2} = \sqrt{(5-0)^2 + (2-y)^2}$

Squaring both sides, we obtain

$\Rightarrow (-4 - 0)^2 + (3 - y)^2 = (5 - 0)^2 + (2 - y)^2$

$\Rightarrow 16 + 9 - 6y + y^2 = 25 + 4 - 4y + y^2$

⇒ 25 – 6y = 29 – 4y

⇒ 2y = -4

⇒ y = -2

Thus, the required point on the y-axis is (0, -2).

Section C

26. We have to find the possible number of ways in which we can give four prizes among five students when no boy gets more than one price which means that there is no repetition.

We will use the concept of multiplication because there are four sub jobs dependent on each other and are performed one after the other.

The thing that is distributed is considered to have choices not the things to which we have to give them, it means that in this problem the prizes have choices more precisely first prize will have five choices, second prize will have four choices and the choices will keep on decreasing by one as we go on giving prizes, and students won't choose any because prizes will have the right to choose.

The number of ways in which we can give four prizes among five students where repetition of distribution is not allowed $5 \times 4 \times 3 \times 2 = 5! = 120$

27. Let P (x, y, z) be any point which is equidistant from A(3, 4, 0) and B(5, 2, -3).

Now PA = PB ⇒ $PA^2 = PB^2$

$\therefore (x - 3)^2 + (y - 4)^2 + (z - 0)^2 = (x - 5)^2 + (y - 2)^2 + (z + 3)^2$

$\Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 = x^2 + 25 - 10x + y^2 + 4 - 4y + z^2 + 9 + 6z$

$\Rightarrow 4x - 4y - 6z - 13 = 0$

28. $(x + 2y)^8 + (x - 2y)^8 = 2[{}^8C_0x^8 + {}^8C_2x^5(2y)^2 + {}^8C_4x^4(2y)^4 + {}^8C_6x^2(2y)^6 + {}^8C_8(2y)^8]$

$\because (x+a)^n + (x-a)^n = 2[{}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 + \ldots\ldots\ldots\ldots]$

$\Rightarrow (x+2y)^8 + (x-2y)^8 = 2[x^8 + 28x^6 \times 4y^2 + 70x^4 \times 16y^4 + 28x^2 \times 64y^6 + 256y^8]$

$= 2[x^8 + 112x^6y^2 + 1120x^4y^4 + 1792x^2y^6 + 256y^8]$

OR

We hand to find value of $(\sqrt{x} + \sqrt{y})^8$

Formula used: ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(\sqrt{x} + \sqrt{y})^8$

We can write $\sqrt{x}$ as $x^{\frac{1}{2}}$ and $\sqrt{y}$ as $y^{\frac{1}{2}}$

Now, we have to solve for $\left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)^8$

$= \left[8C_0\left(x^{\frac{2}{2}}\right)^{8-0}\right] + \left[8C_1\left(x^{\frac{1}{2}}\right)^{8-1}\left(y^{\frac{2}{2}}\right)^1\right] + \left[8C_2\left(x^{\frac{1}{2}}\right)^{8-2}\left(y^{\frac{1}{2}}\right)^2\right] + \left[{}^8C_3\left(x^{\frac{1}{2}}\right)^{8-3}\left(y^{\frac{1}{2}}\right)^3\right]$

$+ \left[{}^8C_4\left(x^{\frac{1}{2}}\right)^{8-4}\left(\frac{1}{y^2}\right)^4\right] + \left[{}^8C_5\left(x^{\frac{1}{2}}\right)^{8-5}\left(y^{\frac{2}{2}}\right)^5\right] + \left[{}^8C_6\left(x^{\frac{1}{2}}\right)^{8-6}\left(y^{\frac{1}{2}}\right)^6\right]$

$+ \left[8C_7\left(x^{\frac{1}{2}}\right)^{8-7}\left(y^{\frac{1}{2}}\right)^7\right] + \left[8C_8\left(y^{\frac{1}{2}}\right)^8\right]$

