

Exercise 14.1**Question 1:**

Find the common factors of the terms

(i) $12x, 36$

(ii) $2y, 22xy$

(iii) $14pq, 28p^2q^2$

(iv) $2x, 3x^2, 4$

(v) $6abc, 24ab^2, 12a^2b$

(vi) $16x^3, -4x^2, 32x$

(vii) $10pq, 20qr, 30rp$

(viii) $3x^2y^3, 10x^3y^2, 6x^2y^2z$

Answer:

(i) $12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

The common factors are 2, 2, 3.

And, $2 \times 2 \times 3 = 12$

(ii) $2y = 2 \times y$

$22xy = 2 \times 11 \times x \times y$

The common factors are 2, y .

And, $2 \times y = 2y$

(iii) $14pq = 2 \times 7 \times p \times q$

$28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$

The common factors are 2, 7, p , q .

And, $2 \times 7 \times p \times q = 14pq$

(iv) $2x = 2 \times x$

$3x^2 = 3 \times x \times x$

$4 = 2 \times 2$

The common factor is 1.

(v) $6abc = 2 \times 3 \times a \times b \times c$

$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$

$$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$$

The common factors are 2, 3, a , b .

$$\text{And, } 2 \times 3 \times a \times b = 6ab$$

$$\text{(vi) } 16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$$

$$-4x^2 = -1 \times 2 \times 2 \times x \times x$$

$$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$$

The common factors are 2, 2, x .

$$\text{And, } 2 \times 2 \times x = 4x$$

$$\text{(vii) } 10pq = 2 \times 5 \times p \times q$$

$$20qr = 2 \times 2 \times 5 \times q \times r$$

$$30rp = 2 \times 3 \times 5 \times r \times p$$

The common factors are 2, 5.

$$\text{And, } 2 \times 5 = 10$$

$$\text{(viii) } 3x^2y^3 = 3 \times x \times x \times y \times y \times y$$

$$10x^3y^2 = 2 \times 5 \times x \times x \times x \times y \times y$$

$$6x^2y^2z = 2 \times 3 \times x \times x \times y \times y \times z$$

The common factors are x , x , y , y .

And,

$$x \times x \times y \times y = x^2y^2$$

Question 2:

Factorise the following expressions

$$\text{(i) } 7x - 42$$

$$\text{(ii) } 6p - 12q$$

$$\text{(iii) } 7a^2 + 14a$$

$$\text{(iv) } -16z + 20z^3$$

$$\text{(v) } 20l^2m + 30alm$$

$$\text{(vi) } 5x^2y - 15xy^2$$

$$\text{(vii) } 10a^2 - 15b^2 + 20c^2$$

$$\text{(viii) } -4a^2 + 4ab - 4ca$$

$$\text{(ix) } x^2yz + xy^2z + xyz^2$$

$$(x) \ ax^2y + bxy^2 + cxyz$$

Answer:

$$(i) \ 7x = 7 \times x$$

$$42 = 2 \times 3 \times 7$$

The common factor is 7.

$$\therefore 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7(x - 6)$$

$$(ii) \ 6p = 2 \times 3 \times p$$

$$12q = 2 \times 2 \times 3 \times q$$

The common factors are 2 and 3.

$$\therefore 6p - 12q = (2 \times 3 \times p) - (2 \times 2 \times 3 \times q)$$

$$= 2 \times 3 [p - (2 \times q)]$$

$$= 6(p - 2q)$$

$$(iii) \ 7a^2 = 7 \times a \times a$$

$$14a = 2 \times 7 \times a$$

The common factors are 7 and a .

$$\therefore 7a^2 + 14a = (7 \times a \times a) + (2 \times 7 \times a)$$

$$= 7 \times a [a + 2] = 7a(a + 2)$$

$$(iv) \ 16z = 2 \times 2 \times 2 \times 2 \times z$$

$$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$$

The common factors are 2, 2, and z .

$$\therefore -16z + 20z^3 = -(2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z)$$

$$= (2 \times 2 \times z) [-(2 \times 2) + (5 \times z \times z)]$$

$$= 4z(-4 + 5z^2)$$

$$(v) \ 20l^2m = 2 \times 2 \times 5 \times l \times l \times m$$

$$30alm = 2 \times 3 \times 5 \times a \times l \times m$$

The common factors are 2, 5, l , and m .

