CBSE Test Paper 04

Chapter 13 Probability

1. If P(A) = $\frac{1}{2}$, P(B) = 0, then P(A|B) is
   a. 0
   b. not defined
   c. $\frac{1}{2}$
   d. 1
2. An instructor has a question bank consisting of 300 easy True / False questions,200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?
   a. $\frac{5}{9}$
   b. $\frac{4}{9}$
   c. $\frac{1}{9}$
   d. $\frac{7}{9}$
3. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable ?
   a. X = 2, 3, 4; no
   b. X = 2, 3, 5; yes
   c. X = 2, 1, 3; yes
   d. X = 0, 1, 2; yes
4. Find the probability of getting 5 exactly twice in 7 throws of a die.
   a. $\frac{5}{12}\left(\frac{5}{6}\right)^5$
   b. $\frac{7}{12}\left(\frac{5}{6}\right)^4$

c. $\frac{7}{12}\left(\frac{1}{6}\right)^5$
d. $\frac{7}{12}\left(\frac{5}{6}\right)^5$

5. Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, find P (E|F) and P(F|E)

   a. P (E|F) = $\frac{2}{3}$,P(F|E) = $\frac{1}{3}$
   b. P (E|F) = $\frac{2}{5}$,P(F|E) = $\frac{1}{4}$
   c. P (E|F) = $\frac{2}{3}$,P(F|E) = $\frac{1}{4}$
   d. P (E|F) = $\frac{2}{4}$,P(F|E) = $\frac{1}{3}$
6. If A and B be two events and P(A | B) = P (A), then A is ________ of B.
7. The possibility of drawing a jack or a spade from a well-shuffled standard deck of 52 playing cards is ________.
8. When two coins are tossed simultaneously, the chances of getting at least one tail is ________.

9. Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Find: P(neither A nor B)

10. A Box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, other wise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

11. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ & $\frac{1}{3}$ respectively If both try to solve the problem independently, find the probability that

    i. the problem is solved
    ii. Exactly one of them solves the problem

12. A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if he gets a total of 7. If A starts the game, find the probability of winning the game by A in third throw of the pair of dice.

13. Two probability distributions of the discrete random variable X and Y are given below.

X	0	1	2	3
P(X)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Y	0	1	2	3
P(Y)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Prove that $E(Y^2) = 2E(X)$.

14. Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the distribution.

15. A bag contains (2n + 1) coins. It is known that n of these coins have a head on both sides where as the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n.

16. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. Write atleast one advantage of coming to school in time.

17. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

18. If X is the number of tails in three tosses of a coin, determine the standard deviation of X.

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Solution

1. b. not defined

   Explanation: We know that :

   $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0}$

   which is not defined

2. a. $\frac{5}{9}$

   Explanation:

Questions	True/False	MCQ
Easy	300	500
Difficult	200	400

   Total = 1400, therefore n(S) = 1400 Let A = event of getting easy question B = event of getting multiple choice question.

   $\Rightarrow n(A) = 800, n(B) = 900, n(A \cap B) = 500$

   $\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{500/1400}{900/1400} = \frac{5}{9}$

3. d. X = 0, 1, 2; yes

   Explanation: An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls.therefore the possible values of X = 0 , 1 ,2 . Yes , X is a random variable.

4. d. $\frac{7}{12}\left(\frac{5}{6}\right)^5$

   Explanation: Probability of getting 5on throwing a die = 1/6 . Therefore , p = 1/6 and q = 5/6 . We have n = 7 , then using binomial distribution , Required probability = P (x = 2) = ${}^7C_2\left(\frac{5}{6}\right)^{7-2}\left(\frac{1}{6}\right)^2 = \frac{7}{12}\left(\frac{5}{6}\right)^5$.