$= \left[\frac{8!}{0!(8-0)!}\left(x^{\frac{5}{2}}\right)\right] + \left[\frac{8!}{1!(8-1)!}\left(x^{\frac{2}{2}}\right)\left(y^{\frac{1}{2}}\right)\right] + \left[\frac{8!}{2!(8-2)!}\left(x^{\frac{6}{2}}\right)\left(y^{\frac{2}{2}}\right)\right]$

$+ \left[\frac{8!}{3!(8-3)!}\left(x^{\frac{5}{2}}\right)\left(y^{\frac{3}{2}}\right)\right] + \left[\frac{8!}{4!(8-4)!}\left(x^{\frac{4}{2}}\right)\left(y^{\frac{4}{2}}\right)\right]$

$+ \left[\frac{8!}{5!(8-5)!}\left(x^{\frac{2}{2}}\right)\left(y^{\frac{5}{2}}\right)\right] + \left[\frac{8!}{6!(8-6)!}\left(x^{\frac{2}{2}}\right)\left(\frac{6}{y^2}\right)\right] + \left[\frac{8!}{7!(8-7)!}\left(x^{\frac{1}{2}}\right)\left(\frac{2}{y^2}\right)\right] + \left[\frac{8!}{8!(8-8)!}\left(y^{\frac{5}{2}}\right)\right]$

$= [1(x^4)] + \left[8\left(x^{\frac{7}{2}}\right)\left(y^{\frac{1}{2}}\right)\right] + [28(x^3)(y)] + \left[56\left(x^{\frac{5}{2}}\right)\left(\frac{2}{y^2}\right)\right]$

$+ [70(x^2)(y^2)] + \left[56\left(x^{\frac{3}{2}}\right)\left(y^{\frac{5}{2}}\right)\right] + [28(x^2)(y^3)] + \left[8\left(x^{\frac{1}{2}}\right)\left(y^{\frac{2}{2}}\right)\right] + [1(y^4)]$

29. Let y = $\sqrt{cosecx}$

Let δy be an increment in y, corresponding to an increment δx in x.

Then, y + δy = $\sqrt{cosec(x+\delta x)}$

$\Rightarrow \delta y$ = $\sqrt{\text{cosed}(x+\delta x)}$ - $\sqrt{\text{cosec}\, x}$

$\Rightarrow \frac{\delta y}{\delta x}$ = $\frac{\sqrt{\text{cosec}(x+\delta x)}-\sqrt{\text{cosec}\, x}}{\delta x}$

$\Rightarrow \frac{dy}{dx}$ = $\lim\limits_{\delta x\to 0}\frac{\delta y}{\delta x}$

= $\lim\limits_{\delta x\to 0}\frac{\sqrt{\text{cosec}(x+\delta x)}-\sqrt{\text{cosec}\, x}}{\delta x}$

= $\lim\limits_{\delta x\to 0}\frac{\left\{\frac{1}{\sqrt{\sin(x+\delta x)}}-\frac{1}{\sqrt{\sin x}}\right\}}{\delta x}$

= $\lim\limits_{\delta x\to 0}\frac{\{\sqrt{\sin x}-\sqrt{\sin(x+\delta x)}\}}{8x\cdot\sqrt{\sin(x+\delta x)}\cdot\sqrt{\sin x}}\times\frac{\{\sqrt{\sin x}+\sqrt{\sin(x+\delta x)}\}}{\{\sqrt{\sin x}+\sqrt{\sin(x+\delta x)}\}}$

= $\lim\limits_{\delta x\to 0}\frac{\{\sin x-\sin(x+\delta x)\}}{\{\sqrt{\sin(x+\delta x)}\cdot\sqrt{\sin x}}\times\frac{1}{\delta x\cdot\{\sqrt{\sin x}+\sqrt{\sin(x+\delta x)}\}}$

= $\lim\limits_{\delta x\to 0}\frac{-2\cos\left(x+\frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\{\sqrt{\sin(x+\delta x)}\}\cdot\sqrt{\sin x}\cdot\delta x\cdot\{\sqrt{\sin x}+\sqrt{\sin(x+\delta x)}\}}$