$$\therefore 20l^2m + 30alm = (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m)$$

$$= (2 \times 5 \times l \times m) [(2 \times l) + (3 \times a)]$$

$$= 10lm(2l + 3a)$$

$$(vi) \ 5x^2y = 5 \times x \times x \times y$$

$$15xy^2 = 3 \times 5 \times x \times y \times y$$

The common factors are 5, x , and y .

$$\begin{aligned}\therefore 5x^2y - 15xy^2 &= (5 \times x \times x \times y) - (3 \times 5 \times x \times y \times y) \\ &= 5 \times x \times y [x - (3 \times y)] \\ &= 5xy (x - 3y)\end{aligned}$$

$$(vii) 10a^2 = 2 \times 5 \times a \times a$$

$$15b^2 = 3 \times 5 \times b \times b$$

$$20c^2 = 2 \times 2 \times 5 \times c \times c$$

The common factor is 5.

$$\begin{aligned}10a^2 - 15b^2 + 20c^2 &= (2 \times 5 \times a \times a) - (3 \times 5 \times b \times b) + (2 \times 2 \times 5 \times c \times c) \\ &= 5 [(2 \times a \times a) - (3 \times b \times b) + (2 \times 2 \times c \times c)] \\ &= 5 (2a^2 - 3b^2 + 4c^2)\end{aligned}$$

$$(viii) 4a^2 = 2 \times 2 \times a \times a$$

$$4ab = 2 \times 2 \times a \times b$$

$$4ca = 2 \times 2 \times c \times a$$

The common factors are 2, 2, and a .

$$\begin{aligned}\therefore -4a^2 + 4ab - 4ca &= - (2 \times 2 \times a \times a) + (2 \times 2 \times a \times b) - (2 \times 2 \times c \times a) \\ &= 2 \times 2 \times a [- (a) + b - c] \\ &= 4a (-a + b - c)\end{aligned}$$

$$(ix) x^2yz = x \times x \times y \times z$$

$$xy^2z = x \times y \times y \times z$$

$$xyz^2 = x \times y \times z \times z$$

The common factors are x , y , and z .

$$\begin{aligned}\therefore x^2yz + xy^2z + xyz^2 &= (x \times x \times y \times z) + (x \times y \times y \times z) + (x \times y \times z \times z) \\ &= x \times y \times z [x + y + z] \\ &= xyz (x + y + z)\end{aligned}$$

$$(x) ax^2y = a \times x \times x \times y$$

$$bxy^2 = b \times x \times y \times y$$

$$cxyz = c \times x \times y \times z$$

The common factors are x and y .

$$\begin{aligned}
 ax^2y + bxy^2 + cxyz &= (a \times x \times x \times y) + (b \times x \times y \times y) + (c \times x \times y \times z) \\
 &= (x \times y) [(a \times x) + (b \times y) + (c \times z)] \\
 &= xy (ax + by + cz)
 \end{aligned}$$

Question 3:

Factorise

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$

(iv) $15pq + 15 + 9q + 25p$

(v) $z - 7 + 7xy - xyz$

Answer:

$$\begin{aligned}
 \text{(i) } x^2 + xy + 8x + 8y &= x \times x + x \times y + 8 \times x + 8 \times y \\
 &= x(x + y) + 8(x + y) \\
 &= (x + y)(x + 8)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } 15xy - 6x + 5y - 2 &= 3 \times 5 \times x \times y - 3 \times 2 \times x + 5 \times y - 2 \\
 &= 3x(5y - 2) + 1(5y - 2) \\
 &= (5y - 2)(3x + 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) } ax + bx - ay - by &= a \times x + b \times x - a \times y - b \times y \\
 &= x(a + b) - y(a + b) \\
 &= (a + b)(x - y)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } 15pq + 15 + 9q + 25p &= 15pq + 9q + 25p + 15 \\
 &= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5 \\
 &= 3q(5p + 3) + 5(5p + 3) \\
 &= (5p + 3)(3q + 5)
 \end{aligned}$$

$$\begin{aligned}
 \text{(v) } z - 7 + 7xy - xyz &= z - x \times y \times z - 7 + 7 \times x \times y \\
 &= z(1 - xy) - 7(1 - xy) \\
 &= (1 - xy)(z - 7)
 \end{aligned}$$

Exercise 14.2

Question 1:

Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$ (**Hint:** Expand $(l + m)^2$ first)

(viii) $a^4 + 2a^2b^2 + b^4$

Answer:

(i) $a^2 + 8a + 16 = (a)^2 + 2 \times a \times 4 + (4)^2$

$= (a + 4)^2 [(x + y)^2 = x^2 + 2xy + y^2]$

(ii) $p^2 - 10p + 25 = (p)^2 - 2 \times p \times 5 + (5)^2$

$= (p - 5)^2 [(a - b)^2 = a^2 - 2ab + b^2]$

(iii) $25m^2 + 30m + 9 = (5m)^2 + 2 \times 5m \times 3 + (3)^2$

$= (5m + 3)^2 [(a + b)^2 = a^2 + 2ab + b^2]$

(iv) $49y^2 + 84yz + 36z^2 = (7y)^2 + 2 \times (7y) \times (6z) + (6z)^2$

$= (7y + 6z)^2 [(a + b)^2 = a^2 + 2ab + b^2]$

(v) $4x^2 - 8x + 4 = (2x)^2 - 2(2x)(2) + (2)^2$

$= (2x - 2)^2 [(a - b)^2 = a^2 - 2ab + b^2]$

$= [(2)(x - 1)]^2 = 4(x - 1)^2$

(vi) $121b^2 - 88bc + 16c^2 = (11b)^2 - 2(11b)(4c) + (4c)^2$

$= (11b - 4c)^2 [(a - b)^2 = a^2 - 2ab + b^2]$

(vii) $(l + m)^2 - 4lm = l^2 + 2lm + m^2 - 4lm$

$= l^2 - 2lm + m^2$

$= (l - m)^2 [(a - b)^2 = a^2 - 2ab + b^2]$

(viii) $a^4 + 2a^2b^2 + b^4 = (a^2)^2 + 2(a^2)(b^2) + (b^2)^2$

$= (a^2 + b^2)^2 [(a + b)^2 = a^2 + 2ab + b^2]$

Question 2:

Factorise

(i) $4p^2 - 9q^2$

(ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$

(v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Answer:

(i) $4p^2 - 9q^2 = (2p)^2 - (3q)^2$

$$= (2p + 3q)(2p - 3q) [a^2 - b^2 = (a - b)(a + b)]$$

(ii) $63a^2 - 112b^2 = 7(9a^2 - 16b^2)$

$$= 7[(3a)^2 - (4b)^2]$$

$$= 7(3a + 4b)(3a - 4b) [a^2 - b^2 = (a - b)(a + b)]$$

(iii) $49x^2 - 36 = (7x)^2 - (6)^2$

$$= (7x - 6)(7x + 6) [a^2 - b^2 = (a - b)(a + b)]$$

(iv) $16x^5 - 144x^3 = 16x^3(x^2 - 9)$

$$= 16x^3[(x)^2 - (3)^2]$$

$$= 16x^3(x - 3)(x + 3) [a^2 - b^2 = (a - b)(a + b)]$$

(v) $(l + m)^2 - (l - m)^2 = [(l + m) - (l - m)][(l + m) + (l - m)]$

$$[\text{Using identity } a^2 - b^2 = (a - b)(a + b)]$$

$$= (l + m - l + m)(l + m + l - m)$$

$$= 2m \times 2l$$

$$= 4ml$$

$$= 4lm$$

(vi) $9x^2y^2 - 16 = (3xy)^2 - (4)^2$

$$= (3xy - 4)(3xy + 4) [a^2 - b^2 = (a - b)(a + b)]$$

(vii) $(x^2 - 2xy + y^2) - z^2 = (x - y)^2 - (z)^2 [(a - b)^2 = a^2 - 2ab + b^2]$

$$\begin{aligned}
 &= (x - y - z)(x - y + z)[a^2 - b^2 = (a - b)(a + b)] \\
 \text{(viii)} \quad &25a^2 - 4b^2 + 28bc - 49c^2 = 25a^2 - (4b^2 - 28bc + 49c^2) \\
 &= (5a)^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2] \\
 &= (5a)^2 - (2b - 7c)^2 \\
 &[\text{Using identity } (a - b)^2 = a^2 - 2ab + b^2] \\
 &= [5a + (2b - 7c)][5a - (2b - 7c)] \\
 &[\text{Using identity } a^2 - b^2 = (a - b)(a + b)] \\
 &= (5a + 2b - 7c)(5a - 2b + 7c)
 \end{aligned}$$

Question 3:

Factorise the expressions

- (i) $ax^2 + bx$
- (ii) $7p^2 + 21q^2$
- (iii) $2x^3 + 2xy^2 + 2xz^2$
- (iv) $am^2 + bm^2 + bn^2 + an^2$
- (v) $(lm + l) + m + 1$
- (vi) $y(y + z) + 9(y + z)$
- (vii) $5y^2 - 20y - 8z + 2yz$
- (viii) $10ab + 4a + 5b + 2$
- (ix) $6xy - 4y + 6 - 9x$

Answer:

- (i) $ax^2 + bx = a \times x \times x + b \times x = x(ax + b)$
- (ii) $7p^2 + 21q^2 = 7 \times p \times p + 3 \times 7 \times q \times q = 7(p^2 + 3q^2)$
- (iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$
- (iv) $am^2 + bm^2 + bn^2 + an^2 = am^2 + bm^2 + an^2 + bn^2$
 $= m^2(a + b) + n^2(a + b)$
 $= (a + b)(m^2 + n^2)$
- (v) $(lm + l) + m + 1 = lm + m + l + 1$
 $= m(l + 1) + 1(l + 1)$
 $= (l + 1)(m + 1)$
- (vi) $y(y + z) + 9(y + z) = (y + z)(y + 9)$

$$(vii) 5y^2 - 20y - 8z + 2yz = 5y^2 - 20y + 2yz - 8z$$

$$= 5y(y - 4) + 2z(y - 4)$$

$$= (y - 4) (5y + 2z)$$

$$(viii) 10ab + 4a + 5b + 2 = 10ab + 5b + 4a + 2$$

$$= 5b(2a + 1) + 2(2a + 1)$$

$$= (2a + 1) (5b + 2)$$

$$(ix) 6xy - 4y + 6 - 9x = 6xy - 9x - 4y + 6$$

$$= 3x(2y - 3) - 2(2y - 3)$$

$$= (2y - 3) (3x - 2)$$

Question 4:

Factorise

$$(i) a^4 - b^4$$

$$(ii) p^4 - 81$$

$$(iii) x^4 - (y + z)^4$$

$$(iv) x^4 - (x - z)^4$$

$$(v) a^4 - 2a^2b^2 + b^4$$

Answer:

$$(i) a^4 - b^4 = (a^2)^2 - (b^2)^2$$

$$= (a^2 - b^2) (a^2 + b^2)$$

$$= (a - b) (a + b) (a^2 + b^2)$$

$$(ii) p^4 - 81 = (p^2)^2 - (9)^2$$

$$= (p^2 - 9) (p^2 + 9)$$

$$= [(p)^2 - (3)^2] (p^2 + 9)$$

$$= (p - 3) (p + 3) (p^2 + 9)$$

$$(iii) x^4 - (y + z)^4 = (x^2)^2 - [(y + z)^2]^2$$

$$= [x^2 - (y + z)^2] [x^2 + (y + z)^2]$$

$$= [x - (y + z)][x + (y + z)] [x^2 + (y + z)^2]$$

$$= (x - y - z) (x + y + z) [x^2 + (y + z)^2]$$

$$(iv) x^4 - (x - z)^4 = (x^2)^2 - [(x - z)^2]^2$$

$$= [x^2 - (x - z)^2] [x^2 + (x - z)^2]$$

$$\begin{aligned} &= [x - (x - z)] [x + (x - z)] [x^2 + (x - z)^2] \\ &= z(2x - z) [x^2 + x^2 - 2xz + z^2] \\ &= z(2x - z) (2x^2 - 2xz + z^2) \\ \text{(v)} \quad a^4 - 2a^2b^2 + b^4 &= (a^2)^2 - 2(a^2)(b^2) + (b^2)^2 \\ &= (a^2 - b^2)^2 \\ &= [(a - b)(a + b)]^2 \\ &= (a - b)^2 (a + b)^2 \end{aligned}$$

Question 5:

Factorise the following expressions

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

Answer:

(i) $p^2 + 6p + 8$

It can be observed that, $8 = 4 \times 2$ and $4 + 2 = 6$

$$\therefore p^2 + 6p + 8 = p^2 + 2p + 4p + 8$$

$$= p(p + 2) + 4(p + 2)$$

$$= (p + 2)(p + 4)$$

(ii) $q^2 - 10q + 21$

It can be observed that, $21 = (-7) \times (-3)$ and $(-7) + (-3) = -10$

$$\therefore q^2 - 10q + 21 = q^2 - 7q - 3q + 21$$

$$= q(q - 7) - 3(q - 7)$$

$$= (q - 7)(q - 3)$$

(iii) $p^2 + 6p - 16$

It can be observed that, $16 = (-2) \times 8$ and $8 + (-2) = 6$

$$p^2 + 6p - 16 = p^2 + 8p - 2p - 16$$

$$= p(p + 8) - 2(p + 8)$$

$$= (p + 8)(p - 2)$$

Exercise 14.3

Question 1:

Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Answer:

(i) $28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x$

$56x = 2 \times 2 \times 2 \times 7 \times x$

$$28x^4 \div 56x = \frac{2 \times 2 \times 7 \times x \times x \times x \times x}{2 \times 2 \times 2 \times 7 \times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

(ii) $36y^3 = 2 \times 2 \times 3 \times 3 \times y \times y \times y$

$9y^2 = 3 \times 3 \times y \times y$

$$-36y^3 \div 9y^2 = \frac{-2 \times 2 \times 3 \times 3 \times y \times y \times y}{3 \times 3 \times y \times y} = -4y$$

(iii) $66pq^2r^3 = 2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r$

$11qr^2 = 11 \times q \times r \times r$

$$66pq^2r^3 \div 11qr^2 = \frac{2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r} = 6pqr$$

(iv) $34x^3y^3z^3 = 2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z$

$51xy^2z^3 = 3 \times 17 \times x \times y \times y \times z \times z \times z$

$$\begin{aligned} 34x^3y^3z^3 \div 51xy^2z^3 &= \frac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z}{3 \times 17 \times x \times y \times y \times z \times z \times z} \\ &= \frac{2}{3}x^2y \end{aligned}$$

(v) $12a^8b^8 = 2 \times 2 \times 3 \times a^8 \times b^8$

$6a^6b^4 = 2 \times 3 \times a^6 \times b^4$

$$12a^8b^8 \div (-6a^6b^4) = \frac{2 \times 2 \times 3 \times a^8 \times b^8}{-2 \times 3 \times a^6 \times b^4} = -2a^2b^4$$

Question 2:

Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$

(ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

Answer:

(i) $5x^2 - 6x = x(5x - 6)$

$$(5x^2 - 6x) \div 3x = \frac{x(5x - 6)}{3x} = \frac{1}{3}(5x - 6)$$

(ii) $3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$$(3y^8 - 4y^6 + 5y^4) \div y^4 = \frac{y^4(3y^4 - 4y^2 + 5)}{y^4} = 3y^4 - 4y^2 + 5$$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) = 8x^2y^2z^2(x + y + z)$

$$8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2 = \frac{8x^2y^2z^2(x + y + z)}{4x^2y^2z^2} = 2(x + y + z)$$

(iv) $x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$$(x^3 + 2x^2 + 3x) \div 2x = \frac{x(x^2 + 2x + 3)}{2x} = \frac{1}{2}(x^2 + 2x + 3)$$

(v) $p^3q^6 - p^6q^3 = p^3q^3(q^3 - p^3)$

$$(p^3q^6 - p^6q^3) \div p^3q^3 = \frac{p^3q^3(q^3 - p^3)}{p^3q^3} = q^3 - p^3$$

Question 3:

Work out the following divisions.

(i) $(10x - 25) \div 5$

$$(ii) (10x - 25) \div (2x - 5)$$

$$(iii) 10y(6y + 21) \div 5(2y + 7)$$

$$(iv) 9x^2y^2(3z - 24) \div 27xy(z - 8)$$

$$(v) 96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$$

Answer:

$$(i) \frac{(10x - 25) \div 5}{5} = \frac{2 \times 5 \times x - 5 \times 5}{5} = \frac{5(2x - 5)}{5} = 2x - 5$$

$$(ii) \frac{(10x - 25) \div (2x - 5)}{(2x - 5)} = \frac{2 \times 5 \times x - 5 \times 5}{(2x - 5)} = \frac{5(2x - 5)}{2x - 5} = 5$$

$$(iii) \frac{10y(6y + 21) \div 5(2y + 7)}{5(2y + 7)} = \frac{2 \times 5 \times y[2 \times 3 \times y + 3 \times 7]}{5(2y + 7)}$$

$$= \frac{2 \times 5 \times y \times 3(2y + 7)}{5(2y + 7)} = 6y$$

$$(iv) \frac{9x^2y^2(3z - 24) \div 27xy(z - 8)}{27xy(z - 8)} = \frac{9x^2y^2[3 \times z - 2 \times 2 \times 2 \times 3]}{27xy(z - 8)}$$

$$= \frac{xy \times 3(z - 8)}{3(z - 8)} = xy$$

$$(v) 96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$$

$$= \frac{96abc(3 \times a - 3 \times 4)(5 \times b - 2 \times 3 \times 5)}{144(a - 4)(b - 6)}$$

$$= \frac{2abc \times 3(a - 4) \times 5(b - 6)}{3(a - 4)(b - 6)} = 10abc$$

Question 4:

Divide as directed.

$$(i) 5(2x + 1)(3x + 5) \div (2x + 1)$$

$$(ii) 26xy(x + 5)(y - 4) \div 13x(y - 4)$$

$$(iii) 52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$$

$$(iv) 20(y + 4) (y^2 + 5y + 3) \div 5(y + 4)$$

$$(v) x(x + 1) (x + 2) (x + 3) \div x(x + 1)$$

Answer:

$$(i) \quad 5(2x+1)(3x+5) \div (2x+1) = \frac{5(2x+1)(3x+1)}{(2x+1)} = 5(3x + 1)$$

$$(ii) \quad 26xy(x+5)(y-4) \div 13x(y-4) = \frac{2 \times 13 \times xy(x+5)(y-4)}{13x(y-4)} = 2y(x + 5)$$

$$(iii) 52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$$

$$= \frac{2 \times 2 \times 13 \times p \times q \times r \times (p+q) \times (q+r) \times (r+p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q+r) \times (r+p)}$$

$$= \frac{1}{2}r(p+q)$$

$$(iv) 20(y + 4) (y^2 + 5y + 3) = 2 \times 2 \times 5 \times (y + 4) (y^2 + 5y + 3)$$

$$20(y+4)(y^2+5y+3) \div 5(y+4) = \frac{2 \times 2 \times 5 \times (y+4) \times (y^2+5y+3)}{5 \times (y+4)} \\ = 4(y^2+5y+3)$$

$$(v) \quad x(x+1)(x+2)(x+3) \div x(x+1) = \frac{x(x+1)(x+2)(x+3)}{x(x+1)}$$

$$= (x + 2) (x + 3)$$

Question 5:

Factorise the expressions and divide them as directed.

$$(i) (y^2 + 7y + 10) \div (y + 5)$$

$$(ii) (m^2 - 14m - 32) \div (m + 2)$$

$$(iii) (5p^2 - 25p + 20) \div (p - 1)$$

$$(iv) 4yz(z^2 + 6z - 16) \div 2y(z + 8)$$

$$(v) 5pq(p^2 - q^2) \div 2p(p + q)$$

$$(vi) 12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$$

$$(vii) 39y^3(50y^2 - 98) \div 26y^2(5y + 7)$$

Answer:

$$\begin{aligned} \text{(i) } (y^2 + 7y + 10) &= y^2 + 2y + 5y + 10 \\ &= y(y + 2) + 5(y + 2) \\ &= (y + 2)(y + 5) \end{aligned}$$

$$(y^2 + 7y + 10) \div (y + 5) = \frac{(y + 5)(y + 2)}{(y + 5)} = y + 2$$

$$\begin{aligned} \text{(ii) } m^2 - 14m - 32 &= m^2 + 2m - 16m - 32 \\ &= m(m + 2) - 16(m + 2) \\ &= (m + 2)(m - 16) \end{aligned}$$

$$(m^2 - 14m - 32) \div (m + 2) = \frac{(m + 2)(m - 16)}{(m + 2)} = m - 16$$

$$\begin{aligned} \text{(iii) } 5p^2 - 25p + 20 &= 5(p^2 - 5p + 4) \\ &= 5[p^2 - p - 4p + 4] \\ &= 5[p(p - 1) - 4(p - 1)] \\ &= 5(p - 1)(p - 4) \end{aligned}$$

$$(5p^2 - 25p + 20) \div (p - 1) = \frac{5(p - 1)(p - 4)}{(p - 1)} = 5(p - 4)$$

$$\begin{aligned} \text{(iv) } 4yz(z^2 + 6z - 16) &= 4yz[z^2 - 2z + 8z - 16] \\ &= 4yz[z(z - 2) + 8(z - 2)] \\ &= 4yz(z - 2)(z + 8) \end{aligned}$$

$$4yz(z^2 + 6z - 16) \div 2y(z + 8) = \frac{4yz(z - 2)(z + 8)}{2y(z + 8)} = 2z(z - 2)$$