5. a. P (E|F) = $\frac{2}{3}$, P(F|E) = $\frac{1}{3}$

   Explanation: We have,

   P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, then ,

$$P(E/F) = \frac{P(E\cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$P(F/E) = \frac{P(E\cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

6. Independent
7. $\frac{4}{13}$
8. $\frac{3}{4}$
9. P (neither A nor B) = P [not $(A \cup B)] = 1 - P(A \cup B)$= 1 - 0.72 = 0.28
10. Let A, B and C be the respective events that the first, second, and third drawn oranges good. Therefore, probability that first drawn orange is good =P(A)=$\frac{12}{15}$. The oranges are not replaced. Therefore, probability of getting second orange good=P(B)= $\frac{11}{14}$. Similarly, probability of getting third orange good=P(C) = $\frac{10}{13}$
    The box is approved for sale, if all the three oranges are good. Required probability $= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$
11. E_1 : A solves the problem

    E_2 : B solves the problem

    P (E_1) = $\frac{1}{2}$ and P (E_2) = $\frac{1}{3}$

    i. P (the problem is solved)

       = 1 – P (the problem is not solved)

       =1-P(A not solve the problem and B not solve the problem)

       $= 1 - \left[\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\right]$

       $= 1 - \frac{1}{3} = \frac{2}{3}$

    ii. P Exactly one of them solves the problem

       =P(A solve the problem and B not)+(B solve the problem and A not)

       = P(E_1) (1 - P(E_2)) + P(E_2) (1 - P(E_1))

       $= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \frac{1}{3}\left(1 - \frac{1}{2}\right) = \frac{1}{2}$
12. Let A = a total of 6 = {(2, 4), (1, 5), (5, 1), (4, 2), (3, 3)}
    And B = a total of 7 = {(2, 5), (1, 6), (6, 1), (5, 2), (3, 4), (4, 3)}
    Let P(A) be the probability, if A wins in a throw $\Rightarrow P(A) = \frac{5}{36}$
    And P(B) be the probability, if B wins in a throw $\Rightarrow P(B) = \frac{1}{36}$
    $\therefore$ Required probability $= P(\overline{A}) \cdot P(\overline{B}) \cdot P(A) = \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{5}{36} = \frac{775}{216.36} = \frac{775}{7776}$
13.

X	0	1	2	3

P(X)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Y	0	1	2	3
P(Y)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Since, we have to prove that, E(Y^2) = 2E(X)

$\therefore E(X) = \sum X\ P(X)$

$= 0 \cdot \frac{1}{5} + 1 \cdot \frac{2}{5} + 2 \cdot \frac{1}{5} + 3 \cdot \frac{1}{5} = \frac{7}{5}$

$\Rightarrow 2E(X) = \frac{14}{5}$ .... (i)

$E(Y)^2 = \sum Y^2 P(Y)$

$= 0 \cdot \frac{1}{5} + 1 \cdot \frac{3}{10} + 4 \cdot \frac{2}{5} + 9 \cdot \frac{1}{10}$

$= \frac{3}{10} + \frac{8}{5} + \frac{9}{10} = \frac{28}{10} = \frac{14}{5}$

$\Rightarrow E(Y)^2 = \frac{14}{5}$ .... (ii)

From Eqs. (i) and (ii),

E(Y)^2 = 2E (X)

Hence proved.

14. Let X is the random variable score obtained when a die is thrown twice.

$\therefore$ X = 1, 2, 3, 4, 5, 6

Here, S = {(1, 1), (1, 2), (2, 1), (2, 2), (1, 3), (2, 3), (3, 1), (3, 2), (3, 3),....., (6, 6)}

$\therefore P(X = 1) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

$P(X = 2) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{3}{36}$

$P(X = 3) = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{5}{36}$

Similarly,

$P(X = 4) = \frac{7}{36}$

$P(X = 5) \frac{9}{36}$

$P(X = 6) = \frac{11}{36}$

So, the required distribution is,

X	1	2	3	4	5	6
P(X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Also, we know that, Mean $\{E(X)\} = \sum X\ P(X)$

$= \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36} = \frac{161}{36}$

15. Given, n coins have head on both sides and (n+1) coins are fair coins.

Let E_1= Event that an unfair coin is selected.

E_2= Event that a fair coin is selected.

E = Event that the toss results in a head.

