

Complex Numbers and Quadratic Equations

Question1

If $S = \{z \in \mathbb{C} : |z - i| = |z + i| = |z - 1|\}$, then, $n(S)$ is:

[27-Jan-2024 Shift 1]

Options:

A.

1

B.

0

C.

3

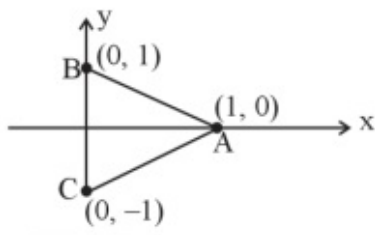
D.

2

Answer: A

Solution:

$$|z - i| = |z + i| = |z - 1|$$



ABC is a triangle. Hence its circum-centre will be the only point whose distance from A, B, C will be same.
So $n(S) = 1$

Question2

If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, $A, B, C \geq 0$, then $5(3A - 2B - C)$ is equal to

[27-Jan-2024 Shift 1]

Answer: 5

Solution:

$$x^2 + x + 1 = 0 \Rightarrow x = \omega, \omega^2 = \alpha$$
$$\text{Let } \alpha = \omega$$
$$\text{Now } (1 + \alpha)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$$
$$A = 1, B = 1, C = 0$$
$$\therefore 5(3A - 2B - C) = 5(3 - 2 - 0) = 5$$

Question3

Let the complex numbers α and $1/\alpha$ lie on the circles $|z - z_0|^2 = 4$ and $|z - z_0|^2 = 16$ respectively, where $z_0 = 1 + i$. Then, the value of $100 |\alpha|^2$ is. ____

[27-Jan-2024 Shift 2]

Answer: 20

Solution:

$$|z - z_0|^2 = 4$$
$$\Rightarrow (\alpha - z_0)(\bar{\alpha} - \bar{z}_0) = 4$$
$$\Rightarrow \alpha\bar{\alpha} - \alpha\bar{z}_0 - z_0\bar{\alpha} + |z_0|^2 = 4$$
$$\Rightarrow |\alpha|^2 - \alpha\bar{z}_0 - z_0\bar{\alpha} = 2 \dots\dots\dots(1)$$
$$|z - z_0|^2 = 16$$
$$\Rightarrow \left(\frac{1}{\alpha} - z_0\right)\left(\frac{1}{\alpha} - \bar{z}_0\right) = 16$$
$$\Rightarrow (1 - \alpha z_0)(1 - \alpha\bar{z}_0) = 16 |\alpha|^2$$
$$\Rightarrow 1 - \alpha z_0 - \alpha\bar{z}_0 + |\alpha|^2 |z_0|^2 = 16 |\alpha|^2$$
$$\Rightarrow 1 - \alpha z_0 - \alpha\bar{z}_0 = 14 |\alpha|^2 \dots\dots\dots(2)$$

From (1) and (2)

$$\Rightarrow 5 |\alpha|^2 = 1$$
$$\Rightarrow 100 |\alpha|^2 = 20$$

Question4

If α, β are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then

[27-Jan-2024 Shift 2]

Options:

A.

$$2S_{12} = S_{11} + S_{10}$$

B.

$$S_{12} = S_{11} + S_{10}$$

C.

$$2S_{11} = S_{12} + S_{10}$$

D.

$$S_{11} = S_{10} + S_{12}$$

Answer: B

Solution:

$$x^2 - x - 1 = 0$$

$$S_n = 2023\alpha^n + 2024\beta^n$$

$$S_{n-1} + S_{n-2} = 2023\alpha^{n-1} + 2024\beta^{n-1} + 2023\alpha^{n-2} + 2024\beta^{n-2}$$

$$= 2023\alpha^{n-2}[1 + \alpha] + 2024\beta^{n-2}[1 + \beta]$$

$$= 2023\alpha^{n-2}[\alpha^2] + 2024\beta^{n-2}[\beta^2]$$

$$= 2023\alpha^n + 2024\beta^n$$

$$S_{n-1} + S_{n-2} = S_n$$

$$\text{Put } n = 12$$

$$S_{11} + S_{10} = S_{12}$$

Question5

If $z = \frac{1}{2} - 2i$, is such that $|z + 1| = \alpha z + \beta(1 + i)$, $i = \sqrt{-1}$ and $\alpha, \beta \in \mathbb{R}$, then $\alpha + \beta$ is equal to

[29-Jan-2024 Shift 1]

Options:

A.

$$-4$$

B.

$$3$$

C.

2

D.

-1

Answer: B

Question6

Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with $\text{Im}(\alpha) > \text{Im}(\beta)$. Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to

[29-Jan-2024 Shift 1]

Answer: 13

Solution:

$$\begin{aligned} & \alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2 \\ &= \alpha^4(\alpha - 2) + \alpha^4 - 5\alpha^2 + (\beta - 2)^2 \\ &= \alpha^5 - \alpha^4 - 5\alpha^2 + \beta^2 - 4\beta + 4 \\ &= \alpha^3(\alpha - 2) - \alpha^4 - 5\alpha^2 + \beta - 2 - 4\beta + 4 \\ &= -2\alpha^3 - 5\alpha^2 - 3\beta + 2 \\ &= -2\alpha(\alpha - 2) - 5\alpha^2 - 3\beta + 2 \\ &= -7\alpha^2 + 4\alpha - 3\beta + 2 \\ &= -7(\alpha - 2) + 4\alpha - 3\beta + 2 \\ &= -3\alpha - 3\beta + 16 = -3(1) + 16 = 13 \end{aligned}$$

Question7

Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i (2 \tan 5\pi/8)$, then (r, θ) is equal to

[29-Jan-2024 Shift 2]

Options:

A.

$$\left(2 \sec \frac{3\pi}{8}, \frac{3\pi}{8} \right)$$

B.

$$\left(2 \sec \frac{3\pi}{8}, \frac{5\pi}{8} \right)$$

C.

$$\left(2 \sec \frac{5\pi}{8}, \frac{3\pi}{8} \right)$$

D.

$$\left(2 \sec \frac{11\pi}{8}, \frac{11\pi}{8} \right)$$

Answer: A

Solution:

$$z = 2 - i \left(2 \tan \frac{5\pi}{8} \right) = x + iy \text{ (let)}$$

$$r = \sqrt{x^2 + y^2} \text{ \& } \theta = \tan^{-1} \frac{y}{x}$$

$$r = \sqrt{(2)^2 + \left(2 \tan \frac{5\pi}{8} \right)^2}$$

$$= \left| 2 \sec \frac{5\pi}{8} \right| = \left| 2 \sec \left(\pi - \frac{3\pi}{8} \right) \right|$$

$$= 2 \sec \frac{3\pi}{8}$$

$$\text{\& } \theta = \tan^{-1} \left(\frac{-2 \tan \frac{5\pi}{8}}{2} \right) .$$

$$= \tan^{-1} \left(\tan^2 \left(\pi - \frac{5\pi}{8} \right) \right)$$

$$= \frac{3\pi}{8}$$

Question8

Let α, β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that $\text{Im}(\alpha) > \text{Im}(\beta)$. Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^n(a + ib), i = \sqrt{-1}$. Then $n + a + b$ is equal to ____

[29-Jan-2024 Shift 2]

Answer: 49

Solution:

$$x^2 - \sqrt{6}x + 6 = 0$$

$$x = \frac{\sqrt{6} \pm i\sqrt{6}}{2} = \frac{\sqrt{6}}{2}(1 \pm i)$$

$$\alpha = \sqrt{3} \left(e^{i\frac{\pi}{4}} \right), \beta = \sqrt{3} \left(e^{-i\frac{\pi}{4}} \right)$$

$$\begin{aligned} \therefore \frac{\alpha^{99}}{\beta} + \alpha^{98} &= \alpha^{98} \left(\frac{\alpha}{\beta} + 1 \right) \\ &= \frac{\alpha^{98}(\alpha + \beta)}{\beta} = 3^{49} \left(e^{i99\frac{\pi}{4}} \right) \times \sqrt{2} \\ &= 3^{49}(-1 + i) \\ &= 3^n(a + ib) \end{aligned}$$

$$\therefore n = 49, a = -1, b = 1$$

$$\therefore n + a + b = 49 - 1 + 1 = 49$$

Question9

Let the set $C = \{(x, y) \mid x^2 - 2^y = 2023, x, y \in \mathbb{N}\}$.

Then $\sum_{(x, y) \in C} (x + y)$ is equal to ____

[29-Jan-2024 Shift 2]

Answer: 46

Solution:

$$x^2 - 2^y = 2023$$

$$\Rightarrow x = 45, y = 1$$

$$\sum_{(x, y) \in C} (x + y) = 46.$$

Question10

If $z = x + iy$, $xy \neq 0$, satisfies the equation $z^2 + i\bar{z} = 0$, then $|z^2|$ is equal to :

[30-Jan-2024 Shift 1]

Options:

A.

9

B.

1

C.

4

D.

1/4

Answer: B

Solution:

$$z^2 = -i\bar{z}$$

$$|z^2| = |i\bar{z}|$$

$$|z^2| = |z|$$

$$|z|^2 - |z| = 0$$

$$|z|(|z| - 1) = 0$$

$$|z| = 0 \text{ (not acceptable)}$$

$$\therefore |z| = 1$$

$$\therefore |z|^2 = 1$$

Question11

If z is a complex number, then the number of common roots of the equation $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to :

[30-Jan-2024 Shift 2]

Options:

A.

1

B.

2

C.

0

D.

3

Answer: B

Solution:

$$z^{1985} + z^{100} + 1 = 0 \text{ \& } z^3 + 2z^2 + 2z + 1 = 0$$

$$(z+1)(z^2 - z + 1) + 2z(z+1) = 0$$

$$(z+1)(z^2 + z + 1) = 0$$

$$\Rightarrow z = -1, \quad z = w, w^2$$

Now putting $z = -1$ not satisfy

Now put $z = w$

$$\Rightarrow w^{1985} + w^{100} + 1$$

$$\Rightarrow w^2 + w + 1 = 0$$

$$\text{Also, } z = w^2$$

$$\Rightarrow w^{3970} + w^{200} + 1$$

$$\Rightarrow w + w^2 + 1 = 0$$

Two common root

Question12

If α denotes the number of solutions of $|1 - i|^x = 2^x$ and $\beta =$

$\left(\frac{|z|}{\arg(z)} \right)$, where $z = \frac{\pi(1+i)^4}{4} \left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i} \right)$, $i = \sqrt{-1}$, then the distance of the point (α, β) from the line $4x - 3y = 7$ is_____

[31-Jan-2024 Shift 1]

Answer: 3

Solution:

$$(\sqrt{2})^x = 2^x \Rightarrow x = 0 \Rightarrow \alpha = 1$$

$$z = \frac{\pi}{4}(1+i)^4 \left[\frac{\sqrt{\pi} - \pi i - i - \sqrt{\pi}}{\pi + 1} + \frac{\sqrt{\pi} - i - \pi i - \sqrt{\pi}}{1 + \pi} \right]$$

$$= -\frac{\pi i}{2}(1 + 4i + 6i^2 + 4i^3 + 1)$$

$$= 2\pi i$$

$$\beta = \frac{2\pi}{\frac{\pi}{2}} = 4$$

Distance from $(1, 4)$ to $4x - 3y = 7$

$$\text{Will be } \frac{15}{5} = 3$$

Question13

Let z_1 and z_2 be two complex number such that

$z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then $|z_1^4 + z_2^4|$ equals-

[31-Jan-2024 Shift 2]

Options:

A.

$$30\sqrt{3}$$

B.

$$75$$

C.

$$15\sqrt{15}$$

D.

$$25\sqrt{3}$$

Answer: B

Solution:

$$z_1 + z_2 = 5$$

$$z_1^3 + z_2^3 = 20 + 15i$$

$$z_1^3 + z_2^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$$

$$z_1^3 + z_2^3 = 125 - 3z_1 \cdot z_2(5)$$

$$\Rightarrow 20 + 15i = 125 - 15z_1z_2$$

$$\Rightarrow 3z_1z_2 = 25 - 4 - 3i$$

$$\Rightarrow 3z_1z_2 = 21 - 3i$$

$$\Rightarrow z_1 \cdot z_2 = 7 - i$$

$$\Rightarrow (z_1 + z_2)^2 = 25$$

$$\Rightarrow z_1^2 + z_2^2 = 25 - 2(7 - i)$$

$$\Rightarrow 11 + 2i$$

$$(z_1^2 + z_2^2)^2 = 121 - 4 + 44i$$

$$\Rightarrow z_1^4 + z_2^4 + 2(7 - i)^2 = 117 + 44i$$

$$\Rightarrow z_1^4 + z_2^4 = 117 + 44i - 2(49 - 1 - 14i)$$

$$\Rightarrow |z_1^4 + z_2^4| = 75$$

Question14

Let $\alpha, \beta \in \mathbb{N}$ be roots of equation $x^2 - 70x + \lambda = 0$, where $\lambda/2, \lambda/3 \notin \mathbb{N}$.

If λ assumes the minimum possible value, then

$\frac{(\sqrt{\alpha-1} + \sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$ is equal to :

[30-Jan-2024 Shift 1]

Answer: 60

Solution:

$$x^2 - 70x + \lambda = 0$$

$$\alpha + \beta = 70$$

$$\alpha\beta = \lambda$$

$$\therefore \alpha(70 - \alpha) = \lambda$$

Since, 2 and 3 does not divide λ

$$\therefore \alpha = 5, \beta = 65, \lambda = 325$$

By putting value of α, β, λ we get the required value 60 .

Question15

The number of real solutions of the equation $x(x^2 + 3x| + 5x - 1| + 6x - 2|) = 0$ is

[30-Jan-2024 Shift 2]

Answer: 1

Solution:

$$x = 0 \text{ and } x^2 + 3x| + 5x - 1| + 6x - 2| = 0$$

Here all terms are +ve except at $x = 0$

So there is no value of x

Satisfies this equation

Only solution $x = 0$

No of solution 1 .

Question16

Let S be the set of positive integral values of a for which

$$\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0, \forall x \in \mathbb{R}.$$

Then, the number of elements in S is :

[31-Jan-2024 Shift 1]

Options:

A.

1

B.

0

C.

∞

D.

3

Answer: B

Solution:

$$ax^2 + 2(a+1)x + 9a + 4 < 0 \quad \forall x \in \mathbb{R}$$

$$\therefore a < 0$$

Question17

For $0 < c < b < a$, let $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements

(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c

(II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c

[31-Jan-2024 Shift 1]

Options:

A.

Both (I) and (II) are true

B.

Neither (I) nor (II) is true

C.

Only (II) is true

D.

Only (I) is true

Answer: A

Solution:

$$f(x) = (a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b)$$

$$f(x) = a + b - 2c + b + c - 2a + c + a - 2b = 0$$

$$f(1) = 0$$

$$\therefore \alpha \cdot 1 = \frac{c + a - 2b}{a + b - 2c}$$

$$\alpha = \frac{c + a - 2b}{a + b - 2c}$$

$$\text{If } -1 < \alpha < 0$$

$$-1 < \frac{c + a - 2b}{a + b - 2c} < 0$$

$$b + c < 2a \text{ and } b > \frac{a + c}{2}$$

therefore, b cannot be G.M. between a and c.

$$\text{If, } 0 < \alpha < 1$$

$$0 < \frac{c + a - 2b}{a + b - 2c} < 1$$

$$b > c \text{ and } b < \frac{a + c}{2}$$

Therefore, b may be the G.M. between a and c.

Question18

The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$ is

[31-Jan-2024 Shift 2]

Options:

A.

2

B.

more than 2

C.

1

D.

0

Answer: D

Solution:

Take $e^{\sin x} = t (t > 0)$

$$\Rightarrow t - \frac{2}{t} = 2$$

$$\Rightarrow \frac{t^2 - 2}{t} = 2$$

$$\Rightarrow t^2 - 2t - 2 = 0$$

$$\Rightarrow t^2 - 2t + 1 = 3$$

$$\Rightarrow (t - 1)^2 = 3$$

$$\Rightarrow t = 1 \pm \sqrt{3}$$

$$\Rightarrow t = 1 \pm 1.73$$

$$\Rightarrow t = 2.73 \text{ or } -0.73 \text{ (rejected as } t > 0)$$

$$\Rightarrow e^{\sin x} = 2.73$$

$$\Rightarrow \log_e e^{\sin x} = \log_e 2.73$$

$$\Rightarrow \sin x = \log_e 2.73 > 1$$

So no solution.

Question 19

Let a, b, c be the length of three sides of a triangle satisfying the condition $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$. If the set of all possible values of x is the interval (α, β) , then $12(\alpha^2 + \beta^2)$ is equal to ____

[31-Jan-2024 Shift 2]

Answer: 36

Solution:

$$(a^2 + b^2)x^2 - 2b(a+c)x + b^2 + c^2 = 0$$

$$\Rightarrow a^2x^2 - 2abx + b^2 + b^2x^2 - 2bcx + c^2 = 0$$

$$\Rightarrow (ax - b)^2 + (bx - c)^2 = 0$$

$$\Rightarrow ax - b = 0, \quad bx - c = 0$$

$$\Rightarrow a + b > c \quad b + c > a \quad c + a > b$$

$$a + ax > bx \quad ax + bx > a \quad ax^2 + a > ax$$

$$a + ax > ax^2 \quad ax + ax^2 > a \quad x^2 - x + 1 > 0$$

$$x^2 - x - 1 < 0 \quad x^2 + x - 1 > 0 \quad \text{always true}$$

$$\frac{1 - \sqrt{5}}{2} < x < \frac{1 + \sqrt{5}}{2}$$

$$x < \frac{-1 - \sqrt{5}}{2}, \text{ or } x > \frac{-1 + \sqrt{5}}{2}$$

$$\Rightarrow \frac{\sqrt{5} - 1}{2} < x < \frac{\sqrt{5} + 1}{2}$$

$$\Rightarrow \alpha = \frac{\sqrt{5} - 1}{2}, \beta = \frac{\sqrt{5} + 1}{2}$$

$$12(\alpha^2 + \beta^2) = 12 \left(\frac{(\sqrt{5} - 1)^2 + (\sqrt{5} + 1)^2}{4} \right) = 36$$

Question20

Let $S = \{z \in \mathbb{C} : |z - 1| = 1 \text{ and } (\sqrt{2} - 1)(z + \bar{z}) - i(z - \bar{z}) = 2\sqrt{2}\}$. Let $z_1, z_2 \in S$ be such that $|z_1| = \max_{z \in S} |z|$ and $z_2 = \min_{z \in S} |z|$. Then $|\sqrt{2}z_1 - z_2|^2$ equals :

[1-Feb-2024 Shift 1]

Options:

A.

1

B.

4

C.

3

D.

2

Answer: D

Solution:

Let $Z = x + iy$

Then $(x - 1)^2 + y^2 = 1 \rightarrow (1)$

$$(\sqrt{2} - 1)(2x) - i(2iy) = 2\sqrt{2}$$

$$\Rightarrow (\sqrt{2} - 1)x + y = \sqrt{2} \rightarrow (2)$$

Solving (1) & (2) we get

$$\text{Either } x = 1 \text{ or } x = \frac{1}{2 - \sqrt{2}} \rightarrow (3)$$

On solving (3) with (2) we get

$$\text{For } x = 1 \Rightarrow y = 1 \Rightarrow Z_2 = 1 + i$$

& for

$$x = \frac{1}{2 - \sqrt{2}} \Rightarrow y = \sqrt{2} - \frac{1}{\sqrt{2}} \Rightarrow Z_1 = \left(1 + \frac{1}{\sqrt{2}}\right) + \frac{i}{\sqrt{2}}$$

Now

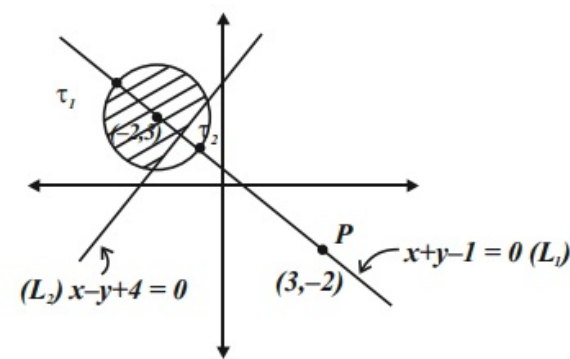
$$\begin{aligned} &|\sqrt{2}z_1 - z_2|^2 \\ &= \left| \left(\frac{1}{\sqrt{2}} + 1\right)\sqrt{2} + i - (1 + i) \right|^2 \\ &= (\sqrt{2})^2 \\ &= 2 \end{aligned}$$

Question21

Let $P = \{z \in C : |z + 2 - 3i| \leq 1\}$ and $Q = \{z \in C : z(1 + i) + \bar{z}(1 - i) \leq -8\}$. Let in $P \cap Q$, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2z_1|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals_____ [1-Feb-2024 Shift 1]

Answer: 8

Solution:



Clearly for the shaded region z_1 is the intersection of the circle and the line passing through $P(L_1)$ and z_2 is intersection of line L_1 & L_2

Circle : $(x+2)^2 + (y-3)^2 = 1$

$L_1 : x + y - 1 = 0$

$L_2 : x - y + 4 = 0$

On solving circle & L_1 we get

$z_1 : \left(-2 - \frac{1}{\sqrt{2}}, 3 + \frac{1}{\sqrt{2}} \right)$

On solving L_1 and z_2 is intersection of line L_1 & L_2 we get $z_2 : \left(\frac{-3}{2}, \frac{5}{2} \right)$

$|z_1|^2 + 2|z_2|^2 = 14 + 5\sqrt{2} + 17$
 $= 31 + 5\sqrt{2}$

So $\alpha = 31$

$\beta = 5$

$\alpha + \beta = 36$

Question22

If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left| z + \frac{1}{2}(3+4i) \right|$ is:

[1-Feb-2024 Shift 2]

Options:

A.

$5/2$

B.

2

C.

$3/2$

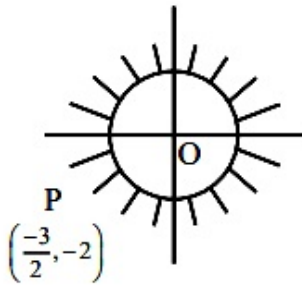
D.

None of above

Answer: D

Solution:

$$|z| \geq 1$$



Min. value of $\left| z + \frac{3}{2} + 2i \right|$ is actually zero.

Question23

$$\text{Let } S = \{x \in R : (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10\}$$

Then the number of elements in S is :

[1-Feb-2024 Shift 1]

Options:

A.

4

B.

0

C.

2

D.

1

Answer: C

Solution:

$$(\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10$$

$$\text{Let } (\sqrt{3} + \sqrt{2})^x = t$$

$$t + \frac{1}{t} = 10$$

$$t^2 - 10t + 1 = 0$$

$$t = \frac{10 \pm \sqrt{100 - 4}}{2} = 5 \pm 2\sqrt{6}$$

$$(\sqrt{3} + \sqrt{2})^x = (\sqrt{3} \pm \sqrt{2})^2$$

$$x = 2 \text{ or } x = -2$$

Number of solutions = 2

Question24

Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non-constant G.P and $1/\alpha + 1/\beta + = 3/4$, then the value of $(\alpha - \beta)^2$ is :

[1-Feb-2024 Shift 2]

Options:

A.

80/9

B.

9

C.

20/3

D.

8

Answer: A

Solution:

$$px^2 + qx - r = 0$$

$$p = A, q = AR, r = AR^2$$

$$Ax^2 + ARx - AR^2 = 0$$

$$x^2 + Rx - R^2 = 0$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$$

$$\therefore \frac{\alpha + \beta}{\alpha\beta} = \frac{3}{4} \Rightarrow \frac{-R}{-R^2} = \frac{3}{4} \Rightarrow R = \frac{4}{3}$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = R^2 - 4(-R^2) = 5 \left(\frac{16}{9} \right)$$

$$= 80/9$$

Question25

Let $p, q \in \mathbb{R}$ and $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$, $i = \sqrt{-1}$ Then $p + q + q^2$ and $p - q + q^2$ are roots of the equation.

[24-Jan-2023 Shift 1]

Options:

A. $x^2 + 4x - 1 = 0$

B. $x^2 - 4x + 1 = 0$

C. $x^2 + 4x + 1 = 0$

D. $x^2 - 4x - 1 = 0$

Answer: B

Solution:

$$\begin{aligned}(1 - \sqrt{3}i)^{200} &= 2^{199}(p + iq) \\ 2^{200} \left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} \right)^{200} &= 2^{199}(p + iq) \\ 2 \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) &= p + iq \\ p = -1, q = -\sqrt{3} \\ \alpha = p + q + q^2 &= 2 - \sqrt{3} \\ \beta = p - q + q^2 &= 2 + \sqrt{3} \\ &= 4 \\ &= 1 \\ \text{equation } x^2 - 4x + 1 &= 0\end{aligned}$$

Question26

The value of $\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}} \right)^3$ is

[24-Jan-2023 Shift 2]

Options:

A. $\frac{-1}{2}(1 - i\sqrt{3})$

B. $\frac{1}{2}(1 - i\sqrt{3})$

C. $\frac{-1}{2}(\sqrt{3} - i)$

D. $\frac{1}{2}(\sqrt{3} + i)$

Answer: C

Solution:

Solution:
Let $\sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9} = z$
 $\left(\frac{1+z}{1+\bar{z}} \right)^3 = \left(\frac{1+z}{1+\frac{1}{z}} \right)^3 = z^3$

$$\begin{aligned} &\Rightarrow \left(i \left(\cos \frac{2\pi}{9} - i \sin \frac{2\pi}{9} \right) \right)^3 \\ &= -i \left(\cos \frac{2\pi}{3} - i \sin \frac{2\pi}{3} \right) = -i \left(\frac{-1}{2} - i \frac{\sqrt{3}}{2} \right) \\ &\Rightarrow \frac{-1}{2}(\sqrt{3} - i). \end{aligned}$$

Question27

Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set

$$S = \left\{ z \in \mathbb{C} : \left| z - z_1 \right|^2 - \left| z - z_2 \right|^2 = \left| z_1 - z_2 \right|^2 \right\}$$

represents a

[25-Jan-2023 Shift 1]

Options:

- A. straight line with sum of its intercepts on the coordinate axes equals 14
- B. hyperbola with the length of the transverse axis 7
- C. straight line with the sum of its intercepts on the coordinate axes equals -18
- D. hyperbola with eccentricity 2

Answer: A

Solution:

Solution:

$$\begin{aligned} &((x-2)^2 + (y-3)^2) - ((x-3)^2 + (y-4)^2) = 1 + 1 \\ &\Rightarrow x + y = 7 \end{aligned}$$

Question28

Let z be a complex number such that $\left| \frac{z-2i}{z+i} \right| = 2$, $z \neq -i$. Then z lies on the circle of radius 2 and centre
[25-Jan-2023 Shift 2]

Options:

- A. (2, 0)
- B. (0, 0)
- C. (0, 2)
- D. (0, -2)

Answer: D

Solution:

Solution:

$$(\underline{z} - 2i)(\overline{z} + 2i) = 4(z + i)(\overline{z} - i)$$

$$z\overline{z} + 4 + 2i(z - \overline{z}) = 4(z\overline{z} + 1 + i(\overline{z} - z))$$

$$3z\overline{z} - 6i(z - \overline{z}) = 0$$

$$x^2 + y^2 - 2i(2iy) = 0$$

$$x^2 + y^2 + 4y = 0$$

Question29

For two non-zero complex number z_1 and z_2 , if $\text{Re}(z_1 z_2) = 0$ and $\text{Re}(z_1 + z_2) = 0$, then which of the following are possible ?

(A) $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) > 0$

(B) $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) > 0$

(C) $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) < 0$

(D) $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) < 0$

Choose the correct answer from the options given below :
[29-Jan-2023 Shift 1]

Options:

A. B and D

B. B and C

C. A and B

D. A and C

Answer: B

Solution:

Solution:

$$z_1 = x_1 + iy_1$$

$$z_2 = x_2 + iy_2$$

$$\text{Re}(z_1 z_2) = x_1 x_2 - y_1 y_2 = 0$$

$$\text{Re}(z_1 + z_2) = x_1 + x_2 = 0$$

x_1 & x_2 are of opposite sign

y_1 & y_2 are of opposite sign

Question30

Let $\alpha = 8 - 14i$, $A = \left\{ z \in \mathbb{C} : \frac{\alpha z - \overline{\alpha z}}{z^2 - (\overline{z})^2 - 112i} = 1 \right\}$ and

$B = \{z \in \mathbb{C} : |z + 3i| = 4\}$

Then $\sum_{z \in A \cap B} (\text{Re } z - \text{Im } z)$ is equal to _____.

[29-Jan-2023 Shift 2]

Answer: 14

Solution:

$$\begin{aligned}\alpha &= 8 - 14i \\ z &= x + iy \\ az &= (8x + 14y) + i(-14x + 8y) \\ z + \bar{z} &= 2x \quad z - \bar{z} = 2iy \\ \text{Set A: } \frac{2i(-14x + 8y)}{i(4xy - 112)} &= 1 \\ (x - 4)(y + 7) &= 0 \\ x = 4 \quad \text{or} \quad y &= -7 \\ \text{Set B: } x^2 + (y + 3)^2 &= 16 \\ \text{when } x = 4 \quad y &= -3 \\ \text{when } y = -7 \quad x &= 0 \\ \therefore A \cap B &= \{4 - 3i, 0 - 7i\} \\ \text{So, } \sum_{z \in A \cap B} (\operatorname{Re} z - \operatorname{Im} z) &= 4 - (-3) + (0 - (-7)) = 14\end{aligned}$$

Question31

Let $z = 1 + i$ and $z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$. Then $\frac{12}{\pi} \arg(z_1)$ is equal to _____.

[30-Jan-2023 Shift 1]

Answer: 9

Solution:

$$\begin{aligned}z_1 &= \frac{1 + i}{\bar{z}(1 - z) + \frac{1}{z}} \\ z_1 &= \frac{1 + i(1 - i)}{(1 - i)(1 - 1 - i) + \frac{1}{1 + i}} \\ &= \frac{1 + i - i^2}{(1 - i)(-i) + \frac{1 - i}{2}} \\ &= \frac{2 + i}{-3i - 1} = \frac{4 + 2i}{-3i - 1} \\ &= \frac{-(4 + 2i)(3i - 1)}{(3i)^2 - (1)^2} \\ \operatorname{Arg}(z_1) &= \frac{3\pi}{4} \\ \therefore \frac{12}{\pi} \arg(z_1) &= \frac{12}{\pi} \times \frac{3\pi}{4} = 9\end{aligned}$$

Question32

For all $z \in \mathbb{C}$ on the curve $C_1: |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2 . Then

[31-Jan-2023 Shift 1]

Options:

- A. the curves C_1 and C_2 intersect at 4 points
- B. the curves C_1 lies inside C_2
- C. the curves C_1 and C_2 intersect at 2 points
- D. the curves C_2 lies inside C_1

Answer: A

Solution:

Solution:

$$\text{Let } w = z + \frac{1}{z} = 4e^{i\theta} + \frac{1}{4}e^{-i\theta}$$

$$\Rightarrow w = \frac{17}{4}\cos\theta + i\frac{15}{4}\sin\theta$$

$$\text{So locus of } w \text{ is ellipse } \frac{x^2}{\left(\frac{17}{4}\right)^2} + \frac{y^2}{\left(\frac{15}{4}\right)^2} = 1$$

$$\text{Locus of } z \text{ is circle } x^2 + y^2 = 16$$

So intersect at 4 points

Question33

The complex number $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ is equal to:

[31-Jan-2023 Shift 2]

Options:

A. $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

B. $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$

C. $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

D. $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

Answer: A

Solution:

$$Z = \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}} = \frac{i-1}{\frac{1}{2} + \frac{\sqrt{3}}{2}i}$$

$$= \frac{i-1}{\frac{1}{2} + \frac{\sqrt{3}}{2}i} \times \frac{\frac{1}{2} - \frac{\sqrt{3}}{2}i}{\frac{1}{2} - \frac{\sqrt{3}}{2}i} = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$$

Apply polar form,

$$r \cos \theta = \frac{\sqrt{3}-1}{2}$$

$$r \sin \theta = \frac{\sqrt{3}+1}{2}$$

$$\text{Now, } \tan \theta = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\text{So, } \theta = \frac{5\pi}{12}$$

Question 34

Let α be a root of the equation

$(a-c)x^2 + (b-a)x + (c-b) = 0$ where a, b, c are distinct real numbers such that the matrix

$$\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$$

is singular. Then the value of $\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$ is
[24-Jan-2023 Shift 1]

Options:

A. 6

B. 3

C. 9

D. 12

Answer: B

Solution:

Solution:

$$\Delta = 0 = \begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$$

$$\Rightarrow \alpha^2(c-b) - \alpha(c-a) + (b-a) = 0$$

It is singular when $\alpha = 1$

$$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$$

$$\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= 3 \frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

Question35

Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda : x \text{ is an integer solution of E}\}$ is
[24-Jan-2023 Shift 1]

Answer: 5

Solution:

$$\begin{aligned} |x|^2 - 2|x| + |\lambda - 3| &= 0 \\ |x|^2 - 2|x| + |\lambda - 3| - 1 &= 0 \\ (|x| - 1)^2 + |\lambda - 3| &= 1 \\ \text{At } \lambda = 3, x &= 0 \text{ and } 2, \\ \text{at } \lambda = 4 \text{ or } 2, &\text{ then} \\ x &= 1 \text{ or } -1 \\ \text{So maximum value of } x + \lambda &= 5 \end{aligned}$$

Question36

The number of real solutions of the equation

$$3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0, \text{ is}$$

[24-Jan-2023 Shift 2]

Options:

- A. 4
- B. 0
- C. 3
- D. 2

Answer: B

Solution:

Solution:

$$\begin{aligned} 3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 &= 0 \\ 3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 &= 0 \\ \text{Let } x + \frac{1}{x} &= t \\ 3t^2 - 2t - 1 &= 0 \\ 3t^2 - 3t + t - 1 &= 0 \\ 3t(t - 1) + 1(t - 1) &= 0 \end{aligned}$$

$$(t - 1)(3t + 1) = 0$$

$$t = 1, -\frac{1}{3}$$

$$x + \frac{1}{x} = 1, -\frac{1}{3} \Rightarrow \text{No solution.}$$

Question37

Let

$$S = \left\{ \alpha : \log_2(9^{2\alpha - 4} + 13) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha - 4} + 1\right) = 2 \right\}.$$

Then the maximum value of β for which the equation

$$x^2 - 2\left(\sum_{\alpha \in S} \alpha\right)^2 x + \sum_{\alpha \in S} (\alpha + 1)^2 \beta = 0 \text{ has real roots, is } \underline{\hspace{2cm}}.$$

[25-Jan-2023 Shift 1]

Answer: 25

Solution:

$$\log_2(9^{2\alpha - 4} + 13) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha - 4} + 1\right) = 2$$

$$\Rightarrow \frac{9^{2\alpha - 4} + 13}{\frac{5}{2} 3^{2\alpha - 4} + 1} = 4$$

$$\sum_{\alpha \in S} \alpha = 5 \text{ and } \sum_{\alpha \in S} (\alpha + 1)^2 = 25$$

$$\Rightarrow x^2 - 50x + 25\beta = 0 \text{ has real roots}$$

$$\Rightarrow \beta \leq 25$$

$$\Rightarrow \beta_{\max} = 25$$

Question38

Let $a \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is .

[25-Jan-2023 Shift 2]

Answer: 45

Solution:

$$\alpha + \beta = -60 \frac{1}{4} \text{ \& } \alpha\beta = a$$

Given $\alpha^4 + \beta^4 = -30$

$$\Rightarrow (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = -30$$

$$\Rightarrow \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2a^2 = -30$$

$$\Rightarrow \left\{ 60 \frac{1}{2} - 2a \right\}^2 - 2a^2 = -30$$

$$\Rightarrow 60 + 4a^2 - 4a \times 60 \frac{1}{2} - 2a^2 = -30$$

$$\Rightarrow 2a^2 - 4.60 \frac{1}{2}a + 90 = 0$$

$$\text{Product} = \frac{90}{2} = 45$$

Question39

Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation :
[29-Jan-2023 Shift 1]

Options:

- A. $7x^2 + 245x - 250 = 0$
- B. $7x^2 - 245x + 250 = 0$
- C. $49x^2 - 245x + 250 = 0$
- D. $49x^2 + 245x + 250 = 0$

Answer: C

Solution:

Solution:

$$14x^2 - 31x + 3\lambda = 0$$

$$\alpha + \beta = \frac{31}{14} \dots (1) \text{ and } \alpha\beta = \frac{3\lambda}{14}$$

$$35x^2 - 53x + 4\lambda = 0$$

$$\alpha + \gamma = \frac{53}{35} \dots (3) \text{ and } \alpha\gamma = \frac{4\lambda}{35} \dots$$

$$\frac{(2)}{(4)} \Rightarrow \frac{\beta}{\gamma} = \frac{3 \times 35}{4 \times 14} = \frac{15}{8} \Rightarrow \beta = \frac{15}{8}\gamma$$

$$(1) - (3) \Rightarrow \beta - \gamma = \frac{31}{14} - \frac{53}{35} = \frac{155 - 106}{70} = \frac{7}{10}$$

$$\frac{15}{8}\gamma - \gamma = \frac{7}{10} \Rightarrow \gamma = \frac{4}{5}$$

$$\Rightarrow \beta = \frac{15}{8} \times \frac{4}{5} = \frac{3}{2}$$

$$\Rightarrow \alpha = \frac{31}{14} - \beta = \frac{31}{14} - \frac{3}{2} = \frac{5}{7}$$

$$\Rightarrow \lambda = \frac{14}{3}\alpha\beta = \frac{14}{3} \times \frac{5}{7} \times \frac{3}{2} = 5$$

$$\text{so, sum of roots } \frac{3\alpha}{\beta} + \frac{4\alpha}{\gamma} = \left(\frac{3\alpha\gamma + 4\alpha\beta}{\beta\gamma} \right)$$

$$= \frac{\left(3 \times \frac{4\lambda}{35} + 4 \times \frac{3\lambda}{14} \right)}{\beta\gamma} = \frac{12\lambda(14 + 35)}{14 \times 35\beta\gamma}$$

$$= \frac{49 \times 12 \times 5}{490 \times \frac{3}{2} \times \frac{4}{5}} = 5$$

Product of roots

$$= \frac{3\alpha}{\beta} \times \frac{4\alpha}{\gamma} = \frac{12\alpha^2}{\beta\gamma} = \frac{12 \times \frac{25}{49}}{\frac{3}{2} \times \frac{4}{5}} = \frac{250}{49}$$

So, required equation is $x^2 - 5x + \frac{250}{49} = 0$

$$\Rightarrow 49x^2 - 245x + 250 = 0$$

Question40

If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real roots is $\frac{3}{\sqrt{2}\beta}$ then β is equal to _____.