$$\text{(v) } 5pq(p^2 - q^2) = 5pq(p - q)(p + q)$$

$$5pq(p^2 - q^2) \div 2p(p + q) = \frac{5pq(p - q)(p + q)}{2p(p + q)} = \frac{5}{2}q(p - q)$$

$$\text{(vi) } 12xy(9x^2 - 16y^2) = 12xy[(3x)^2 - (4y)^2] = 12xy(3x - 4y)(3x + 4y)$$

$$\begin{aligned}12xy(9x^2 - 16y^2) \div 4xy(3x + 4y) &= \frac{2 \times 2 \times 3 \times x \times y \times (3x - 4y) \times (3x + 4y)}{2 \times 2 \times x \times y \times (3x + 4y)} \\&= 3(3x - 4y)\end{aligned}$$

$$\begin{aligned}(\text{vii}) \quad 39y^3(50y^2 - 98) &= 3 \times 13 \times y \times y \times y \times 2[(25y^2 - 49)] \\&= 3 \times 13 \times 2 \times y \times y \times y \times [(5y)^2 - (7)^2] \\&= 3 \times 13 \times 2 \times y \times y \times y (5y - 7)(5y + 7) \\26y^2(5y + 7) &= 2 \times 13 \times y \times y \times (5y + 7) \\39y^3(50y^2 - 98) \div 26y^2(5y + 7)\end{aligned}$$

Question 3:

Find and correct the errors in the statement: $2x + 3y = 5xy$

Answer:

$$\text{L.H.S} = 2x + 3y \neq \text{R.H.S.}$$

The correct statement is $2x + 3y = 2x + 3y$

Question 4:

Find and correct the errors in the statement: $x + 2x + 3x = 5x$

Answer:

$$\text{L.H.S} = x + 2x + 3x = 1x + 2x + 3x = x(1 + 2 + 3) = 6x \neq \text{R.H.S.}$$

The correct statement is $x + 2x + 3x = 6x$

Question 5:

Find and correct the errors in the statement: $5y + 2y + y - 7y = 0$

Answer:

$$\text{L.H.S.} = 5y + 2y + y - 7y = 8y - 7y = y \neq \text{R.H.S}$$

The correct statement is $5y + 2y + y - 7y = y$

Question 6:

Find and correct the errors in the statement: $3x + 2x = 5x^2$

Answer:

$$\text{L.H.S.} = 3x + 2x = 5x \neq \text{R.H.S}$$

The correct statement is $3x + 2x = 5x$

Question 7:

Find and correct the errors in the statement: $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$

Answer:

$$\text{L.H.S} = (2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7 \neq \text{R.H.S}$$

$$\text{The correct statement is } (2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$$

Question 8:

$$\text{Find and correct the errors in the statement: } (2x)^2 + 5x = 4x + 5x = 9x$$

Answer:

$$\text{L.H.S} = (2x)^2 + 5x = 4x^2 + 5x \neq \text{R.H.S.}$$

$$\text{The correct statement is } (2x)^2 + 5x = 4x^2 + 5x$$

Question 9:

$$\text{Find and correct the errors in the statement: } (3x + 2)^2 = 3x^2 + 6x + 4$$

Answer:

$$\begin{aligned} \text{L.H.S.} &= (3x + 2)^2 = (3x)^2 + 2(3x)(2) + (2)^2 [(a + b)^2 = a^2 + 2ab + b^2] \\ &= 9x^2 + 12x + 4 \neq \text{R.H.S} \end{aligned}$$

$$\text{The correct statement is } (3x + 2)^2 = 9x^2 + 12x + 4$$

Question 10:

Find and correct the errors in the following mathematical statement. Substituting $x = -3$ in

$$(a) x^2 + 5x + 4 \text{ gives } (-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$$

$$(b) x^2 - 5x + 4 \text{ gives } (-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$$

$$(c) x^2 + 5x \text{ gives } (-3)^2 + 5(-3) = -9 - 15 = -24$$

Answer:

(a) For $x = -3$,

$$x^2 + 5x + 4 = (-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = 13 - 15 = -2$$

(b) For $x = -3$,

$$x^2 - 5x + 4 = (-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28$$

(c) For $x = -3$,

$$x^2 + 5x = (-3)^2 + 5(-3) = 9 - 15 = -6$$

Question 11:

$$\text{Find and correct the errors in the statement: } (y - 3)^2 = y^2 - 9$$

Answer:

$$\begin{aligned}\text{L.H.S} &= (y - 3)^2 = (y)^2 - 2(y)(3) + (3)^2 [(a - b)^2 = a^2 - 2ab + b^2] \\ &= y^2 - 6y + 9 \neq \text{R.H.S}\end{aligned}$$