$\therefore P(E_1) = \frac{n}{2n+1}$ and $P(E_2) = \frac{n+1}{2n+1}$

Also, $P\left(\frac{E}{E_1}\right) = 1$ and $P\left(\frac{E}{E_2}\right) = \frac{1}{2}$

$\therefore P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) = \frac{n}{2n+1} \cdot 1 + \frac{n+1}{2n+1} \cdot \frac{1}{2}$

$\Rightarrow \frac{31}{42} = \frac{2n+n+1}{2(n+1)} \Rightarrow \frac{31}{42} = \frac{3n+1}{4n+2}$

$\Rightarrow$ 124n + 62 = 126n + 42

$\Rightarrow$ 2n = 20 $\Rightarrow$ n = 10

16. Given, probability of student A coming in time,

$P(A) = \frac{3}{7}$

Then, probability of student A not coming in time,

$P(\overline{A}) = 1 - \frac{3}{7} = \frac{4}{7}$

and probability of student B coming in time,

$P(B) = \frac{5}{7}$

Then, probability of student B not coming in time,

$P(\overline{B}) = 1 - \frac{5}{7} = \frac{2}{7}$

Now, required probability$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$= P(A) \cdot P(\overline{B}) + P(\overline{A})P(B)$

[events A and Bare independent events]

$= \frac{3}{7} \times \frac{2}{7} + \frac{4}{7} \times \frac{5}{7}$

$= \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$

Regular attendance and punctuality are important, if children want to take advantage of the learning opportunities offered by the school. Also, it is vital to the educational process and encourages for a good pattern of work.

17. A patient has options to have the treatment of yoga and meditation and that of prescription of drugs.

Let these events be denoted by E_1 and E_2 i.e.,

E_1 = Treatment of yoga and meditation

E_2 = Treatment of prescription of certain drugs

P (E_1) $= \frac{1}{2}$ and P (E_2) = $= \frac{1}{2}$

Let A denotes that a person has heart attack, then P (A) = 40% = 0.40

Yoga and meditation reduces heart attack by 30.

$\Rightarrow$ Inspite of getting yoga and meditation treatment heart risk is 70% of 0.40

$\Rightarrow P(A|E_1) = 0.40 \times 0.70 = 0.28$

Also, Drug prescription reduces the heart attack rick by 25%

Even after adopting the drug prescription hear rick is 75% of 0.40

$\Rightarrow P(A|E_2) = 0.40 \times 0.75 = 0.30$

$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$= \frac{\frac{1}{2}\times 0.28}{\frac{1}{2}\times 0.28+\frac{1}{2}\times 0.30}$

$= \frac{0.28}{0.28+0.30} = \frac{0.28}{0.58} = \frac{28}{58} = \frac{14}{29}$

18. Given that, random variable X is the number of tails in three tosses of a coin.

So, X = 0,1 , 2 , 3

$\Rightarrow P(X-x) = {}^nC_x (p)^x q^{n-x}$

Where n = 3, p = $\frac{1}{2}$, q = $\frac{1}{2}$ and x = 0, 1, 2 , 3

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
XP(X)	0	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{3}{8}$
X^2P(X)	0	$\frac{3}{8}$	$\frac{3}{2}$	$\frac{9}{8}$

We know that, Var(X) = E(X^2) - $[E(X)]^2$ Eq...(i)

Where, E(X^2) = $\sum_{i=1}^{n} x_i^2 P(X_i)$ and E(X) = $\sum_{i=1}^{n} x_i P(X_i)$

$\therefore E(X^2) = \sum_{i=1}^{n} x^2 P(X_i) = 0 + \frac{3}{8} + \frac{3}{2} + \frac{9}{8} = \frac{24}{8} = 3$

And $\therefore [E(X^2)]^2 = \left[\sum_{i=1}^{n} x^2 P(X_i)\right]^2 = \left[0 + \frac{3}{8} + \frac{3}{4} + \frac{9}{8}\right]^2 = \left[\frac{12}{8}\right]^2 = \frac{9}{4}$

$\therefore$ Var (X) = 3 - $\frac{9}{4}$ = $\frac{3}{4}$ [using Eq. (i)]

And standard deviation of X = $\sqrt{Var(X)} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$