[30-Jan-2023 Shift 2]

Answer: 13

Solution:

Solution:

Two equations have common root

$$\therefore (4a)(26a) = (-6)^2 = 36$$

$$\Rightarrow a^2 = \frac{9}{26} \quad \therefore a = \frac{3}{\sqrt{26}} \Rightarrow \beta = 13$$

Question41

The number of real roots of the equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}, \text{ is:}$$

[31-Jan-2023 Shift 1]

Options:

A. 0

B. 1

C. 3

D. 2

Answer: B

Solution:

$$\sqrt{(x-1)(x-3)} + \sqrt{(x-3)(x+3)}$$

$$= \sqrt{4\left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right)}$$

$$\Rightarrow \sqrt{x-3} = 0 \Rightarrow x = 3 \text{ which is in domain}$$

$$\sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$$

$$2\sqrt{(x-1)(x+3)} = 2x-4$$

$$x^2 + 2x - 3 = x^2 - 4x + 4$$

$$6x = 7$$

$$x = 7/6 \text{ or}$$

Question42

The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0$, $x \in \mathbb{R}$ has:
[31-Jan-2023 Shift 2]

Options:

- A. two solutions and both are negative
- B. no solution
- C. four solutions two of which are negative
- D. two solutions and only one of them is negative

Answer: A

Solution:

Solution:

$$e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0$$

$$\text{Let } e^x = t$$

$$\text{Now, } t^4 + 8t^3 + 13t^2 - 8t + 1 = 0$$

Dividing equation by t^2 ,

$$t^2 + 8t + 13 - \frac{8}{t} + \frac{1}{t^2} = 0$$

$$t^2 + \frac{1}{t^2} + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

$$\left(t - \frac{1}{t}\right)^2 + 2 + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

$$\text{Let } t - \frac{1}{t} = z$$

$$z^2 + 8z + 15 = 0$$

$$(z+3)(z+5) = 0$$

$$z = -3 \text{ or } z = -5$$

$$\text{So, } t - \frac{1}{t} = -3 \text{ or } t - \frac{1}{t} = -5$$

$$t^2 + 3t - 1 = 0 \text{ or } t^2 + 5t - 1 = 0$$

$$t = \frac{-3 \pm \sqrt{13}}{2} \text{ or } t = \frac{-5 \pm \sqrt{29}}{2}$$

as $t = e^x$ so t must be positive,

$$t = \frac{\sqrt{13} - 3}{2} \text{ or } \frac{\sqrt{29} - 5}{2}$$

$$\text{So, } x = \ln\left(\frac{\sqrt{13} - 3}{2}\right) \text{ or } x = \ln\left(\frac{\sqrt{29} - 5}{2}\right)$$

Hence two solution and both are negative.

Question43

If the center and radius of the circle $\left| \frac{z-2}{z-3} \right| = 2$ are respectively (α, β) and γ , then $3(\alpha + \beta + \gamma)$ is equal to
[1-Feb-2023 Shift 1]

Options:

- A. 11
- B. 9
- C. 10
- D. 12

Answer: D

Solution:

Solution:

$$\begin{aligned} \sqrt{(x-2)^2 + y^2} &= 2\sqrt{(x-3)^2 + y^2} \\ &= x^2 + y^2 - 4x + 4 = 4x^2 + 4y^2 - 24x + 36 \\ &= 3x^2 + 3y^2 - 20x + 32 = 0 \\ &= x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0 \\ &= (\alpha, \beta) = \left(\frac{10}{3}, 0 \right) \\ \gamma &= \sqrt{\frac{100}{9} - \frac{32}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3} \\ 3(\alpha, \beta, \gamma) &= 3 \left(\frac{10}{3} + \frac{2}{3} \right) \\ &= 12 \end{aligned}$$

Question44

Let a, b be two real numbers such that $ab < 0$. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and $a+ib$ lies on the circle $|z-1| = |2z|$, then a possible value of $\frac{1+[a]}{4b}$, where $[t]$ is greatest integer function, is :
[1-Feb-2023 Shift 2]

Options:

- A. $-\frac{1}{2}$
- B. -1
- C. 1
- D. $\frac{1}{2}$
- E. 0

Answer: E

Solution:

Solution:

$$\left| \frac{1+ai}{b+i} \right| = 1$$

$$|1+ia| = |b+i|$$

$$a^2 + 1 = b^2 + 1 \Rightarrow a = \pm b \Rightarrow b = -a \quad \text{as } ab < 0$$

$$(a+ib) \text{ lies on } |z-1| = |2z|$$

$$|a+ib-1| = 2|a+ib|$$

$$(a-1)^2 + b^2 = 4(a^2 + b^2)$$

$$(a-1)^2 = a^2 = 4(2a^2)$$

$$1-2a = 6a^2 \Rightarrow 6a^2 + 2a - 1 = 0$$

$$a = \frac{-2 \pm \sqrt{28}}{12} = \frac{-1 \pm \sqrt{7}}{6}$$

$$a = \frac{\sqrt{7}-1}{6} \text{ and } b = \frac{1-\sqrt{7}}{6}$$

$$[a] = 0$$

$$\therefore \frac{1+[a]}{4b} = \frac{6}{4(1-\sqrt{7})} = -\left(\frac{1+\sqrt{7}}{4}\right)$$

$$\text{Similarly when } a = \frac{-1-\sqrt{7}}{6} \text{ and } b = \frac{1+\sqrt{7}}{6} \text{ then } [a] = -1$$

$$\therefore \frac{1+[a]}{4b} = \frac{1-1}{4 \times \frac{1+\sqrt{7}}{6}} = 0$$

Question45

Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then [1-Feb-2023 Shift 2]

Options:

- A. the number of favourable cases of the event $(A \cup B) \cap C$ is 6
- B. A and B are mutually exclusive
- C. The number of favourable cases of the events A, B and C are 15, 6 and 6 respectively
- D. B and C are independent

Answer: A

Solution:

Solution:

A : no. on 1st die < no. on 2nd die

A : no. on 1st die = even & no. of 2nd die = odd

C : no. on 1st die = odd & no. on 2nd die = even

$$n(A) = 5 + 4 + 3 + 2 + 1 = 15$$

$$n(B) = 9$$

$$n(C) = 9$$

$$n((A \cup B) \cap C) = (A \cap C) \cup (B \cap C)$$

$$= (3 + 2 + 1) + 0 = 6.$$

Question46

Let

$$S = \{ x : x \in \mathbb{R} \text{ and } (\sqrt{3} + \sqrt{2})^{x^2 - 4} + (\sqrt{3} - \sqrt{2})^{x^2 - 4} = 10 \}.$$

Then $n(S)$ is equal to

[1-Feb-2023 Shift 1]

Options:

A. 2

B. 4

C. 6

D. 0

Answer: B

Solution:

Solution:

Sol. Let $(\sqrt{3} + \sqrt{2})^{x^2 - 4} = t$

$$t + \frac{1}{t} = 10$$

$$\Rightarrow t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$$

$$\Rightarrow (\sqrt{3} + \sqrt{2})^{x^2 - 4} = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$$

$$\Rightarrow x^2 - 4 = 2, -2 \quad \text{or} \quad x^2 = 6, 2$$

$$\Rightarrow x = \pm\sqrt{2}, \pm\sqrt{6}$$

Question47

Let $a \neq b$ be two-zero real numbers. Then the number of elements in the set $X = \{ z \in \mathbb{C} : \operatorname{Re}(az^2 + bz) = a \text{ and } \operatorname{Re}(bz^2 + az) = b \}$ is equal to :
[6-Apr-2023 shift 2]

Options:

A. 0

B. 2

C. 1

D. 3

Answer: A

Solution:

Solution:

(1) Bonus

$$\because z + \bar{z} = 2 \operatorname{Re}(z) \quad \text{If } z = x + iy$$

$$\Rightarrow z + \bar{z} = 2x$$

$$z^2 + (\bar{z})^2 = 2(x^2 - y^2)$$

$(az^2 + bz) + (a\overline{z}^2 + b\overline{z}) = 2a \dots (1)$
 $(bz^2 + az) + (b\overline{z}^2 + a\overline{z}) = 2b \dots (2)$
 add (1) and (2)
 $(a + b)z^2 + (a + b)z + (a + b)\overline{z}^2 + (a + b)\overline{z} = 2(a + b)$
 $(a + b)[z^2 + z + (\overline{z})^2 + \overline{z}] = 2(a + b)$
 sub. (1) and (2)
 $(a - b)[z^2 - z + \overline{z}^2 - \overline{z}] = 2(a - b) \dots (3)$
 $z^2 + \overline{z}^2 - z - \overline{z} = 2 \dots (4)$
 Case I: If $a + b \neq 0$
 From (3) & (4)
 $2x + 2(x^2 - y^2) = 2 \Rightarrow x^2 - y^2 + x = 1 \dots (5)$
 $2(x^2 - y^2) - 2x = 2 \Rightarrow x^2 - y^2 - x = 1 \dots (6)$
 $(5) - (6)$
 $2x = 0 \Rightarrow x = 0$
 from (5) $y^2 = -1 \Rightarrow$ not possible
 $\therefore \text{Ans} = 0$
 Case II: If $a + b = 0$ then infinite number of solution.
 So, the set X have infinite number of elements.

Question48

For $\alpha, \beta, z \in \mathbb{C}$ and $\lambda > 1$, if $\sqrt{\lambda - 1}$ is the radius of the circle $|z - \alpha|^2 + |z - \beta|^2 = 2\lambda$, then $|\alpha - \beta|$ is equal to _____ :
[6-Apr-2023 shift 2]

Answer: 2

Solution:

$$\begin{aligned}
 |z - z_1|^2 + |z - z_2|^2 &= |z_1 - z_2|^2 \\
 z_1 &= \alpha, z_2 = \beta \\
 |\alpha - \beta|^2 &= 2\lambda \\
 |\alpha - \beta| &= \sqrt{2\lambda} \\
 2r &= \sqrt{2\lambda} \\
 2\sqrt{\lambda - 1} &= \sqrt{2\lambda} \\
 \Rightarrow 4(\lambda - 1) &= 2\lambda \\
 \lambda &= 2 \\
 |\alpha - \beta| &= 2
 \end{aligned}$$

Question49

If for $z = \alpha + i\beta$, $|z + 2| = z + 4(1 + i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation
[8-Apr-2023 shift 1]

Options:

- A. $x^2 + 3x - 4 = 0$
- B. $x^2 + 7x + 12 = 0$

C. $x^2 + x - 12 = 0$

D. $x^2 + 2x - 3 = 0$

Answer: B

Solution:

Solution:

$$|z + 2| = |\alpha + i\beta + 2|$$

$$= \alpha + i\beta + 4 + 4i$$

$$\sqrt{(\alpha + 2)^2 + \beta^2} = (\alpha + 4) + i(\beta + 4) \quad \beta + 4 = 0$$

$$(\alpha + 2)^2 + 16 = (\alpha + 4)^2$$

$$\alpha^2 + 4 + 4\alpha + 16 = \alpha^2 + 16 + 8\alpha$$

$$4 = 4\alpha$$

$$\alpha = 1$$

$$\alpha = 1, \beta = -4$$

$$\alpha + \beta = -3, \alpha\beta = -4$$

$$\text{Sum of roots} = -7$$

$$\text{Product of roots} = 12$$

$$x^2 + 7x + 12 = 0$$

Question 50

Let $A = \left\{ \theta \in (0, 2\pi) : \frac{1 + 2i\sin \theta}{1 - i\sin \theta} \text{ is purely imaginary} \right\}$. Then the sum of the elements in A is.
[8-Apr-2023 shift 2]

Options:

A. π

B. 3π

C. 4π

D. 2π

Answer: C

Solution:

Solution:

$$z = \frac{1 + 2i\sin \theta}{1 - i\sin \theta} \times \frac{1 + i\sin \theta}{1 + i\sin \theta}$$

$$z = \frac{1 - 2\sin^2 \theta + i(3\sin \theta)}{1 + \sin^2 \theta}$$

$$\text{Re}(z) = 0$$

$$\frac{1 - 2\sin^2 \theta}{1 + \sin^2 \theta} = 0$$

$$\sin \theta = \frac{\pm 1}{\sqrt{2}}$$

$$A = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

$$\text{sum} = 4\pi \text{ (Option 3)}$$

Question51

Let the complex number $z = x + iy$ be such that $\frac{2z - 3i}{2z + i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to :
[10-Apr-2023 shift 1]

Options:

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Answer: D

Solution:

Solution:

$$z = x + iy$$

$$\frac{(2z - 3i)}{2z + i} = \text{purely imaginary}$$

$$\text{Means } \operatorname{Re} \left(\frac{2z - 3i}{2z + i} \right) = 0$$

$$\Rightarrow \frac{(2z - 3i)}{(2z + i)} = \frac{2(x + iy) - 3i}{2(x + iy) + i}$$

$$= \frac{2x + 2yi - 3i}{2x + i2y + i}$$

$$= \frac{2x + i(2y - 3)}{2x + i(2y + 1)} \times \frac{2x - i(2y + 1)}{2x - i(2y + 1)}$$

$$\operatorname{Re} \left[\frac{2z - 3i}{2z + i} \right] = \frac{4x^2 + (2y - 3)(2y + 1)}{4x^2 + (2y + 1)^2} = 0$$

$$\Rightarrow 4x^2 + (2y - 3)(2y + 1) = 0$$

$$\Rightarrow 4x^2 + [4y^2 + 2y - 6y - 3] = 0$$

$$\because x + y^2 = 0 \Rightarrow x = -y^2$$

$$\Rightarrow 4(-y^2)^2 + 4y^2 - 4y - 3 = 0$$

$$\Rightarrow 4y^4 + 4y^2 - 4y - 3 = 0$$

$$\Rightarrow y^4 + y^2 - y = \frac{3}{4}$$

Therefore, correct answer is option (4).

Question52

Let $S = \left\{ z = x + iy : \frac{2z - 3i}{4z + 2i} \text{ is a real number} \right\}$. Then which of the following is NOT correct?
[10-Apr-2023 shift 2]

Options:

A. $y \in \left(-\infty, -\frac{1}{2} \right) \cup \left(-\frac{1}{2}, \infty \right)$

B. $(x, y) = \left(0, -\frac{1}{2}\right)$

C. $x = 0$

D. $y + x^2 + y^2 \neq -\frac{1}{4}$

Answer: B

Solution:

Solution:

$$\frac{2z - 3i}{4z + 2i} \in \mathbb{R}$$

$$\frac{2(x + iy) - 3i}{4(x + iy) + 2i} = \frac{2x + (2y - 3)i}{4x + (4y + 2)i} \times \frac{4x - (4y + 2)i}{4x - (4y + 2)i}$$

$$4x(2y - 3) - 2x(4y + 2) = 0$$

$$x = 0 \quad y \neq -\frac{1}{2}$$

Ans. = 2

Question53

Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to :

[11-Apr-2023 shift 1]

Options:

A. $\pi - \tan^{-1} \frac{8}{9}$

B. $-\pi + \tan^{-1} \frac{8}{9}$

C. $\pi - \tan^{-1} \frac{33}{5}$

D. $-\pi + \tan^{-1} \frac{33}{5}$

Answer: A

Solution:

Solution:

$$W_1 = z_1 i = (5 + 4i)i = -4 + 5i \dots (i)$$

$$W_1 = z_2(-i) = (3 + 5i)(-i) = 5 - 3i \dots (2)$$

$$W_1 - W_2 = -9 + 8i$$

$$\text{Principal argument} = \pi - \tan^{-1} \left(\frac{8}{9} \right)$$

Question54

For $a \in \mathbb{C}$, let $A = \{z \in \mathbb{C} : \operatorname{Re}(a + \bar{z}) > \operatorname{Im}(\bar{a} + z)\}$ and $B = \{z \in \mathbb{C} : \operatorname{Re}(a + \bar{z}) < \operatorname{Im}(\bar{a} + z)\}$. The among the two statements:
(S1) : If $\operatorname{Re}(a), \operatorname{Im}(a) > 0$, then the set A contains all the real numbers
(S2) : If $\operatorname{Re}(a), \operatorname{Im}(a) < 0$, then the set B contains all the real numbers,
[11-Apr-2023 shift 2]

Options:

- A. only (S1) is true
- B. both are false
- C. only (S2) is true
- D. both are true

Answer: B

Solution:

Let $a = x_1 + iy_1, z = x + iy$

Now $\operatorname{Re}(a + \bar{z}) > \operatorname{Im}(\bar{a} + z)$

$$\therefore x_1 + x > -y_1 + y$$

$$x_1 = 2, y_1 = 10, x = -12, y = 0$$

Given inequality is not valid for these values.

S1 is false.

Now $\operatorname{Re}(a + \bar{z}) < \operatorname{Im}(\bar{a} + z)$

$$x_1 + x < -y_1 + y$$

$$x_1 = -2, y_1 = -10, x = 12, y = 0$$

Given inequality is not valid for these values. S2 is false.

Question55

Let $S = \left\{ z \in \mathbb{C} - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in \mathbb{R} \right\}$. If $\alpha - \frac{13}{11}i \in S, a \in \mathbb{R} - \{0\}$, then $242\alpha^2$ is equal to _____.
[11-Apr-2023 shift 2]

Answer: 1680

Solution:

Solution:

$$\left(\frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \right) \in \mathbb{R}$$

$$\Rightarrow 1 + \frac{(11iz - 13)}{(z^2 - 3iz - 2)} \in \mathbb{R}$$

$$\text{Put } Z = \alpha - \frac{13}{11}i$$

$\Rightarrow (z^2 - 3iz - 2)$ is imaginary

$$\text{Put } z = x + iy$$

$$\Rightarrow (x^2 - y^2 + 2xyi - 3ix + 3y - 2) \in \text{Imaginary}$$

$$\Rightarrow \text{Re}(x^2 - y^2 + 3y - 2 + (2xy - 3x)i) = 0$$

$$\Rightarrow x^2 - y^2 + 3y - 2 = 0$$

$$x^2 = y^2 - 3y + 2$$

$$x^2 = (y - 1)(y - 2) \therefore z = \alpha - \frac{13}{11}i$$

$$\text{Put } x = \alpha, y = \frac{-13}{11}$$

$$\alpha^2 = \left(\frac{-13}{11} - 11 \right) \left(\frac{-13}{11} - 2 \right)$$

$$\alpha^2 = \frac{(24 \times 35)}{121}$$

$$242\alpha^2 = 48 \times 35 = 1680$$

Question56

Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1 + 3i)$ and radius $r = 1$. Let $z_1 = 1 + i$ and the complex number z_2 be outside the circle C such that $|z_1 - z_0||z_2 - z_0| = 1$. If $z_0 \cdot z_1$ and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to
[12-Apr-2023 shift 1]

Options:

A. $\frac{7}{2}$

B. $\frac{13}{2}$

C. $\frac{5}{2}$

D. $\frac{3}{2}$

Answer: C

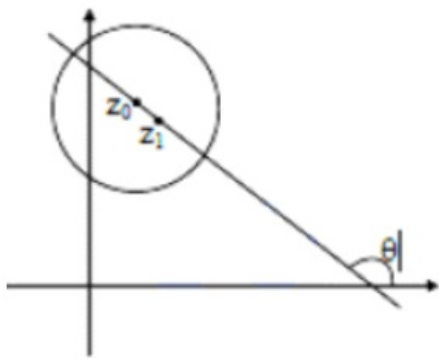
Solution:

Solution:

$$|z_1 - z_0| = \left| \frac{1-i}{2} \right| = \frac{1}{2}$$

$$\Rightarrow |z_2 - z_0| = \sqrt{2} : \text{ centre } \left(\frac{1}{2}, \frac{3}{2} \right)$$

$$z_0 \left(\frac{1}{2}, \frac{3}{2} \right) \text{ and } z_1(1, 1)$$



$$\tan \theta = -1 \Rightarrow \theta = 135^\circ$$

$$z_2 \left(\frac{1}{2} + \sqrt{2} \cos 135^\circ, \frac{3}{2} + \sqrt{2} \sin 135^\circ \right)$$

or

$$\left(\frac{1}{2} - \sqrt{2} \cos 135^\circ, \frac{3}{2} - \sqrt{2} \sin 135^\circ \right)$$

$$\Rightarrow z_2 \left(-\frac{1}{2}, \frac{5}{2} \right) \text{ or } z_2 \left(\frac{3}{2}, \frac{1}{2} \right)$$

$$\Rightarrow |z_3|^2 = \frac{26}{4}, \frac{5}{2}$$

$$\Rightarrow |z_2|_{\min}^2 = \frac{5}{2}$$

Question57

Let $S = \{z \in \mathbb{C} : \bar{z} = i(z^2 + \operatorname{Re}(\bar{z}))\}$. Then $\sum_{z \in S} |z|^2$ is equal to
[13-Apr-2023 shift 2]

Options:

- A. 4
- B. $\frac{7}{2}$
- C. 3
- D. $\frac{5}{2}$

Answer: A

Solution:

$$\text{Let } z = x + iy$$

$$\bar{z} = i(z^2 + \operatorname{Re}(\bar{z}))$$

$$\Rightarrow x - iy = i(x^2 - y^2 + 2ixy + x)$$

$$\Rightarrow x - iy = -2xy + i(x^2 - y^2 + x)$$

$$x + 2xy = 0 \text{ and } x^2 - y^2 + x + y = 0$$

$$x(1 + 2y) = 0 \text{ and } x^2 - y^2 + x + y = 0$$

If $x = 0$ then $-y^2 + y = 0$

$$\Rightarrow y = 1, 0$$

If $y = -\frac{1}{2}$ then $x^2 - \frac{1}{4} + x - \frac{1}{2} = 0$

$$\Rightarrow x = -\frac{3}{2}, \frac{1}{2}$$

$$= \left\{ 0 + i0, 0 + i, -\frac{3}{2} - \frac{1}{2}i, \frac{1}{2} - \frac{1}{2}i \right\}$$

$$\sum_{z \in S} |z|^2 = 0^2 + 1^2 + \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 4$$

Question58

If the set $\left\{ \operatorname{Re} \left(\frac{z - \bar{z} + z\bar{z}}{2 - 3z + 5\bar{z}} \right) : z \in \mathbb{C}, \operatorname{Re}(z) = 3 \right\}$ is equal to the interval $(\alpha, \beta]$, then $24(\beta - \alpha)$ is equal to
[15-Apr-2023 shift 1]

Options:

A. 36

B. 27

C. 30

D. 42

Answer: C

Solution:

Solution:

$$\text{Let } z_1 = \left(\frac{z - \bar{z} + z\bar{z}}{2 - 3z + 5\bar{z}} \right)$$

$$\text{Let } z = 3 + iy$$

$$\bar{z} = 3 - iy$$

$$z_1 = \frac{2iy + (9 + y^2)}{2 - 3(3 + iy) + 5(3 - iy)}$$

$$= \frac{9 + y^2 + i(2y)}{8 - 8iy}$$

$$= \frac{(9 + y^2) + i(2y)}{8(1 - iy)}$$

$$\operatorname{Re}(z_1) = \frac{(9 + y^2) - 2y^2}{8(1 + y^2)}$$

$$= \frac{9 - y^2}{8(1 + y^2)}$$

$$= \frac{1}{8} \left[\frac{10 - (1 + y^2)}{(1 + y^2)} \right]$$

$$= \frac{1}{8} \left[\frac{10}{(1 + y^2)} - 1 \right]$$

$$1 + y^2 \in [1, \infty]$$

$$\frac{1}{1 + y^2} \in (0, 1]$$

$$\frac{10}{1 + y^2} \in (0, 10]$$

$$\frac{10}{1 + y^2} - 1 \in (-1, 9]$$

$$\operatorname{Re}(z_1) \in \left(-\frac{1}{8}, \frac{9}{8} \right]$$

$$\alpha = -\frac{1}{8}, \beta = \frac{9}{8}$$

$$24(\beta - \alpha) = 24 \left(\frac{9}{8} + \frac{1}{8} \right) = 30$$

Question59

The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is :
[6-Apr-2023 shift 1]

Options:

A. $11 - \sqrt{3}$

B. $9 - \sqrt{3}$

C. $9 + \sqrt{3}$

D. $11 + \sqrt{3}$

Answer: C

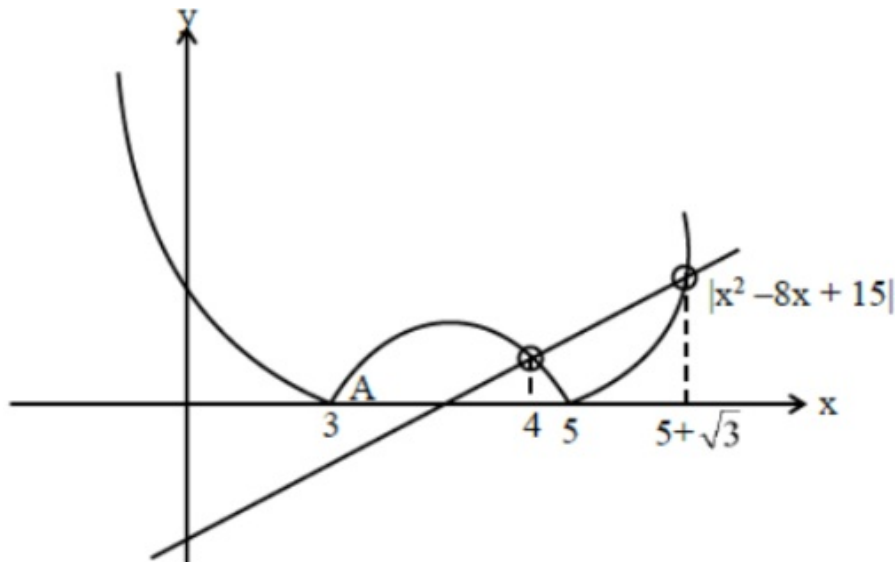
Solution:

Solution:

$$|x^2 - 8x + 15| = 2x - 7$$
$$x^2 - 8x + 15 = 2x - 7 \text{ \& } x^2 - 8x + 15 = 7 - 2x$$
$$x^2 - 10x + 22 = 0 \text{ \& } x^2 - 6x + 8 = 0$$

$$x_1 = 5 + \sqrt{3} \quad x_2 = 5 - \sqrt{3} \text{ (reject)} \quad x_3 = 4 \quad x_4 = 2 \text{ (reject)}$$

$$\text{Sum of roots is } = 5 + \sqrt{3} + 4 = 9 + \sqrt{3}$$



Question60

Let α, β, γ , be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to
[8-Apr-2023 shift 1]

Options:

A. $\frac{155}{8}$

B. 21

C. 19

D. $\frac{169}{8}$

Answer: C

Solution:

Solution:

$$\begin{array}{c} \nearrow \alpha \\ x^3 + bx + c = 0 \longrightarrow \beta \\ \searrow \gamma \end{array}$$

$$\beta\gamma = 1$$

$$\alpha = -1$$

$$\text{Put } \alpha = -1$$

$$-1 - b + c = 0$$

$$c - b = 1$$

also

$$\alpha \cdot \beta \cdot \gamma = -c$$

$$-1 = -c \Rightarrow c = 1$$

$$\therefore b = 0$$

$$x^3 + 1 = 0$$

$$\alpha = -1, \beta = -\omega, \gamma = -\omega^2$$

$$\therefore b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$$

$$0 + 2 + 3 + 6 + 8 = 19$$

Question61

Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x + 2| - 4 = 0$ respectively, where $[x]$ denotes the greatest integer less than or equal to x . Then $m^2 + mn + n^2$ is equal to
[8-Apr-2023 shift 2]

Answer: 9

Solution:

$$\begin{aligned}
 x^2 - 12x + [x] + 31 &= 0 \\
 \{x\} &= x^2 - 11x + 31 \\
 0 \leq x^2 - 11x + 31 &< 1 \\
 x^2 - 11x + 30n &< 0 \\
 x &\in (5, 6) \\
 \text{so } [x] &= 5 \\
 x^2 - 12x + 5 + 31 &= 0 \\
 x^2 - 12x + 36 &= 0 \\
 x = 6 \text{ but } x &\in (5, 6) \\
 \text{so } x &\in \varnothing \\
 m &= 0
 \end{aligned}$$

Now

$$\begin{array}{cc}
 x^2 - 5|x+2| - 4 = 0 & \\
 \swarrow & \searrow \\
 x \geq -2 & x < -2 \\
 x^2 - 5x - 14 = 0 & x^2 + 5x + 6 = 0 \\
 (x-7)(x+2) = 0 & (x+3)(x+2) = 0 \\
 x = 7, -2 & x = -3, -2
 \end{array}$$

$$\begin{aligned}
 x &= \{7, -2, -3\} \\
 n &= 3 \\
 m^2 + mn + n^2 &= n^2 = 9
 \end{aligned}$$

Question62

If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to _____.

[11-Apr-2023 shift 1]

Answer: 51

Solution:

By newton's theorem

$$\begin{aligned}
 S_{n+2} - 7S_{n+1} - S_n &= 0 \\
 S_{21} - 7S_{20} - S_{19} &= 0 \\
 S_{20} - 7S_{19} - S_{18} &= 0 \\
 S_{19} - 7S_{18} - S_{17} &= 0 \\
 \frac{S_{21} + S_{17}}{S_{19}} &= \frac{S_{21} + (S_{19} - 7S_{18})}{S_{19}} \\
 &= \frac{50S_{19} + (S_{21} - 7S_{20})}{S_{19}} \\
 &= 51 \cdot \frac{S_{19}}{S_{19}} = 51
 \end{aligned}$$

Question63

The number of points where the curve

$f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in \mathbb{R}$ cuts x-axis, is equal to _____
[11-Apr-2023 shift 2]

Answer: 2

Solution:

$$\begin{aligned}\text{Let } e^{2x} &= t \\ \Rightarrow t^4 - t^3 - 3t^2 - t + 1 &= 0 \\ \Rightarrow t_2 + \frac{1}{t_2} - \left(t + \frac{1}{t}\right) - 3 &= 0 \\ \Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 5 &= 0 \\ \Rightarrow t + \frac{1}{t} &= \frac{1 + \sqrt{21}}{2}\end{aligned}$$

Two real values of t.

Question64

Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$ is equal to
[12-Apr-2023 shift 1]

Options:

- A. 9
- B. 729
- C. 72
- D. 81

Answer: D

Solution:

Solution:

$$\begin{aligned}\alpha, \beta &= \frac{-\sqrt{6} \pm \sqrt{6-12}}{2} = \frac{-\sqrt{6} \pm \sqrt{6}i}{2} \\ &= \sqrt[3]{3e^{\pm \frac{3\pi i}{4}}}\end{aligned}$$

Required expression

$$\begin{aligned}&\frac{(\sqrt{3})^{23} \left(2 \cos \frac{69\pi}{4}\right) + (\sqrt{3})^{14} \left(2 \cos \frac{42\pi}{4}\right)}{(\sqrt{3})^{15} \left(2 \cos \frac{45\pi}{4}\right) + (\sqrt{3})^{10} \left(2 \cos \frac{30\pi}{4}\right)} \\ &(\sqrt{3})^8 = 81\end{aligned}$$

Question65

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$, Then $\alpha^{14} + \beta^{14}$ is equal to

[13-Apr-2023 shift 2]

Options:

A. $-128\sqrt{2}$

B. $-64\sqrt{2}$

C. -128

D. -64

Answer: C

Solution:

Solution:

$$x^2 - \sqrt{2}x + 2 = 0$$

$$x = \frac{\sqrt{2} \pm \sqrt{-6}}{2}$$

$$= \sqrt{2} \left(\frac{1 \pm i\sqrt{3}}{2} \right)$$

$$= -\sqrt{2}\omega, -\sqrt{2}\omega^2$$

$$\Rightarrow \alpha = -\sqrt{2}\omega, \beta = -\sqrt{2}\omega^2$$

$$\alpha^{14} + \beta^{14} = 2^7(\omega^{14} + \omega^{28}) = 2^7(\omega^2 + \omega) = -128$$

Question66

The number of real roots of the equation $x |x| - 5 |x + 2| + 6 = 0$, is [15-Apr-2023 shift 1]

Options:

A. 5

B. 6

C. 4

D. 3

Answer: D

Solution:

Solution:

$$x |x| - 5 |x + 2| + 6 = 0$$

$$C - 1 : -x \in [0, \infty]$$

$$x^2 - 5x - 4 = 0$$

$$x = \frac{5 \pm \sqrt{25 + 16}}{2} = \frac{5 + \sqrt{41}}{2}$$

$$x = \frac{5 \pm \sqrt{41}}{2}$$

$$C - 2 : - : -x \in [-2, 0)$$

$$-x^2 - 5x - 4 = 0$$

$$x^2 + 5x + 4 = 0$$

$$x = -1, -4$$

$$x = -1$$

$$C - 3 : x \in [-\infty, -2)$$

$$-x^2 + 5x + 16 = 0$$

$$x^2 - 5x - 16 = 0$$

$$x = \frac{5 \pm \sqrt{25 + 64}}{2}$$

$$x = \frac{5 \pm \sqrt{89}}{2}$$

$$x = \frac{5 - \sqrt{89}}{2}$$

Question67

Let the point $(p, p + 1)$ lie inside the region

$E = \{ (x, y) : 3 - x \leq y \leq \sqrt{9 - x^2}, 0 \leq x \leq 3 \}$. If the set of all values of p is the interval (a, b) , then $b^2 + b - a^2$ is equal to _____.

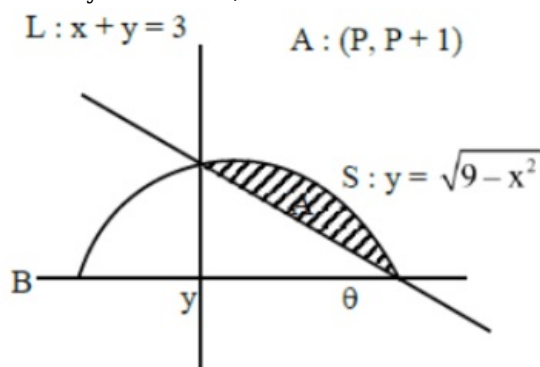
[6-Apr-2023 shift 1]

Answer: 3

Solution:

Solution:

$$3 - x \leq y \leq \sqrt{9 - x^2}; 0 \leq x \leq 3$$



$$L(A) > 0 \Rightarrow P + P + 1 - 3 > 0 \Rightarrow P > 1 \dots (1)$$

$$S(A) < 0 \Rightarrow P + 1 - \sqrt{9 - P^2} < 0$$

$$\Rightarrow P + 1 < \sqrt{9 - P^2}$$

$$\Rightarrow P + 2P + 1 < 9 - P^2$$

$$\Rightarrow 2P^2 + 2P - 8 < 0$$

$$\Rightarrow P^2 + P - 4 < 0$$

$$\Rightarrow P \in \left(\frac{-1 - \sqrt{17}}{2}, \frac{-1 + \sqrt{17}}{2} \right) \dots (2)$$

$$(1) \cap (2) P \in \left(1, \frac{\sqrt{17} - 1}{2} \right) \equiv (a, b)$$

$$b^2 + b - a^2 = 4 - 1 = 3$$

Question68

Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$. Then $6a + 5bc$ is equal to _____.
[10-Apr-2023 shift 1]

Answer: 8

Solution:

Solution:

$$\begin{aligned}(2a)^{\ln a} &= (bc)^{\ln b} \quad 2a > 0, bc > 0 \\ \ln a(\ln 2 + \ln a) &= \ln b(\ln b + \ln c) \\ \ln 2 \cdot \ln b &= \ln c \cdot \ln a \\ \ln 2 = \alpha, \ln a = x, \ln b = y, \ln c = z \\ \alpha y &= xz \\ x(\alpha + x) &= y(y + z) \\ \alpha &= \frac{xz}{y} \\ x\left(\frac{xz}{y} + x\right) &= y(y + z) \\ x^2(z + y) &= y^2(y + z) \\ y + z = 0 \text{ or } x^2 &= y^2 \Rightarrow x = -y \\ bc = 1 \text{ or } ab &= 1 \\ bc = 1 \text{ or } ab &= 1\end{aligned}$$

$$(1) \text{ if } bc = 1 \Rightarrow (2a)^{\ln a} = 1 \begin{cases} a = 1 \\ a = 1/2 \end{cases}$$

$$(a, b, c) = \left(\frac{1}{2}, \lambda, \frac{1}{\lambda}\right), \lambda \neq 1, 2, \frac{1}{2}$$

then

$$6a + 5bc = 3 + 5 = 8$$

$$(II) (a, b, c) = \left(\lambda, \frac{1}{\lambda}, \frac{1}{2}\right), \lambda \neq 1, 2, \frac{1}{2}$$

In this situation infinite answer are possible

So, Bonus.