$=-\lim\limits_{\delta x\to 0}\cos\left(x+\frac{\delta x}{2}\right)\lim\limits_{\delta x\to 0}\frac{\sin\left(\frac{\delta x}{2}\right)}{\left(\frac{\delta x}{2}\right)}$

= $\lim\limits_{\delta x\to 0}\frac{1}{\sqrt{\sin(x+\delta x)}\cdot\sqrt{\sin x}}\lim\limits_{\delta x\to 0}\frac{1}{\{\sqrt{\sin x}+\sqrt{\sin(x+\delta x)}\}}$

= - cos x $\times$ 1 $\times$ $\frac{1}{\sqrt{\sin x}\cdot\sqrt{\sin x}}\frac{1}{(\sqrt{\sin x}+\sqrt{\sin x})}$

= $\frac{-\cos x}{\sin x}\frac{1}{2\sqrt{\sin x}}$ = $-\frac{1}{2}\sqrt{\text{cosec}\, x}$ cot x

Hence, $\frac{d}{dx}(\sqrt{cosecx})=-\frac{1}{2}\sqrt{cosecx}$ cot x

OR

Let $f(x)=\frac{ax+b}{cx+d}$ ...(i)

Taking increment ,then we have,

$f(x+\Delta x)=\frac{a(x+\Delta x)+b}{c(x+\Delta x)+d}$ ...(ii)

Subtracting eq. (i) from eq. (ii) we have,

$f(x+\Delta x)-f(x)=\frac{a(x+\Delta x)+b}{c(x+\Delta x)+d}-\frac{ax+b}{cx+d}$

Dividing both sides by $\Delta x$and take the limit, we get

$\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{\frac{a(x+\Delta x)+b}{c(x+\Delta x)+d}-\frac{ax+b}{cx+d}}{\Delta x}$

$\Rightarrow f'(x)=\lim\limits_{\Delta x\to 0}\frac{(ax+a\Delta x+b)(cx+d)-(ax+b)(cx+c\Delta x+d)}{[c(x+\Delta x)+d](cx+d)\cdot\Delta x}$ [Using definition of differentiation

$=\lim\limits_{\Delta x\to 0}\frac{acx^2+ac\Delta x\cdot x+bcx+adx+ad\Delta x+bd-acx^2-ac\Delta x\cdot x-adx-bcx-bc\cdot\Delta x-bd}{(cx+c\Delta x+d)(cx+d)\cdot\Delta x}$

$=\lim\limits_{\Delta x\to 0}\frac{(ad-bc)\Delta x}{(cx+c\Delta x+d)(cx+d)\cdot\Delta x}$

$=\lim\limits_{\Delta x\to 0}\frac{(ad-bc)}{(cx+c\cdot\Delta x+d)(cx+d)}$

Taking limits, we have

$=\frac{(ad-bc)}{(cx+d)(cx+d)}=\frac{ad-bc}{(cx+d)^2}$

30. Let first term be A and common ratio be R of a GP.

Given, p^{th} term, T_p = q and q^{th} term, T_q = p

Then, AR^{p-1} = q and AR^{q-1} = p ...(i)

$\therefore\ \frac{AR^{p-1}}{AR^{q-1}}=\frac{q}{p}$

$\Rightarrow R^{p-q}=\frac{q}{p}\Rightarrow$ R = $\left(\frac{q}{p}\right)^{\frac{1}{\rho-q}}$ [$\because$ rasisint power $\frac{1}{p-q}$ on both sides]

On putting the value of R in Eq. (i), we get

$A \cdot \left(\frac{q}{p}\right)^{\frac{p-1}{p-q}}$ = q

$\Rightarrow$ A = $q \cdot \left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}$

Now, (p + q)th term,

$T_{p+q} = AR^{p+q-1} = q \cdot \left(\frac{p}{q}\right)^{\frac{p-1}{p-q}} \times \left(\frac{q}{p}\right)^{\frac{p+q-1}{p-q}}$

$= \frac{q^{1-\frac{p-1}{P-q}+\frac{p+q-1}{p-q}}}{p^{\frac{p+q-1}{p-q}-\frac{p-1}{p-q}}} = \frac{q^{\frac{p-q-p+1+p+q-1}{p-q}}}{p^{\frac{p+q-1-p+1}{p-q}}}$

$= \frac{q^{\frac{p}{p-q}}}{p^{\frac{q}{p-q}}} = \left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$

Hence proved.