The correct statement is $(y - 3)^2 = y^2 - 6y + 9$

Question 12:

Find and correct the errors in the statement: $(z + 5)^2 = z^2 + 25$

Answer:

$$\begin{aligned}\text{L.H.S} &= (z + 5)^2 = (z)^2 + 2(z)(5) + (5)^2 [(a + b)^2 = a^2 + 2ab + b^2] \\ &= z^2 + 10z + 25 \neq \text{R.H.S}\end{aligned}$$

The correct statement is $(z + 5)^2 = z^2 + 10z + 25$

Question 13:

Find and correct the errors in the statement: $(2a + 3b)(a - b) = 2a^2 - 3b^2$

Answer:

$$\begin{aligned}\text{L.H.S.} &= (2a + 3b)(a - b) = 2a \times a + 3b \times a - 2a \times b - 3b \times b \\ &= 2a^2 + 3ab - 2ab - 3b^2 = 2a^2 + ab - 3b^2 \neq \text{R.H.S.}\end{aligned}$$

The correct statement is $(2a + 3b)(a - b) = 2a^2 + ab - 3b^2$

Question 14:

Find and correct the errors in the statement: $(a + 4)(a + 2) = a^2 + 8$

Answer:

$$\begin{aligned}\text{L.H.S.} &= (a + 4)(a + 2) = (a)^2 + (4 + 2)(a) + 4 \times 2 \\ &= a^2 + 6a + 8 \neq \text{R.H.S}\end{aligned}$$

The correct statement is $(a + 4)(a + 2) = a^2 + 6a + 8$

Question 15:

Find and correct the errors in the statement: $(a - 4)(a - 2) = a^2 - 8$

Answer:

$$\begin{aligned}\text{L.H.S.} &= (a - 4)(a - 2) = (a)^2 + [(-4) + (-2)](a) + (-4)(-2) \\ &= a^2 - 6a + 8 \neq \text{R.H.S.}\end{aligned}$$

The correct statement is $(a - 4)(a - 2) = a^2 - 6a + 8$

Question 16:

Find and correct the errors in the statement: $\frac{3x^2}{3x^2} = 0$

Answer:

$$\text{L.H.S} = \frac{3x^2}{3x^2} = \frac{3 \times x \times x}{3 \times x \times x} = 1 \neq \text{R.H.S.}$$

$$\text{The correct statement is } \frac{3x^2}{3x^2} = 1$$

Question 17:

$$\text{Find and correct the errors in the statement: } \frac{3x^2+1}{3x^2} = 1+1=2$$

Answer:

$$\frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2} = 1 + \frac{1}{3x^2} \neq \text{R.H.S.}$$

$$\text{The correct statement is } \frac{3x^2+1}{3x^2} = 1 + \frac{1}{3x^2}$$

Question 18:

$$\text{Find and correct the errors in the statement: } \frac{3x}{3x+2} = \frac{1}{2}$$

Answer:

$$\text{L.H.S} = \frac{3x}{3x+2} \neq \text{R.H.S.}$$

$$\text{The correct statement is } \frac{3x}{3x+2} = \frac{3x}{3x+2}$$

Question 19:

$$\text{Find and correct the errors in the statement: } \frac{3}{4x+3} = \frac{1}{4x}$$

Answer:

$$\text{L.H.S.} = \frac{3}{4x+3} \neq \text{R.H.S.}$$

The correct statement is $\frac{3}{4x+3} = \frac{3}{4x+3}$

Question 20:

Find and correct the errors in the statement: $\frac{4x+5}{4x} = 5$

Answer:

$$\text{L.H.S.} = \frac{4x+5}{4x} = \frac{4x}{4x} + \frac{5}{4x} = 1 + \frac{5}{4x} \neq \text{R.H.S.}$$

The correct statement is $\frac{4x+5}{4x} = 1 + \frac{5}{4x}$

Question 21:

Find and correct the errors in the statement: $\frac{7x+5}{5} = 7x$

Answer:

$$\text{L.H.S.} = \frac{7x+5}{5} = \frac{7x}{5} + \frac{5}{5} = \frac{7x}{5} + 1 \neq \text{R.H.S.}$$

The correct statement is $\frac{7x+5}{5} = \frac{7x}{5} + 1$