Question69

The number of integral solutions x of $\log_{\left(x + \frac{7}{2}\right)} \left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is :

[11-Apr-2023 shift 1]

Options:

A. 5

B. 7

C. 8

D. 6

Answer: D

Solution:

Solution:

$$\log_{x+\frac{7}{2}} \left(\frac{x-7}{2x-3} \right)^2 \geq 0$$

$$\text{Feasible region: } x + \frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$$

$$\text{And } x + \frac{7}{2} \neq 1 \Rightarrow x \neq \frac{-5}{2}$$

$$\text{Taking intersection: } x \in \left(-\frac{7}{2}, \infty \right) - \left\{ -\frac{5}{2}, \frac{3}{2}, 7 \right\}$$

Now $\log_a b \geq 0$ if $a > 1$ and $b \geq 1$

$a \in (0, 1)$ and $b \in (0, 1)$

$$C - I; x + \frac{7}{2} > 1 \text{ and } \left(\frac{x-7}{2x-3} \right)^2 \geq 1$$

$$x > -\frac{5}{2}; (2x-3)^2 - (x-7)^2 \leq 0$$

$$(2x-3+x-7)(2x-3-x+7) \leq 0$$

$$(3x-10)(x+4) \leq 0$$

$$x \in \left[-4, \frac{10}{3} \right]$$

$$\text{Intersection: } x \in \left(-\frac{5}{2}, \frac{10}{3} \right]$$

$$C - II: x + \frac{7}{2} \in (0, 1) \text{ and } \left(\frac{x-7}{2x-3} \right)^2 \in (0, 1)$$

$$0 < x + \frac{7}{2} < 1; \left(\frac{x-7}{2x-3} \right)^2 < 1$$

$$-\frac{7}{2} < x < \frac{-5}{2}; (x-7)^2 < (2x-3)^2$$

$$x \in (-\infty, -4) \cup \left(\frac{10}{3}, \infty \right)$$

No common values of x .

Hence intersection with feasible region

$$\text{We get } x \in \left(-\frac{5}{2}, \frac{10}{3} \right] - \left\{ \frac{3}{2} \right\}$$

Integral value of x are $\{-2, -1, 0, 1, 2, 3\}$

No. of integral values = 6

Question70

If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to:

[24-Jun-2022-Shift-1]

Options:

A. 18

B. 24

C. 36

D. 96

Answer: B

Solution:

$$3x^2 + \lambda x - 1 = 0$$

Given, two roots are α and β .

$$\therefore \text{Sum of roots} = \alpha + \beta = \frac{-\lambda}{3}$$

$$\text{And product of roots} = \alpha\beta = \frac{-1}{3}$$

Given that,

Sum of square of reciprocal of roots α and β is 15 .

$$\therefore \frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$$

$$\Rightarrow \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} = 15$$

$$\Rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = 15$$

$$\Rightarrow \frac{\frac{\lambda^2}{9} + 2 \times \frac{1}{3}}{\frac{1}{9}} = 15$$

$$\Rightarrow \frac{\frac{\lambda^2 + 6}{9}}{\frac{1}{9}} = 15$$

$$\Rightarrow \lambda^2 + 6 = 15$$

$$\Rightarrow \lambda^2 = 9$$

$$\text{Now, } 6(\alpha^3 + \beta^3)^2$$

$$= 6\{(\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)\}^2$$

$$= 6(\alpha + \beta)^2[(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta]^2$$

$$= 6\left(\frac{-\lambda}{3}\right)^2\left[\left(\frac{-\lambda}{3}\right)^2 - 3 \cdot \frac{-1}{3}\right]^2$$

$$= 6 \times \frac{\lambda^2}{9} \times \left[\frac{\lambda^2}{9} + 1\right]$$

$$= 6 \times \frac{9}{9} \times \left[\frac{9}{9} + 1\right]^2$$

$$= 6 \times (2)^2$$

$$= 6 \times 4 = 24$$

Question71

Let $S = \{ z \in \mathbb{C} : |z - 3| \leq 1 \text{ and } z(4 + 3i) + \bar{z}(4 - 3i) \leq 24 \}$. If $\alpha + i\beta$ is the point in S which is closest to $4i$, then $25(\alpha + \beta)$ is equal to____
[24-Jun-2022-Shift-2]

Answer: 80

Solution:

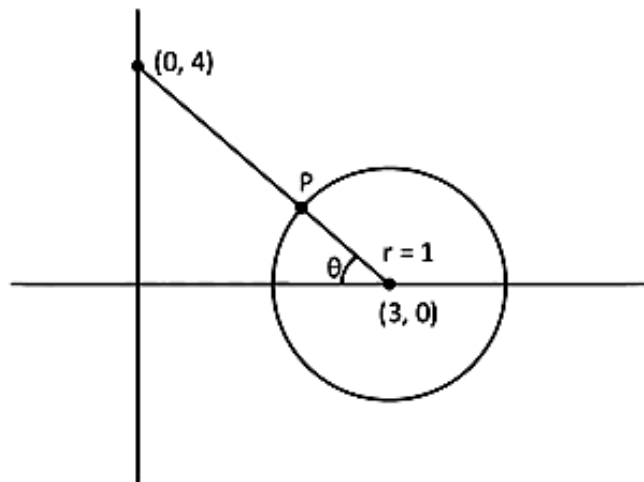
Here $|z - 3| \leq 1$

$$\Rightarrow (x - 3)^2 + y^2 \leq 1$$

and $z(4 + 3i) + \bar{z}(4 - 3i) \leq 24$

$$\Rightarrow 4x - 3y \leq 12$$

$$\tan \theta = \frac{4}{3}$$



$\therefore$ Coordinate of $P = (3 - \cos \theta, \sin \theta)$

$$= \left(3 - \frac{3}{5}, \frac{4}{5} \right)$$

$$\therefore \alpha + i\beta = \frac{12}{5} + \frac{4}{5}i$$

$$\therefore 25(\alpha + \beta) = 80$$

Question72

Let a circle C in complex plane pass through the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z (\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3 , then $\arg(z)$ is equal to:

[25-Jun-2022-Shift-1]

Options:

A. $\tan^{-1} \left(\frac{2}{\sqrt{5}} \right) - \pi$

B. $\tan^{-1} \left(\frac{24}{7} \right) - \pi$

C. $\tan^{-1}(3) - \pi$

D. $\tan^{-1} \left(\frac{3}{4} \right) - \pi$

Answer: B

Solution:

Solution:

$$z_1 = 3 + 4i, z_2 = 4 + 3i \text{ and } z_3 = 5i$$

$$\text{Clearly, } C \equiv x^2 + y^2 = 25$$

Let $z(x, y)$

$$\Rightarrow \left(\frac{y-4}{x-3} \right) \left(\frac{2}{-4} \right) = -1$$

$$\Rightarrow y = 2x - 2 \equiv L$$

$\therefore z$ is intersection of C & L

$$\Rightarrow z \equiv \left(\frac{-7}{5}, \frac{-24}{5} \right)$$

$$\therefore \text{Arg}(z) = -\pi + \tan^{-1} \left(\frac{24}{7} \right)$$

Question73

Let z_1 and z_2 be two complex numbers such that $\overline{z_1} = iz_2$ and

$\arg\left(\frac{z_1}{z_2}\right) = \pi$. Then

[25-Jun-2022-Shift-2]

Options:

A. $\arg z_2 = \frac{\pi}{4}$

B. $\arg z_2 = -\frac{3\pi}{4}$

C. $\arg z_1 = \frac{\pi}{4}$

D. $\arg z_1 = -\frac{3\pi}{4}$

Answer: C

Solution:

Solution:

$$\because \frac{z_1}{z_2} = -i \Rightarrow z_1 = -iz_2$$

$$\Rightarrow \arg(z_1) = -\frac{\pi}{2} + \arg(z_2) \dots\dots (i)$$

$$\text{Also } \arg(z_1) - \arg(\bar{z}_2) = \pi$$

$$\Rightarrow \arg(z_1) + \arg(z_2) = \pi \dots\dots (ii)$$

$$\text{From (i) and (ii), we get } \arg(z_1) = \frac{\pi}{4} \text{ and } \arg(z_2) = \frac{3\pi}{4}$$

Question74

Let $A = \left\{ z \in \mathbb{C} : \left| \frac{z+1}{z-1} \right| < 1 \right\}$ and $B = \left\{ z \in \mathbb{C} : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3} \right\}$. Then

$A \cap B$ is :

[26-Jun-2022-Shift-1]

Options:

A. a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

B. a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

C. an empty

D. a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

Answer: B

Solution:

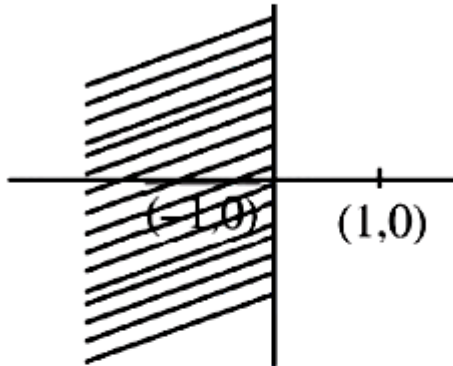
Set A

$$\Rightarrow \left| \frac{z+1}{z-1} \right| < 1$$

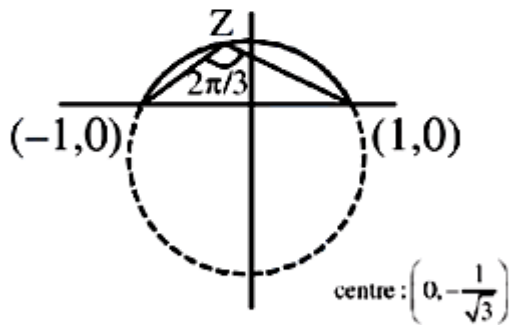
$$\Rightarrow |z+1| < |z-1|$$

$$\Rightarrow (x+1)^2 + y^2 < (x-1)^2 + y^2$$

$$\Rightarrow x < 0$$



Set B



$$\Rightarrow \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow x^2 + y^2 + \frac{2y}{\sqrt{3}} - 1 = 0$$

$$A \cap B$$

$$\Rightarrow \text{Centre} \left(0, -\frac{1}{\sqrt{3}}\right)$$

Question 75

If $z^2 + z + 1 = 0$, $z \in \mathbb{C}$, then

$$\left| \sum_{n=1}^{15} \left(z^n + (-1)^n \frac{1}{z^n} \right)^2 \right| \text{ is equal to } \underline{\hspace{2cm}}$$

[26-Jun-2022-Shift-2]

Answer: 2

Solution:

$$\because z^2 + z + 1 = 0$$

$$\Rightarrow \omega \text{ or } \omega^2$$

$$\begin{aligned} &\because \left| \sum_{n=1}^{15} \left(z^n + (-1)^n \frac{1}{z^n} \right)^2 \right| \\ &= \left| \sum_{n=1}^{15} z^{2n} + \sum_{n=1}^{15} z^{-2n} + 2 \cdot \sum_{n=1}^{15} (-1)^n \right| \\ &= |0 + 0 - 2| \\ &= 2 \end{aligned}$$

Question76

The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :
[27-Jun-2022-Shift-1]

Options:

- A. $\frac{3\sqrt{3}}{4}$
- B. $\frac{3\sqrt{3}}{2}$
- C. $\frac{3}{2}$
- D. $\frac{3}{4}$

Answer: A

Solution:

$$\begin{aligned} \bar{z} &= iz^2 \\ \text{Let } z &= x + iy \\ x - iy &= i(x^2 - y^2 + 2ixy) \\ x - iy &= i(x^2 - y^2) - 2xy \\ \therefore x &= -2yx \text{ or } x^2 - y^2 = -y \\ x &= 0 \text{ or } y = -\frac{1}{2} \\ \text{Case - I} \\ x &= 0 \\ -y^2 &= -y \\ y &= 0, 1 \\ \text{Case - II} \\ y &= -\frac{1}{2} \\ \Rightarrow x^2 - \frac{1}{4} &= \frac{1}{2} \Rightarrow x = \pm \frac{\sqrt{3}}{2} \\ x &= \left\{ 0, i, \frac{\sqrt{3}}{2} - \frac{i}{2}, \frac{-\sqrt{3}}{2} - \frac{i}{2} \right\} \end{aligned}$$

$$\begin{aligned} \text{Area of polygon} &= \frac{1}{2} \begin{vmatrix} 0 & 1 & 1 \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} & 1 \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 1 \end{vmatrix} \\ &= \frac{1}{2} \left| -\sqrt{3} - \frac{\sqrt{3}}{2} \right| = \frac{3\sqrt{3}}{4} \end{aligned}$$

Question77

The number of points of intersection of $|z - (4 + 3i)| = 2$ and $|z| + |z - 4| = 6, z \in \mathbb{C}$, is
[27-Jun-2022-Shift-2]

Options:

- A. 0
- B. 1
- C. 2
- D. 3

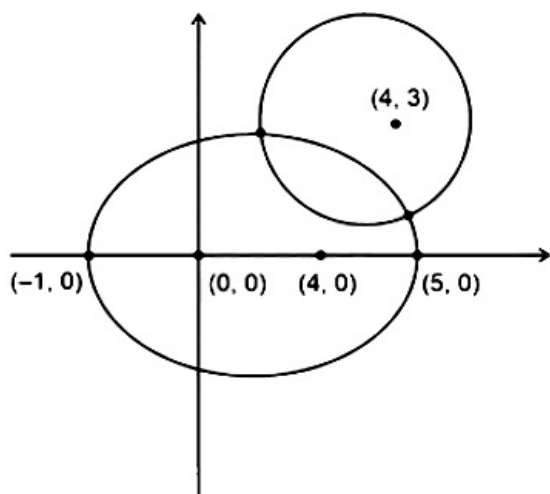
Answer: C

Solution:

Solution:

$C_1: |z - (4 + 3i)| = 2$ and $C_2: |z| + |z - 4| = 6, z \in \mathbb{C}$

C_1 represents a circle with centre $(4, 3)$ and radius 2 and C_2 represents an ellipse with foci at $(0, 0)$ and $(4, 0)$ and length of major axis = 6, and length of semi-major axis $2\sqrt{5}$ and $(4, 2)$ lies inside the both C_1 and C_2 and $(4, 3)$ lies outside the C_2



$\therefore$ number of intersection points = 2

Question78

The number of elements in the set $\{z = a + ib \in \mathbb{C} : a, b \in \mathbb{Z} \text{ and } 1 < |z - 3 + 2i| < 4\}$ is__

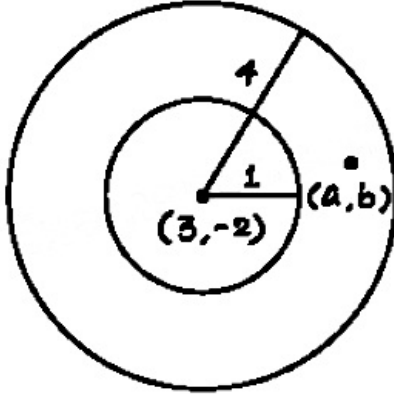
[28-Jun-2022-Shift-1]

Answer: 40

Solution:

Solution:

$$1 < |Z - 3 + 2i| < 4$$



$$1 < (a - 3)^2 + (b + 2)^2 < 16$$

$$(0, \pm 2), (\pm 2, 0), (\pm 1, \pm 2), (\pm 2, \pm 1)$$

$$(\pm 2, \pm 3), (3 \pm, \pm 2), (\pm 1, \pm 1), (2 \pm, \pm 2)$$

$$(\pm 3, 0), (0, \pm 3), (\pm 3 \pm 1), (\pm 1, \pm 3)$$

Total 40 points

Question 79

Sum of squares of modulus of all the complex numbers z satisfying

$$\bar{z} = iz^2 + z^2 - z \text{ is equal to } \underline{\hspace{2cm}}$$

[28-Jun-2022-Shift-2]

Answer: 2

Solution:

$$\text{Let } z = x + iy$$

$$\text{So } 2x = (1 + i)(x^2 - y^2 + 2xyi)$$

$$\Rightarrow 2x = x^2 - y^2 - 2xy$$

(i) and

$$x^2 - y^2 + 2xy = 0$$

From (i) and (ii) we get

$$x = 0 \text{ or } y = -\frac{1}{2}$$

When $x = 0$ we get $y = 0$

$$\text{When } y = -\frac{1}{2} \text{ we get } x^2 - x - \frac{1}{4} = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{2}}{2}$$

So there will be total 3 possible values of z , which are 0 , $\left(\frac{-1+\sqrt{2}}{2}\right) - \frac{1}{2}i$ and $\left(\frac{-1-\sqrt{2}}{2}\right) - \frac{1}{2}i$

Sum of squares of modulus

$$= 0 + \left(\frac{\sqrt{2}-1}{2}\right)^2 + \frac{1}{4} + \left(\frac{\sqrt{2}+1}{2}\right)^2 = +\frac{1}{4}$$
$$= 2$$

Question80

Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to:
[29-Jun-2022-Shift-1]

Options:

- A. 50
- B. 250
- C. 1250
- D. 1500

Answer: A

Solution:

Solution:

Given equation,

$$x^2 + (2i - 1) = 0$$

$$\Rightarrow x^2 = 1 - 2i$$

Let α and β are the two roots of the equation.

As, we know roots of a equation satisfy the equation so

$$\alpha^2 = 1 - 2i$$

$$\text{and } \beta^2 = 1 - 2i$$

$$\therefore \alpha^2 = \beta^2 = 1 - 2i$$

$$\therefore |\alpha^2| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

$$\text{Now, } |\alpha^8 + \beta^8|$$

$$|\alpha^8 + \alpha^8|$$

$$= 2 |\alpha^8|$$

$$= 2 |\alpha^2|^4$$

$$= 2(\sqrt{5})^4$$

$$= 2 \times 25$$

$$= 50$$

Question81

Let $S = \{z \in \mathbb{C} : |z - 2| \leq 1, z(1 + i) + \bar{z}(1 - i) \leq 2\}$. Let $|z - 4i|$ attains minimum and maximum values, respectively, at $z_1 \in S$ and $z_2 \in S$. If $5(z_1|^2 + z_2|^2) = \alpha + \beta\sqrt{5}$, where α and β are integers, then the value of

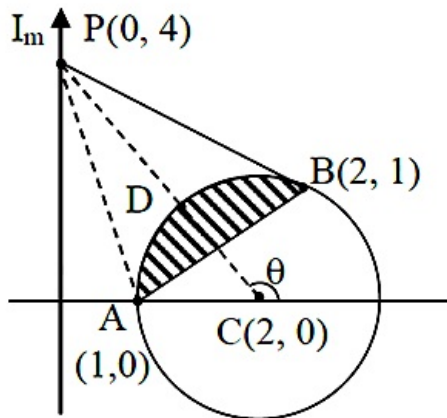
$\alpha + \beta$ is equal to _____
[29-Jun-2022-Shift-1]

Answer: 26

Solution:

Solution:

$$|z - 2| \leq 1$$



$$(x - 2)^2 + y^2 \leq 1$$

&

$$z(1 + i) + \bar{z}(1 - i) \leq 2$$

Put $z = x + iy$

$$\therefore x - y \leq 1 \dots \dots (2)$$

$$PA = \sqrt{17}, PB = \sqrt{13}$$

Maximum is PA & Minimum is PD

Let $D(2 + \cos \theta, 0 + \sin \theta)$

$$\therefore m_{cp} = \tan \theta = -2$$

$$\cos \theta = -\frac{1}{\sqrt{5}}, \sin \theta = \frac{2}{\sqrt{5}}$$

$$\therefore D\left(2 - \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow z_1 = \left(2 - \frac{1}{\sqrt{5}}\right) + \frac{2i}{\sqrt{5}}$$

$$\left|z_1\right| = \frac{25 - 4\sqrt{5}}{5} z_2 = 1$$

$$\therefore \left|z_2\right|^2 = 1$$

$$\therefore 5\left(\left|z_1\right|^2 + \left|z_2\right|^2\right) = 30 - 4\sqrt{5}$$

$$\therefore \alpha = 30$$

$$\beta = -4$$

$$\therefore \alpha + \beta = 26$$

Question 82

Let $\arg(z)$ represent the principal argument of the complex number z .

Then, $|z| = 3$ and $\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$ intersect

[29-Jun-2022-Shift-2]

Options:

A. exactly at one point.

B. exactly at two points.

C. nowhere.

D. at infinitely many points.

Answer: C

Solution:

Solution:

Let $z = x + iy$

$$\therefore |z| = \sqrt{x^2 + y^2}$$

Given, $|z| = 3$

$$\therefore \sqrt{x^2 + y^2} = 3$$

$$\Rightarrow x^2 + y^2 = 9 = 3^2$$

This represents a circle with center at $(0, 0)$ and radius $= 3$

Now, given

$$\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$$

$$\Rightarrow \arg(x + iy - 1) - \arg(x + iy + 1) = \frac{\pi}{4}$$

$$\Rightarrow \arg(x - 1 + iy) - \arg(x + 1 + iy) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1 + \frac{y}{x-1} \times \frac{y}{x+1}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{xy + y - xy + y}{x^2 - 1}}{\frac{x^2 - 1 + y^2}{x^2 - 1}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{xy + y - xy + y}{x^2 - 1 + y^2}\right) = \frac{\pi}{4}$$

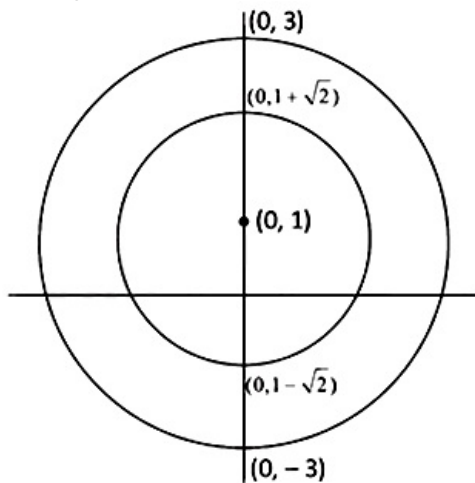
$$\Rightarrow \frac{2y}{x^2 - 1 + y^2} = \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow 2y = x^2 + y^2 - 1$$

$$\Rightarrow x^2 + y^2 - 2y - 1 = 0$$

$$\Rightarrow x^2 + (y - 1)^2 = (\sqrt{2})^2$$

This represents a circle with center at $(0, 1)$ and radius $\sqrt{2}$.



From diagram you can see both the circles do not cut anywhere.

The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$ is

[24-Jun-2022-Shift-2]

Options:

A. $\log_e 3$

B. $-\log_e 3$

C. $\log_e 6$

D. $-\log_e 6$

Answer: B

Solution:

Solution:

$$(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$$

$$\text{Let } e^x = t$$

$$\therefore (t^2 - 4)(6t^2 - 5t + 1) = 0$$

$$\Rightarrow (t^2 - 4)(2t - 1)(3t - 1) = 0$$

$$\therefore t = 2, -2, \frac{1}{2}, \frac{1}{3}$$

$$\therefore e^x = 2 \Rightarrow x = \ln 2$$

$$e^x = -2 \text{ (not possible)}$$

$$e^x = \frac{1}{2} \Rightarrow x = -\ln 2$$

$$e^x = \frac{1}{3} \Rightarrow x = -\ln 3$$

$$\therefore \text{Sum of all real roots}$$

$$= \ln 2 - \ln 2 - \ln 3$$

$$= -\ln 3$$

Question84

For a natural number n , let $\alpha_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is ____

[25-Jun-2022-Shift-1]

Answer: 4

Solution:

Solution:

$$\alpha_n = 19^n - 12^n$$

Let equation of roots 12&19 i.e.

$$x^2 - 31x + 228 = 0$$

$$\Rightarrow (31 - x) = \frac{228}{x} \text{ (where x can be 19 or 12)}$$

$$\therefore \frac{31\alpha_9 - \alpha_{10}}{57\alpha_8} = \frac{31(19^9 - 12^9) - (19^{10} - 12^{10})}{57(19^8 - 12^8)}$$

$$= \frac{19^9(31 - 19) - 12^9(31 - 12)}{57(19^8 - 12^8)}$$

$$= \frac{228(19^8 - 12^8)}{57(19^8 - 12^8)} = 4$$

Question85

Let $a, b \in \mathbb{R}$ be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α . If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to :

[25-Jun-2022-Shift-2]

Options:

A. 37

B. 58

C. 68

D. 92

Answer: B

Solution:

Solution:

$ax^2 - 2bx + 15 = 0$ has repeated root so $b^2 = 15a$ and $\alpha = \frac{15}{b}$

$\therefore \alpha$ is a root of $x^2 - 2bx + 21 = 0$

So $\frac{225}{b^2} = 9 \Rightarrow b^2 = 25$

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4b^2 - 42 = 100 - 42 = 58$

Question86

The sum of the cubes of all the roots of the equation

$x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ is ____
[26-Jun-2022-Shift-1]

Answer: 36

Solution:

Solution:

$$x^4 - 3x^3 - x^2 - x^2 + 3x + 1 = 0$$

$$(x^2 - 1)(x^2 - 3x - 1) = 0$$

Let the root of $x^2 - 3x - 1 = 0$ be α and β and other two roots of given equation are 1 and -1

So sum of cubes of roots

$$= 1^3 + (-1)^3 + \alpha^3 + \beta^3$$

$$= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= (3)^3 - 3(-1)(3)$$

$$= 36$$

Question87

If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to ____
[27-Jun-2022-Shift-1]

Answer: 45

Solution:

Let $e^x = t$ then equation reduces to

$$t^2 - 11t - \frac{45}{t} + \frac{81}{2} = 0$$

$$\Rightarrow 2t^3 - 22t^2 + 81t - 45 = 0 \dots (i)$$

if roots of $e^{2xt} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ are α, β, γ then roots of (i) will be $e^{\alpha_1}e^{\alpha_2}e^{\alpha_3}$ using product of roots

$$e^{\alpha_1 + \alpha_2 + \alpha_3} = 45$$

$$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 = \ln 45 \Rightarrow p = 45$$

Question88

Let α, β be the roots of the equation $x^2 - 4\lambda x + 5 = 0$ and α, γ be the roots of the equation $x^2 - (3\sqrt{2} + 2\sqrt{3})x + 7 + 3\lambda\sqrt{3} = 0, \lambda > 0$. If $\beta + \gamma = 3\sqrt{2}$, then $(\alpha + 2\beta + \gamma)^2$ is equal to ____
[27-Jun-2022-Shift-2]

Answer: 98

Solution:

$$\because \alpha, \beta \text{ are roots of } x^2 - 4\lambda x + 5 = 0$$

$$\therefore \alpha + \beta = 4\lambda \text{ and } \alpha\beta = 5$$

Also, α, γ are roots of

$$x^2 - (3\sqrt{2} + 2\sqrt{3})x + 7 + 3\lambda\sqrt{3} = 0, \lambda > 0$$

$$\therefore \alpha + \gamma = 3\sqrt{2} + 2\sqrt{3}, \alpha\gamma = 7 + 3\lambda\sqrt{3}$$

$\because \alpha$ is common root

$$\therefore \alpha^2 - 4\lambda\alpha + 5 = 0$$

$$\text{and } \alpha^2 - (3\sqrt{2} + 2\sqrt{3})\alpha + 7 + 3\lambda\sqrt{3} = 0$$

$$\text{From (i) - (ii) : we get } \alpha = \frac{2 + 3\sqrt{3}\lambda}{3\sqrt{2} + 2\sqrt{3} - 4\lambda}$$

$$\because \beta + \gamma = 3\sqrt{2}$$

$$\therefore 4\lambda + 3\sqrt{2} + 2\sqrt{3} - 2\alpha = 3\sqrt{2}$$

$$\Rightarrow 3\sqrt{2} = 4\lambda + 3\sqrt{2} + 2\sqrt{3} - \frac{4 + 6\sqrt{3}\lambda}{3\sqrt{2} + 2\sqrt{3} - 4\lambda}$$

$$\Rightarrow 8\lambda^2 + 3(\sqrt{3} + 2\sqrt{2})\lambda - 4 - 3\sqrt{6} = 0$$

$$\therefore \lambda = \frac{6\sqrt{2} - 3\sqrt{2} \pm \sqrt{9(11 - 4\sqrt{6}) + 32(4 + 3\sqrt{6})}}{16}$$

$$\therefore \lambda = \sqrt{2}$$

$$\therefore (\alpha + 2\beta + \gamma)^2 = (\alpha + \beta + \beta + \gamma)^2$$

$$= (4\sqrt{2} + 3\sqrt{2})^2$$

$$= (7\sqrt{2})^2 = 98$$

Question89

The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is ____
[28-Jun-2022-Shift-1]

Answer: 2

Solution:

Dividing by e^{2x}

$$e^{2x} + 4e^x - 58 + 4e^{-x} + e^{-2x} = 0$$

$$\Rightarrow (e^x + e^{-x})^2 + 4(e^x + e^{-x}) - 60 = 0$$

$$\begin{aligned} \text{Let } e^x + e^{-x} &= t \in [2, \infty) \\ \Rightarrow t^2 + 4t - 60 &= 0 \\ \Rightarrow t = 6 &\text{ is only possible solution} \\ e^x + e^{-x} = 6 &\Rightarrow e^{2x} - 6e^x + 1 = 0 \\ \text{Let } e^x &= p \\ p^2 - 6p + 1 &= 0 \\ \Rightarrow p &= \frac{3 + \sqrt{5}}{2} \text{ or } \frac{3 - \sqrt{5}}{2} \\ \text{So } x &= \ln\left(\frac{3 + \sqrt{5}}{2}\right) \text{ or } \ln\left(\frac{3 - \sqrt{5}}{2}\right) \end{aligned}$$

Question90

Let $f(x)$ be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1 , then the sum of the roots of $f(x) = 0$ is equal to :

[28-Jun-2022-Shift-2]

Options:

- A. $\frac{11}{3}$
- B. $\frac{7}{3}$
- C. $\frac{13}{3}$
- D. $\frac{14}{3}$

Answer: A

Solution:

Solution:

$$\begin{aligned} \because x = -1 &\text{ be the roots of } f(x) = 0 \\ \therefore \text{ Let } f(x) &= A(x + 1)(x - 1) \dots (i) \\ \text{Now, } f(-2) + f(3) &= 0 \\ \Rightarrow A[-1(-2 - b) + 4(3 - b)] &= 0 \\ b &= \frac{14}{3} \\ \therefore \text{ Second root of } f(x) = 0 &\text{ will be } \frac{14}{3} \\ \therefore \text{ Sum of roots } &= \frac{14}{3} - 1 = \frac{11}{3} \end{aligned}$$

Question91

Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then, the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to :

[29-Jun-2022-Shift-2]

Options:

- A. 1

- B. α
- C. $1 + \alpha$
- D. $1 + 2\alpha$

Answer: A

Solution:

Given, α is a root of the equation $1 + x^2 + x^4 = 0$

$\therefore \alpha$ will satisfy the equation.

$$\therefore 1 + \alpha^2 + \alpha^4 = 0$$

$$\alpha^2 = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{3}i}{2}$$

$$\therefore \alpha^2 = \omega \text{ or } \omega^2$$

Now,

$$\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$$

$$= \alpha \cdot (\alpha^2)^{505} + (\alpha^2)^{1011} - \alpha \cdot (\alpha^2)^{1516}$$

$$= \alpha(\omega)^{505} + (\omega)^{1011} - \alpha \cdot (\omega)^{1516}$$

$$= \alpha \cdot (\omega^3)^{168} \cdot \omega + (\omega^3)^{337} - \alpha \cdot (\omega^3)^{505} \cdot \omega$$

$$= \alpha\omega + 1 - \alpha\omega$$

$$= 1$$

Question92

Let $x, y > 0$. If $x^3y^2 = 2^{15}$, then the least value of $3x + 2y$ is [24-Jun-2022-Shift-2]

Options:

- A. 30
- B. 32
- C. 36
- D. 40

Answer: D

Solution:

$$x, y > 0 \text{ and } x^3 y^2 = 2^{15}$$

$$\text{Now, } 3x + 2y = (x + x + x) + (y + y)$$

So, by $A \cdot M \geq$ G.M inequality

$$\frac{3x + 2y}{5} \geq \sqrt[5]{x^3 \cdot y^2}$$

$$\therefore 3x + 2y \geq 5 \sqrt[5]{2^{15}} \geq 40$$

$$\therefore \text{Least value of } 3x + 4y = 40$$

Question93

Let p and q be two real numbers such that $p + q = 3$ and $p^4 + q^4 = 369$.

Then $\left(\frac{1}{p} + \frac{1}{q} \right)^{-2}$ is equal to ____

[26-Jun-2022-Shift-2]

Answer: 4

Solution:

Solution:

$$\therefore p + q = 3 \dots\dots (i)$$

$$\text{and } p^4 + q^4 = 369 \dots\dots (ii)$$

$$\{(p + q)^2 - 2pq\}^2 - 2p^2 q^2 = 369$$

$$\text{or } (9 - 2pq)^2 - 2(pq)^2 = 369$$

$$\text{or } (pq)^2 - 18pq - 144 = 0$$

$$\therefore pq = -6 \text{ or } 24$$

But $pq = 24$ is not possible

$$\therefore pq = -6$$

$$\text{Hence, } \left(\frac{1}{p} + \frac{1}{q} \right)^{-2} = \left(\frac{pq}{p + q} \right)^2 = (-2)^2 = 4$$

Question94

If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to :

[25-Jul-2022-Shift-1]

Options:

A. -4

B. -1

C. 1

D. 4

Answer: B

Solution:

Solution:

When, $x^5 = 1$

then $x^5 - 1 = 0$

$\Rightarrow (x - 1)(x^4 + x^3 + x^2 + x + 1) = 0$

Given, $x^4 + x^3 + x^2 + x + 1 = 0$ has roots α, β, γ and 8 .

$\therefore$ Roots of $x^5 - 1 = 0$ are $1, \alpha, \beta, \gamma$ and 8

We know, Sum of p^{th} power of n^{th} roots of unity $= 0$. (If p is not multiple of n) or n (If p is multiple of n)

$\therefore$ Here, Sum of p^{th} power of n^{th} roots of unity

Here, $p = 2021$, which is not multiple of 5 .

$\therefore 1^{2021} + \alpha^{2021} + \beta^{2021} + \gamma^{2021} + 8^{2021} = 0$

$\Rightarrow \alpha^{2021} + \beta^{2021} + \gamma^{2021} + 8^{2021} = -1$

Question95

For $n \in \mathbb{N}$, let $S_n = \left\{ z \in \mathbb{C} : |z - 3 + 2i| = \frac{n}{4} \right\}$ and

$T_n = \left\{ z \in \mathbb{C} : |z - 2 + 3i| = \frac{1}{n} \right\}$.

Then the number of elements in the set $\{n \in \mathbb{N} : S_n \cap T_n = \varnothing\}$ is :

[25-Jul-2022-Shift-1]

Options:

A. 0

B. 2

C. 3

D. 4

Answer: D

Solution:

Solution:

$S_n = \left\{ z \in \mathbb{C} : |z - 3 + 2i| = \frac{n}{4} \right\}$ represents a circle with centre $C_1(3, -2)$ and radius $r_1 = \frac{n}{4}$

Similarly T_n represents circle with centre $C_2(2, -3)$ and radius $r_2 = \frac{1}{n}$

As $S_n \cap T_n = \varnothing$

$C_1C_2 > r_1 + r_2$ OR $C_1C_2 < |r_1 - r_2|$

$\sqrt{2} > \frac{n}{4} + \frac{1}{n}$ OR $\sqrt{2} < \left| \frac{n}{4} - \frac{1}{n} \right|$

$n = 1, 2, 3, 4$ n may take infinite values

Question96

For $z \in \mathbb{C}$ if the minimum value of $(|z - 3\sqrt{2}| + |z - p\sqrt{2}i|)$ is $5\sqrt{2}$, then a value of p is

Question: of p is

[25-Jul-2022-Shift-2]

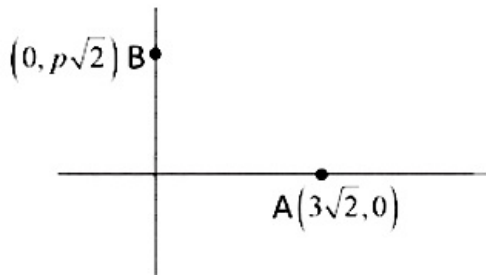
Options:

- A. 3
- B. $\frac{7}{2}$
- C. 4
- D. $\frac{9}{2}$

Answer: C

Solution:

Solution:



It is sum of distance of z from $(3\sqrt{2}, 0)$ and $(0, p\sqrt{2})$ For minimising, z should lie on AB and $AB = 5\sqrt{2}$
 $(AB)^2 = 18 + 2p^2$
 $p = \pm 4$

Question97

Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2 , $\text{Re}(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true?