OR

Suppose a be the first term and r be the common ratio of the given GP. Then,

$S_1(S_3 - S_2) = \frac{a(1-r^n)}{(1-r)} \times \left\{\frac{a(1-r^{3n})}{(1-r)} - \frac{a(1-r^{2n})}{(1-r)}\right\}$

$= \frac{a(1-r^n)}{(1-r)} \times \frac{(a-ar^{3n}-a+ar^{2n})}{(1-r)}$

$= \frac{a(1-r^n)}{(1-r)} \times \frac{ar^{2n}(1-r^n)}{(1-r)}$

$= \frac{a^2r^{2n}(1-r^n)^2}{(1-r)^2}$

And, $(S_2 - S_1)^2 = \left\{\frac{a(1-r^{2n})}{(1-r)} - \frac{a(1-r^n)}{(1-r)}\right\}^2$

$= \frac{(a-ar^{2n}-a+ar^n)^2}{(1-r)^2}$

$= \frac{(ar^n(1-r^n)\}^2}{(1-r)^2}$

$= \frac{a^2r^{2n}(1-r^n)^2}{(1-r)^2}$

Therefore, $S_1(S_3 - S_2) = (S_2 - S_1)^2$

31. We know that $A = A \cap (A \cup B)$ and $A = A \cup (A \cap B)$

Now $A \cap B = A \cap C$ and $A \cup B = A \cup C$

$\therefore B = B \cup (B \cap A) = B \cup (A \cap B) = B \cup (A \cap C)$ $[\because A \cap B = A \cap C]$

$= (B \cup A) \cap (B \cup C)$ (By distributive law)

$= (A \cup C) \cap (B \cup C)$

$= (A \cup C) \cap (B \cup C)$ $[\because A \cup B = A \cup C]$

$= (C \cup A) \cap (C \cup B)$

$= C \cup (A \cap B)$ (by distributive law)

$= C \cup (A \cap C)$ $[\because A \cap B = A \cap C]$

$= C \cup (C \cap A) = C$

Hence B = C.

Section D

32. To find: the mean and standard deviation of the marks.

It is given there are 60 students in the class, so

$\sum f_i = 60$

$\Rightarrow (x - 2) + x + x^2 + (x + 1)^2 + 2x + x + 1 = 60$

$\Rightarrow 5x - 1 + x^2 + x^2 + 2x + 1 = 60$

$\Rightarrow 2x^2 + 7x = 60$

$\Rightarrow 2x^2 + 7x - 60 = 0$

Splitting the middle term, we get

$\Rightarrow 2x^2 + 15x – 8x – 60 = 0$

$\Rightarrow x(2x + 15) – 4(2x + 15) = 0$

$\Rightarrow (2x + 15) (x – 4) = 0$

$\Rightarrow$ 2x + 15 = 0 or x - 4 = 0

$\Rightarrow$ 2x = -15 or x = 4

Given x is a positive number, so x can take 4 as the only value.

And let assumed mean, a = 3.

Now put x = 4 and a = 3 in the frequency distribution table and add other columns after calculations, we get

Marks(x_i)	**Frequency (f_i)**	**$d_i = x_i - a$**	**$f_i d_i$**	$f_i x_i^2$
0	x - 2 = 4 - 2 = 2	0 - 3 = -3	2 × -3 = -6	2 × -3^2 = -6
1	x = 4	1 - 3 = -2	4 × -2 = -8	4 × -2^2 = 16
2	$x^2 = 4^2$ = 16	2 - 3 = -1	16 × -1 = -16	16 × -1^2 = -16
3	$(x = 1)^2 = (4 + 1)^2$ = 25	3 - 3 = 0	25 × 0 = 0	25 × 0^2 = 0
4	2x = 2x 4 = 8	4 - 3 = 1	8 × 1 = 4	8 × 1^2 = 4
5	x + 1 = 4 + 1 = 5	5 - 3 = 2	5 × 2 = 10	5 × 2^2 = 10
Total	n = 60	$\Sigma f_i d_i$ = 12		$\Sigma f_i d_i^2$ = 78