Question: of p is

[26-Jul-2022-Shift-1]

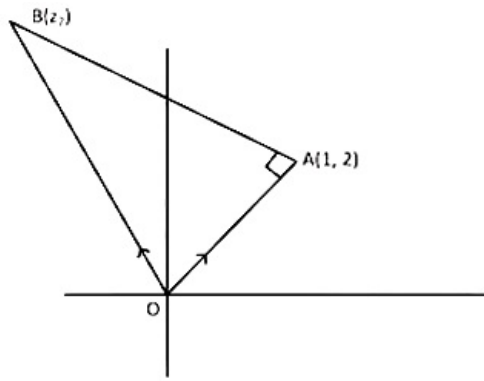
Options:

- A. $\arg z_2 = \pi - \tan^{-1} 3$
- B. $\arg(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$
- C. $|z_2| = \sqrt{10}$
- D. $|2z_1 - z_2| = 5$

Answer: D

Solution:

Solution:



$$\frac{z_2 - 0}{(1 + 2i) - 0} = \frac{|OB|}{|OA|} e^{i\frac{\pi}{4}}$$

$$\Rightarrow \frac{z_2}{1 + 2i} = \sqrt{2} e^{i\frac{\pi}{4}}$$

$$\text{OR } z_2 = (1 + 2i)(1 + i)$$

$$= -1 + 3i$$

$$\arg z_2 = \pi - \tan^{-1} 3$$

$$|z_2| = \sqrt{10}$$

$$z_1 - 2z_2 = (1 + 2i) + 2 - 6i = 3 - 4i$$

$$\arg(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$$

$$|2z_1 - z_2| = |2 + 4i + 1 - 3i| = |3 + i| = \sqrt{10}$$

Question98

If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z - i| - |z + 5i| = 0$, then
[26-Jul-2022-Shift-2]

Options:

A. $x + 2y - 4 = 0$

B. $x^2 + y - 4 = 0$

C. $x + 2y + 4 = 0$

D. $x^2 - y + 3 = 0$

Answer: C

Solution:

Solution:

$$|z - i| = |z + 5i|$$

So, z lies on $\perp^r$ bisector of $(0, 1)$ and $(0, -5)$

i.e., line $y = -2$

$$\text{as } |z| = 2$$

$$\Rightarrow z = -2i$$

$$x = 0 \text{ and } y = -2$$

$$\text{so, } x + 2y + 4 = 0$$

Question99

Let the minimum value v_0 of $v = |z|^2 + |z - 3|^2 + |z - 6i|^2, z \in \mathbb{C}$ is attained at $z = z_0$. Then $\left| 2z_0^2 - \bar{z}_0^3 + 3 \right|^2 + v_0^2$ is equal to
[27-Jul-2022-Shift-1]

Options:

- A. 1000
- B. 1024
- C. 1105
- D. 1196

Answer: A

Solution:

Solution:

Let $z = x + iy$

$$\begin{aligned} v &= x^2 + y^2 + (x - 3)^2 + y^2 + x^2 + (y - 6)^2 \\ &= (3x^2 - 6x + 9) + (3y^2 - 12y + 36) \\ &= 3(x^2 + y^2 - 2x - 4y + 15) \\ &= 3[(x - 1)^2 + (y - 2)^2 + 10] \end{aligned}$$

$v_{\min}$ at $z = 1 + 2i = z_0$ and $v_0 = 30$

$$\begin{aligned} \text{so } &|2(1 + 2i)^2 - (1 - 2i)^3 + 3|^2 + 900 \\ &= |2(-3 + 4i) - (1 - 8i^3 - 6i(1 - 2i)) + 3|^2 + 900. \\ &= |-6 + 8i - (1 + 8i - 6i - 12) + 3|^2 + 900 \\ &= |8 + 6i|^2 + 900 \end{aligned}$$

$$\begin{aligned} &= |8 + 6i|^2 + 900 \\ &= 1000 \end{aligned}$$

Question100

Let $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} (\text{Re}(z) + \text{Im}(z))$ is equal to _____.
[27-Jul-2022-Shift-1]

Answer: 0

Solution:

$$\because z^2 + \bar{z} = 0$$

Let $z = x + iy$

$$\because x^2 - y^2 + 2ixy + x - iy = 0$$

$$(x^2 - y^2 + x) + i(2xy - y) = 0$$

$$\because x^2 + y^2 = 0 \text{ and } (2x - 1)y = 0$$

$$\text{if } x = +\frac{1}{2} \text{ then } y = \pm \frac{\sqrt{3}}{2}$$

And if $y = 0$ then $x = 0, -1$

$$\therefore z = 0 + 0i, -1 + 0i, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$\therefore \sum (R_e(z) + m(z)) = 0$$

Question101

Let S be the set of all (α, β) , $\pi < \alpha, \beta < 2\pi$, for which the complex number $\frac{1 - i \sin \alpha}{1 + 2i \sin \alpha}$ is purely imaginary and $\frac{1 + i \cos \beta}{1 - 2i \cos \beta}$ is purely real. Let

$Z_{\alpha\beta} = \sin 2\alpha + i \cos 2\beta$, $(\alpha, \beta) \in S$. Then $\sum_{(\alpha, \beta) \in S} \left(iZ_{\alpha\beta} + \frac{1}{iZ_{\alpha\beta}} \right)$ is equal to :

[27-Jul-2022-Shift-2]

Options:

A. 3

B. 3i

C. 1

D. $2 - i$

Answer: C

Solution:

Solution:

$\therefore \frac{1 - i \sin \alpha}{1 + 2i \sin \alpha}$ is purely imaginary

$$\therefore \frac{1 - i \sin \alpha}{1 + 2i \sin \alpha} + \frac{1 + i \sin \alpha}{1 - 2i \sin \alpha} = 0$$

$$\Rightarrow 1 - 2 \sin^2 \alpha = 0$$

$$\therefore \alpha = \frac{5\pi}{4}, \frac{7\pi}{4}$$

and $\frac{1 + i \cos \beta}{1 - 2i \cos \beta}$ is purely real

$$\frac{1 + i \cos \beta}{1 - 2i \cos \beta} - \frac{1 - i \cos \beta}{1 + 2i \cos \beta} = 0$$

$$\Rightarrow \cos \beta = 0$$

$$\therefore \beta = \frac{3\pi}{2}$$

$$\therefore S = \left\{ \left(\frac{5\pi}{4}, \frac{3\pi}{2} \right), \left(\frac{7\pi}{4}, \frac{3\pi}{2} \right) \right\}$$

$$Z_{\alpha\beta} = 1 - i \text{ and } Z_{\alpha\beta} = -1 - i$$

$$\therefore \sum_{(\alpha, \beta) \in S} \left(iZ_{\alpha\beta} + \frac{1}{iZ_{\alpha\beta}} \right) = i(-2i) + \frac{1}{i} \left[\frac{1}{1+i} + \frac{1}{-1+i} \right]$$

$$= 2 + \frac{1}{i} \frac{2i}{-2} = 1$$

Question102

Let $S_1 = \left\{ z_1 \in \mathbb{C} : |z_1 - 3| = \frac{1}{2} \right\}$ and

$S_2 = \{z_2 \in \mathbb{C} : |z_2 - 1| = |z_2 + 1|\}$. Then, for $z_1 \in S_1$ and

$z_2 \in S_2$, the least value of $|z_2 - z_1|$ is :

[28-Jul-2022-Shift-1]

Options:

A. 0

B. $\frac{1}{2}$

C. $\frac{3}{2}$

D. $\frac{5}{2}$

Answer: C

Solution:

Solution:

$$\because |Z_2 + |Z_2 - 1||^2 = |Z_2 - |Z_2 + 1||^2$$

$$\Rightarrow (Z_2 + |Z_2 - 1|)(\overline{Z_2} + |Z_2 - 1|) = (Z_2 - |Z_2 + 1|)(\overline{Z_2} - |Z_2 + 1|)$$

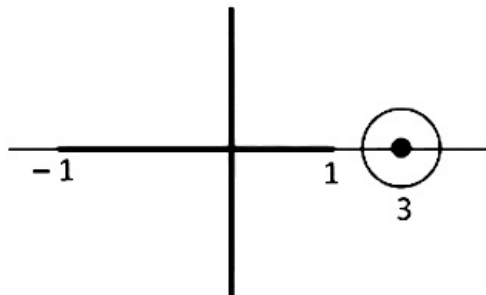
$$\Rightarrow Z_2(|Z_2 - 1| + |Z_2 + 1|) + \overline{Z_2}(|Z_2 - 1| + |Z_2 + 1|) = |Z_2 + 1|^2 - |Z_2 - 1|^2$$

$$\Rightarrow (Z_2 + \overline{Z_2})(|Z_2 + 1| + |Z_2 - 1|) = 2(Z_2 + \overline{Z_2})$$

$$\Rightarrow \text{Either } Z_2 + \overline{Z_2} = 0 \text{ or } |Z_2 + 1| + |Z_2 - 1| = 2$$

So, Z_2 lies on imaginary axis or on real axis within $[-1, 1]$

Also $|Z_1 - 3| = \frac{1}{2} \Rightarrow Z_1$ lies on the circle having center 3 and radius $\frac{1}{2}$.



Clearly $|Z_1 - Z_2|_{\min} = \frac{3}{2}$

Question103

Let $z = a + ib$, $b \neq 0$ be complex numbers satisfying $z^2 = \overline{z} \cdot 2^{1-|z|}$. Then the least value of $n \in \mathbb{N}$, such that $z^n = (z + 1)^n$, is equal to _____.

[28-Jul-2022-Shift-2]

Answer: 6

Solution:

Solution:

$$\because z^2 = \overline{z} \cdot 2^{1-|z|} \dots\dots (1)$$

$$\Rightarrow |z|^2 = |\bar{z}| \cdot 2^{1-|z|}$$

$$\Rightarrow |z| = 2^{1-|z|}$$

$$\because b \neq 0 \Rightarrow |z| \neq 0$$

$$\therefore |z| = 1 \dots \dots (2)$$

$$\because z = a + ib \text{ then } \sqrt{a^2 + b^2} = 1 \dots \dots (3)$$

Now again from equation (1), equation (2), equation (3) we get :

$$a^2 - b^2 + i2ab = (a - ib)2^0$$

$$\therefore a^2 - b^2 = a \text{ and } 2ab = -b$$

$$\therefore a = -\frac{1}{2} \text{ and } b = \pm \frac{\sqrt{3}}{2}$$

$$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \text{ or } z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$z^n = (z + 1)^n \Rightarrow \left(\frac{z+1}{z} \right)^n = 1$$

$$\left(1 + \frac{1}{z} \right)^n = 1$$

$$\left(\frac{1 + \sqrt{3}i}{2} \right)^n = 1, \text{ then minimum value of } n \text{ is } 6.$$

Question104

If $z = 2 + 3i$, then $z^5 + \left(\bar{z} \right)^5$ is equal to :

[29-Jul-2022-Shift-1]

Options:

A. 244

B. 224

C. 245

D. 265

Answer: A

Solution:

Solution:

$$z = (2 + 3i)$$

$$\Rightarrow z^5 = (2 + 3i)((2 + 3i)^2)^2$$

$$= (2 + 3i)(-5 + 12i)^2$$

$$= (2 + 3i)(-119 - 120i)$$

$$= -238 - 240i - 357i + 360$$

$$= 122 - 597i$$

$$\bar{z}^5 = 122 + 597i$$

$$z^5 + \bar{z}^5 = 244$$

Question105

If $z \neq 0$ be a complex number such that $\left| z - \frac{1}{z} \right| = 2$, then the maximum value of $|z|$ is:

[29-Jul-2022-Shift-2]

Options:

A. $\sqrt{2}$

B. 1

C. $\sqrt{2} - 1$

D. $\sqrt{2} + 1$

Answer: D

Solution:

Solution:

We know,

$$||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$$

$$\therefore \left| |z| - \frac{1}{|z|} \right| \leq \left| z - \frac{1}{z} \right|$$

$$\Rightarrow \left| |z| - \frac{1}{|z|} \right| \leq 2 \left[\text{Given } \left| z - \frac{1}{z} \right| = 2 \right]$$

$$\Rightarrow \left| \frac{|z|^2 - 1}{|z|} \right| \leq 2$$

$$\Rightarrow -2 \leq \frac{|z|^2 - 1}{|z|} \leq 2$$

$$\therefore \frac{|z|^2 - 1}{|z|} \leq 2$$

$$\Rightarrow |z|^2 - 1 \leq 2|z|$$

$$\Rightarrow |z|^2 - 2|z| - 1 \leq 0$$

$$\Rightarrow |z|^2 - 2|z| + 1 - 2 \leq 0$$

$$\Rightarrow (|z| - 1)^2 - 2 \leq 0$$

$$\Rightarrow -\sqrt{2} \leq |z| - 1 \leq \sqrt{2}$$

$$\Rightarrow 1 - \sqrt{2} \leq |z| \leq 1 + \sqrt{2} \dots \dots (1)$$

or

$$-2 \leq \frac{|z|^2 - 1}{|z|}$$

$$\Rightarrow |z|^2 - 1 \leq -2|z|$$

$$\Rightarrow |z|^2 + 2|z| - 1 \leq 0$$

$$\Rightarrow |z|^2 + 2|z| + 1 - 2 \leq 0$$

$$\Rightarrow (|z| + 1)^2 - 2 \leq 0$$

$$\Rightarrow -\sqrt{2} \leq |z| + 1 \leq +\sqrt{2}$$

$$\Rightarrow -\sqrt{2} - 1 \leq |z| \leq \sqrt{2} - 1 \dots \dots (2)$$

From (1) and (2) we get,

Maximum value of $|z| = \sqrt{2} + 1$ and minimum value of $|z| = -\sqrt{2} - 1$

Question 106

Let $S = \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x , for which $u = 2x + iy \in S$ for some $y \in \mathbb{R}$, is [29-Jul-2022-Shift-2]

Options:

A. $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}} \right]$

B. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4} \right]$

C. $\left(-\sqrt{2}, \frac{1}{2}\right]$

D. $\left[-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

Answer: B

Solution:

Question107

If the numbers appeared on the two throws of a fair six faced die are α and β , then the probability that $x^2 + \alpha x + \beta > 0$, for all $x \in \mathbb{R}$, is :
[25-Jul-2022-Shift-1]

Options:

A. $\frac{17}{36}$

B. $\frac{4}{9}$

C. $\frac{1}{2}$

D. $\frac{19}{36}$

Answer: A

Solution:

Solution:

For $x^2 + \alpha x + \beta > 0 \forall x \in \mathbb{R}$ to hold, we should have $\alpha^2 - 4\beta < 0$

If $\alpha = 1$, β can be 1, 2, 3, 4, 5, 6 i.e., 6 choices

If $\alpha = 2$, β can be 2, 3, 4, 5, 6 i.e., 5 choices

If $\alpha = 3$, β can be 3, 4, 5, 6 i.e., 4 choices

If $\alpha = 4$, β can be 5 or 6 i.e., 2 choices

If $\alpha = 6$, No possible value for β i.e., 0 choices

Hence total favourable outcomes

$$= 6 + 5 + 4 + 2 + 0 + 0$$

$$= 17$$

Total possible choices for α and $\beta = 6 \times 6 = 36$

$$\text{Required probability} = \frac{17}{36}$$

Question108

Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is

equal to
[25-Jul-2022-Shift-1]

Answer: 38

Solution:

Solution:

$\therefore$ Roots of $2ax^2 - 8ax + 1 = 0$ are $\frac{1}{p}$ and $\frac{1}{r}$ and roots of $6bx^2 + 12bx + 1 = 0$ are $\frac{1}{q}$ and $\frac{1}{s}$

Let $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ as $\alpha - 3\beta, \alpha - \beta, \alpha + \beta, \alpha + 3\beta$

So sum of roots $2\alpha - 2\beta = 4$ and $2\alpha + 2\beta = -2$

Clearly $\alpha = \frac{1}{2}$ and $\beta = -\frac{3}{2}$

Now product of roots, $\frac{1}{p} \cdot \frac{1}{r} = \frac{1}{2a} = -5 \Rightarrow \frac{1}{a} = -10$

and $\frac{1}{q} \cdot \frac{1}{s} = \frac{1}{6b} = -8 \Rightarrow \frac{1}{b} = -48$

So, $\frac{1}{a} - \frac{1}{b} = 38$

Question109

If for some $p, q, r \in \mathbb{R}$, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2 + r^2}{p^2}$ is equal to _____.

[26-Jul-2022-Shift-1]

Answer: 272

Solution:

$$(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$\therefore \alpha + \beta > 0$ and $\alpha\beta > 0$

Also, it has a common root with $x^2 + 2x - 8 = 0$

$\therefore$ The common root between above two equations is 4 .

$$\Rightarrow 16(p^2 + q^2) - 8q(p + r) + q^2 + r^2 = 0$$

$$\Rightarrow (16p^2 - 8pq + q^2) + (16q^2 - 8qr + r^2) = 0$$

$$\Rightarrow (4p - q)^2 + (4q - r)^2 = 0$$

$$\Rightarrow q = 4p \text{ and } r = 16p$$

$$\therefore \frac{q^2 + r^2}{p^2} = \frac{16p^2 + 256p^2}{p^2} = 272$$

Question110

The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is _____.

[26-Jul-2022-Shift-1]

Answer: 3

Solution:

Solution:

$$x^8 - x^7 - x^6 + x^5 + 3x^4 - 4x^3 - 2x^2 + 4x - 1 = 0$$

$$\Rightarrow x^7(x-1) - x^5(x-1) + 3x^3(x-1) - x(x^2-1) + 2x(1-x) + (x-1) = 0$$

$$\Rightarrow (x-1)(x^7 - x^5 + 3x^3 - x(x+1) - 2x + 1) = 0$$

$$\Rightarrow (x-1)(x^7 - x^5 + 3x^3 - x^2 - 3x + 1) = 0$$

$$\Rightarrow (x-1)(x^5(x^2-1) + 3x(x^2-1) - 1(x^2-1)) = 0$$

$$\Rightarrow (x-1)(x^2-1)(x^5 + 3x - 1) = 0$$

$\therefore x = \pm 1$ are roots of above equation and $x^5 + 3x - 1$ is a monotonic term hence vanishes at exactly one value of x other than 1 or -1.

$\therefore$ 3 real roots.

Question111

The minimum value of the sum of the squares of the roots of $x^2 + (3-a)x + 1 = 2a$ is:

[26-Jul-2022-Shift-2]

Options:

A. 4

B. 5

C. 6

D. 8

Answer: C

Solution:

Solution:

$$x^2 + (3-a)x + 1 = 2a \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$$\alpha + \beta = a - 3, \alpha\beta = 1 - 2a$$

$$\Rightarrow \alpha^2 + \beta^2 = (a-3)^2 - 2(1-2a)$$

$$= a^2 - 6a + 9 - 2 + 4a$$

$$= a^2 - 2a + 7$$

$$= (a-1)^2 + 6$$

So, $\alpha^2 + \beta^2 \geq 6$

Question112

Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of $(a + b - c)$ is _____.
[26-Jul-2022-Shift-2]

Options:

- A. 12
- B. 13
- C. 14
- D. 16

Answer: A

Solution:

Solution:

Abscissae of PQ are roots of $x^2 - 4x - 6 = 0$
Ordinates of PQ are roots of $y^2 + 2y - 7 = 0$
and PQ is diameter
 $\Rightarrow$ Equation of circle is
 $x^2 + y^2 - 4x + 2y - 13 = 0$
But, given $x^2 + y^2 + 2ax + 2by + c = 0$
By comparison $a = -2$, $b = 1$, $c = -13$
 $\Rightarrow a + b - c = -2 + 1 + 13 = 12$

Question113

If α, β are the roots of the equation

$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right) + 3 \left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$$

then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is :

[27-Jul-2022-Shift-2]

Options:

- A. $3x^2 - 20x - 12 = 0$
- B. $3x^2 - 10x - 4 = 0$
- C. $3x^2 - 10x + 2 = 0$
- D. $3x^2 - 20x + 16 = 0$

Answer: B

Solution:

Solution:

$$3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}} = 3^{\sqrt{\log_3 5}} - (3^{\log_3 5})^{\sqrt{\log_5 3}}$$
$$3^{(\log_3 5) \frac{1}{3}} - 5^{(\log_5 3) \frac{2}{3}} = 5^{(\log_5 3) \frac{2}{3}} - 5^{(\log_5 3) \frac{2}{3}} = 0$$

Note: In the given equation 'x' is missing.

So

$$x^2 - 5x + 3(-1) = 0 \quad \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$$\alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} = (\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta}$$

$$= 5 - \frac{5}{3} = \frac{10}{3}$$

$$\left(\alpha + \frac{1}{\beta}\right) \left(\beta + \frac{1}{\alpha}\right) = 2 + \alpha\beta + \frac{1}{\alpha\beta} = 2 - 3 - \frac{1}{3} = \frac{-4}{3}$$

So Equation must be option (B).

Question 114

The sum of all real values of x for which $\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} - \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$ is equal to

 .
[28-Jul-2022-Shift-1]

Answer: 6

Solution:

Solution:

$$\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} = \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$$

$$\Rightarrow \frac{3x^2 - 9x + 17}{5x^2 - 7x + 19} = \frac{x^2 + 3x + 10}{3x^2 + 5x + 12}$$

$$\frac{-2x^2 - 2x - 2}{5x^2 - 7x + 19} = \frac{-2x^2 - 2x - 2}{3x^2 + 5x + 12}$$

$$\text{Either } x^2 + x + 1 = 0 \text{ or No real roots } \Rightarrow 5x^2 - 7x + 19 = 3x^2 + 5x + 12$$

$$2x^2 - 12x + 7 = 0$$

$$\text{sum of roots} = 6$$

Question 115

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2} + 1, \frac{1}{\beta^2} + 1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are :

[28-Jul-2022-Shift-2]

Options:

- A. non-real complex numbers
- B. real and both negative
- C. real and both positive
- D. real and exactly one of them is positive

Answer: B

Solution:

Solution:

$$\alpha + \beta = \sqrt{2}, \alpha\beta = \sqrt{6}$$

$$\frac{1}{\alpha^2} + 1 + \frac{1}{\beta^2} + 1 = 2 + \frac{\alpha^2 + \beta^2}{6}$$

$$= 2 + \frac{2 - 2\sqrt{6}}{6} = -a$$

$$\left(\frac{1}{\alpha^2} + 1 \right) \left(\frac{1}{\beta^2} + 1 \right) = 1 + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\alpha^2\beta^2}$$

$$= \frac{7}{6} + \frac{2 - 2\sqrt{6}}{6} = b$$

$$\Rightarrow a + b = \frac{-5}{6}$$

$$\text{So, equation is } x^2 + \frac{17x}{6} + \frac{7}{6} = 0$$

$$\text{OR } 6x^2 + 17x + 7 = 0$$

Both roots of equation are -ve and distinct

Question 116

Let $f(x) = ax^2 + bx + c$ be such that $f(1) = 3$, $f(-2) = \lambda$ and $f(3) = 4$. If $f(0) + f(1) + f(-2) + f(3) = 14$, then λ is equal to :
[28-Jul-2022-Shift-2]

Options:

- A. -4
- B. $\frac{13}{2}$
- C. $\frac{23}{2}$
- D. 4

Answer: D

Solution:

Solution:

$$f(1) = a + b + c = 3 \dots (i)$$

$$f(3) = 9a + 3b + c = 4 \dots (ii)$$

$$f(0) + f(1) + f(-2) + f(3) = 14$$

$$\text{OR } c + 3 + (4a - 2b + c) + 4 = 14$$

$$\text{OR } 4a - 2b + 2c = 7 \dots (iii)$$

$$\text{From (i) and (ii) } 8a + 2b = 1 \dots (iv)$$

$$\text{From (iii) } -(2) \times (i)$$

$$\Rightarrow 2a - 4b = 1 \dots \dots (v)$$

$$\text{From (iv) and (v) } a = \frac{1}{6}, b = \frac{-1}{6} \text{ and } c = 3$$

$$f(-2) = 4a - 2b + c$$

$$= \frac{4}{6} + \frac{2}{6} + 3 = 4$$

Question 117

Let $\alpha, \beta (\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If

$P_n = \alpha^n - \beta^n, n \in \mathbb{N}$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to

[29-Jul-2022-Shift-2]

Options:

Answer: 16

Solution:

Solution:

α and β are the roots of the quadratic equation $x^2 - x - 4 = 0$.

$\therefore \alpha$ and β satisfy the given equation.

$$\alpha^2 - \alpha - 4 = 0$$

$$\Rightarrow \alpha^{n+1} - \alpha^n - 4\alpha^{n-1} = 0 \dots \dots (i)$$

$$\text{and } \beta^2 - \beta - 4 = 0$$

$$\Rightarrow \beta^{n+1} - \beta^n - 4\beta^{n-1} = 0 \dots \dots (2) \text{ Substituting (2) from (1), we get,}$$

$$(\alpha^{n+1} - \beta^{n+1}) - (\alpha^n - \beta^n) - 4(\alpha^{n-1} - \beta^{n-1}) = 0$$

$$\Rightarrow P_{n+1} - P_n - 4P_{n-1} = 0$$

$$\Rightarrow P_{n+1} = P_n + 4P_{n-1}$$

$$\Rightarrow P_{n+1} - P_n = 4P_{n-1}$$

$$\text{For } n = 14, P_{15} - P_{14} = 4P_{13}$$

$$\text{For } n = 15, P_{16} - P_{15} = 4P_{14}$$

$$\text{Now, } \frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$$

$$= \frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$$

$$= \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}}$$

$$= \frac{(4P_{13})(4P_{14})}{P_{13}P_{14}}$$

$$= 16$$

Question 118

Let $S = \left\{ x \in [-6, 3] - \{-2, 2\} : \frac{|x+3|-1}{|x|-2} \geq 0 \right\}$ and

$T = \{x \in \mathbb{Z} : x^2 - 7|x| + 9 \leq 0\}$

Then the number of elements in $S \cap T$ is :

[28-Jul-2022-Shift-2]

Options:

A. 7

B. 5

C. 4

D. 3

Answer: D

Solution:

Solution:

$$|x^2| - 7|x| + 9 \leq 0$$

$$\Rightarrow |x| \in \left[\frac{7 - \sqrt{13}}{2}, \frac{7 + \sqrt{13}}{2} \right]$$

As $x \in \mathbb{Z}$

So, x can be $\pm 2, \pm 3, \pm 4, \pm 5$

Out of these values of x ,

$$x = 3, -4, -5$$

satisfy S as well

$$n(S \cap T) = 3$$

Question 119

Let $i = \sqrt{-1}$. If $\frac{(-1 + i\sqrt{3})^{21}}{(1 - i)^{24}} + \frac{(1 + i\sqrt{3})^{21}}{(1 + i)^{24}} = k$ and $n = [|k|]$ be the greatest integral part of $|k|$. Then, $\sum_{j=0}^{n+5} (j+5)^2 - \sum_{j=0}^{n+5} (j+5)$ is equal to [2021, 24 Feb. Shift-II]

Answer: 310

Solution:

Solution:

$$\text{Given, } \frac{(-1 + i\sqrt{3})^{21}}{(1 - i)^{24}} + \frac{(1 + i\sqrt{3})^{21}}{(1 + i)^{24}} = k$$

$$\therefore -1 + i\sqrt{3} = 2e^{i2\pi/3}$$

$$1 + i\sqrt{3} = 2e^{i\pi/3}$$

$$1 - i = \sqrt{2}e^{-i\pi/4}$$

$$1 + i = \sqrt{2}e^{i\pi/4}$$

$$\text{Now, } \frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{(\sqrt{2}e^{-i\pi/4})^{24}} + \frac{(2e^{i\pi/3})^{21}}{(\sqrt{2}e^{i\pi/4})^{24}}$$

$$= \frac{2^{21} \cdot e^{i14\pi}}{2^{12} \cdot e^{-i6\pi}} + \frac{2^{21} \cdot e^{i7\pi}}{2^{12} \cdot e^{i6\pi}}$$

$$= 2^9 \cdot e^{i20\pi} + 2^9 \cdot e^{i\pi}$$

$$= 2^9(1) + 2^9(-1)$$

$$\Rightarrow 2^9 - 2^9 = 0 = k \text{ (given)}$$

$$\therefore n = [|k|] = [101] = 0$$

$$\text{Now, } \sum_{j=0}^5 (j+5)^2 - \sum_{j=0}^5 (j+5) \quad [\because n = 0]$$

$$= [5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2]$$

$$- [5 + 6 + 7 + 8 + 9 + 10]$$

$$= [(1^2 + 2^2 + 3^2 + \dots + 10^2) -$$

$$(1^2 + 2^2 + \dots + 4^2)] - [(1 + 2 + 3 + \dots + 10)]$$

$$\begin{aligned}
 & -(1 + 2 + 3 + 4)] \\
 & = \left[\frac{10 \times 11 \times 21}{6} - \frac{4 \times 5 \times 9}{6} \right] - \left[\frac{10 \times 11}{2} - \frac{4 \times 5}{2} \right] \\
 & = (385 - 30) - (55 - 10) \\
 & = 385 - 45 = 310
 \end{aligned}$$

Question 120

Let z be those complex numbers which satisfy $|z + 5| \leq 4$ and $z(1 + i) + \bar{z}(1 - i) \geq -10$, $i = \sqrt{-1}$. If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is [2021, 26 Feb. Shift-II]

Answer: 48

Solution:

Solution:

Given, $|z + 5| \leq 4$, which is equation of circle.

$$|z + 5| \leq 4$$

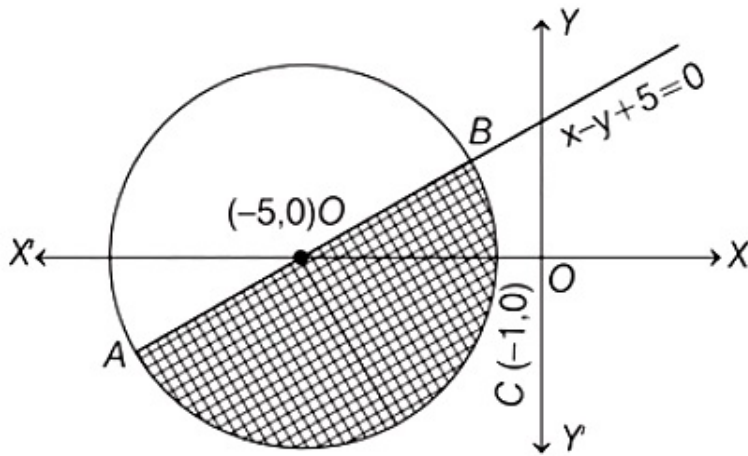
$$\Rightarrow (x + 5)^2 + y^2 \leq 16$$

$$\text{and } z(1 + i) + \bar{z}(1 - i) \geq -10$$

$$\Rightarrow (z + \bar{z}) + i(z - \bar{z}) \geq -10$$

$$\Rightarrow x - y + 5 \geq 0$$

From Eqs. (i) and (ii), region bounded by inequalities are



$$\text{Now, } |z + 1|^2 = |z - (-1)|^2$$

Maximum value of $|z + 1|^2$ will be equal to $(AC)^2$.

$$\text{Now, } (x + 5)^2 + y^2 = 16$$

$$\text{and } x - y + 5 = 0$$

$$\text{Given, } y = \pm 2\sqrt{2}$$

$$\text{and } x = \pm 2\sqrt{2} - 5$$

$\therefore$ Coordinates are

$$A(-2\sqrt{2} - 5, -2\sqrt{2})$$

$$B(2\sqrt{2} - 5, 2\sqrt{2})$$

$$C(-1, 0)$$

Then,

$$AC^2 = (2\sqrt{2} + 4)^2 + (2\sqrt{2})^2$$

$$= 32 + 16\sqrt{2}$$

Given, that maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$

$$\Rightarrow \alpha + \beta\sqrt{2} = 32 + 16\sqrt{2}$$

$$\Rightarrow \alpha = 32, \beta = 16$$

$$\therefore \alpha + \beta = 32 + 16 = 48$$

Question121

Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is
[2021, 25 Feb. Shift-1]

Options:

A. $\frac{3}{\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $3\sqrt{2}$

D. $\frac{3}{2\sqrt{2}}$

Answer: D

Solution:

Solution:

Given, $(2 - i)z = (2 + i)\bar{z}$

Let $z = x + iy$, then $\bar{z} = x - iy$

$$\Rightarrow (2 - i)(x + iy) = (2 + i)(x - iy)$$

$$\Rightarrow 2x - ix + 2iy + y = 2x + ix - 2iy + y$$

$$\Rightarrow 2ix - 4iy = 0$$

$$\therefore \text{Equation of line } L_1 \Rightarrow x - 2y = 0 \quad \dots\dots\dots (i)$$

Also, $(2 + i)z + (i - 2)\bar{z} - 4i = 0$

$$\Rightarrow (2 + i)(x + iy) + (i - 2)(x - iy) - 4i = 0$$

$$\Rightarrow 2x + ix + 2iy - y + ix - 2x + y$$

$$+ 2iy - 4i = 0$$

$$\Rightarrow 2ix + 4iy - 4i = 0$$

$$\therefore \text{Equation of line } L_2 \Rightarrow x + 2y - 2 = 0 \dots (ii)$$

From Eqs. (i) and (ii),

$$4y = 2 \text{ or } y = 1/2 \text{ and } x = 1$$

Hence, centre = $(1, 1/2)$

Equation of third line

$$L_3 \Rightarrow iz + \bar{z} + 1 + i = 0$$

$$\Rightarrow i(x + iy) + (x - iy) + 1 + i = 0$$

$$\Rightarrow ix - y + x - iy + 1 + i = 0$$

$$\Rightarrow (x - y + 1) + i(x - y + 1) = 0$$

$$\therefore \text{Radius} = \text{Distance of point } (1, 1/2) \text{ to the line } x - y + 1 = 0$$

$$\therefore r = \frac{\left|1 - \frac{1}{2} + 1\right|}{\sqrt{1^2 + 1^2}} = \frac{3}{2\sqrt{2}}$$

Question122

Let α and β be two real numbers, such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$, for some integer $n \geq 1$. Then, the value of p_n^2 is

[2021, 26 Feb. Shift-III]

Answer: 324

Solution:

Given that, $\alpha + \beta = 1$, $\alpha\beta = -1$

Let α, β be roots of quadratic equation, then the quadratic equation be

$$x^2 - x - 1 = 0$$

$$\text{Now, } \alpha^2 - \alpha - 1 = 0$$

$$\Rightarrow \alpha^2 = \alpha + 1 \quad \dots\dots\dots (i)$$

$$\text{Similarly, } \beta^2 = \beta + 1 \quad \dots\dots\dots (ii)$$

Multiply α^{n-1} in Eq. (i), we get

$$\alpha^{n+1} = \alpha^n + \alpha^{n-1} \quad \dots\dots\dots (iii)$$

Multiply β^{n-1} in Eq. (ii), we get

$$\beta^{n+1} = \beta^n + \beta^{n-1} \quad \dots\dots\dots (iv)$$

Add Eqs. (iii) and (iv), we get

$$\alpha^{n+1} + \beta^{n+1} = (\alpha^n + \beta^n) + (\alpha^{n-1} + \beta^{n-1})$$

$$p_{n+1} = p_n + p_{n-1}$$

$$29 = p_n + 11$$

$$\Rightarrow p_n = 18$$

$$p_n^2 = (18)^2 = 324$$

Question123

The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is

[2021, 26 Feb. Shift-1]

Answer: 1

Solution:

$$\log_4(x - 1) = \log_2(x - 3) \text{ (given)}$$

$$\Rightarrow \log_2 2(x - 1) = \log_2(x - 3)$$

Using property of logarithm,

$$\log_b c^a = \frac{1}{c} \log_b a$$

$$\Rightarrow \frac{1}{2} \log_2(x - 1) = \log_2(x - 3)$$

$$\Rightarrow \log_2(x - 1) = 2 \log_2(x - 3)$$

$$\Rightarrow \log_2(x - 1) = \log_2(x - 3)^2$$

$$\text{On comparing, } x - 1 = (x - 3)^2$$

$$\text{or } x - 1 = x^2 + 9 - 6x$$

$$\Rightarrow x^2 - 7x + 10 = 0$$

$$\Rightarrow x^2 - 5x - 2x + 10 = 0$$

$$\Rightarrow (x - 5)(x - 2) = 0$$

$$\Rightarrow x = 2, 5$$

$$x = 2 \text{ (rejected) as } x > 1$$

$\therefore x = 5$ is only solution i.e. number of solution is 1.

Question124

Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is
[2021, 25 Feb. Shift-II]

Options:

- A. 4
- B. 3
- C. 2
- D. 1

Answer: C

Solution:

Solution:

We have, $x^2 - 6x - 2 = 0$

Given, α and β are roots of above quadratic equation, then

$$\alpha^2 - 6\alpha - 2 = 0$$

$$\beta^2 - 6\beta - 2 = 0$$

Also, given $a_n = \alpha^n - \beta^n$, then

$$\begin{aligned} & \frac{a_{10} - 2a_8}{3a_9} \\ &= \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)} \\ &= \frac{\alpha^{10} - 2\alpha^8 - \beta^{10} + 2\beta^8}{3(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)} \\ & \text{[from Eqs. (i) and (ii) } \alpha^2 - 2 = 6\alpha, \beta^2 - 2 = 6\beta \text{]} \\ &= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)} \\ &= \frac{6\alpha^9 - 6\beta^9}{3(\alpha^9 - \beta^9)} = \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)} \\ &= 2 \end{aligned}$$

Question125

If $\alpha, \beta \in \mathbb{R}$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to
[2021, 25 Feb. Shift-II]

Options:

- A. 3
- B. -3

C. 7

D. -7

Answer: D

Solution:

Solution:

Given, root of $z^2 + \alpha z + \beta = 0$ is $1 - 2i$.

Since, it is quadratic equation and one root is complex in nature, its another root is complex conjugate.

$\therefore$ Two roots are $1 - 2i$ and $1 + 2i$.

Now, sum of roots $= -\frac{\alpha}{1} = -\alpha$

$= (1 - 2i) + (1 + 2i) = 2$

Gives, $\alpha = -2$

Product of roots $= \frac{\beta}{1} = \beta$

$= (1 - 2i)(1 + 2i) = 1 + 4 = 5$

Gives, $\beta = 5$

$\therefore \alpha - \beta = -2 - 5 = -7$

Question126

The integer ' k ', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is
[2021, 25 Feb. Shift-1]

Options:

A. 3

B. 2

C. 0

D. 4

Answer: A

Solution:

Given, $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$

Here, $a > 0$

$\therefore D < 0$

$\Rightarrow [2(3k - 1)]^2 - 4(8k^2 - 7) < 0$

$\Rightarrow 4(9k^2 + 1 - 6k) - 4(8k^2 - 7) < 0$

$\Rightarrow k^2 - 6k + 8 < 0$

$\Rightarrow (k - 4)(k - 2) < 0$



$k \in (2, 4)$

$\therefore$ Required integer, $k = 3$

Question127

The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is
[2021, 26 Feb. Shift-1]

Answer: 3

Solution:

$$\begin{aligned}\text{Given, } x^3 - 2x^2 + 2x - 1 &= 0 \\ \text{i.e. } (x^3 - 1) - (2x^2 - 2x) &= 0 \\ \Rightarrow (x - 1)(x^2 + x + 1) - 2x(x - 1) &= 0 \\ \Rightarrow (x - 1)(x^2 + x + 1 - 2x) &= 0 \\ \Rightarrow (x - 1)(x^2 - x + 1) &= 0 \\ \therefore x = 1 \text{ and } x = \frac{-(-1) \pm \sqrt{1 - 4}}{2} &= \frac{1 \pm \sqrt{3}i}{2}\end{aligned}$$

$\therefore$ Roots are $1, -\omega_1 - \omega^2$.

$$\begin{aligned}\text{Then, sum of } 162^{\text{th}} \text{ power of the roots} &= (1)^{162} + (-\omega)^{162} + (-\omega^2)^{162} \\ &= 1 + \omega^{162} + \omega^{324} \\ &= 1 + (\omega^3)^{54} + (\omega^3)^{108} \\ &= 1 + (1)^{54} + (1)^{108} \quad [\because \omega^3 = 1] \\ &= 1 + 1 + 1 = 3\end{aligned}$$

Question128

Let a, b, c be in an arithmetic progression. Let the centroid of the triangle with vertices $(a, c), (2, b)$ and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is
[2021, 24 Feb. Shift-II]

Options:

- A. $\frac{71}{256}$
- B. $\frac{69}{256}$
- C. $-\frac{69}{256}$
- D. $-\frac{71}{256}$

Answer: D

Solution:

Given, a, b, c are in AP.

(a, c), (2, b), (a, b) are vertices of triangle.

$$\text{Centroid} = \left(\frac{10}{3}, \frac{7}{3} \right)$$

α and β are the roots of equation $ax^2 + bx + 1 = 0$

$\therefore$ a, b, c are in AP.

$$\therefore 2b = a + c$$

$$\text{Centroid} = \left(\frac{a + 2 + a}{3}, \frac{c + b + b}{3} \right)$$

$$= \left(\frac{2a + 2}{3}, \frac{c + 2b}{3} \right) = \left(\frac{10}{3}, \frac{7}{3} \right)$$

$$\Rightarrow \frac{2a + 2}{3} = \frac{10}{3} \text{ and } \frac{c + 2b}{3} = \frac{7}{3}$$

$$\Rightarrow a = 4$$

$$\Rightarrow c + a + c = 7 \quad [\because 2b = a + c]$$

$$\Rightarrow 2c = 7 - 4 \quad [\because a = 4]$$

$$c = 3/2$$

$$\text{Also, } 2b = a + c = 4 + \frac{3}{2}$$

$$\Rightarrow b = 11/4$$

Now, α and β are roots of $ax^2 + bx + 1 = 0$

$$\therefore \alpha + \beta = \frac{-b}{a} = \frac{-11/4}{4}$$

$$\Rightarrow \alpha + \beta = \frac{-11}{16}$$

$$\Rightarrow \alpha\beta = \frac{1}{a} = \frac{1}{4}$$

$$\Rightarrow \alpha\beta = \frac{1}{4}$$

Now, $\alpha^2 + \beta^2 - \alpha\beta$

$$= (\alpha + \beta)^2 - 3\alpha\beta$$

$$= \left(\frac{-11}{16} \right)^2 - 3 \times \frac{1}{4}$$

$$= \frac{121 - 192}{256} = \frac{-71}{256}$$

Question 129

The number of the real roots of the equation $(x + 1)^2 + |x - 5| = \frac{27}{4}$ is [2021, 24 Feb. Shift-II]

Answer: 2

Solution:

$$\text{Given, equation } (x + 1)^2 + |x - 5| = \frac{27}{4}$$

Case I For $x \geq 5$

$$|x - 5| \Rightarrow (x + 1)^2 + (x - 5) = \frac{27}{4}$$

$$\Rightarrow x^2 + 3x - 4 = \frac{27}{4}$$

$$\Rightarrow 4x^2 + 12x - 43 = 0$$

$$\therefore x = \frac{-12 \pm \sqrt{144 + 688}}{8}$$

$$= \frac{-12 \pm \sqrt{832}}{8} = \frac{-12 \pm 28.8}{8}$$

$$x = \frac{-3 \pm 7.2}{8}$$

$$x = \frac{-3 + 7.2}{8}, \frac{-3 - 7.2}{8}$$

Both the values are less than 5.

∴ No solution from here.

Case II $x < 5$

$$\Rightarrow (x+1)^2 + (x-5) = \frac{27}{4}$$

$$\Rightarrow x^2 + x - 6 - \frac{27}{4} = 0$$

$$\Rightarrow 4x^2 + 4x - 3 = 0$$

$$\Rightarrow x = \frac{-4 \pm \sqrt{16 + 48}}{8}$$

$$= \frac{-4 \pm 8}{8}$$

$$\Rightarrow x = \frac{-12}{8}, \frac{4}{8}, \text{ both are less than 5.}$$

∴ These values must be the solution. Hence, here 2 real roots are possible.

Question 130

If the least and the largest real values of α , for which the equation $z + \alpha |z - 1| + 2i = 0$ ($z \in \mathbb{C}$ and $i = \sqrt{-1}$) has a solution, are p and q respectively, then $4(p^2 + q^2)$ is equal to
[2021, 24 Feb. Shift-I]

Answer: 10

Solution:

Given, $\alpha_{\text{least}} = p$

$\alpha_{\text{max}} = q$

Equation given is $z + \alpha |z - 1| + 2i = 0$;

$z \in \mathbb{C}$ and $i = \sqrt{-1}$

Let $z = x + iy$

Then, $z + \alpha |z - 1| + 2i = 0$

$$\Rightarrow x + iy + \alpha \sqrt{(x-1)^2 + y^2} + 2i = 0$$

$$\Rightarrow (x + \alpha \sqrt{(x-1)^2 + y^2}) + i(y+2) = 0$$

$$\therefore y+2 = 0 \text{ and } x + \alpha \sqrt{(x-1)^2 + y^2} = 0$$

$$y = -2 \text{ and } x^2 = \alpha^2(x^2 + 1 - 2x + y^2)$$

$$x^2 = \alpha^2(x^2 - 2x + 5) \quad (\because y = -2)$$

$$\Rightarrow \alpha^2 = \frac{x^2}{x^2 - 2x + 5}$$

$$\therefore \alpha^2 \in \left[0, \frac{5}{4}\right]$$

$$\therefore \alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$$

$$\text{Now, } 4(p^2 + q^2) = 4[(\alpha_{\text{least}})^2 + (\alpha_{\text{max}})^2]$$

$$= 4\left[\left(-\frac{\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2\right]$$

$$= 4 \times \left[\frac{5}{4} + \frac{5}{4}\right] = 10$$

Question131

Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation:
[24-Feb-2021 Shift 1]

Options:

- A. $x^2 - 2x + 2 = 0$
- B. $x^2 - 2x + 8 = 0$
- C. $x^2 - 2x + 136 = 0$
- D. $x^2 - 2x + 16 = 0$

Answer: D

Solution:

Solution:

We have

$$(p^2 + q^2)^2 - 2p^2q^2 = 272$$

$$((p+q)^2 - 2pq)^2 - 2p^2q^2 = 272$$

$$(4 - 2pq)^2 - 2p^2q^2 = 272$$

$$16 - 16pq + 2p^2q^2 = 272$$

$$(pq)^2 - 8pq - 128 = 0$$

$$pq = \frac{8 \pm 24}{2} = 16, -8$$

$$\therefore pq = 16 \quad (\because p, q > 0)$$

$\therefore$ Required equation :

$$x^2 - (2)x + 16 = 0$$

Question132

If the equation $a|z|^2 + \overline{\alpha z} + \alpha z + d = 0$ represents a circle, where a, d are real constants, then which of the following condition is correct?
[2021, 18 March Shift-I]

Options:

- A. $|\alpha|^2 - ad \neq 0$
- B. $|\alpha|^2 - ad > 0$ and $a \in \mathbb{R} - \{0\}$
- C. $|\alpha|^2 - ad \geq 0$ and $a \in \mathbb{R}$
- D. $\alpha = 0, a, d \in \mathbb{R}^+$

Answer: B**Solution:****Solution:**

Given, $a|z|^2 + \overline{\alpha z} + \alpha \overline{z} + d = 0$

$$\Rightarrow a|z|^2 + \overline{\alpha z} + \alpha \overline{z} + d = 0 \quad \dots(i)$$

Putting $z = x + iy$ and $\alpha = p + iq$ in Eq. (i),

we get

$$a(x^2 + y^2) + (p + iq)(x - iy) + (p - iq)(x + iy) + d = 0$$

$$\Rightarrow (x + iy) + d = 0$$

$$a(x^2 + y^2) + px + qy - ipy + iqx + px + qy - iqx + ipy + d = 0$$

$$\Rightarrow a(x^2 + y^2) + 2px + 2qy + d = 0$$

$$\Rightarrow x^2 + y^2 + \left(\frac{2p}{a}\right)x + \left(\frac{2q}{a}\right)y + \frac{d}{a} = 0 \text{ be a circle}$$

$$\text{If } a \neq 0 \text{ and } r^2 = \left(\frac{p^2}{a^2} + \frac{q^2}{a^2} - \frac{d}{a}\right) > 0 \text{ If } a \neq 0 \text{ and } r^2 = \left(\frac{p^2}{a^2} + \frac{q^2}{a^2} - \frac{d}{a}\right) > 0$$

$$\Rightarrow p^2 + q^2 - ad > 0$$

$$\Rightarrow |\alpha|^2 - ad > 0$$

$$\text{and } a \in \mathbb{R} - \{0\}$$

Question 133

Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is
[2021, 18 March Shift-I]

Answer: 6**Solution:****Solution:**

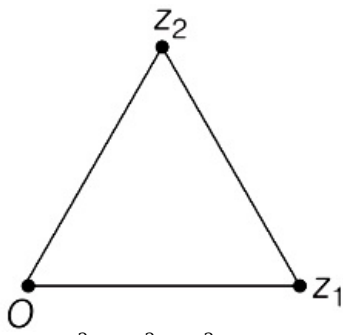
Given, z_1, z_2 are the roots of

$$z^2 + az + 12 = 0$$

$$\therefore z_1 + z_2 = \frac{-a}{1} = -a$$

$$\text{and } z_1 z_2 = \frac{12}{1} = 12$$

Now, z_1, z_2 and origin forms an equilateral triangle.



$$\begin{aligned}
 \text{I c } \therefore z_1^2 + z_2^2 + 0^2 &= z_1 z_2 + 0 + 0 \\
 \Rightarrow z_1^2 + z_2^2 &= z_1 z_2 \\
 \Rightarrow z_1^2 + z_2^2 + 2z_1 z_2 &= z_1 z_2 + 2z_1 z_2 \\
 \Rightarrow (z_1 + z_2)^2 &= 3z_1 z_2 \\
 \Rightarrow (-a)^2 &= 3 \times (12) \\
 \Rightarrow a^2 = 36 \Rightarrow |a|^2 &= 36 \\
 \Rightarrow |a| &= \pm 6 \\
 \text{But } |a| &\geq 0 \\
 \therefore |a| &= 6
 \end{aligned}$$

Question134

Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w , is equal to [2021, 18 March Shift-III]

Options:

A. 4

B. $\frac{1}{2}$

C. $\frac{1}{4}$

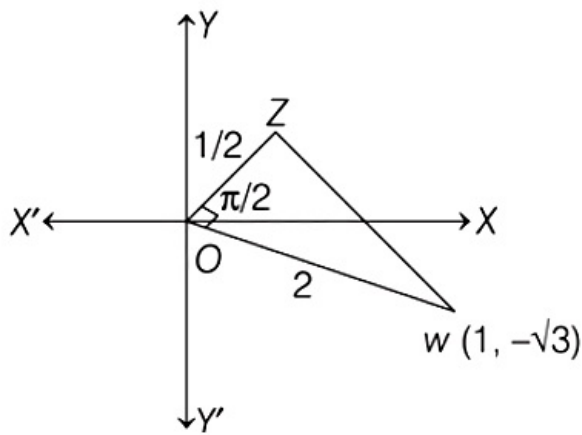
D. 2

Answer: B

Solution:

Solution:

$$\begin{aligned}
 \text{Given, } w &= 1 - \sqrt{3}i \\
 \Rightarrow |w| &= \sqrt{(1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2 \\
 \text{and } |zw| &= 1 \Rightarrow |z| |w| = 1 \\
 \Rightarrow |z| &= \frac{1}{|w|} = \frac{1}{2}
 \end{aligned}$$



$$\therefore \text{Area of } \Delta = \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{2}$$

Question 135

Let S_1 , S_2 and S_3 be three sets defined as

$$S_1 = \{z \in \mathbb{C} : |z - 1| \leq \sqrt{2}\}$$

$$S_2 = \{z \in \mathbb{C} : \operatorname{Re}[(1 - i)z] \geq 1\}$$

$$S_3 = \{z \in \mathbb{C} : \operatorname{Im}(z) \leq 1\}$$

Then, the set $S_1 \cap S_2 \cap S_3$

[2021, 17 March Shift-II]

Options:

- A. is a singleton
- B. has exactly two elements
- C. has infinitely many elements
- D. has exactly three elements

Answer: C

Solution:

Solution:

For $|z - 1| \leq \sqrt{2}$, ... (i)

z lies on and inside the circle of radius $\sqrt{2}$ units and centre $(1, 0)$.

For S_2 , let $z = x + iy$

$$\text{Now } (1 - i)(z) = (1 - i)(x + iy)$$

$$= x + iy - ix + y = (x + y) + i(y - x)$$

$$\therefore \operatorname{Re}[(1 - i)z] = (x + y), \text{ which is greater than or equal to one.}$$

$$\text{i.e., } x + y \geq 1 \quad \dots\dots\dots (ii)$$

Also, for S_3

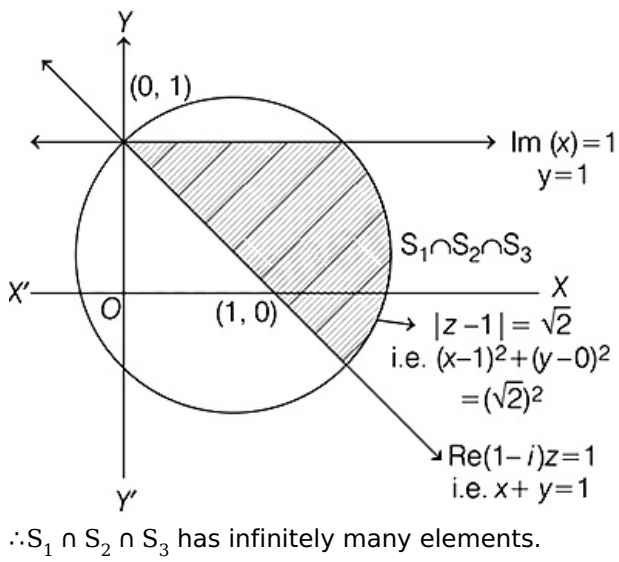
$$\text{Let } z = x + iy$$

$$\therefore \operatorname{Im}(z) = y, \text{ which is less than or equal to}$$

one.

$$\text{i.e., } y \leq 1 \quad \dots\dots\dots (iii)$$

Concept Draw the graph of Eqs. (i), (ii) and (iii) and then select the common region bounded by Eqs. (i), (ii) and (iii) for $S_1 \cap S_2 \cap S_3$.



Question136

The area of the triangle with vertices $A(z)$, $B(iz)$ and $C(z + iz)$ is [2021, 17 March Shift-I]

Options:

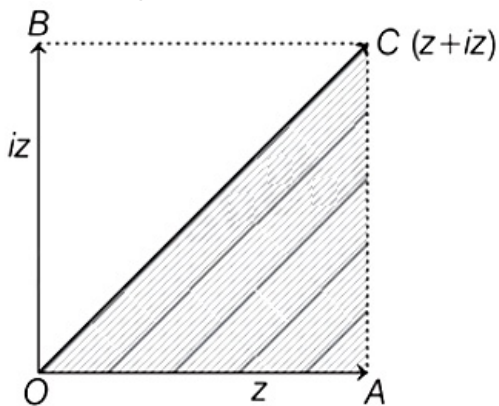
- A. 1
- B. $\frac{1}{2} |z|^2$
- C. $\frac{1}{2}$
- D. $\frac{1}{2} |z + iz|^2$

Answer: B

Solution:

Solution:

Area of triangle whose vertices are $A(z)$, $B(iz)$, $C(z + iz)$



Area of the triangle

$$= \frac{1}{2} |z| |iz| = \frac{1}{2} |z|^2$$

Question137

The value of $4 + \frac{1}{5 + \frac{1}{4 + \frac{1}{5 + \frac{1}{4 + \dots \infty}}}}$ is

[2021, 17 March Shift-I]

Options:

A. $2 + \frac{2}{5}\sqrt{30}$

B. $2 + \frac{4}{\sqrt{5}}\sqrt{30}$

C. $4 + \frac{4}{\sqrt{5}}\sqrt{30}$

D. $5 + \frac{2}{5}\sqrt{30}$

Answer: A

Solution:

Solution:

$$\text{Let } x = 4 + \frac{1}{5 + \frac{1}{4 + \frac{1}{5 + \frac{1}{4 + \dots \infty}}}}$$

$$x = 4 + \frac{1}{5 + \frac{1}{x}}$$

$$\Rightarrow (x - 4)(5x + 1) = x$$

$$\Rightarrow 5x^2 - 19x - 4 = x$$

$$\Rightarrow 5x^2 - 20x - 4 = 0$$

$$\Rightarrow x = \frac{20 \pm \sqrt{400 + 80}}{10}$$

$$\Rightarrow x = \frac{20 \pm \sqrt{480}}{10}$$

$$\Rightarrow x = 2 \pm \sqrt{\frac{480}{100}}$$

$$= 2 \pm \frac{2}{5}\sqrt{30}$$

$\therefore x \neq 0$

$$\text{So, } x = 2 + \frac{2}{5}\sqrt{30}$$

Question138

The number of elements in the set $\{x \in \mathbb{R} : (|x| - 3) | x + 4 | = 6\}$ is equal to

[2021, 16 March Shift-1]

Options:

A. 3

B. 2

C. 4

D. 1

Answer: B**Solution:****Solution:**Given, set = $\{x \in \mathbb{R} : (|x| - 3) |x + 4| = 6\}$

As, we already know

$$|x| = \begin{cases} x_1 & x \geq 0 \\ -x_1 & x < 0. \end{cases} \text{ and}$$

$$|x + 4| = \begin{cases} x + 4 & x \geq -4 \\ -(x + 4) & x < -4. \end{cases}$$

Case I $x < -4$

$$r(-x - 3)(-x - 4) = 6$$

$$(x + 3)(x + 4) = 6$$

$$x^2 + 7x + 12 = 6$$

$$x^2 + 7x + 6 = 0$$

$$(x + 6)(x + 1) = 0$$

$$x = -6 \text{ or } x = -1$$

We will reject $x = -1$ as, $-1 > -4$ $\therefore$ When $x < -4$, $x = -6$ is the solution.**Case II** $-4 \leq x < 0$

$$(-x - 3)(x + 4) = 6$$

$$\Rightarrow -(x + 3)(x + 4) = 6$$

$$\Rightarrow -(x^2 + 7x + 12) = 6$$

$$\Rightarrow x^2 + 7x + 18 = 0$$

As, the discriminant of this quadratic

$$\text{equation is } D = 7^2 - 4 \cdot 18 = 49 - 72 = -23$$

$$\therefore D = -23 \text{ and } D < 0$$

So, no real roots and as per the question,

 $x \in \mathbb{R}$.No solution when $-4 \leq x < 0$.**Case III** $x \geq 0$

$$(|x| - 3) |x + 4| = 6$$

$$\Rightarrow (x - 3)(x + 4) = 6$$

$$\Rightarrow x^2 + x - 12 = 6$$

$$\Rightarrow x^2 + x - 18 = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 72}}{2} = \frac{-1 \pm \sqrt{73}}{2}$$

$$\text{We will reject } x = \frac{-1 - \sqrt{73}}{2} \text{ as } \frac{-1 - \sqrt{73}}{2} < 0 \text{ and here, } x \geq 0.$$

$$\text{So, } x = \frac{-1 + \sqrt{73}}{2}, \text{ when } x \geq 0.$$

$$\therefore x = -6 \text{ and } x = \frac{-1 + \sqrt{73}}{2}$$

are the two solutions which belong to the set.

Hence, number of solutions = 2

Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients, such that $\int_0^1 P(x) dx = 1$ and $P(x)$ leaves remainder 5 when it is divided by $(x - 2)$. Then, the value of $g(b + c)$ is
[2021, 16 March Shift-II]

Options:

- A. 9
- B. 15
- C. 7
- D. 11

Answer: C

Solution:

Solution:

$$P(x) = x^2 + bx + c$$

$$\Rightarrow \int_0^1 (x^2 + bx + c) dx = 1$$

$$\Rightarrow \left[\frac{x^3}{3} + \frac{bx^2}{2} + cx \right]_0^1 = 1$$

$$\Rightarrow \frac{1}{3} + \frac{b}{2} + c = 1$$

$$\Rightarrow b + 2c = 4/3 \quad \dots\dots (i)$$

$$\text{And, } P(x) = (x - 2) \cdot Q(x) + 5$$

$$\text{When, } x = 2$$

$$P(2) = 5$$

$$4 + 2b + c = 5$$

$$c = 1 - 2b \quad \dots\dots (ii)$$

$$\text{Putting } c = 1 - 2b \text{ in Eq. (i),}$$

$$b + 2(1 - 2b) = 4/3$$

$$\Rightarrow -3b + 2 = 4/3$$

$$\Rightarrow b = 2/9$$

$$\therefore c = 1 - 4/9 = 5/9$$

$$9(b + c) = 9 \left(\frac{2}{9} + \frac{5}{9} \right) = 7$$

Question140

Let z and w be two complex numbers, such that $w = \overline{z\overline{z}} - 2z + 2$,

$\left| \frac{z+i}{z-3i} \right| = 1$ and $\text{Re}(w)$ has minimum value. Then, the minimum value of

$n \in \mathbb{N}$ for which w^n is real, is equal to

[2021, 16 March Shift-1]

Answer: 4

Solution:

Solution:

Given, $w = \bar{z}z - 2z + z$

$$\left| \frac{z+i}{z-3i} \right| = 1$$

$$\Rightarrow |z+i| = |z-3i|$$

Let $z = x + iy$

$$\Rightarrow |x + i(y+1)| = |x + i(y-3)|$$

$$\Rightarrow x^2 + (y+1)^2 = x^2 + (y-3)^2$$

$$\Rightarrow 2y + 1 = -6y + 9$$

$$\therefore y = 1$$

Now, $w = \bar{z}z - 2z + 2$

$$w = |z|^2 - 2z + 2$$

$$\Rightarrow w = x^2 + y^2 - 2(x + iy) + 2$$

$$\Rightarrow w = (x^2 + y^2 - 2x + 2) + i(-2y)$$

$$\Rightarrow w = (x^2 + 1 - 2x + 2) + i(-2)$$

$$w = (x-1)^2 + 2 - 2i$$

Re(w) has minimum value.

So, $(x-1)^2 + 2$ is minimum when $x = 1$

$$\therefore w = 2 - 2i$$

$$= 2(1 - i)$$

$$= 2\sqrt{2} \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)$$

$$w = 2\sqrt{2}e^{-i\pi/4}$$

$$\text{Now, } w^n = (2\sqrt{2})^n e^{\frac{-in\pi}{4}}$$

$$= (2\sqrt{2})^n \left[\cos\left(\frac{n\pi}{4}\right) - i \sin\left(\frac{n\pi}{4}\right) \right]$$

This has to be zero for w^n to be real.

$$\text{So, } \sin\left(\frac{n\pi}{4}\right) = 0$$

$$\Rightarrow \frac{n\pi}{4} = 0, \pi, 2\pi, 3\pi \dots$$

$$\Rightarrow n = 0, 4, 8, 12 \dots$$

The minimum value of n is $4(n \in \mathbb{N})$.

Question 141

The least value of $|z|$, where z is a complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{|z|+1} \log_e 2\right) \geq \log_{\sqrt{2}} |5\sqrt{7} + 9i|,$$

$i = \sqrt{-1}$, is equal to:

[2021, 16 March Shift-II]

Options:

A. 3

B. $\sqrt{5}$

C. 2

D. 8

Answer: A

Solution:

Solution:

$$\begin{aligned} \exp \left[\frac{(|z|+3)(|z|-1)}{(|z|+1)} \times \log_e 2 \right] &\geq \log_{\sqrt{2}} |5\sqrt{7} + 9i| \\ \exp \left[\frac{(|z|+3)(|z|-1)}{(|z|+1)} \times \log_e 2 \right] &\geq \log_{\sqrt{2}} 16 \\ \Rightarrow \log_{\sqrt{2}} |5\sqrt{7} + 9i| & \\ \Rightarrow \frac{(|z|+3)(|z|-1)}{(|z|+1)} &\geq 3 \\ \Rightarrow |z|^2 - 3|z| + 3 & \\ \Rightarrow |z|^2 - 3|z| + 3 &\geq 0 \\ \Rightarrow |z| - 3 &\leq 0 \\ \Rightarrow |z| &\leq 3 \end{aligned}$$

Question 142

If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to [2021, 18 March Shift-II]

Answer: 0

Solution:

Solution:

Method (1)

Given, $P(x) = f(x^3) + xg(x^3)$ (i)

$\therefore P(1) = f(1) + g(1)$ (ii)

Given, $P(x)$ is divisible by $(x^2 + x + 1)$.

$\therefore P(x) = Q(x) \cdot (x^2 + x + 1)$

As, we know that ω and ω^2 are non-real cube roots of unity and this is also root

of $x^2 + x + 1 = 0$

$\therefore P(\omega) = P(\omega^2) = 0$

As, we know that ω and ω^2 are non-real cube roots of unity and this is also root of $x^2 + x + 1 = 0$

$\therefore P(\omega) = P(\omega^2) = 0$... (iii)

From Eq. (i),

$P(\omega) = f(\omega^3) + \omega[g(\omega^3)] = 0$ [from Eq. (iii)]

$\Rightarrow f(1) + \omega g(1) = 0$... (iv)

and $P(\omega^2) = 0$ [from Eq. (iii)]

$\Rightarrow f(\omega^6) + \omega^2 \cdot g(\omega^6) = 0$

$\Rightarrow f(1) + \omega^2 g(1) = 0$ (v)

Now, adding Eqs. (iv) and (v), we get

$2f(1) + (\omega + \omega^2)g(1) = 0$

$\Rightarrow 2f(1) - 1g(1) = 0$ ($\because 1 + \omega + \omega^2 = 0$)

$\Rightarrow 2f(1) = g(1)$... (vi)

Subtracting Eq. (iv) from Eq. (v), we get

$0 + (\omega - \omega^2)g(1) = 0$

$\Rightarrow g(1) = 0$

$f(1) = \frac{g(1)}{2} = \frac{0}{2}$ [from Eq. (vi)]

$\Rightarrow f(1) = 0$

From Eq. (ii), $P(1) = f(1) + g(1) = 0 + 0 = 0$

Method (2)

$\because P(\omega) = 0$

$\Rightarrow f(1) + \omega g(1) = 0$

$\Rightarrow f(1) + \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)g(1) = 0$

$$\Rightarrow \left(f(1) - \frac{g(1)}{2} \right) + i \left(\frac{\sqrt{3}}{2} g(1) \right) = 0$$

On comparing real and imaginary parts from both sides, we have

$$\text{I.I } f(1) - \frac{g(1)}{2} = 0, \quad \frac{\sqrt{3}}{2} g(1) = 0$$

$$\Rightarrow f(1) = \frac{g(1)}{2}, \quad \Rightarrow g(1) = 0$$

$$\therefore f(1) = \frac{0}{2} = 0$$

$$\therefore P(1) = f(1) + g(1) = 0 + 0 = 0$$

Question 143

The value of $3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$

is equal to

[2021, 18 March shift-I]

Options:

A. $1.5 + \sqrt{3}$

B. $2 + \sqrt{3}$

C. $3 + 2\sqrt{3}$

D. $4 + \sqrt{3}$

Answer: A

Solution:

Solution:

$$\text{Let } x = 3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$$

$$\text{So, } x = 3 + \frac{1}{4 + \frac{1}{x}} = 3 + \frac{1}{\frac{4x+1}{x}} = 3 + \frac{x}{4x+1}$$

$$\Rightarrow (x-3) = \frac{x}{4x+1}$$

$$\Rightarrow (4x+1)(x-3) = x$$

$$\text{I.I } \Rightarrow 4x^2 - 12x - 3 = 0$$

$$\Rightarrow x = \frac{3 \pm 2\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}$$

But from above, $x > 0$

$\therefore$ Only positive value of x is accepted

$$\therefore x = 1.5 + \sqrt{3}$$

Question 144

Let C be the set of all complex numbers. Let

$$S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\},$$

$$S_2 = \{z \in C \mid \operatorname{Re}(z) \geq 5\} \text{ and}$$

$$S_3 = \{z \in C \mid |z - \bar{z}| \geq 8\}.$$

Then, the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to

[2021, 27 July Shift-1]

Options:

A. 1

B. 0

C. 2

D. Infinite

Answer: A

Solution:

Solution:

$$S_1: |z - 3 - 2i|^2 = 8$$

$$\Rightarrow |(x + iy) - (3 + 2i)|^2 = 8$$

$$\Rightarrow |(x - 3) + i(y - 2)|^2 = 8$$

$$\Rightarrow (x - 3)^2 + (y - 2)^2 = 8$$

$$S_2: \operatorname{Re}(z) \geq 5$$

$$x \geq 5$$

$$S_3: |z - \bar{z}| \geq 8$$

$$|(x + iy) - (x - iy)| \geq 8$$

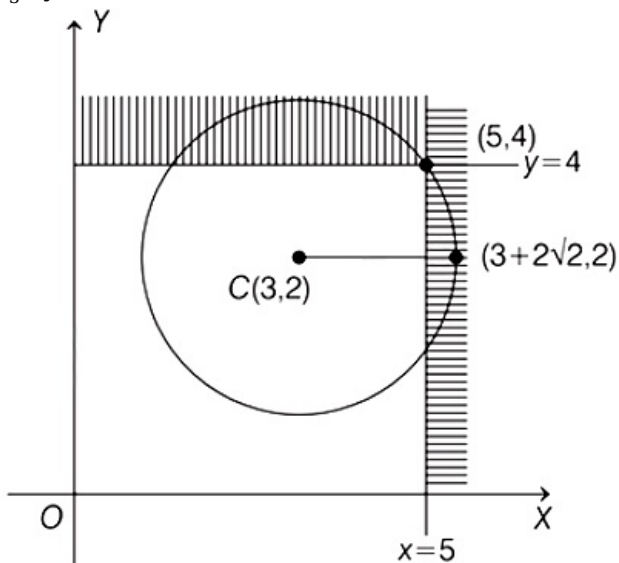
$$\Rightarrow 2y \geq 8$$

$$\Rightarrow y \geq 4$$

$$S_1: (x - 3)^2 + (y - 2)^2 = 8$$

$$S_2: x \geq 5$$

$$S_3: y \geq 4$$



Circle passes through $(5, 4)$ as shown in the figure.

$\Rightarrow$ There is exactly one point $(5, 4)$ in $S_1 \cap S_2 \cap S_3$.

The point $P(a, b)$ undergoes the following three transformations successively

(A) Reflection about the line $y = x$.

(B) Translation through 2 units along the positive direction of X-axis.

(C) Rotation through angle $\frac{\pi}{4}$

about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$,

then the value of $2a + b$ is equal to

[2021, 27 July Shift-II]

Options:

A. 13

B. 9

C. 5

D. 7

Answer: B

Solution:

Solution:

The image of $P(a, b)$ along $y = x$ is $Q(b, a)$. Translating it 2 units along the positive direction of X-axis, it becomes $R(b + 2, a)$. Then, rotation through $\frac{\pi}{4}$ about the origin in the anticlockwise direction, the final position of the point P is

$$\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right).$$

Now, applying rotational theorem,

$$-\frac{1}{\sqrt{2}} + \frac{7}{\sqrt{2}}i = [(b + 2) + ai] \cdot \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$\Rightarrow -\frac{1}{\sqrt{2}} + \frac{7}{\sqrt{2}}i = \left(\frac{b + 2}{\sqrt{2}} - \frac{a}{\sqrt{2}}\right) + i\left(\frac{b + 2}{\sqrt{2}} + \frac{a}{\sqrt{2}}\right)$$

$$\Rightarrow -\frac{1}{\sqrt{2}} + \frac{7}{\sqrt{2}}i = \left(\frac{b - a + 2}{\sqrt{2}}\right) + i\left(\frac{a + b + 2}{\sqrt{2}}\right)$$

$$\text{|| So, } \frac{b - a + 2}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow b - a = -3 \quad \dots\dots (i)$$

$$\text{and } \frac{a + b + 2}{\sqrt{2}} = \frac{7}{\sqrt{2}}$$

$$\Rightarrow a + b = 5 \quad \dots\dots (ii)$$

Adding Eqs. (i) and (ii),

$$2b = 2 \Rightarrow b = 1$$

Substitute the value of b in Eq. (ii), $a = 4$

$$\text{Now, } 2a + b = 2 \times 4 + 1 = 9$$

Question 146

Let C be the set of all complex numbers.

Let $S_1 = \{z \in C : |z - 2| \leq 1\}$ and $S_2 = \{z \in C : z(1 + i) + \bar{z}(1 - i) \geq 4\}$.