And we know standard deviation is

$$\sigma = \sqrt{\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n}\right)^2}$$

Substituting values from above table, we get

$$\sigma = \sqrt{\frac{78}{60} - \left(\frac{-12}{60}\right)^2}$$

$$\sigma = \sqrt{1.3 - (0.2)^2}$$

$$\sigma = \sqrt{1.3 - 0.04}$$

$\Rightarrow \sigma$ = 1.12

Hence the standard deviation is 1.12

Now mean is

$$\bar{x} = A + \frac{\sum f_i d_i}{N}$$

= 3 + $\left(-\frac{12}{60}\right)$

= 3 - $\frac{1}{5}$

= $\frac{14}{5}$

= 2.8

Hence the mean and standard deviation of the marks are 2.8 and 1.12 respectively.

33. Given: $x^2 + 4y^2 = 100$

After dividing by 400 to both the sides, we get

$\frac{x^2}{100} + \frac{4y^2}{100}$ = 1

$\frac{x^2}{100} + \frac{y^2}{25}$ = 1 ...(i)

Now, above equation is of the form,

$\frac{x^2}{a^2} + \frac{y^2}{b^2}$ = 1 ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 100$ and $b^2 = 25$

$\Rightarrow$ a = $\sqrt{100}$ and b = $\sqrt{25}$

$\Rightarrow$ a = 10 and b = 5

i. Length of major axes

$\therefore$ Length of major axes = 2a = 2 × 5 = 10 units

ii. Coordinates of the Vertices

$\therefore$ Coordinate of vertices = (a, 0) and (-a, 0) = (10, 0) and (-10, 0)

iii. Coordinates of foci = ($\pm$c, 0)

Now $c^2 = a^2 - b^2$ = 100 -25

$\Rightarrow c^2 = 75 \Rightarrow c = \sqrt{75} \;\; \Rightarrow c = 5\sqrt{3}$ ...(iii)

$\therefore$ Coordinates of foci = $(\pm 5\sqrt{3}, 0)$

iv. Eccentricity

As we know that, Eccentricity $= \frac{c}{a} \Rightarrow e = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$ [from (iii)]

v. Length of the Latus Rectum

As we know, Length of Latus Rectum = $\frac{2b^2}{a} = \frac{2\times(4)^2}{5} = \frac{32}{5}$

OR

Let the equation of circle whose centre (- g, - f) be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since, is passes through points (3, - 2) and (- 2, 0)

$\therefore (3)^2 + (-2)^2 + 2g(3) + 2f(-2) + c = 0$

and $(-2)^2 + (0)^2 + 2g(-2) + 2f(0) + c = 0$

$\Rightarrow 9 + 4 + 6g - 4f + c = 0$

and $4 + 0 - 4g + 0 + c = 0$

$\Rightarrow 6g - 4f + c = -13$

and $c = 4g - 4$ ...(ii)

$\therefore 6g - 4f + (4g - 4) = -13$

$\Rightarrow 10g - 4f = -9$ ...(iii)

Also, centre (- g, - f) lies on the line 2x - y = 3

$\therefore -2g + f = 3$ ...(iv)

On solving Eqs. (iii) and (iv), we get

$g = \frac{3}{2}$ and $f = 6$

On putting the values of g and f in Eq. (ii), we get

$c = 4\left(\frac{3}{2}\right) - 4 = 6 - 4 = 2$

On putting the values of g, f and c in Eq. (i), we get

$x^2 + y^2 + 2\left(\frac{3}{2}\right)x + 2(6)x + 2 = 0$

$\Rightarrow x^2 + y^2 + 3x + 12x + 2 = 0$

which is the required equation of the path

The message which he wants to give to the public is 'Keep your place clean'.