Then, the maximum value of $|z - \frac{5}{2}|^2$ for $z \in S_1 \cap S_2$ is equal to

Options:

A. $\frac{3 + 2\sqrt{2}}{4}$

B. $\frac{5 + 2\sqrt{2}}{2}$

C. $\frac{3 + 2\sqrt{2}}{2}$

D. $\frac{5 + 2\sqrt{2}}{4}$

Answer: D**Solution:****Solution:**

Let $S_1 = \{z \in \mathbb{C} : |z - 2| \leq 1\}$

and $S_2 = \{z \in \mathbb{C} : z(1 + i) + \bar{z}(1 - i) \geq 4\}$

Now $|z - 2| \leq 1$

Let $z = x + iy$

$\Rightarrow |x + iy - 2| \leq 1$

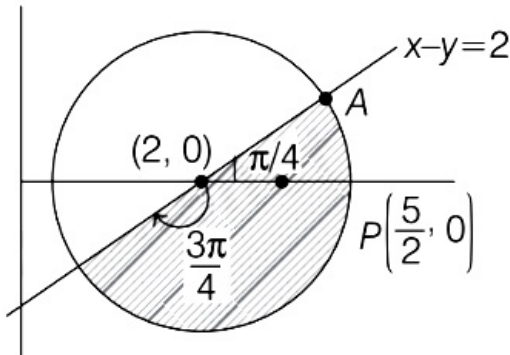
$\Rightarrow (x - 2)^2 + y^2 \leq 1$

Also, $z(1 + i) + \bar{z}(1 - i) \geq 4$

$\Rightarrow (x + iy)(1 + i) + (x - iy)(1 - i) \geq 4$

$\Rightarrow 2x - 2y \geq 4$

$\Rightarrow x - y \geq 2$

Let point on circle be $A(2 + \cos \theta, \sin \theta)$,

$\theta \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

$(AP)^2 = \left(2 + \cos \theta - \frac{5}{2}\right)^2 + \sin^2 \theta$

$\Rightarrow (AP)^2 = \cos^2 \theta + \frac{1}{4} - \cos \theta + \sin^2 \theta$

$\Rightarrow (AP)^2 = \frac{5}{4} - \cos \theta$

For $(AP)^2$ to be maximum, $\theta = -\frac{3\pi}{4}$

$\Rightarrow (AP)^2 = \frac{5}{4} + \frac{1}{\sqrt{2}}$

$\Rightarrow (AP)^2 = \frac{5 + 2\sqrt{2}}{4}$

Question 147Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then,

**$\alpha^8 + \beta^8$ is equal to
[2021, 27 July Shift-I]**

Options:

- A. 10
- B. 50
- C. 100
- D. 160

Answer: B

Solution:

Solution:

$$x^2 + (20)^{\frac{1}{4}} \cdot x + (5)^{\frac{1}{2}} = 0.$$

roots α & β .

$$\alpha + \beta = -(20)^{\frac{1}{4}}$$

$$\alpha\beta = (5)^{\frac{1}{2}}.$$

$$\alpha^8 + \beta^8 = (\alpha^4)^2 + (\beta^4)^2$$

$$= (\alpha^4 - \beta^4)^2 + 2(\alpha\beta)^4. \dots\dots (i)$$

$$\Rightarrow (\alpha + \beta)^2 = (\alpha^2 + \beta^2) + 2\alpha\beta.$$

$$\Rightarrow (20)^{\frac{1}{2}} = (\alpha^2 + \beta^2) + 2 \cdot 5^{\frac{1}{2}}$$

$$\Rightarrow 2 \cdot (5)^{\frac{1}{2}} = (\alpha^2 + \beta^2) + 2 \cdot 5^{\frac{1}{2}}$$

$$\Rightarrow 0 = (\alpha^2 + \beta^2)$$

From eqn (1)

$$\alpha^8 + \beta^8 = ((\alpha^2 + \beta^2) \cdot (\alpha^2 - \beta^2))^2 + 2 \cdot (5)^{1/2}$$

$$= 0 + 2 \times 5^2$$

$$= 2 \times 25$$

$$= 50 \text{ (Ans)}$$

Question 148

**The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to
[2021, 27 July Shift-II]**

Answer: 2

Solution:

Given equation,

$$e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$$

$$\text{Let } e^x = t > 0$$

$$t^4 - t^3 - 4t^2 - t + 1 = 0$$

$$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$$

$$\Rightarrow t^2 + \frac{1}{t^2} + 2 - \left(t + \frac{1}{t}\right) - 6 = 0$$

$$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 6 = 0$$

$$\text{Let } \alpha = t + \frac{1}{t} \geq 2$$

$$\text{I.C.} \Rightarrow \alpha^2 - \alpha - 6 = 0$$

$$\Rightarrow \alpha^2 - 3\alpha + 2\alpha - 6 = 0$$

$$\Rightarrow \alpha(\alpha - 3) + 2(\alpha - 3) = 0$$

$$\Rightarrow (\alpha - 3)(\alpha + 2) = 0$$

$$\Rightarrow \alpha = 3 \text{ or } \alpha = -2 \quad (\text{not possible})$$

$$\Rightarrow t + \frac{1}{t} = 3$$

$$\Rightarrow t^2 - 3t + 1 = 0$$

$$\therefore \text{The number of real roots} = 2$$

Question 149

The number of real roots of the equation

$$e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0 \text{ is}$$

[2021, 25 July Shift-1]

Options:

A. 2

B. 4

C. 6

D. 1

Answer: A

Solution:

Solution:

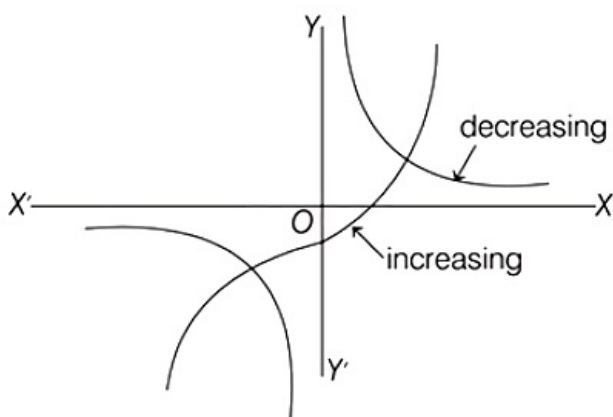
$$e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$$

$$\Rightarrow (e^{3x} - 1)^2 - e^x(e^{3x} - 1) = 12e^{2x}$$

$$\Rightarrow (e^{3x} - 1)(e^{3x} - e^x - 1) = 12e^{2x}$$

$$\Rightarrow (e^{3x} - 1)(e^x - e^{-x} - e^{-2x}) = 12$$

$$\Rightarrow e^x - e^{-x} - e^{-2x} = \frac{12}{e^{3x} - 1}$$



Hence, the number of real roots is 2.

Question150

If α, β are roots of the equation.

$$x^2 + 5(\sqrt{2})x + 10 = 0, \alpha > \beta \text{ and}$$

$P_n = \alpha^n - \beta^n$ for each positive

integer n , then the value of

$$\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} \right) \text{ is equal to}$$

[2021, 25 July Shift-1]

Answer: 1

Solution:

$$x^2 + 5\sqrt{2}x + 10 = 0$$

$$P_n = \alpha^n - \beta^n$$

$$\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} = \frac{P_{17}(5\sqrt{2}P_{19} + P_{20})}{P_{18}(5\sqrt{2}P_{18} + P_{19})}$$

$$\Rightarrow x^{18}(x^2 + 5\sqrt{2}x + 10) = 0$$

$$\Rightarrow x^{20} + 5\sqrt{2}x^{19} + x^{18} = 0$$

$$(\alpha^{20} - \beta^{20}) + 5\sqrt{2}(\alpha^{19} - \beta^{19}) + (\alpha^{18} - \beta^{18}) = 0$$

$$P_{20} + 5\sqrt{2}P_{19} + P_{18} = 0$$

Similarly,

$$P_{19} + 5\sqrt{2}P_{18} + P_{17} = 0$$

$$\text{So, } \frac{P_{17}(5\sqrt{2}P_{19} + P_{20})}{P_{18}(5\sqrt{2}P_{18} + P_{19})} = \frac{P_{17}(-P_{18})}{P_{18}(-P_{17})} = 1$$

Question151

The number of real solutions of the equation $x^2 - |x| - 12 = 0$ is

[2021, 25 July Shift-II]

Options:

A. 2

B. 3

C. 1

D. 4

Answer: A

Solution:

Solution:

Given equation,

$$cx^2 - |x| - 12 = 0$$

$$\Rightarrow |x|^2 - |x| - 12 = 0$$

$$\Rightarrow |x|^2 - 4|x| + 3|x| - 12 = 0$$

$$\Rightarrow (|x| - 4)(|x| + 3) = 0$$

$$\text{So } |x| - 4 = 0 \text{ or } |x| + 3 = 0$$

$$|x| = 4 \text{ or } |x| = -3 \text{ (not possible)}$$

$$x = \pm 4$$

Hence, the number of real solutions = 2

Question152

Let $[x]$ denote the greatest integer less than or equal to x . Then, the values of $x \in \mathbb{R}$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval

[2021, 22 July Shift-II]

Options:

A. $\left[0, \frac{1}{e}\right)$

B. $[\log_e 2, \log_e 3)$

C. $[1, e)$

D. $[0, \log_e 2)$

Answer: D

Solution:

Solution:

$$[e^x]^2 + [e^x + 1] - 3 = 0$$

$$\Rightarrow [e^x]^2 + [e^x] + 1 - 3 = 0$$

$$\Rightarrow [e^x]^2 + [e^x] - 2 = 0$$

$$\Rightarrow ([e^x] - 1)([e^x] + 2) = 0$$

$$[e^x] = 1 \text{ or } [e^x] = -2$$

Not possible as $e^x > 0$.

$$\Rightarrow [e^x] = 1$$

$$\Rightarrow 1 \leq e^x < 2$$

$$\Rightarrow 0 \leq x < \log_e 2$$

Question153

If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of

$\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to

[2021, 20 July Shift-1]

Options:

A. 56×3^{25}

B. 56×3^{24}

C. 52×3^{24}

D. 28×3^{25}

Answer: C

Solution:

Solution:

$$\begin{aligned} x^2 + 3\frac{1}{4}x + 3\frac{1}{2} &= 0 \\ \therefore x &= \frac{-3^{1/4} \pm \sqrt{3^{1/2} - 4 \cdot 3^{1/2}}}{2} \\ &= \frac{3^{1/4}(-1 \pm \sqrt{3}i)}{2} \\ &= 3^{1/4} \left(\frac{-1 + \sqrt{3}i}{2} \right) \text{ or } 3^{1/4} \left(\frac{-1 - \sqrt{3}i}{2} \right) \\ &= 3^{1/4}\omega \text{ or } 3^{1/4}\omega^2 \end{aligned}$$

$$\begin{aligned} &= 3^{1/4}\omega \text{ or } 3^{1/4}\omega^2 \\ \text{Now, } \alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1) \\ &= \alpha^{108} - \alpha^{96} + \beta^{108} - \beta^{96} \\ &= (\alpha^{108} + \beta^{108}) - (\alpha^{96} + \beta^{96}) \\ &= \{(3^{1/4}\omega)^{108} + (3^{1/4}\omega^2)^{108}\} \\ &\quad - \{(3^{1/4}\omega)^{96} + (3^{1/4}\omega^2)^{96}\} \\ &= 3^{27}(\omega^{108} + \omega^{216}) - 3^{24}(\omega^{96} + \omega^{192}) \\ &= 3^{27}(2) - 3^{24}(2) = 3^{24}(54) - 3^{24}(2) \\ &= 3^{24}(52) = 52 \times 3^{24} \end{aligned}$$

Question154

The number of solutions of the equation

$$\log_{(x+1)}(2x^2 + 7x + 5) +$$

$$\log_{(2x+5)}(x+1)^2 - 4 = 0$$

$x > 0$, is

[2021, 20 July Shift-II]

Answer: 1

Solution:

$$\begin{aligned} &\log_{(x+1)}(2x^2 + 7x + 5) \\ &+ \log_{(2x+5)}(x+1)^2 - 4 = 0 \\ &= \log_{(x+1)}\{(2x+5)(x+1)\} \\ &+ 2\log_{(2x+5)}(x+1) - 4 = 0 \end{aligned}$$

$$\begin{aligned}
&= \log_{(x+1)}(2x+5) + \log_{(x+1)}(x+1) \\
&+ 2\log_{(2x+5)}(x+1) - 4 = 0 \\
&= \log_{(x+1)}(2x+5) + 2\log_{(2x+5)}(x+1) - 3 = 0 \\
&[\because \log_a a = 1]
\end{aligned}$$

$$= \log_{(x+1)}(2x+5) + 2 \frac{\log_{(x+1)}(x+1)}{\log_{(x+1)}(2x+5)} = 3$$

$$\text{Let } \log_{(x+1)}(2x+5) = t$$

$$t + \frac{2}{t} = 3 \Rightarrow t^2 - 3t + 2 = 0$$

$$(t-1)(t-2) = 0$$

$$\Rightarrow t = 1, t = 2$$

$$\Rightarrow \log_{(x+1)}(2x+5) = 1 \text{ and}$$

$$\log_{(x+1)}(2x+5) = 2$$

$$2x+5 = (x+1)$$

$$\text{and } 2x+5 = (x+1)^2$$

$$x = -4$$

$$\text{and } 2x+5 = x^2 + 1 + 2x$$

$$\text{i.e., } x^2 = 4$$

$$\Rightarrow x = +2, -2$$

Given, $x > 0$

$x = -4, x = -2$ are discarded

$\therefore x = 2$ is only solution.

Question 155

If the real part of the complex number $z = \frac{3 + 2i \cos \theta}{1 - 3i \cos \theta}$, $\theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2 \theta$ is equal to
[2021, 27 July Shift-11]

Answer: 1

Solution:

Solution:

We have,

$$\begin{aligned}
\operatorname{Re} z &= \frac{3 + 2i \cos \theta}{1 - 3i \cos \theta} = \frac{3 + 2i \cos \theta}{1 - 3i \cos \theta} \times \frac{1 + 3i \cos \theta}{1 + 3i \cos \theta} \\
&= \frac{(3 - 6 \cos^2 \theta) + i(9 \cos \theta + 2 \cos \theta)}{1 + 9 \cos^2 \theta}
\end{aligned}$$

$$z = \frac{(3 - 6 \cos^2 \theta) + (11 \cos \theta)i}{1 + 9 \cos^2 \theta}$$

Given, $\operatorname{Re}(z) = 0$

$$\Rightarrow \frac{3 - 6 \cos^2 \theta}{1 + 9 \cos^2 \theta} = 0$$

$$\Rightarrow 3 - 6 \cos^2 \theta = 0$$

$$\Rightarrow \cos^2 \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4} \left\{ \theta \in \left(0, \frac{\pi}{2}\right) \right\}$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} = 1$$

Hence, $\sin^2 3\theta + \cos^2 \theta$

$$= \sin^2 \frac{3\pi}{4} + \cos^2 \frac{\pi}{4} = \frac{1}{2} + \frac{1}{2} = 1$$

Question156

Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then, the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to
[2021, 22 July Shift-11]

Options:

A. 1

B. $\frac{4}{3}$

C. $\frac{3}{2}$

D. 2

Answer: B

Solution:

Solution:

$$z^2 + 3z = 0$$

$$z = x + iy$$

$$\Rightarrow (x^2 - y^2) + i(2xy) + 3(x - iy) = 0$$

$$\Rightarrow (x^2 - y^2 + 3x) + i(2xy - 3y) = 0$$

$$\begin{cases} x^2 - y^2 + 3x = 0 \\ y(2x - 3) = 0. \end{cases}$$

$$y = 0 \text{ or } x = \frac{3}{2}$$

$$\text{If } y = 0,$$

$$\Rightarrow x(x + 3) = 0$$

$$\Rightarrow x = 0, -3$$

$$\Rightarrow \text{So, } (0, 0) \text{ and } (-3, 0) \text{ are solutions, when}$$

$$y = 0.$$

$$\text{When } x = \frac{3}{2}, \quad \frac{9}{4} - y^2 + \frac{9}{2} = 0 \Rightarrow y^2 = \frac{27}{4}$$

$$\Rightarrow y = \pm \frac{3\sqrt{3}}{2}$$

$$\therefore \left(\frac{3}{2}, \frac{3\sqrt{3}}{2} \right) \text{ and } \left(\frac{3}{2}, -\frac{3\sqrt{3}}{2} \right)$$

There are 4 solutions.

$$\sum_{k=0}^{\infty} \left(\frac{1}{n^k} \right) = 1 + \frac{1}{4} + \frac{1}{4^2} + \dots \dots \infty$$

$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

Question157

If the real part of the complex number $(1 - \cos \theta + 2i \sin \theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral

$\int_0^{\theta} \sin x \, dx$ is equal to
[2021, 22 July Shift-II]

Options:

A. 1

B. 2

C. -1

D. 0

Answer: A

Solution:

Solution:

Let $z = (1 - \cos \theta + 2i \sin \theta)^{-1}$

$$\begin{aligned} \Rightarrow z &= \frac{1}{1 - \cos \theta + 2i \sin \theta} \\ &= \frac{1}{1 - \cos \theta + 2i \sin \theta} \times \frac{1 - \cos \theta - 2i \sin \theta}{1 - \cos \theta - 2i \sin \theta} \\ &= \frac{(1 - \cos \theta) - 2i \sin \theta}{(1 - \cos \theta)^2 - (2i \sin \theta)^2} \\ &= \frac{2\sin^2 \frac{\theta}{2} - 4i \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{4\sin^4 \frac{\theta}{2} + 16\sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2}} \\ &= \frac{2 \sin \frac{\theta}{2} \left(\sin \frac{\theta}{2} - 2i \cos \frac{\theta}{2} \right)}{4\sin^2 \frac{\theta}{2} \left(\sin^2 \frac{\theta}{2} + 4\cos^2 \frac{\theta}{2} \right)} \\ &= \frac{\sin \frac{\theta}{2} - 2i \cos \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \left(\sin^2 \frac{\theta}{2} + 4\cos^2 \frac{\theta}{2} \right)} \end{aligned}$$

$$\begin{aligned} \text{Now, Re}(z) &= \frac{\sin \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \left(\sin^2 \frac{\theta}{2} + 4\cos^2 \frac{\theta}{2} \right)} \\ &= \frac{1}{2 \left(1 + 3\cos^2 \frac{\theta}{2} \right)} \end{aligned}$$

$$\text{Given, Re}(z) = \frac{1}{5}$$

$$\Rightarrow \frac{1}{2 \left(1 + 3\cos^2 \frac{\theta}{2} \right)} = \frac{1}{5}$$

$$\Rightarrow 1 + 3\cos^2 \frac{\theta}{2} = \frac{5}{2} \Rightarrow \cos^2 \frac{\theta}{2} = \frac{1}{2}$$

$$\Rightarrow \cos \frac{\theta}{2} = \pm \frac{1}{\sqrt{2}}$$

$$\therefore \frac{\theta}{2} = n\pi \pm \frac{\pi}{4}$$

$$\therefore \theta = 2n\pi \pm \frac{\pi}{2}$$

Given, range is $\theta \in (0, \pi)$.

$$\therefore \theta = \frac{\pi}{2}$$

$$\text{Now, } \int_0^{\theta} \sin x \, dx = \int_0^{\frac{\pi}{2}} \sin x \, dx$$

$$\int_0^{\frac{\pi}{2}} \sin x \, dx = -\cos x \Big|_0^{\pi/2}$$

$$= -\left(\cos \frac{\pi}{2} - \cos 0\right)$$

$$= -(0 - 1) = 1$$

Question 158

If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg$

$(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1 - 2\bar{z}\omega}{1 + 3z\omega}\right)$ is

(Here, $\arg(z)$ denotes the principal argument of complex number z)
[2021, 20 July Shift-1]

Options:

A. $\frac{\pi}{4}$

B. $-\frac{3\pi}{4}$

C. $-\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

Answer: B

Solution:

Solution:

$$|zW| = 1, \arg(z) - \arg(w) = \frac{3\pi}{2}$$

$$\text{Let } z = re^{i\theta}$$

$$w = \frac{1}{r}e^{i\left(\theta - \frac{3\pi}{2}\right)} \Rightarrow \bar{z} = re^{-i\theta}$$

$$w\bar{z} = \frac{1}{r}e^{i\left(\theta - \frac{3\pi}{2}\right)} \cdot re^{-i\theta}$$

$$\Rightarrow w\bar{z} = e^{i\left(\theta - \frac{3\pi}{2} - \theta\right)} = e^{-i\frac{3\pi}{2}}$$

$$\Rightarrow w\bar{z} = \cos\left(\frac{-3\pi}{2}\right) + i\sin\left(\frac{-3\pi}{2}\right)$$

$$\Rightarrow w\bar{z} = 0 + i$$

$$\Rightarrow w\bar{z} = i$$

$$\left(\frac{1 - 2w\bar{z}}{1 + 3w\bar{z}}\right) = \left(\frac{1 - 2i}{1 + 3i} \times \frac{1 - 3i}{1 - 3i}\right)$$

$$= \frac{1 - 2i - 3i + 6i^2}{10} = \frac{-5 - 5i}{10}$$

$$\therefore \arg = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}$$

Question 159

Let Z_1 and Z_2 be two complex numbers such that $\arg(Z_1 - Z_2) = \frac{\pi}{4}$ and Z_1, Z_2 satisfy the equation $|Z - 3| = \operatorname{Re}(Z)$. Then, the imaginary part of

$Z_1 + Z_2$ is equal to
[2021, 27 Aug. Shift-11]

Answer: 6

Solution:

Solution:

$$\text{Let } Z_1 = a_1 + ib_1, Z_2 = a_2 + ib_2$$

$$Z_1 - Z_2 = (a_1 - a_2) + i(b_1 - b_2)$$

$$\arg(Z_1 - Z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1} \left(\frac{b_1 - b_2}{a_1 - a_2} \right) = \frac{\pi}{4}$$

$$\text{I.I.} \Rightarrow b_1 - b_2 = a_1 - a_2$$

$$\text{Also, } |Z_1 - 3| = \text{Re}(Z_1)$$

$$\Rightarrow (a_1 - 3)^2 + b_1^2 = a_1^2$$

$$\text{and } |Z_2 - 3| = \text{Re}(Z_2)$$

$$\Rightarrow (a_2 - 3)^2 + b_2^2 = a_2^2$$

$$\Rightarrow (a_1 - 3)^2 - (a_2 - 3)^2 + b_1^2 - b_2^2$$

$$= a_1^2 - a_2^2$$

$$\Rightarrow (a_1 - a_2)(a_1 + a_2 - 6) + (b_1 - b_2)(b_1 + b_2)$$

$$= (a_1 - a_2)(a_1 + a_2)$$

$$\Rightarrow a_1 + a_2 - 6 + b_1 + b_2 = a_1 + a_2$$

$$\Rightarrow b_1 + b_2 = 6$$

$$\Rightarrow \text{Im}(Z_1 + Z_2) = 6$$

[using Eq. (i).]

Question 160

**The sum of the roots of the equation $x + 1 - 2\log_2(3 + 2^x)$
 $+ 2\log_4(10 - 2^{-x}) = 0$ is**
[2021, 31 Aug. Shift-II]

Options:

A. $\log_2 14$

B. $\log_2 11$

C. $\log_2 12$

D. $\log_2 13$

Answer: B

Solution:

Solution:

$$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$$

$$\begin{aligned}
&\Rightarrow \\
&x + 1 - 2\log_2(3 + 2^x) + \log_2\left(\frac{10 \cdot 2^x - 1}{2^x}\right) = 0 \\
&\Rightarrow x + 1 - 2\log_2(3 + 2^x) + \log_2(10 \cdot 2^x - 1) \\
&\Rightarrow 1 + \log_2\left(\frac{10 \cdot 2^x - 1}{(3 + 2^x)^2}\right) = 0 \\
&\Rightarrow \frac{10 \cdot 2^x - 1}{9 + (2^x)^2 + 6 \cdot 2^x} = \frac{1}{2} \\
&\Rightarrow (2^x)^2 - 14 \cdot 2^x + 11 = 0 \\
&\text{Let } 2^x = y \\
&\Rightarrow y^2 - 14y + 11 = 0 \\
&\text{Let } 2^x = y \\
&\Rightarrow y^2 - 14y + 11 = 0 \\
&y = \frac{14 \pm \sqrt{152}}{2} = 7 \pm \frac{\sqrt{152}}{2} \\
&y_1 = 7 + \frac{\sqrt{152}}{2}, \\
&y_2 = 7 - \frac{\sqrt{152}}{2} \\
&\Rightarrow 2^{x_1} = 7 + \frac{\sqrt{152}}{2}, \\
&2^{x_2} = 7 - \frac{\sqrt{152}}{2} \\
&\Rightarrow x_1 = \log_2\left(7 + \frac{\sqrt{152}}{2}\right) \\
&x_2 = \log_2\left(7 - \frac{\sqrt{152}}{2}\right) \\
&\therefore \text{Sum of roots} = x_1 + x_2 \\
&= \log_2\left(49 - \frac{152}{4}\right) = \log_2 11
\end{aligned}$$

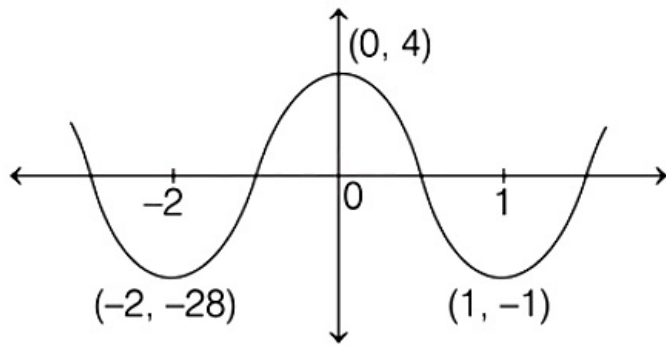
Question161

The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is [2021, 27 Aug. Shift-I]

Answer: 4

Solution:

$$\begin{aligned}
&\text{Let } f(x) = 3x^4 + 4x^3 - 12x^2 + 4 = 0 \\
&\text{Differentiating w.r.t. } x_1 \\
&f'(x) = 12x^3 + 12x^2 - 24x = 0 \\
&\Rightarrow 12x(x^2 + x - 2) = 0 \\
&\Rightarrow x(x + 2)(x - 1) = 0 \\
&\text{Critical point } x = 0, 1, -2
\end{aligned}$$



Graph of $y = f(x)$

Number of real roots = 4

Question 162

The set of all values of $k > -1$, for which the equation $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3) + k(3x^2 + 4x + 2) = 0$ has real roots, is
[2021, 27 Aug. Shift-II]

Options:

- A. $\left(1, \frac{5}{2}\right]$
- B. $[2, 3)$
- C. $\left[-\frac{1}{2}, 1\right)$
- D. $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

Answer: A

Solution:

Solution:

Given,

$$(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3) + k(3x^2 + 4x + 2) = 0$$

$$(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$$

$$\text{Let } y = 3x^2 + 4x + 2$$

Then, given equation becomes

$$(y + 1)^2 - (k + 1)y(y + 1) + ky^2 = 0$$

$$\Rightarrow y^2 + 2y + 1 - ky^2 - ky - y^2 - y + ky^2 = 0$$

$$\Rightarrow y + 1 - ky = 0$$

$$\Rightarrow y(1 - k) = -1$$

$$\Rightarrow y = \frac{1}{k - 1}$$

$$\Rightarrow 3x^2 + 4x + 2 - \frac{1}{k - 1} = 0$$

For real roots, $D \geq 0$

$$\Rightarrow 16 - 4 \cdot 3 \cdot \left(2 - \frac{1}{k - 1}\right) \geq 0$$

$$\Rightarrow -8 + \frac{12}{k - 1} \geq 0 \Rightarrow \frac{3}{k - 1} \geq 2$$

$$\Rightarrow \frac{3 - 2k + 2}{k - 1} \geq 0 \Rightarrow \frac{2k - 5}{k - 1} \leq 0$$

$$\Rightarrow k \in \left(1, \frac{5}{2}\right] \quad [\because k \neq 1]$$

Question 163

The sum of all integral values of k ($k \neq 0$) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is
[2021, 26 Aug. Shift-I]

Answer: 66

Solution:

Solution:

$$\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k} \Rightarrow x \in \mathbb{R} - \{1, 2\}$$

$$k(2x - 4 - x + 1) = 2(x^2 - 3x + 2)$$

$$k(x - 3) = 2(x^2 - 3x + 2)$$

$$2x^2 - (6 + k)x + 3k + 4 = 0$$

$$\text{For no real roots } b^2 - 4ac < 0$$

$$\therefore (k + 6)^2 - 8 \cdot (3k + 4) < 0$$

$$\Rightarrow k^2 - 12k - 4 < 0$$

$$\Rightarrow (k - 6)^2 - 32 < 0$$

$$\Rightarrow (k - 6)^2 < 32$$

$$\Rightarrow -4\sqrt{2} < k - 6 < 4\sqrt{2}$$

$$\Rightarrow 6 - 4\sqrt{2} < k < 6 + 4\sqrt{2}$$

$$\text{Integral } k \in \{1, 2, 3, 4, \dots, 11\}$$

$$\text{Sum} = 66$$

Question 164

If $(\sqrt{3} + i)^{100} = 2^{99}(p + iq)$, then p and q are roots of the equation
[2021, 26 Aug. Shift-III]

Options:

A. $x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0$

B. $x^2 + (\sqrt{3} + 1)x + \sqrt{3} = 0$

C. $x^2 + (\sqrt{3} - 1)x - \sqrt{3} = 0$

D. $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

Answer: A

Solution:

Solution:

$$(\sqrt{3} + i)^{100} = 2^{99}(p + iq)$$

$$2^{100} e^{i100 \frac{\pi}{6}} = 2^{99}(p + iq)$$

$$\Rightarrow 2e^{i \frac{2\pi}{3}} = p + iq$$

$$\Rightarrow 2 \left[\cos \left(\pi - \frac{\pi}{3} \right) + i \sin \left(\pi - \frac{\pi}{3} \right) \right] = p + iq$$

$$\Rightarrow (-1 + i\sqrt{3}) = p + iq$$

$$\Rightarrow p = -1 \text{ and } q = \sqrt{3}$$

Equation whose roots are -1 and $\sqrt{3}i$ is

$$\Rightarrow (x + 1)(x - \sqrt{3}i) = 0$$

$$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0$$

Question 165

Let $\lambda \neq 0$ be in $\mathbb{R}$. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$ and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to
[2021, 26 Aug. Shift-II]

Answer: 18

Solution:

Solution:

We have, α is common root of the equations $x^2 - x + 2\lambda = 0$ and $3x^2 - 10x + 27\lambda = 0$.

Now, common root of these equations is $(3\alpha^2 - 10\alpha + 27\lambda) - (3\alpha^2 - 3\alpha + 6\lambda) = 0 \Rightarrow -7\alpha + 21\lambda = 0$

$$\Rightarrow \alpha = 3\lambda$$

Again, α is root of $x^2 - x + 2\lambda = 0$

$$\therefore \alpha^2 - \alpha + 2\lambda = 0$$

$$\Rightarrow (3\lambda)^2 - 3\lambda + 2\lambda = 0$$

$$\Rightarrow 9\lambda^2 - \lambda = 0$$

$$\Rightarrow \lambda(9\lambda - 1) = 0$$

$$\Rightarrow \lambda = 0, \frac{1}{9}$$

$$\Rightarrow \lambda = \frac{1}{9} [\because \lambda \neq 0]$$

$$\therefore \alpha = 3\lambda = 3 \times \frac{1}{9} = \frac{1}{3}$$

Again, α and β are roots of the equation

$$x^2 - x + 2\lambda = 0$$

$$\therefore \alpha + \beta = \frac{-(-1)}{1} = 1$$

$$\Rightarrow \beta = 1 - \alpha = 1 - \frac{1}{3} = \frac{2}{3}$$

And α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$

$$\therefore \alpha + \gamma = \frac{-(-10)}{3} = \frac{10}{3}$$

$$\Rightarrow \gamma = \frac{10}{3} - \alpha = \frac{10}{3} - \frac{1}{3} = \frac{9}{3} = 3$$

$$\therefore \frac{\beta\gamma}{\lambda} = \frac{\left(\frac{2}{3}\right) \times (3)}{\left(\frac{1}{9}\right)} = 18$$

Question 166

If $S = \left\{ z \in \mathbb{C} : \frac{z-i}{z+2i} \in \mathbb{R} \right\}$, then
[2021, 27 Aug. Shift-1]

Options:

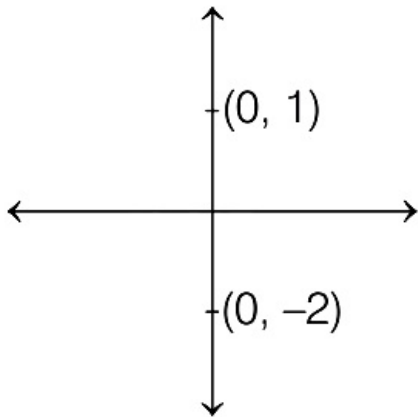
- A. S contains exactly two elements.
- B. S contains only one element.
- C. S is a circle in the complex plane.
- D. S is a straight line in the complex plane.

Answer: D

Solution:

Solution:

Given, $\frac{z-i}{z+2i} \in \mathbb{R}$



$$\Rightarrow \arg\left(\frac{z-i}{z+2i}\right) = 0 \text{ or } \pi$$

$\Rightarrow i, -2i, z$ are collinear.

$\Rightarrow S$ is a straight line in the complex plane.

Question 167

Let $z = \frac{1-i\sqrt{3}}{2}$ and $i = \sqrt{-1}$. Then the value of $21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \dots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$ is
[2021, 26 Aug. Shift-1]

Answer: 13

Solution:

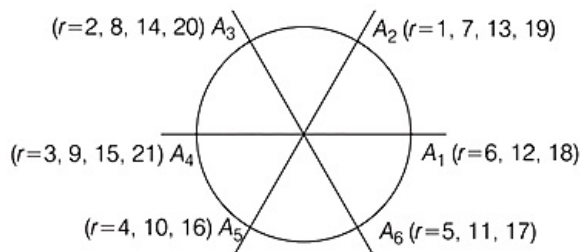
Solution:

$$z = \frac{1}{2} - \frac{\sqrt{3}}{2}i = e^{-i\pi/3}$$

$$\text{Again, } z^r + \frac{1}{z^r} = z^r + \bar{z}^r = 2\operatorname{Re}(z^r)$$

$$[\because |z^r| = 1] = 2 \cos\left(\frac{r\pi}{3}\right)$$

$$21 + \sum_{r=1}^{21} \left(z^r + \frac{1}{z^r}\right)^3 = 21 + \sum_{r=1}^{21} 8 \cos^3\left(\frac{r\pi}{3}\right)$$



Now, all the diametric ends will cancel out each other. Only a single value at A_L will remain which is -1 .

$$\text{So, } 21 + 8(-1) = 13$$

Question 168

The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with
[2021, 26 Aug. Shift-I]

Options:

A. centre at $(0, -1)$ and radius $\sqrt{2}$

B. centre at $(0, 1)$ and radius $\sqrt{2}$

C. centre at $(0, 0)$ and radius $\sqrt{2}$

D. centre at $(0, 1)$ and radius 2

Answer: B

Solution:

Solution:

$$\text{We have, } \arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4} \Rightarrow \arg(z-1) - \arg(z+1) = \frac{\pi}{4}$$

$$\text{Let } z = x + iy$$

$$\arg[(x-1) + iy] - \arg[(x+1) + iy] = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{\pi}{4}$$

$$\Rightarrow \left(\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1 + \frac{y}{x-1} \cdot \frac{y}{x+1}}\right) = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{y(x+1) - y(x-1)}{(x^2-1) + y^2} = 1$$

$$\Rightarrow 2y = x^2 + y^2 - 1$$

$$\Rightarrow x^2 + y^2 - 2y - 1 = 0$$

$$\Rightarrow x^2 + (y - 1)^2 = 2$$

$$\Rightarrow x^2 + (y - 1)^2 = (\sqrt{2})^2$$

Which is a circle with Centre (0, 1) and Radius = $\sqrt{2}$ units

Question 169

A point z moves in the complex plane such that $\arg \left(\frac{z-2}{z+2} \right) = \frac{\pi}{4}$, then the minimum value of $|z - 9\sqrt{2} - 2i|^2$ equal to
[2021, 31 Aug. Shift-I]

Answer: 98

Solution:

Solution:

$$\arg \left(\frac{z-2}{z+2} \right) = \frac{\pi}{4}$$

If

$$z = x + iy$$

$$\arg[(x-2) + iy] - \arg[(x+2) + iy] = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x-2} \right) - \tan^{-1} \left(\frac{y}{x+2} \right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \frac{y}{x-2} \cdot \frac{y}{x+2}} = \tan \left(\frac{\pi}{4} \right)$$

$$1 \Rightarrow \frac{xy + 2y - xy + 2y}{x^2 + y^2 - 4} = 1$$

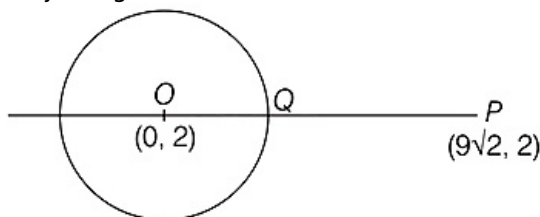
$$\Rightarrow 4y = x^2 + y^2 - 4$$

$$\Rightarrow x^2 + y^2 - 4y - 4 = 0$$

z is a circle.

Centre = (0, 2), Radius = $(2\sqrt{2})$

$|z - 9\sqrt{2} - 2i|^2$ is the distance of $(9\sqrt{2}, 2)$ from any point on circle. Distance will be minimum when $(9\sqrt{2}, 2)$ will lie on the line joining the centre.



$$PQ = OP - OQ$$

$$= 9\sqrt{2} - 2\sqrt{2} = 7\sqrt{2}$$

$$PO^2 = (7\sqrt{2})^2 = 98$$

Question 170

If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is

Options:

A. $2\sqrt{2} - 1$

B. $6\sqrt{2}$

C. $3\sqrt{2}$

D. $2\sqrt{2}$

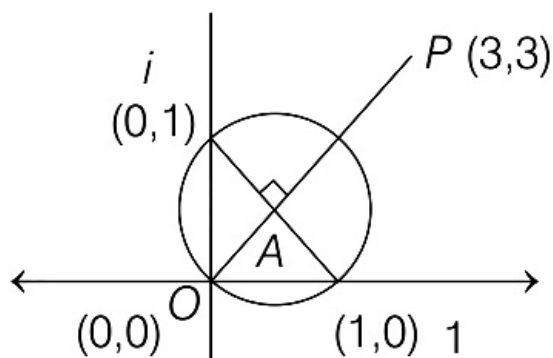
Answer: D**Solution:****Solution:**Let $z = x + iy$

$$\begin{aligned} c \frac{z-i}{z-1} &= \frac{x+i(y-1)}{(x-1)+iy} \times \frac{(x-1)-iy}{(x-1)-iy} \\ &= \frac{x(x-1)+y(y-1)}{(x-1)^2+y^2} + i \left[\frac{(x-1)(y-1)-xy}{(x-1)^2+y^2} \right] \end{aligned}$$

As $\frac{z-i}{z-1}$ is purely imaginary,

$$x^2 + y^2 - x - y = 0$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 0$$

This is a circle with centre $\left(\frac{1}{2}, \frac{1}{2}\right)$, radius $= \frac{1}{2}$ which passes through origin as shown in the figure.