34. The given system of linear inequalities is

$2(2x + 3) - 10 < 6(x - 2)$ ....(i)

and $\frac{2x-3}{4} + 6 \geq 2 + \frac{4x}{3}$ ...(ii)

From inequality (i), we get

$2(2x + 3) - 10 < 6(x - 2)$

$\Rightarrow 4x + 6 - 10 < 6x - 12$

$\Rightarrow 4x - 4 < 6x - 12$

$\Rightarrow 4x - 4 + 4 < 6x - 12 + 4$ [adding 4 on both sides]

$\Rightarrow 4x < 6x - 8$

$\Rightarrow 4x - 6x < 6x - 8 - 6x$ [subtracting 6x from both sides]

$\Rightarrow -2x < -8$

$\Rightarrow 2x > 8$ [dividing both sides by - 1 and then inequality sign will change]

$\Rightarrow \frac{2x}{2} > \frac{8}{2}$ [dividing both sides by 2]

$\therefore x > 4$ ...(iii)

Thus, any value of x greater than 4 satisfies the inequality.

$\therefore$ Solution set is $x \in (4, \infty)$

The representation of solution of inequality (i) is

From inequality (ii), we get

$\frac{2x-3}{4} + 6 \geq 2 + \frac{4x}{3} \Rightarrow \frac{2x-3+24}{4} \geq \frac{6+4x}{3}$

$\Rightarrow \quad \frac{2x+21}{4} \geq \frac{6+4x}{3} \Rightarrow 3(2x+21) \geq 4(6+4x)$

$\Rightarrow \quad 6x + 63 \geq 24 + 16x$

$\Rightarrow -10x \geq -39 \Rightarrow 10x \leq 39$

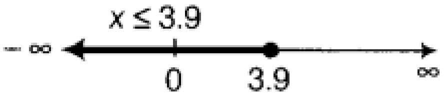
$\Rightarrow \quad \frac{10x}{10} \leq \frac{39}{10}$

$\Rightarrow \quad x \leq 3.9$ ...(iv)

Thus, any value of x less than or equal to 3.9 satisfies the inequality.

$\therefore$ Solution set is $x \in (-\infty, 3.9]$.

Its representation on number line is



From Eqs. (iii) and (iv), it is clear, that there is no common value of x, which satisfies both inequalities (iii) and (iv).

Hence, the given system of inequalities has no solution.

35. If 2 $\tan\frac{\alpha}{2} = \tan\frac{\beta}{2}$, prove that $\cos\alpha = \frac{3+5\cos\beta}{5+3\cos\beta}$

LHS = $\cos\alpha$

$= \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$ ... $[\because \tan\frac{\alpha}{2} = \frac{1}{2}\tan\frac{\beta}{2}]$

$= \frac{1-\left(\frac{1}{2}\tan\frac{\beta}{2}\right)^2}{1+\left(\frac{1}{2}\tan\frac{\beta}{2}\right)^2}$

$= \frac{1-\frac{1}{4}\tan^2\frac{\beta}{2}}{1+\frac{1}{4}\tan^2\frac{\beta}{2}}$

$= \frac{\frac{4-\tan^2\frac{\beta}{2}}{4}}{\frac{4+\tan^2\frac{\beta}{2}}{4}}$

$= \frac{4-\tan^2\frac{\beta}{2}}{4+\tan^2\frac{\beta}{2}}$

Now,

Take RHS = $\frac{3+5\cos\beta}{5+3\cos\beta}$

$= \frac{3+5\left(\frac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}\right)}{5+3\left(\frac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}\right)}$ ... $[\because \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}]$

$= \frac{\frac{3\left(1+\tan^2\frac{\beta}{2}\right)+5\left(1-\tan^2\frac{\beta}{2}\right)}{1+\tan^2\frac{\beta}{2}}}{\frac{5\left(1+\tan^2\frac{\beta}{2}\right)+3\left(1-\tan^2\frac{\beta}{2}\right)}{1+\tan^2\frac{\beta}{2}}}$