Minimum $|z - (3 + 3i)| = \overline{OP - OA}$

$$\begin{aligned} &\sqrt{(3-0)^2 + (3-0)^2} - \sqrt{\left(\frac{1}{2}-0\right)^2 + \left(\frac{1}{2}-0\right)^2} \\ &= 3\sqrt{2} - \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

Question171

The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}$, $i = \sqrt{-1}$, is a positive integer, is

[2021, 26 Aug. Shift-11]**Answer: 6**

Solution:

Solution:

We have,

$$\begin{aligned}\frac{(2i)^n}{(1-i)^{n-2}} &= \frac{(2i)^n}{(1-i)^n(1-i)^{-2}} \\&= \left(\frac{2i}{1-i} \right)^n (1-i)^2 \\&= \left[\frac{2i(1+i)}{(1-i)(1+i)} \right]^n (1+i^2-2i) \\&= \left(\frac{2i-2}{2} \right)^n (1-1-2i) \\&= (i-1)^n(-2i)\end{aligned}$$

If $n = 1$, $(i-1)(-2i) = -2i^2 + 2i = 2 + 2i$

If $n = 2$, $-2i(i-1)^2 = -2i(-2i) = -4$

If $n = 4$, $-2i(i-1)^4 = -2i(-2i)(-2i) = 8i$

If $n = 6$, $-2i(i-1)^6 = -2i(-2i)(-2i)(-2i) = 16$

So, least value of n for which given complex is positive is 6 .

Question 172

The numbers of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is

[2021, 01 Sep. Shift-II]

Options:

A. 6

B. 2

C. 4

D. 8

Answer: A

Solution:

Solution:

Given equation $x^2 + ax + b = 0$

It has two roots (not necessarily real α and β)

$\Rightarrow$ Either $\alpha = \beta$ or $\alpha \neq \beta$

1. If $\alpha = \beta \Rightarrow \alpha = \alpha^2 - 2 \Rightarrow \alpha = -1, 2$

When $\alpha = -1$, then $(a, b) = (2, 1)$

When $\alpha = 2$, then $(a, b) = (-4, 4)$

II. If $\alpha \neq \beta$, then

(a) $\alpha = \alpha^2 - 2$ and $\beta = \beta^2 - 2$

Here, $(\alpha, \beta) = (2, -1)$ or $(-1, 2)$

Hence $(a, b) = (-\alpha - \beta, \alpha\beta) = (-1, -2)$

(b) $\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$

Then $\alpha - \beta = \beta^2 - \alpha^2 = (\beta - \alpha)(\beta + \alpha)$

$\therefore \alpha \neq \beta$

$\Rightarrow \alpha + \beta = \beta^2 + \alpha^2 - 4$

or $\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta - 4$

$\Rightarrow -1 = 1 - 2\alpha\beta - 4 \Rightarrow \alpha\beta = -1$

$\Rightarrow (a, b) = (-\alpha - \beta, \alpha\beta) = (1, -1)$

(c) $\alpha = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta \Rightarrow \alpha = -\beta$
 Thus, $\alpha = 2, \beta = -2$
 or $\alpha = -1, \beta = 1$
 $\therefore (a, b) = (0, -4)$ and $(0, -1)$
 (d) $\beta = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$ (as in (c))
 $\Rightarrow$ We get 6 pairs of (a, b)
 They are $(2, 1), (-4, 4), (-1, -2), (1, -1), (0, -4)$, and $(0, -1)$.

Question 173

If $\frac{3 + i\sin\theta}{4 - i\cos\theta}$, $\theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is:

[Jan. 7, 2020 (II)]

Options:

A. $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

B. $\pi - \tan^{-1}\left(\frac{3}{4}\right)$

C. $-\tan^{-1}\left(\frac{3}{4}\right)$

D. $\tan^{-1}\left(\frac{4}{3}\right)$

Answer: A

Solution:

Solution:

Let $z = \frac{3 + i\sin\theta}{4 - i\cos\theta}$, after rationalising

$$z = \frac{(3 + i\sin\theta)}{(4 - i\cos\theta)} \times \frac{(4 + i\cos\theta)}{(4 + i\cos\theta)}$$

As z is purely real

$$\Rightarrow 3\cos\theta + 4\sin\theta = 0 \Rightarrow \tan\theta = -\frac{3}{4}$$

$$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$= \pi + \tan^{-1}\left(-\frac{4}{3}\right) = \pi - \tan^{-1}\left(\frac{4}{3}\right)$$

Question 174

Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z + 3i|$ is:

[Jan. 9, 2020 (I)]

Options:

A. $\sqrt{10}$

B. $\frac{7}{2}$

C. $\frac{15}{4}$

D. $2\sqrt{3}$

Answer: B

Solution:

Solution:

Let $z = x + iy$

$$\text{Then, } \left| \frac{z-i}{z+2i} \right| = 1 \Rightarrow x^2 + (y-1)^2$$

$$= x^2 + (y+2)^2 \Rightarrow -2y + 1 = 4y + 4$$

$$\Rightarrow 6y = -3 \Rightarrow y = -\frac{1}{2}$$

$$\therefore |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$$

$$\Rightarrow x^2 = \frac{24}{4} = 6$$

$$\therefore z = x + iy \Rightarrow z = \pm\sqrt{6} - \frac{i}{2}$$

$$|z + 3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}}$$

$$\Rightarrow |z + 3i| = \frac{7}{2}$$

Question 175

If z be a complex number satisfying $|\operatorname{Re}(z)| + |\operatorname{Im}(z)| = 4$, then $|z|$ cannot be:

[Jan. 9, 2020 (II)]

Options:

A. $\sqrt{\frac{17}{2}}$

B. $\sqrt{10}$

C. $\sqrt{7}$

D. $\sqrt{8}$

Answer: C

Solution:

Solution:

$$z = x + iy \quad |x| + |y| = 4$$

$$|z| = \sqrt{x^2 + y^2}$$

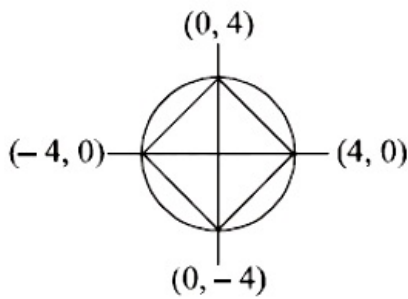
Minimum value of

$$|z| = 2\sqrt{2}$$

Maximum value of

$$|z| = 4 \quad |z| \in [\sqrt{8}, \sqrt{16}]$$

So, $|z|$ can't be $\sqrt{7}$.



Question176

Let $\alpha = \frac{-1 + i\sqrt{3}}{2}$. If $a = (1 + \alpha) \sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then a and b are the roots of the quadratic equation:
[Jan. 8, 2020 (II)]

Options:

- A. $x^2 + 101x + 100 = 0$
- B. $x^2 - 102x + 101 = 0$
- C. $x^2 - 101x + 100 = 0$
- D. $x^2 + 102x + 101 = 0$

Answer: B

Solution:

Solution:

$$\begin{aligned} \text{Let } \alpha = \omega, b &= 1 + \omega^3 + \omega^6 + \dots = 101 \\ a &= (1 + \omega)(1 + \omega^2 + \omega^4 + \dots + \omega^{198} + \omega^{200}) \\ &= (1 + \omega) \frac{(1 - (\omega^2)^{101})}{1 - \omega^2} = \frac{(\omega + 1)(\omega^{202} - 1)}{(\omega^2 - 1)} \\ \Rightarrow a &= \frac{(1 + \omega)(1 - \omega)}{1 - \omega^2} = 1 \end{aligned}$$

$$\begin{aligned} \text{Required equation} &= x^2 - (101 + 1)x + (101) \times 1 = 0 \\ \Rightarrow x^2 - 102x + 101 &= 0 \end{aligned}$$

Question177

If $\text{Re} \left(\frac{z-1}{2z+i} \right) = 1$, where $z = x + iy$, then the point (x, y) lies on a :
[Jan. 7, 2020 (I)]

Options:

- A. circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2} \right)$.
- B. straight line whose slope is $-\frac{2}{3}$.

C. straight line whose slope is $\frac{3}{2}$.

D. circle whose diameter is $\frac{\sqrt{5}}{2}$.

Answer: D

Solution:

Solution:

$\because z = x + iy$

$$\left(\frac{z-1}{2z+i} \right) = \frac{(x-1) + iy}{2(x+iy) + i}$$

$$= \frac{(x-1) + iy}{2x + (2y+1)i} \times \frac{2x - (2y+1)i}{2x - (2y+1)i}$$

$$\operatorname{Re} \left(\frac{z-1}{2z+i} \right) = \frac{2x(x-1) + y(2y+1)}{(2x)^2 + (2y+1)^2} = 1$$

$$\Rightarrow \left(x + \frac{1}{2} \right)^2 + \left(y + \frac{3}{4} \right)^2 = \left(\frac{\sqrt{5}}{4} \right)^2$$

Question 178

The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is:
[Jan. 9, 2020 (I)]

Options:

A. 1

B. 3

C. 2

D. 4

Answer: A

Solution:

Solution:

Let $e^x = t \in (0, \infty)$

Given equation

$$t^4 + t^3 - 4t^2 + t + 1 = 0$$

$$\Rightarrow t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0$$

$$\Rightarrow \left(t^2 + \frac{1}{t^2} \right) + \left(t + \frac{1}{t} \right) - 4 = 0$$

$$\text{Let } t + \frac{1}{t} = y$$

$$(y^2 - 2) + y - 4 = 0 \Rightarrow y^2 + y - 6 = 0$$

$$y^2 + y - 6 = 0 \Rightarrow y = -3, 2$$

$$\Rightarrow y = 2 \Rightarrow t + \frac{1}{t} = 2$$

$$\Rightarrow e^x + e^{-x} = 2$$

$x = 0$, is the only solution of the equation

Hence, there only one solution of the given equation.

Question179

The least positive value of ' a ' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is _____

[Jan. 8, 2020 (I)]

Answer: 8

Solution:

Solution:

Since, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots,

$\therefore D \geq 0$

$$\Rightarrow (a - 10)^2 - 4(2) \left(\frac{33}{2} - 2a \right) \geq 0$$

$$\Rightarrow (a - 10)^2 - 4(33 - 4a) \geq 0$$

$$\Rightarrow a^2 - 4a - 32 \geq 0$$

$$\Rightarrow (a - 8)(a + 4) \geq 0$$

$$\Rightarrow a \leq -4 \cup a \geq 8$$

$$\Rightarrow a \in (-\infty, -4] \cup [8, \infty)$$

Question180

If the equation, $x^2 + bx + 45 = 0 (b \in \mathbb{R})$ has conjugate complex roots and they satisfy $|z + 1| = 2\sqrt{10}$, then:

[Jan. 8, 2020 (I)]

Options:

A. $b^2 - b = 30$

B. $b^2 + b = 72$

C. $b^2 - b = 42$

D. $b^2 + b = 12$

Answer: A

Solution:

Solution:

Let $z = \alpha \pm i\beta$ be the complex roots of the equation

So, sum of roots $= 2\alpha = -b$ and

Product of roots $= \alpha^2 + \beta^2 = 45$

$$(\alpha + 1)^2 + \beta^2 = 40$$

Given, $|z + 1| = 2\sqrt{10}$

$$\Rightarrow (\alpha + 1)^2 - \alpha^2 = -5 \quad [\because \beta^2 = 45 - \alpha^2]$$

$$\Rightarrow 2\alpha + 1 = -5 \Rightarrow 2\alpha = -6$$

Hence, $b = 6$ and $b^2 - b = 30$

Question181

Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true?

[Jan. 7, 2020 (II)]

Options:

A. $p_3 = p_5 - p_4$

B. $p_5 = 11$

C. $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

D. $p_5 = p_2 \cdot p_3$

Answer: D

Solution:

Solution:

$$\alpha^5 = 5\alpha + 3$$

$$\beta^5 = 5\beta + 3$$

$$p_5 = 5(\alpha + \beta) + 6 = 5(1) + 6$$

$$\left[\because \text{from } x^2 - x - 1 = 0, \alpha + \beta = \frac{-b}{a} = 1 \right]$$

$$p_5 = 11 \text{ and } p_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$$

$$p_2 = 3 \text{ and } p_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1$$

$$= 2(1) + 2 = 4$$

$$p_2 \times p_3 = 12 \text{ and } p_5 = 11 \Rightarrow p_5 \neq p_2 \times p_3$$

Question182

Let α and $\bar{\beta}$ be two real roots of the equation $(k + 1)\tan^2 x - \sqrt{2} \cdot \lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is:

[Jan. 7, 2020 (I)]

Options:

A. $10\sqrt{2}$

B. 10

C. 5

D. $5\sqrt{2}$

Answer: B

Solution:

Solution:

$$(k+1)\tan^2 x - \sqrt{2}\lambda \tan x + (k-1) = 0$$

$$\tan \alpha + \tan \beta = \frac{\sqrt{2}\lambda}{k+1} \quad [\text{Sum of roots}]$$

$$\tan \alpha \cdot \tan \beta = \frac{k-1}{k+1} \quad [\text{Product of roots}]$$

$$\therefore \tan(\alpha + \beta) = \frac{\frac{\sqrt{2}\lambda}{k+1}}{1 - \frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$$

$$\tan^2(\alpha + \beta) = \frac{\lambda^2}{2} = 50$$

$$\lambda = 10$$

Question183

Let $a, b \in \mathbb{R}$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α , which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to :
[Jan. 9, 2020 (II)]

Options:

A. 25

B. 26

C. 28

D. 24

Answer: A

Solution:

Solution:

$$ax^2 - 2bx + 5 = 0$$

If α and α are roots of equations, then sum of roots

$$2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

$$\text{and product of roots} = \alpha^2 = \frac{5}{a} \Rightarrow \frac{b^2}{a^2} = \frac{5}{a}$$

$$\Rightarrow b^2 = 5a \quad (a \neq 0) \dots (i)$$

$$\text{For } x^2 - 2bx - 10 = 0$$

$$\alpha + \beta = 2b \dots (ii)$$

$$\text{and } \alpha\beta = -10 \dots (iii)$$

$$\alpha = \frac{b}{a} \text{ is also root of } x^2 - 2bx - 10 = 0$$

$$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$$

$$\text{By eqn. (i)} \Rightarrow 5a - 10a^2 - 10a^2 = 0$$

$$\Rightarrow 20a^2 = 5a$$

$$\Rightarrow a = \frac{1}{4} \text{ and } b^2 = \frac{5}{4}$$

$$\alpha^2 = 20 \text{ and } \beta^2 = 5$$

$$\text{Now, } \alpha^2 + \beta^2 = 5 + 20 = 25$$

Question184

If the four complex numbers z , $\bar{z}$, $\bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to :

[Sep. 05, 2020 (I)]

Options:

A. $4\sqrt{2}$

B. 4

C. $2\sqrt{2}$

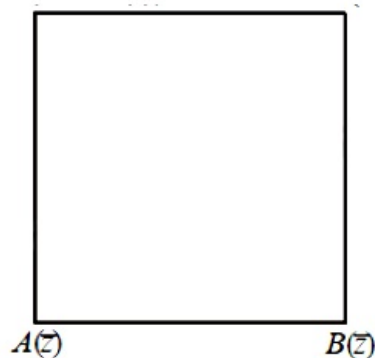
D. 2

Answer: C

Solution:

Solution:

$$D(z - 2\text{Re}(z)) \quad C(\bar{z} - 2\text{Re}(\bar{z}))$$



Let $z = x + iy$

$\therefore$ Length of side of square = 4 units

Then, $|z - \bar{z}| = 4 \Rightarrow |2iy| = 4 \Rightarrow |y| = 2$

Also, $|z - (z - 2\text{Re}(z))| = 4$

$\Rightarrow |2\text{Re}(z)| = 4 \Rightarrow |2x| = 4 \Rightarrow |x| = 2$

$\therefore |z| = \sqrt{x^2 + y^2} = \sqrt{4 + 4} = 2\sqrt{2}$

Question185

The value of $\left(\frac{-1 + i\sqrt{3}}{1 - i} \right)^{30}$ is:

[Sep. 05, 2020 (II)]

Options:

A. -2^{15}

B. $2^{15}i$

C. $-2^{15}i$

D. 6^5

Answer: C

Solution:

Solution:

$$\begin{aligned}\because -1 + \sqrt{3}i &= 2 \cdot e^{\frac{2\pi i}{3}} \text{ and } 1 - i = \sqrt{2} \cdot e^{-\frac{i\pi}{4}} \\ \therefore \left(\frac{-1 + \sqrt{3}i}{1 - i} \right)^{30} &= \left(\sqrt{2} e^{\left(\frac{2\pi}{3} + \frac{\pi}{4} \right)i} \right)^{30} \\ &= 2^{15} \cdot e^{-\frac{\pi i}{2}} = -2^{15} \cdot i.\end{aligned}$$

Question 186

If $\left(\frac{1+i}{1-i} \right)^{m/2} = \left(\frac{1+i}{i-1} \right)^{n/3} = 1$, ($n, m \in \mathbb{N}$), then the greatest common divisor of the least values of m and n is _____.
[Sep. 03, 2020 (I)]

Answer: 4

Solution:

Solution:

$$\begin{aligned}\text{Given that } \left(\frac{1+i}{1-i} \right)^{m/2} &= \left(\frac{1+i}{i-1} \right)^{n/3} = 1 \\ \Rightarrow \left(\frac{(1+i)^2}{2} \right)^{m/2} &= \left(\frac{(1+i)^2}{-2} \right)^{n/3} = 1 \\ \Rightarrow i^{m/2} &= (-i)^{n/3} = 1 \\ m(\text{ least }) &= 8, n(\text{ least }) = 12 \\ \text{GCD}(8, 12) &= 4\end{aligned}$$

Question 187

If Z_1, Z_2 are complex numbers such that

$\text{Re}(z_1) = |Z_1 - 1|$, $\text{Re}(Z_2) = |Z_2 - 1|$ and $\arg(Z_1 - Z_2) = \frac{\pi}{6}$, then

$\text{Im}(Z_1 + Z_2)$ is equal to:

[Sep. 03, 2020 (II)]

Options:

A. $\frac{2}{\sqrt{3}}$

B. $2\sqrt{3}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Solution:

Solution:

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\because |z_1 - 1| = \text{Re}(z_1)$$

$$\Rightarrow (x_1 - 1)^2 + y_1^2 = x_1^2$$

$$\Rightarrow y_1^2 - 2x_1 + 1 = 0 \dots\dots(i)$$

$$|z_2 - 1| = \text{Re}(z_2) \Rightarrow (x_2 - 1)^2 + y_2^2 = x_2^2$$

$$\Rightarrow y_2^2 - 2x_2 + 1 = 0 \dots\dots(ii)$$

From eqn. (i) - (ii)

$$y_1^2 - y_2^2 - 2(x_1 - x_2) = 0$$

$$\Rightarrow y_1 + y_2 = 2 \left(\frac{x_1 - x_2}{y_1 - y_2} \right) \dots\dots(iii)$$

$$\because \arg(z_1 - z_2) = \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1} \left(\frac{y_1 - y_2}{x_1 - x_2} \right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{2}{y_1 + y_2} = \frac{1}{\sqrt{3}} \left[\text{From, } \frac{y_1 - y_2}{x_1 - x_2} = \frac{2}{y_1 + y_2} \right]$$

$$\therefore y_1 + y_2 = 2\sqrt{3} \Rightarrow \text{Im}(z_1 + z_2) = 2\sqrt{3}$$

Question 188

Let $z = x + iy$ be a non-zero complex number such that $z^2 = i |z|^2$, where $i = \sqrt{-1}$, then z lies on the:

[Sep. 06, 2020 (II)]

Options:

A. line, $y = -x$

B. imaginary axis

C. line, $y = x$

D. real axis

Answer: C

Solution:

Solution:

Let $z = x + iy$

$$\because z^2 = i |z|^2$$

$$\therefore x^2 - y^2 + 2ixy = i(x^2 + y^2)$$

$$\Rightarrow x^2 - y^2 = 0 \text{ and } 2xy = x^2 + y^2$$

$$\Rightarrow (x - y)(x + y) = 0 \text{ and } (x - y)^2 = 0$$

$$\Rightarrow x = y$$

Question189

If a and b are real numbers such that $(2 + \alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1 + i\sqrt{3}}{2}$, then a + b is equal to :
[Sep. 04, 2020 (II)]

Options:

- A. 9
- B. 24
- C. 33
- D. 57

Answer: A

Solution:

Solution:

Given that, $\alpha = \frac{-1 + \sqrt{3}i}{2} = \omega$

$$\therefore (2 + \omega)^4 = a + b\omega \Rightarrow (4 + \omega^2 + 4\omega)^2 = a + b\omega$$

$$\Rightarrow (\omega^2 + 4(1 + \omega))^2 = a + b\omega$$

$$\Rightarrow (\omega^2 - 4\omega^2)^2 = a + b\omega$$

$$[\because 1 + \omega = -\omega^2]$$

$$\Rightarrow (-3\omega^2)^2 = a + b\omega \Rightarrow 9\omega^4 = a + b\omega$$

$$\Rightarrow 9\omega = a + b\omega (\because \omega^3 = 1)$$

$$\text{On comparing, } a = 0, b = 9$$

$$\Rightarrow a + b = 0 + 9 = 9$$

Question190

The value of $\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}} \right)^3$ is :

[Sep. 02, 2020 (I)]

Options:

- A. $\frac{1}{2}(1 - i\sqrt{3})$
- B. $\frac{1}{2}(\sqrt{3} - i)$
- C. $-\frac{1}{2}(\sqrt{3} - i)$
- D. $-\frac{1}{2}(1 - i\sqrt{3})$

Answer: C

Solution:

Solution:

$$\begin{aligned} & \left(\frac{1 + \cos \frac{5\pi}{18} + i \sin \frac{5\pi}{18}}{1 + \cos \frac{5\pi}{18} - i \sin \frac{5\pi}{18}} \right)^3 \\ &= \left(\frac{2\cos^2 \frac{5\pi}{36} + i2 \sin \frac{5\pi}{36} \cdot \cos \frac{5\pi}{36}}{2\cos^2 \frac{5\pi}{36} - i2 \sin \frac{5\pi}{36} \cdot \cos \frac{5\pi}{36}} \right)^3 \\ &= \left(\frac{\cos \frac{5\pi}{36} + i \sin \frac{5\pi}{36}}{\cos \frac{5\pi}{36} - i \sin \frac{5\pi}{36}} \right)^3 = \left(\cos \frac{5\pi}{36} + i \sin \frac{5\pi}{36} \right)^6 \\ &= \cos \left(6 \times \frac{5\pi}{36} \right) + i \sin \left(6 \times \frac{5\pi}{36} \right) = \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \\ &= -\frac{\sqrt{3}}{2} + i \frac{1}{2} = -\frac{1}{2}(\sqrt{3} - i) \end{aligned}$$

Question191

The imaginary part of $(3 + 2\sqrt{-54})^{1/2} - (3 - 2\sqrt{-54})^{1/2}$ can be:
[Sep. 02, 2020 (II)]

Options:

- A. $-\sqrt{6}$
- B. $-2\sqrt{6}$
- C. 6
- D. $\sqrt{6}$

Answer: B

Solution:

Solution:

$$\begin{aligned} 3 + 2\sqrt{-54} &= 3 + 6\sqrt{6}i \\ \text{Let } \sqrt{3 + 6\sqrt{6}i} &= a + ib \\ \Rightarrow a^2 - b^2 &= 3 \text{ and } ab = 3\sqrt{6} \\ \Rightarrow a^2 + b^2 &= \sqrt{(a^2 - b^2)^2 + 4a^2b^2} = 15 \\ \text{So, } a &= \pm 3 \text{ and } b = \pm\sqrt{6} \\ \sqrt{3 + 6\sqrt{6}i} &= \pm(3 + \sqrt{6}i) \\ \text{Similarly, } \sqrt{3 - 6\sqrt{6}i} &= \pm(3 - \sqrt{6}i) \\ \text{Im}(\sqrt{3 + 6\sqrt{6}i} - \sqrt{3 - 6\sqrt{6}i}) &= \pm 2\sqrt{6} \end{aligned}$$

Question192

If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is:

[Sep. 06, 2020 (I)]

Options:

A. 2

B. 3

C. 1

D. 4

Answer: A

Solution:

Solution:

$\because \alpha + \beta = 64, \alpha\beta = 256$

$$\frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} = \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{(2^8)^{5/8}} = \frac{64}{32} = 2$$

Question193

If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to:

[Sep. 06, 2020 (II)]

Options:

A. $2\alpha(\alpha + 1)$

B. $-2\alpha(\alpha + 1)$

C. $2\alpha(\alpha - 1)$

D. $2\alpha^2$

Answer: B

Solution:

Solution:

Let α and β be the roots of the given quadratic equation,

$$2x^2 + 2x - 1 = 0$$

$$\text{Then, } \alpha + \beta = -\frac{1}{2} \Rightarrow -1 = 2\alpha + 2\beta$$

$$\text{and } 4\alpha^2 + 2\alpha - 1 = 0 \quad [\because \alpha \text{ is root of eq. (i)}]$$

$$\Rightarrow 4\alpha^2 + 2\alpha + 2\alpha + 2\beta = 0 \Rightarrow \beta = -2\alpha(\alpha + 1)$$

Question194

**The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is:
[Sep. 05, 2020 (I)]**

Options:

- A. $\frac{5}{9}$
- B. $\frac{25}{81}$
- C. $\frac{5}{27}$
- D. $\frac{25}{9}$

Answer: B

Solution:

Solution:

Let $|x| = y$ then

$$9y^2 - 18y + 5 = 0$$

$$\Rightarrow 9y^2 - 15y - 3y + 5 = 0$$

$$\Rightarrow (3y - 1)(3y - 5) = 0$$

$$\Rightarrow y = \frac{1}{3} \text{ or } \frac{5}{3} \Rightarrow |x| = \frac{1}{3} \text{ or } \frac{5}{3}$$

Roots are $\pm \frac{1}{3}$ and $\pm \frac{5}{3}$

$$\text{Product} = \frac{25}{81}$$

Question 195

**If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$ the the value of $\frac{\alpha}{1 - \alpha^2} + \frac{\beta}{1 - \beta^2}$ is equal to :
[Sep. 05, 2020 (II)]**

Options:

- A. $\frac{27}{32}$
- B. $\frac{1}{24}$
- C. $\frac{3}{8}$
- D. $\frac{27}{16}$

Answer: D

Solution:

Solution:

Let α and β be the roots of the quadratic equation $7x^2 - 3x - 2 = 0$

$$\begin{aligned}\therefore \alpha + \beta &= \frac{3}{7}, \alpha\beta = \frac{-2}{7} \\ \text{Now, } \frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2} &= \frac{\alpha - \alpha\beta(\alpha + \beta) + \beta}{1 - (\alpha^2 + \beta^2) + (\alpha\beta)^2} \\ &= \frac{(\alpha + \beta) - \alpha\beta(\alpha + \beta)}{1 - (\alpha + \beta)^2 + 2\alpha\beta + (\alpha\beta)^2} \\ &= \frac{\frac{3}{7} + \frac{2}{7} \times \frac{3}{7}}{1 - \frac{9}{49} + 2 \times \frac{-2}{7} + \frac{4}{49}} = \frac{27}{16}\end{aligned}$$

Question 196

Let $u = \frac{2z+i}{z-ki}$, $z = x + iy$ and $k > 0$. If the curve represented by $\text{Re}(u) + \text{Im}(u) = 1$ intersects the y -axis at the points P and Q where $PQ = 5$, then the value of k is :
[Sep. 04, 2020 (I)]

Options:

- A. $3/2$
- B. $1/2$
- C. 4
- D. 2

Answer: D

Solution:

Solution:

$$\begin{aligned}u &= \frac{2(x+iy)+i}{(x+iy)-ki} = \frac{2x+i(2y+1)}{x+i(y-k)} \\ \text{Real part of } u &= \text{Re}(u) = \frac{2x^2 + (y-k)(2y+1)}{x^2 + (y-k)^2} \\ \text{Imaginary part of } u &= \text{Im}(u) = \frac{-2x(y-k) + x(2y+1)}{x^2 + (y-k)^2} \\ \therefore \text{Re}(u) + \text{Im}(u) &= 1 \\ \Rightarrow 2x^2 + 2y^2 - 2Ky + y - K - 2xy + 2Kx + 2xy + x &= x^2 + y^2 + K^2 - 2Ky \\ \text{Since, the curve intersect at } y\text{-axis} & \\ \therefore x &= 0 \\ \Rightarrow y^2 + y - K(K+1) &= 0 \\ \text{Let } y_1 \text{ and } y_2 \text{ are roots of equations if } x &= 0 \\ \therefore y_1 + y_2 &= -1 \\ y_1 y_2 &= -(K^2 + K) \\ \therefore (y_1 - y_2)^2 &= (1 + 4K^2 + 4K) \\ \text{Given } PQ = 5 \Rightarrow |y_1 - y_2| &= 5 \\ \Rightarrow 4K^2 + 4K - 24 &= 0 \Rightarrow K = 2 \text{ or } -3 \\ \text{as } K > 0, \therefore K &= 2\end{aligned}$$

Question 197

Let $\lambda \neq 0$ be in $\mathbb{R}$. If α and β are roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to
[Sep. 04, 2020 (II)]

Options:

A. 27

B. 18

C. 9

D. 36

Answer: B

Solution:

Solution:

Since α is common root of $x^2 - x + 2\lambda = 0$ and $3x^2 - 10x + 27\lambda = 0$

$\therefore 3\alpha^2 - 10\alpha + 27\lambda = 0$ (i)

$3\alpha^2 - 3\alpha + 6\lambda = 0$ (ii)

$\therefore$ On subtract, we get $\alpha = 3\lambda$

Now, $\alpha\beta = 2\lambda \Rightarrow 3\lambda \cdot \beta = 2\lambda \Rightarrow \beta = \frac{2}{3}$

$\Rightarrow \alpha + \beta = 1 \Rightarrow 3\lambda + \frac{2}{3} = 1 \Rightarrow \lambda = \frac{1}{9}$ and

$\alpha\gamma = 9\lambda \Rightarrow 3\lambda \cdot \gamma = 9\lambda \Rightarrow \gamma = 3$

$\therefore \frac{\beta\gamma}{\lambda} = 18$

Question198

If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$ then

$\left(\alpha - \frac{1}{\alpha}\right) \left(\beta - \frac{1}{\beta}\right) \left(\alpha + \frac{1}{\beta}\right) \left(\beta + \frac{1}{\alpha}\right)$ is equal to

[Sep. 03, 2020 (I)]

Options:

A. $\frac{9}{4}(9 + q^2)$

B. $\frac{9}{4}(9 - q^2)$

C. $\frac{9}{4}(9 + p^2)$

D. $\frac{9}{4}(9 - p^2)$

Answer: D

Solution:

Solution:

$$\alpha \cdot \beta = 2 \text{ and } \alpha + \beta = -p \text{ also } \frac{1}{\alpha} + \frac{1}{\beta} = -q$$

$$\Rightarrow p = 2q$$

$$\text{Now } \left(\alpha - \frac{1}{\alpha} \right) \left(\beta - \frac{1}{\beta} \right) \left(\alpha + \frac{1}{\beta} \right) \left(\beta + \frac{1}{\alpha} \right)$$

$$= \left[\alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha} \right] \left[\alpha\beta + \frac{1}{\alpha\beta} + 2 \right]$$

$$= \frac{9}{2} \left[\frac{5}{2} - \frac{\alpha^2 + \beta^2}{2} \right] = \frac{9}{4} [5 - (p^2 - 4)]$$

$$= \frac{9}{4} (9 - p^2) \quad [\because \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta]$$

Question199

The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0,1)$ is :

[Sep. 03, 2020 (II)]

Options:

A. $(0,2)$

B. $(2,4]$

C. $(1,3]$

D. $(-3,-1)$

Answer: C

Solution:

Solution:

The given quadratic equation is

$$(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$$

$\therefore$ One root is in the interval $(0,1)$

$$\therefore f(0)f(1) \leq 0$$

$$\Rightarrow 2(\lambda^2 + 1 - 4\lambda + 2) \leq 0$$

$$\Rightarrow 2(\lambda^2 - 4\lambda + 3) \leq 0$$

$$(\lambda - 1)(\lambda - 3) \leq 0 \Rightarrow \lambda \in [1, 3]$$

But at $\lambda = 1$, both roots are 1 so $\lambda \neq 1$

$$\therefore \lambda \in (1, 3]$$

Question200

Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$.

If $S_n = \alpha^n + \beta^n$, $n = 1, 2, 3, \dots$, then :

[Sep. 02, 2020 (I)]

Options:

A. $6S_6 + 5S_5 = 2S_4$

B. $6S_6 + 5S_5 + 2S_4 = 0$

C. $5S_6 + 6S_5 = 2S_4$

D. $5S_6 + 6S_5 + 2S_4 = 0$

Answer: C

Solution:

Solution:

Since, α and β are the roots of the equation

$$5x^2 + 6x - 2 = 0$$

$$\text{Then, } 5\alpha^2 + 6\alpha - 2 = 0, 5\beta^2 + 6\beta - 2 = 0$$

$$5\alpha^2 + 6\alpha = 2$$

$$5S_6 + 6S_5 = 5(\alpha^6 + \beta^6) + 6(\alpha^5 + \beta^5)$$

$$= (5\alpha^4 + 6\alpha^5) + (5\beta^6 + 6\beta^5)$$

$$= \alpha^4(5\alpha^2 + 6\alpha) + \beta^4(5\beta^2 + 6\beta)$$

$$= 2(\alpha^4 + \beta^4) = 2S_4$$

Question 201

If λ be the ratio of the roots of the quadratic equation in x , $3m^2x^2 + m(m - 4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is:

[Jan. 12, 2019 (I)]

Options:

A. $2 - \sqrt{3}$

B. $4 - 3\sqrt{2}$

C. $-2 + \sqrt{2}$

D. $4 - 2\sqrt{3}$

Answer: B

Solution:

Solution:

Let roots of the quadratic equation are α, β .

$$\text{Given, } \lambda = \frac{\alpha}{\beta} \text{ and } \lambda + \frac{1}{\lambda} = 1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$$

$$\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = 1 \dots (i)$$

$$\text{The quadratic equation is, } 3m^2x^2 + m(m - 4)x + 2 = 0$$

$$\therefore \alpha + \beta = \frac{m(4 - m)}{3m^2} = \frac{4 - m}{3m} \text{ and } \alpha\beta = \frac{2}{3m^2}$$

Put these values in eq (1)

$$\frac{\left(\frac{4 - m}{3m}\right)^2}{\frac{2}{3m^2}} = 3$$

$$\Rightarrow (m - 4)^2 = 18 \Rightarrow m = 4 \pm \sqrt{18}$$

Therefore, least value is

$$4 - \sqrt{18} = 4 - 3\sqrt{2}$$

Question202

If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is :

[Jan. 11, 2019 (I)]

Options:

- A. -81
- B. 100
- C. 144
- D. -300

Answer: D

Solution:

Solution:

Let α and β be the roots of the equation,
 $81x^2 + kx + 256 = 0$

$$\text{Given } (\alpha)^{\frac{1}{3}} = \beta \Rightarrow \alpha = \beta^3$$

$$\therefore \text{Product of the roots} = \frac{256}{81}$$

$$\therefore (\alpha)(\beta) = \frac{256}{81}$$

$$\Rightarrow \beta^4 = \left(\frac{4}{3}\right)^4 \Rightarrow \beta = \frac{4}{3} \Rightarrow \alpha = \frac{64}{27}$$

$$\text{Sum of the roots} = -\frac{k}{81}$$

$$\therefore \alpha + \beta = -\frac{k}{81} \Rightarrow \frac{4}{3} + \frac{64}{27} = -\frac{k}{81}$$

$$\Rightarrow k = -300$$

Question203

Consider the quadratic equation $(c - 5)x^2 - 2cx + (c - 4) = 0$ $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3) .

Then the number of elements in S is:

[Jan. 10, 2019 (I)]

Options:

- A. 18
- B. 12
- C. 10

D. 11

Answer: D

Solution:

Solution:

Consider the quadratic equation

$$(c - 5)x^2 - 2cx + (c - 4) = 0$$

Now, $f(0), f(3) > 0$ and $f(0) \cdot f(2) < 0$

$$\Rightarrow (c - 4)(4c - 49) > 0 \text{ and } (c - 4)(c - 24) < 0$$

$$\Rightarrow c \in (-\infty, 4) \cup \left(\frac{49}{4}, \infty \right) \text{ and } c \in (4, 24)$$

$$\Rightarrow c \in \left(\frac{49}{4}, 24 \right)$$

Integral values in the interval $\left(\frac{49}{4}, 24 \right)$ are 13, 14, ..., 23

$$\therefore S = \{13, 14, \dots, 23\}$$

Question204

The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is:
[Jan. 10, 2019 (II)]

Options:

A. $\frac{15}{8}$

B. 1

C. $\frac{4}{9}$

D. 2

Answer: D

Solution:

Solution:

The given quadratic equation is

$$x^2 + (3 - \lambda)x + 2 = \lambda$$

$$\text{Sum of roots} = \alpha + \beta = \lambda - 3$$

$$\text{Product of roots} = \alpha\beta = 2 - \lambda$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (\lambda - 3)^2 - 2(2 - \lambda)$$

$$= \lambda^2 - 4\lambda + 5 = (\lambda - 2)^2 + 1$$

$$\text{For least } (\alpha^2 + \beta^2)\lambda = 2$$

Question205

Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to:

[Jan. 9, 2019 (I)]

Options:

- A. -256
- B. 512
- C. -512
- D. 256

Answer: A

Solution:

Solution:

Consider the equation

$$x^2 + 2x + 2 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm i$$

$$\text{Let } \alpha = -1 + i, \beta = -1 - i$$

$$\alpha^{15} + \beta^{15} = (-1 + i)^{15} + (-1 - i)^{15}$$

$$= \left(\sqrt{2} e^{i \frac{3\pi}{4}} \right)^{15} + \left(\sqrt{2} e^{-i \frac{3\pi}{4}} \right)^{15}$$

$$= (\sqrt{2})^{15} \left[e^{\frac{i45\pi}{4}} + e^{\frac{-i45\pi}{4}} \right]$$

$$= (\sqrt{2})^{15} \cdot 2 \cos \frac{45\pi}{4} = (\sqrt{2})^{15} \cdot 2 \cos \frac{3\pi}{4}$$

$$= \frac{-2}{\sqrt{2}} (\sqrt{2})^{15}$$

$$= -2(\sqrt{2})^{14} = -256$$

Question206

The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is:

[Jan. 09, 2019 (II)]

Options:

- A. 3
- B. 2
- C. 4
- D. 5

Answer: A

Solution:

Solution:

The roots of $6x^2 - 11x + \alpha = 0$ are rational numbers.