$= \frac{3+3\tan^2\frac{\beta}{2}+5-5\tan^2\frac{\beta}{2}}{5+5\tan^2\frac{\beta}{2}+3-3\tan^2\frac{\beta}{2}}$

$= \frac{8-2\tan^2\frac{\beta}{2}}{8+2\tan^2\frac{\beta}{2}}$

$= \frac{2\left(4-\tan^2\frac{\beta}{2}\right)}{2\left(4+\tan^2\frac{\beta}{2}\right)}$

$= \frac{4-\tan^2\frac{\beta}{2}}{4+\tan^2\frac{\beta}{2}}$ ... $[\because \cos\alpha = \frac{4-\tan^2\frac{\beta}{2}}{4+\tan^2\frac{\beta}{2}}]$

= $\cos\alpha$

Hence Proved

OR

LHS = $\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{16\pi}{15}$

$= cos\ \frac{2\pi}{15}\ cos2\left(\frac{2\pi}{15}\right)cos4\left(\frac{2\pi}{15}\right)\ cos8\left(\frac{2\pi}{15}\right)$

Put $\frac{2\pi}{15}=\alpha$

$\Rightarrow$LHS = $\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha$

= $\frac{2\sin\alpha[\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha]}{2\sin\alpha}$ [multiplying numerator and denominator by $2sin\ \alpha$]

= $\frac{(2\sin\alpha\cdot\cos\alpha)\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha}{2\sin\alpha}$

= $\frac{2(\sin 2\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha)}{2(2\sin\alpha)}$ [$\because 2sin\alpha\ cos\alpha = sin2\alpha$ and multiplying numerator and denominator by 2]

= $\frac{(2\sin 2\alpha\cdot\cos 2\alpha)\cdot\cos 4\alpha\cdot\cos 8\alpha}{4\sin\alpha}$

= $\frac{2(\sin 4\alpha\cdot\cos 4\alpha)\cos 8\alpha}{2(4\sin\alpha)}$ [$\because 2sin\alpha\ cos\alpha = sin2\alpha$ and multiplying numerator and denominator by 2]

= $\frac{2(\sin 8\alpha\cdot\cos 8\alpha)}{2(8\sin\alpha)}$

= $\frac{\sin 16\alpha}{16\sin\alpha}=\frac{\sin(15\alpha+\alpha)}{16\sin\alpha}$

Now, $15\alpha = 2\pi$,

= $\frac{\sin(2\pi+\alpha)}{16\sin\alpha}=\frac{\sin\alpha}{16\sin\alpha}$ = $\frac{1}{16}$ = RHS

$\therefore$ **LHS = RHS**

Hence proved.

Section E

36. **Read the text carefully and answer the questions:**

Ordered Pairs The ordered pair of two elements a and b is denoted by (a, b): a is first element (or first component) and b is second element (or second component).

Two ordered pairs are equal if their corresponding elements are equal.

i.e. (a, b) = (c, d) $\Rightarrow$ a = c and b = d

Cartesian Product of Two Sets For two non-empty sets A and B, the cartesian product A $\times$ B is the set of all ordered pairs of elements from sets A and B.

In symbolic form, it can be written as

A $\times$ B = {(a, b): a $\in$ A, b $\in$ B}

(i) Since {(1, 4), (2, 6), (3, 6)} are the elements of A $\times$ B.
It follows that the elements of set A = {1, 2, 3} and B = {4, 6}
Hence A $\times$ B = {(1, 4), (1, 6), (2, 4), (2, 6), (3, 4), (3, 6)}
B $\times$ A = {(4, 1), (4, 2), (4, 3), (6, 1), (6, 2), (6, 3)}

(ii) (x + 2, 4) = (5, 2x + y)
Then, x + 2 = 5
x = 5 - 2 = 3
Then, putting the value of x in 2x + y = 4
2 $\times$ 3 + y = 4
6 + y = 4
y = -2
The value of x and y are 3 and -2.

(iii) (x + 6, y - 2) = (0, 6)
x + 6 = 0
x = -6
y - 2 = 6
y = 8

OR

(a - 3, b + 7) = (3, 7)
a - 3 = 3
a = 6
b + 7 = 7
b = 0

37. **Read the text carefully and answer the questions:**

On her vacation, Priyanka visits four cities. Delhi, Lucknow, Agra, Meerut in a random order.


Meerut


New Delhi


Agra


Lucknow

(i) Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24

$\therefore$ n(S) = 24

Clearly, sample space for this experiment is

$$S = \left\{ \begin{array}{l} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB \\ BACD, BADC, BCAD, BCDA, BDAC, BDCA \\ CABD, CADB, CBAD, CBDA, CDAB, CDBA, \\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{array} \right\}$$

Let E_1 be the event that Priyanka visits A before B.

Then,

E_1 = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, CABD, CADB, CDAB, DABC, DACB, DCAB}

$\Rightarrow n(E_1) = 12$

$\therefore$ P(she visits A before B) = $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{12}{24} = \frac{1}{2}$

(ii) Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24

$\therefore$ n(S) = 24

Clearly, sample space for this experiment is

$$S = \left\{ \begin{array}{l} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB \\ BACD, BADC, BCAD, BCDA, BDAC, BDCA \\ CABD, CADB, CBAD, CBDA, CDAB, CDBA, \\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{array} \right\}$$

E_1 = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, CABD, CADB, CDAB, DABC, DACB, DCAB}

$\Rightarrow n(E_1) = 12$

$\therefore$ P(she visits A before B) $= P(E_1) = \frac{n(E_1)}{n(S)} = \frac{12}{24} = \frac{1}{2}$

(iii) Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24

$\therefore$ n(S) = 24

Clearly, sample space for this experiment is

$$S = \left\{ \begin{array}{l} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB \\ BACD, BADC, BCAD, BCDA, BDAC, BDCA \\ CABD, CADB, CBAD, CBDA, CDAB, CDBA, \\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{array} \right\}$$

Let E_3 be the event that she visits A first and B last.

Then,

E_3 = {ACDB, ADCB}

$n(E_3) = 2$

$\because$ P(she visits A first and B last) = P(E_3)

$= \frac{n(E_3)}{n(S)} = \frac{2}{24} = \frac{1}{12}$

OR

Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24

$\therefore$ n(S) = 24

Clearly, sample space for this experiment is

S = $\left\{\begin{array}{l} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB \\ BACD, BADC, BCAD, BCDA, BDAC, BDCA \\ CABD, CADB, CBAD, CBDA, CDAB, CDBA, \\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{array}\right\}$

Let E_4 be the event that she visits A either first or second. Then,

E_4 = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, CABD, CADB, DABC, DACB}

$\Rightarrow$ n(E_4) = 12

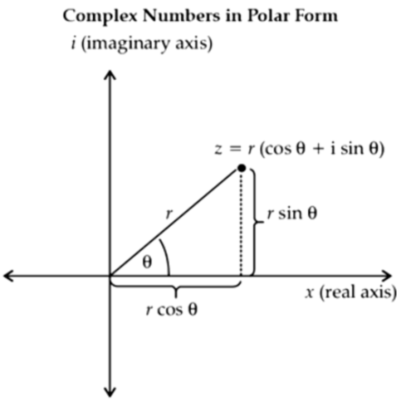
Hence, P(she visits A either first or second)

$= P(E_4) = \frac{n(E_4)}{n(S)} = \frac{12}{24} = \frac{1}{2}$

38. **Read the text carefully and answer the questions:**

Consider the complex number Z = 2 - 2i.

Complex Number in Polar Form



(i) r = $|Z| = 2\sqrt{2}$

x = 2, y = -2

cosθ = $\frac{x}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

sinθ = $\frac{y}{r} = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}}$

Arg(Z) = $\frac{-\pi}{4}$

(ii) $z\bar{z} = |z|^2 = (2\sqrt{2})^2$ = 8