$\therefore$ Discriminant D must be perfect square number.

$$D = (-11)^2 - 4 \cdot 6 \cdot \alpha$$

$= 121 - 24\alpha$ must be a perfect square
Hence, possible values for α are
 $\alpha = 3, 4, 5$
 $\therefore$ 3 positive integral values are possible.

Question207

If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval $[1, 5]$, then m lies in the interval:
[Jan. 09, 2019 (II)]

Options:

- A. (-5,-4)
- B. (4,5)
- C. (5,6)
- D. (3,4)

Answer: B

Solution:

Solution:

Given quadratic equation is: $x^2 - mx + 4 = 0$

Both the roots are real and distinct.

So, discriminant $B^2 - 4AC > 0$

$$\therefore m^2 - 4 \cdot 1 \cdot 4 > 0$$

$$\therefore (m - 4)(m + 4) > 0$$

$$\therefore m \in (-\infty, -4) \cup (4, \infty) \dots\dots(i)$$

Since, both roots lies in $[1, 5]$

$$\therefore -\frac{-m}{2} \in (1, 5)$$

$$\Rightarrow m \in (2, 10)$$

$$\text{And } 1 \cdot (1 - m + 4) > 0 \Rightarrow m < 5$$

$$\therefore m \in (-\infty, 5) \dots (iii)$$

$$\text{And } 1 \cdot (25 - 5m + 4) > 0 \Rightarrow m < \frac{29}{5}$$

$$\therefore m \in \left(-\infty, \frac{29}{5}\right) \dots (iv)$$

From (i), (ii), (iii) and (iv), $m \in (4, 5)$

Question208

Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then $\arg z$ is equal to:
[Jan. 09, 2019 (II)]

Options:

- A. $\frac{\pi}{4}$
- B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. 0

Answer: A

Solution:

Solution:

$\therefore z_0$ is a root of quadratic equation

$$x^2 + x + 1 = 0$$

$$\therefore z_0 = \omega \text{ or } \omega^2 \Rightarrow z_0^3 = 1$$

$$\begin{aligned} \therefore z &= 3 + 6iz_0^{81} - 3iz_0^{93} \\ &= 3 + 6i((z_0)^3)^{27} - 3i((z_0)^3)^{31} \\ &= 3 + 6i - 3i \\ &= 3 + 3i \end{aligned}$$

$$\therefore \arg(z) \tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$$

Question209

If 5, 5r, 5r² are the lengths of the sides of a triangle, then r cannot be equal to:

[Jan. 10, 2019 (I)]

Options:

A. $\frac{3}{4}$

B. $\frac{5}{4}$

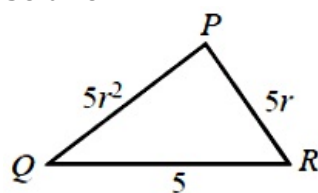
C. $\frac{7}{4}$

D. $\frac{3}{2}$

Answer: C

Solution:

Solution:



ΔPQR is possible if

$$5 + 5r > 5r^2$$

$$\Rightarrow 1 + r > r^2$$

$$\Rightarrow r^2 - r - 1 < 0$$

$$\Rightarrow \left(r - \frac{1}{2} + \frac{\sqrt{5}}{2}\right) \left(r - \frac{1}{2} - \frac{\sqrt{5}}{2}\right) < 0$$

$$\Rightarrow r \in \left(\frac{-\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2}\right)$$

$$\because \frac{7}{4} \notin \left(\frac{-\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2} \right) \therefore r \neq \frac{7}{4}$$

Question210

If $\frac{z-\alpha}{z+\alpha}$ ($\alpha \in \mathbb{R}$) is a purely imaginary number and $|z| = 2$, then a value of α is:

[Jan. 12, 2019 (I)]

Options:

A. 2

B. 1

C. $\frac{1}{2}$

D. $\sqrt{2}$

Answer: A

Solution:

Solution:

$$\text{Let } t = \frac{z-\alpha}{z+\alpha}$$

$\because$ t is purely imaginary number.

$$\therefore t + \bar{t} = 0$$

$$\Rightarrow \frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$$

$$\Rightarrow (z-\alpha)(\bar{z}+\alpha) + (\bar{z}-\alpha)(z+\alpha) = 0$$

$$\Rightarrow z\bar{z} - \alpha^2 + z\bar{z} - \alpha^2 = 0$$

$$\Rightarrow z\bar{z} - \alpha^2 = 0$$

$$\Rightarrow |z|^2 - \alpha^2 = 0$$

$$\Rightarrow \alpha^2 = 4$$

$$\Rightarrow \alpha = \pm 2$$

Question211

Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2| - |3| - |4i| = |4|$. Then the minimum value of $|z_1 - z_2|$ is :

[Jan. 12, 2019 (II)]

Options:

A. 0

B. $\sqrt{2}$

C. 1

D. 2

Answer: A

Solution:

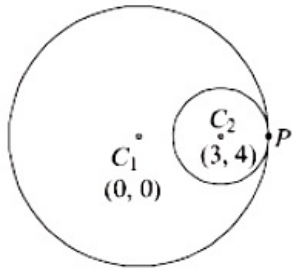
Solution:

$$|z_1| = 9, |z_2 - 3 - 4i| = 4$$

z_1 lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$

z_2 lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$

So, minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. A)



Question212

Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$)

Then $|z|$ is equal to :

[Jan. 11, 2019 (II)]

Options:

A. $\frac{\sqrt{34}}{3}$

B. $\frac{5}{3}$

C. $\frac{\sqrt{41}}{4}$

D. $\frac{5}{4}$

Answer: B

Solution:

Solution:

Since, $|z| + z = 3 + i$

Let $z = a + ib$, then

$$|z| + z = 3 + i \Rightarrow \sqrt{a^2 + b^2} + a + ib = 3 + i$$

Compare real and imaginary coefficients on both sides

$$b = 1, \sqrt{a^2 + b^2} + a = 3$$

$$\sqrt{a^2 + 1} = 3 - a$$

$$a^2 + 1 = a^2 + 9 - 6a$$

$$6a = 8 \Rightarrow a = \frac{4}{3}$$

Then

$$|z| = \sqrt{\left(\frac{4}{3}\right)^2 + 1} = \sqrt{\frac{16}{9} + 1} = \frac{5}{3}$$

Question213

Let z_1 and z_2 be any two non-zero complex numbers such that

$3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then

[Jan. 10 2019 (II)]

Options:

A. $\operatorname{Re}(z) = 0$

B. $|z| = \sqrt{\frac{5}{2}}$

C. $|z| = \frac{1}{2} \sqrt{\frac{17}{2}}$

D. $\operatorname{Im}(z) = 0$

E. None of Above

Answer: E

Solution:

Solution:
(none)

Let $z_1 = r_1 e^{i\theta}$ and $z_2 = r_2 e^{i\varphi}$

$$3|z_1| = 4|z_2| \Rightarrow 3r_1 = 4r_2$$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = \frac{3r_1}{2r_2} e^{i(\theta - \varphi)} + \frac{2}{3} \frac{r_2}{r_1} e^{i(\varphi - \theta)}$$

$$= \frac{3}{2} \times \frac{4}{3} (\cos(\theta - \varphi) + i \sin(\theta - \varphi)) +$$

$$\frac{2}{3} \times \frac{3}{4} [\cos(\theta - \varphi) - i \sin(\theta - \varphi)]$$

$$z = \left(2 + \frac{1}{2}\right) \cos(\theta - \varphi) + i \left(2 - \frac{1}{2}\right) \sin(\theta - \varphi)$$

$$\therefore |z| = \sqrt{\frac{25}{4} \cos^2(\theta - \varphi) + \frac{9}{4} \sin^2(\theta - \varphi)}$$

$$= \sqrt{\frac{16}{4} \cos^2(\theta - \varphi) + \frac{9}{4}} \Rightarrow \frac{3}{2} \leq |z| \leq \frac{5}{2}$$

Question 214

Let $A = \left\{ \theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3 + 2i \sin \theta}{1 - 2i \sin \theta} \text{ is purely imaginary} \right\}$.

Then the sum of the elements in A is:

[Jan. 9 2019 (I)]

Options:

A. $\frac{5\pi}{6}$

B. π

C. $\frac{3\pi}{4}$

D. $\frac{2\pi}{3}$

Answer: D

Solution:

Solution:

$$\text{Suppose } z = \frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$$

Since, z is purely imaginary, then $z + \bar{z} = 0$

$$\Rightarrow \frac{3 + 2i \sin \theta}{1 - 2i \sin \theta} + \frac{3 - 2i \sin \theta}{1 + 2i \sin \theta} = 0$$

$$\Rightarrow \frac{(3 + 2i \sin \theta)(1 + 2i \sin \theta) + (3 - 2i \sin \theta)(1 - 2i \sin \theta)}{1 + 4\sin^2 \theta}$$

$$= 0$$

$$\Rightarrow \sin^2 \theta = \frac{3}{4} \Rightarrow \sin \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\text{Now, the sum of elements in } A = -\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}$$

Question215

Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x+iy}{27}$ ($i = \sqrt{-1}$), where x and y are real numbers then

$y - x$ equals:

[Jan. 11, 2019 (I)]

Options:

A. 91

B. -85

C. 85

D. -91

Answer: A

Solution:

Solution:

$$-(6 + i)^3 = x + iy$$

$$\Rightarrow -[216 + i^3 + 18i(6 + i)] = x + iy$$

$$\Rightarrow -[216 - i + 108i - 18] = x + iy$$

$$\Rightarrow -216 + i - 108i + 18 = x + iy$$

$$\Rightarrow -198 - 107i = x + iy$$

$$\Rightarrow x = -198, y = -107$$

$$\Rightarrow y - x = -107 + 198 = 91$$

Question216

Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2} \right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2} \right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z , then:
[Jan. 10, 2019 (II)]

Options:

- A. $I(z) = 0$
- B. $R(z) > 0$ and $I(z) > 0$
- C. $R(z) < 0$ and $I(z) > 0$
- D. $R(z) = -(c)$

Answer: A

Solution:

Solution:

$$\begin{aligned}
 z &= \left(\frac{\sqrt{3}}{2} + \frac{i}{2} \right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2} \right)^5 \\
 &= \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)^5 + \left(\cos \frac{\pi}{6} - i \sin \frac{\pi}{6} \right)^5 \\
 &= \left(e^{i \frac{\pi}{6}} \right)^5 + \left(e^{-i \frac{\pi}{6}} \right)^5 = 2 \cos \frac{\pi}{6} = \sqrt{3} \\
 \Rightarrow I(z) &= 0, \text{Re}(z) = \sqrt{3}
 \end{aligned}$$

Question217

Let $z \in \mathbb{C}$ with $\text{Im}(Z) = 10$ and it satisfies $\frac{2z - n}{2z + n} = 2i - 1$ for some natural number n .Then:
[April 12, 2019 (II)]

Options:

- A. $n = 20$ and $\text{Re}(z) = -10$
- B. $n = 40$ and $\text{Re}(z) = 10$
- C. $n = 40$ and $\text{Re}(z) = -10$
- D. $n = 20$ and $\text{Re}(z) = 10$

Answer: C

Solution:

Solution:

$$\begin{aligned}
 \text{Let } \text{Re}(z) &= x \text{ i.e., } z = x + 10i \\
 2z - n &= (2i - 1)(2z + n) \\
 (2x - n) + 20i &= (2i - 1)((2x + n) + 20i) \\
 \text{On comparing real and imaginary parts,} \\
 -(2x + n) - 40 &= 2x - n \text{ and } 20 = 4x + 2n - 20 \\
 \Rightarrow 4x &= -40 \text{ and } 40 = -40 + 2n \\
 \Rightarrow x &= -10 \text{ and } n = 40
 \end{aligned}$$

Hence, $\text{Re}(z) = -10$

Question 218

The equation $|Z - i| = |Z - 1|$, $i = \sqrt{-1}$, represents:
[April 12, 2019 (I)]

Options:

- A. a circle of radius $\frac{1}{2}$
- B. the line through the origin with slope 1.
- C. a circle of radius 1.
- D. the line through the origin with slope -1.

Answer: B

Solution:

Solution:

Given equation is, $|z - 1| = |z - i|$
 $\Rightarrow (x - 1)^2 + y^2 = x^2 + (y - 1)^2$ [Here, $z = x + iy$]
 $\Rightarrow 1 - 2x = 1 - 2y \Rightarrow x - y = 0$
Hence, locus is straight line with slope 1.

Question 219

if $a > 0$ and $Z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to:
[April 10, 2019 (I)]

Options:

- A. $-\frac{1}{2} - \frac{3}{5}i$
- B. $-\frac{3}{5} - \frac{1}{5}i$
- C. $\frac{1}{5} - \frac{3}{5}i$
- D. $-\frac{1}{5} + \frac{3}{5}i$

Answer: A

Solution:

Solution:

$$z = \frac{(1+i)^2}{a-i} \times \frac{a+i}{a+i}$$

$$z = \frac{(1 - 1 + 2i)(a + i)}{a^2 + 1} = \frac{2ai - 2}{a^2 + 1}$$

$$|z| = \sqrt{\left(\frac{-2}{a^2 + 1}\right)^2 + \left(\frac{2a}{a^2 + 1}\right)^2} = \sqrt{\frac{4 + 4a^2}{(a^2 + 1)^2}}$$

$$\Rightarrow |z| = \sqrt{\frac{4(1 + a^2)}{(1 + a^2)^2}} = \frac{2}{\sqrt{1 + a^2}} \dots\dots\dots(i)$$

Since, it is given that $|z| = \sqrt{\frac{2}{5}}$

Then, from equation (i),

$$\sqrt{\frac{2}{5}} = \frac{2}{\sqrt{1 + a^2}}$$

Now, square on both side; we get

$$\Rightarrow 1 + a^2 = 10 \Rightarrow a = \pm 3$$

Since, it is given that $a > 0 \Rightarrow a = 3$ Then, $z = \frac{(1 + i)^2}{a - i} = \frac{1 + i^2 + 2i}{3 - i} = \frac{2i}{3 - i}$

$$= \frac{2i(3 + i)}{10} = \frac{-1 + 3i}{5}$$

$$\text{Hence, } \bar{z} = \frac{-1}{5} - \frac{3}{5}i$$

Question220

If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then:

[April 10, 2019 (II)]

Options:

A. $\bar{z}\omega = i$

B. $\bar{z}\omega = \frac{-1 + i}{\sqrt{2}}$

C. $\bar{z}\omega = -i$

D. $\bar{z}\omega = \frac{1 - i}{\sqrt{2}}$

Answer: C

Solution:

Solution:

Given $|z\omega| = 1 \dots (i)$

$$\text{and } \arg\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$$

$$\therefore \frac{z}{\omega} + \frac{\bar{z}}{\bar{\omega}} = 0 \left[\because \operatorname{Re}\left(\frac{z}{\omega}\right) = 0 \right]$$

$$\Rightarrow z\bar{\omega} = -\bar{z}\omega$$

from equation (i), $z\bar{z}\omega\bar{\omega} = 1$ [using $z\bar{z} = |z|^2$]

$$(z\omega)^2 = -1 \Rightarrow z\omega = \pm i$$

$$\text{from equation (ii), } -\arg(\bar{z}) - \arg \omega = \frac{\pi}{2} - \arg(\bar{z}\omega) = \frac{-\pi}{2}$$

$$\text{Hence, } \bar{z}\omega = -i$$

Question221

Let $z \in \mathbb{C}$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then :
[April 09, 2019 (II)]

Options:

- A. $5\operatorname{Re}(\omega) > 4$
- B. $4\operatorname{Im}(\omega) > 5$
- C. $5\operatorname{Re}(\omega) > 1$
- D. $5\operatorname{Im}(\omega) < 1$

Answer: C

Solution:

Solution:

$$\begin{aligned}\omega &= \frac{5+3z}{5-5z} \Rightarrow 5\omega - 5\omega z = 5 + 3z \\ \Rightarrow 5\omega - 5 &= z(3 + 5\omega) \Rightarrow z = \frac{5(\omega - 1)}{3 + 5\omega} \\ \because |z| &\leq 1, \therefore 5|\omega - 1| < |3 + 5\omega| \\ \Rightarrow 25(\omega\bar{\omega} - \omega - \bar{\omega} + 1) &< 9 + 25\omega\bar{\omega} + 15\omega + 15\bar{\omega} \\ (\because |z|^2 &= z\bar{z}) \\ \Rightarrow 16 &< 40\omega + 40\bar{\omega} \Rightarrow \omega + \bar{\omega} > \frac{2}{5} \Rightarrow 2\operatorname{Re}(\omega) > \frac{2}{5} \\ \Rightarrow \operatorname{Re}(\omega) &> \frac{1}{5}\end{aligned}$$

Question 222

If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}(i = \sqrt{-1})$, then $(1 + iz + z^5 + iz^8)^9$ is equal to:
[April 08, 2019 (II)]

Options:

- A. 0
- B. 1
- C. $(-1 + 2i)^9$
- D. -1

Answer: D

Solution:

Solution:

$$\begin{aligned}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) &= -i\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -i\omega \\ \text{where } \omega &\text{ is imaginary cube root of unity.} \\ \text{Now, } (1 + iz + z^5 + iz^8)^9 &= (1 + \omega - i\omega^2 + i\omega^2)^9 = (1 + \omega)^9 \\ &= (-\omega^2)^9 = -\omega^{18} = -1 \quad (\because 1 + \omega + \omega^2 = 0)\end{aligned}$$

Question223

If α and β are the roots of the quadratic equation, $x^2 + x \sin \theta - 2 \sin \theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12})(\alpha - \beta)^{24}}$ is equal to :
[April 10, 2019 (I)]

Options:

- A. $\frac{2^{12}}{(\sin \theta - 4)^{12}}$
B. $\frac{2^{12}}{(\sin \theta + 8)^{12}}$
C. $\frac{2^{12}}{(\sin \theta - 8)^6}$
D. $\frac{2^6}{(\sin \theta + 8)^{12}}$

Answer: B

Solution:

Solution:

Given equation is,

$$x^2 + x \sin \theta - 2 \sin \theta = 0$$

$$\alpha + \beta = -\sin \theta \text{ and } \alpha\beta = -2 \sin \theta$$

$$\frac{(\alpha^{12} + \beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12} + \beta^{12})(\alpha - \beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}}$$

$$\therefore |\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{\sin^2 \theta + 8 \sin \theta}$$

$$\therefore \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}} = \frac{(2 \sin \theta)^{12}}{\sin^{12} \theta (\sin \theta + 8)^{12}} = \frac{2^{12}}{(\sin \theta + 8)^{12}}$$

Question224

The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is:
[April 10, 2019 (II)]

Options:

- A. 3
B. 2
C. 4
D. 1

Answer: D

Solution:

Solution:

$$\text{Let } 2^x - 1 = t$$

$$5 + |t| = (t + 1)(t - 1) \Rightarrow |t| = t^2 - 6$$

$$\text{When } t > 0, t^2 - t - 6 = 0 \Rightarrow t = 3 \text{ or } -2$$

$$t = -2 \text{ (rejected)}$$

$$\text{When } t < 0, t^2 + t - 6 = 0 \Rightarrow t = -3 \text{ or } 2 \text{ (both rejected)}$$

$$\therefore 2^x - 1 = 3 \Rightarrow 2^x = 4 \Rightarrow x = 2$$

Question225

Let $p, q \in \mathbb{R}$. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then:

[April 10, 2019 (II)]

Options:

A. $p^2 - 4q + 12 = 0$

B. $q^2 - 4p - 16 = 0$

C. $q^2 + 4p + 14 = 0$

D. $p^2 - 4q - 12 = 0$

Answer: D

Solution:**Solution:**

Since $2 - \sqrt{3}$ is a root of the quadratic equation

$$x^2 + px + q = 0$$

$\therefore 2 + \sqrt{3}$ is the other root

$$\begin{aligned} \Rightarrow x^2 + px + q &= [x - (2 - \sqrt{3})][x - (2 + \sqrt{3})] \\ &= x^2 - (2 + \sqrt{3})x - (2 - \sqrt{3})x + (2^2 - (\sqrt{3})^2) \\ &= x^2 - 4x + 1 \end{aligned}$$

Now, by comparing $p = -4, q = 1$

$$\Rightarrow p^2 - 4q - 12 = 16 - 4 - 12 = 0$$

Question226

If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:

[April 09, 2019 (II)]

Options:

A. $10\sqrt{5}$

B. $8\sqrt{3}$

C. $8\sqrt{5}$

D. $4\sqrt{3}$

Answer: C

Solution:

Solution:

$$\text{Sum of roots} = \frac{3}{m^2 + 1}$$

$\therefore$ sum of roots is greatest. $\therefore m = 0$

Hence equation becomes $x^2 - 3x + 1 = 0$

Now, $\alpha + \beta = 3$, $\alpha\beta = 1 \Rightarrow |-\alpha - \beta| = \sqrt{5}$

$$|\alpha^3 - \beta^3| = |(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)| = \sqrt{5}(9 - 1) = 8\sqrt{5}$$

Question227

The sum of the solutions of the equation

$|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, ($x > 0$) is equal to:

[April 8, 2019 (I)]

Options:

A. 9

B. 12

C. 4

D. 10

Answer: D

Solution:

Solution:

Let $\sqrt{x} = a$

$\therefore$ given equation will become:

$$|a - 2| + a(a - 4) + 2 = 0$$

$$\Rightarrow |a - 2| + a^2 - 4a + 4 - 2 = 0$$

$$\Rightarrow |a - 2| + (a - 2)^2 - 2 = 0$$

Let $|a - 2| = y$ (Clearly $y \geq 0$)

$$\Rightarrow y + y^2 - 2 = 0$$

$$\Rightarrow y = 1 \text{ or } -2 \text{ (rejected)} \Rightarrow |a - 2| = 1 \Rightarrow a = 1, 3$$

When $\sqrt{x} = 1 \Rightarrow x = 1$

When $\sqrt{x} = 3 \Rightarrow x = 9$

Hence, the required sum of solutions of the equation
= 10

Question228

If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is:

[April 8, 2019 (I)]

Options:

A. 2

B. 5

C. 4

D. 3

Answer: C

Solution:

Solution:

The given quadratic equation is $x^2 - 2x + 2 = 0$

Then, the roots of the this equation are $\frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$

$$\text{Now, } \frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = i$$

$$\text{or } \frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = -i \quad \text{So, } \frac{\alpha}{\beta} = \pm i$$

$$\text{Now, } \left(\frac{\alpha}{\beta} \right)^n = 1 \Rightarrow (\pm i)^n = 1$$

$\Rightarrow n$ must be a multiple of 4

Hence, the required least value of $n = 4$

Question 229

The set of all $\alpha \in \mathbb{R}$, for which $w = \frac{1 + (1 - 8\alpha)z}{1 - z}$ is a purely imaginary number, for all $z \in \mathbb{C}$ satisfying $|z| = 1$ and $\operatorname{Re} z \neq 1$, is [Online April 15, 2018]

Options:

A. $\{0\}$

B. an empty set

C. $\left\{ 0, \frac{1}{4}, -\frac{1}{4} \right\}$

D. equal to $\mathbb{R}$

Answer: A

Solution:

Solution:

$\because |z| = 1$ & $\operatorname{Re} z \neq 1$

Suppose $z = x + iy \Rightarrow x^2 + y^2 = 1$ (i)

$$\text{Now, } w = \frac{1 + (1 - 8\alpha)z}{1 - z}$$

$$\Rightarrow w = \frac{1 + (1 - 8\alpha)(x + iy)}{1 - (x + iy)}$$

$$\Rightarrow w = \frac{1 + (1 - 8\alpha)(x + iy)((1 - x) + iy)}{1 - (x + iy)((1 - x) + iy)}$$

$$\Rightarrow w = \frac{[(1 + x(1 - 8\alpha)(1 - x) - (1 - 8\alpha)y^2)]}{(1 - x)^2 + y^2}$$

$$+ i \frac{[(1+x(1-8\alpha))y - (1-8\alpha)y(1-x)]}{(1-x)^2 + y^2}$$

As, w is purely imaginary. So,

$$\text{Re}w = \frac{[(1+x(1-8\alpha))(1-x) - (1-8\alpha)y^2]}{(1-x)^2 + y^2} = 0$$

$$\Rightarrow (1-x) + x(1-8\alpha)(1-x) = (1-8\alpha)y^2$$

$$\Rightarrow (1-x) + x(1-8\alpha) - x^2(1-8\alpha) = (1-8\alpha)y^2$$

$$\Rightarrow (1-x) + x(1-8\alpha) = 1-8\alpha \quad [\text{From (i), } x^2 + y^2 = 1]$$

$$\Rightarrow 1-8\alpha = 1 \Rightarrow \alpha = 0$$

$$\therefore \alpha \in \{0\}$$

Question230

The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is

[Online April 16, 2018]

Options:

A. 2

B. 6

C. 5

D. 3

Answer: D

Solution:

Solution:

$$\text{Let } 1 = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$$

$$\therefore 1 = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{1+i\sqrt{3}}{1+i\sqrt{3}}\right)$$

$$= \left(\frac{-2+i2\sqrt{3}}{4}\right) = \left(\frac{1-i\sqrt{3}}{-2}\right)$$

$$\text{Also, } 1 = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{1-i\sqrt{3}}{1-i\sqrt{3}}\right)$$

$$= \left(\frac{4}{-2-i2\sqrt{3}}\right) = \left(\frac{-2}{1+i\sqrt{3}}\right)$$

$$\text{Now, } \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^3 = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$$

$$= \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{-2}{1+i\sqrt{3}}\right) \times \left(\frac{1-i\sqrt{3}}{-2}\right) = 1$$

$\therefore$ least positive integer n is 3 .

Question231

Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to.

[Online April 16, 2018]

Options:

A. $p^2 + q^2 + r^2$

B. $p^2 + q^2$

C. $2(p^2 + q^2)$

D. $\frac{p^2 + q^2}{2}$

Answer: B

Solution:

Solution:

$$\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$$

$$\frac{x+p+x+q}{(x+p)(x+q)} = \frac{1}{r}$$

$$(2x+p+q)r = x^2 + px + qx + pq$$

$$x^2 + (p+q-2r)x + pq - pr - qr = 0$$

Let α and β be the roots.

$$\therefore \alpha + \beta = -(p+q-2r) \dots\dots\dots(i)$$

$$\&\alpha\beta = pq - pr - qr \dots\dots\dots(ii)$$

$$\because \alpha = -\beta \text{ (given)}$$

$\therefore$ in eq. (1), we get

$$\Rightarrow -(p+q-2r) = 0 \dots\dots\dots(iii)$$

$$\text{Now, } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-(p+q-2r))^2 - 2(pq - pr - qr) \dots\dots\dots \text{ (from (i) and (ii))}$$

$$= p^2 + q^2 + 4r^2 + 2pq - 4pr - 4qr - 2pq + 2pr + 2qr$$

$$= p^2 + q^2 + 4r^2 - 2pr - 2qr$$

$$= p^2 + q^2 + 2r(2r - p - q) \dots\dots \text{ (from (iii))}$$

$$= p^2 + q^2 + 0$$

$$= p^2 + q^2$$

Question232

If an angle A of a ΔABC satisfies $5 \cos A + 3 = 0$, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are.

[Online April 16, 2018]

Options:

A. $\sin A, \sec A$

B. $\sec A, \tan A$

C. $\tan A, \cos A$

D. $\sec A, \cot A$

Answer: B

Solution:

Solution:

$$\text{Here, } 9x^2 + 27x + 20 = 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-27 \pm \sqrt{27^2 - 4 \times 9 \times 20}}{2 \times 9}$$

$$\Rightarrow x = -\frac{4}{3}, -\frac{5}{3}$$

Given, $\cos A = -\frac{3}{5}$

$$\therefore \sec A = \frac{1}{\cos A} = -\frac{5}{3}$$

Here, A is an obtuse angle.

$$\therefore \tan A = -\sqrt{\sec^2 A - 1} = -\frac{4}{3}$$

Hence, roots of the equation are $\sec A$ and $\tan A$

Question233

If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$ then the value of $3\sin^2(A + B) - 10\sin(A + B) \cdot \cos(A + B) - 25\cos^2(A + B)$ is [Online April 15, 2018]

Options:

- A. 25
- B. -25
- C. -10
- D. 10

Answer: B

Solution:

Solution:

As $\tan A$ and $\tan B$ are the roots of $3x^2 - 10x - 25 = 0$,

$$\text{So, } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{10}{3}}{1 + \frac{25}{3}} = \frac{10 / 3}{28 / 3} = \frac{5}{14}$$

$$\text{Now, } \cos 2(A + B) = -1 + 2\cos^2(A + B)$$

$$= \frac{1 - \tan^2(A + B)}{1 + \tan^2(A + B)} \Rightarrow \cos^2(A + B) = \frac{196}{221}$$

$$\begin{aligned} \therefore 3\sin^2(A + B) - 10\sin(A + B)\cos(A + B) - 25\cos^2(A + B) \\ = \cos^2(A + B)[3\tan^2(A + B) - 10\tan(A + B) - 25] \\ = \frac{75 - 700 - 4900}{196} \times \frac{196}{221} = -\frac{5525}{196} \times \frac{196}{221} = -25 \end{aligned}$$

Question234

If $f(x)$ is a quadratic expression such that $f + f' = 0$ and -1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is [Online April 15, 2018]

Options:

A. $-\frac{5}{8}$

B. $-\frac{8}{5}$

C. $\frac{5}{8}$

D. $\frac{8}{5}$

Answer: D

Solution:

Solution:

If a and -1 are the roots of the polynomial, then we get

$$f(x) = x^2 + (1 - a)x - a$$

$$\therefore f(1) = 2 - 2a$$

$$\text{and } f(2) = 6 - 3a$$

$$\text{As, } f(1) + f(2) = 0 \Rightarrow 2 - 2a + 6 - 3a = 0 \Rightarrow a = \frac{8}{5}$$

Therefore, the other root is $\frac{8}{5}$

Question235

If $\alpha, \beta \in \mathbb{C}$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to :
[2018]

Options:

A. 0

B. 1

C. 2

D. -1

Answer: B

Solution:

Solution:

α, β are roots of $x^2 - x + 1 = 0$

$$\therefore \alpha = -\omega \text{ and } \beta = -\omega^2$$

where ω is cube root of unity

$$\begin{aligned} \therefore \alpha^{101} + \beta^{107} &= (-\omega)^{101} + (-\omega)^{107} \\ &= -[\omega^2 + \omega] = -[-1] = 1 \end{aligned}$$

Question236

If $\lambda \in \mathbb{R}$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is
[Online April 15, 2018]

Options:

- A. 20
- B. $2\sqrt{5}$
- C. $2\sqrt{7}$
- D. $4\sqrt{2}$

Answer: B

Solution:

Solution:

Let, the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ are α and β
Also roots of the given equation are

$$\frac{\lambda - 2 \pm \sqrt{4 - 4\lambda + \lambda^2 - 40 + 4\lambda}}{2} = \frac{\lambda - 2 \pm \sqrt{\lambda^2 - 36}}{2}$$

The magnitude of the difference of the roots is $|\sqrt{\lambda^2 - 36}|$

$$\begin{aligned} \text{So, } \alpha^3 + \beta^3 &= \frac{(\lambda - 2)^3}{4} + \frac{3(\lambda - 2)(\lambda^2 - 36)}{4} \\ &= \frac{(\lambda - 2)(4\lambda^2 - 4\lambda - 104)}{4} = (\lambda - 2)(\lambda^2 - \lambda - 26) = f(\lambda) \end{aligned}$$

As $f(\lambda)$ attains its minimum value at $\lambda = 4$

Therefore, the magnitude of the difference of the roots is
 $|\pm\sqrt{20}| = 2\sqrt{5}$

Question237

If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is
[Online April 15, 2018]

Options:

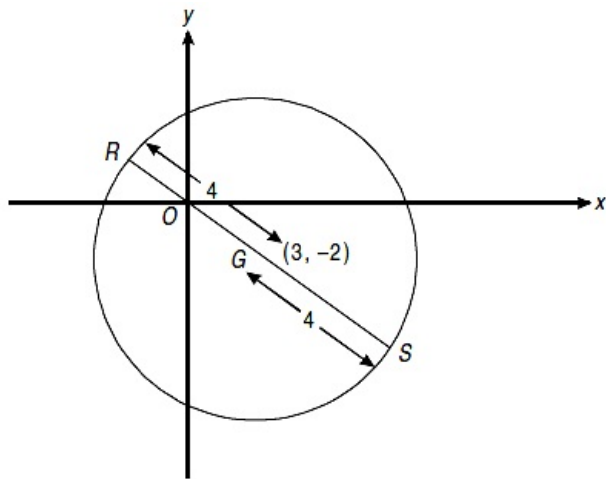
- A. $\sqrt{13}$
- B. $2\sqrt{13}$
- C. 8
- D. $4 + \sqrt{13}$

Answer: B

Solution:

Solution:

$|z - (3 - 2i)| \leq 4$ represents a circle whose centre is (3,-2) and radius = 4
 $|z| = |z - 0|$ represents the distance of point z ' from origin (0,0)



Suppose RS is the normal of the circle passing through origin ' O ' and G is its center (3,-2) .

Here, OR is the least distance and OS is the greatest distance

OR = RG – OG and OS = OG + GS

As, RG = GS = 4

OG = $\sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13}$

From (i), OR = $4 - \sqrt{13}$ and OS = $4 + \sqrt{13}$

So, required difference = $(4 + \sqrt{13}) - (4 - \sqrt{13})$
 $= \sqrt{13} + \sqrt{13} = 2\sqrt{13}$

Question238

If, for a positive integer n, the quadratic equation,
 $x(x + 1) + (x + 1)(x + 2) + \dots + (x + n - 1)(x + n) = 10n$ has two
consecutive integral solutions, then n is equal to:
[2017]

Options:

- A. 11
- B. 12
- C. 9
- D. 10

Answer: A

Solution:

Solution:

We have, $\sum_{r=1}^n (x + r - 1)(x + r) = 10n$

$\sum_{r=1}^n (x^2 + xr + (r - 1)x + r^2 - r) = 10n$

$\Rightarrow \sum_{r=1}^n (x^2 + (2r - 1)x + r(r - 1)) = 10n$

$\Rightarrow nx^2 + \{1 + 3 + 5 + \dots + (2n - 1)\}x$
 $+ \{1.2 + 2.3 + \dots + (n - 1)n\} = 10n$

$\Rightarrow nx^2 + n^2x + \frac{(n - 1)n(n + 1)}{3} = 10n$

$\Rightarrow x^2 + nx + \frac{n^2 - 31}{3} = 0$

Let α and $\alpha + 1$ be its two solutions

($\because$ it has two consecutive integral solutions)

$$\Rightarrow \alpha + (\alpha + 1) = -n$$

$$\Rightarrow \alpha = \frac{-n-1}{2} \dots\dots(i)$$

$$\text{Also } \alpha(\alpha + 1) = \frac{n^2 - 31}{3} \dots\dots(ii)$$

Putting value of (i) in (ii), we get

$$-\left(\frac{n+1}{2}\right)\left(\frac{1-n}{2}\right) = \frac{n^2 - 31}{3}$$

$$\Rightarrow n^2 = 121 \Rightarrow n = 11$$

Question239

The sum of all the real values of x satisfying the equation

$$2^{(x-1)(x^2+5x-50)} = 1 \text{ is:}$$

[Online April 9, 2017]

Options:

A. 16

B. 14

C. -4

D. -5

Answer: C

Solution:

Solution:

$$(x-1)(x^2+5x-50) = 0$$

$$\Rightarrow (x-1)(x+10)(x-5) = 0$$

$$\Rightarrow x = 1, 5, -10$$

$$\text{Sum} = -4$$

Question240

Let p(x) be a quadratic polynomial such that p(0) = 1. If p(x) leaves remainder 4 when divided by x – 1 and it leaves remainder 6 when divided by x + 1; then

[Online April 8, 2017]

Options:

A. $p(b) = 11$

B. $p(b) = 19$

C. $p(-2) = 19$

D. $p(-2) = 11$

Answer: C

Solution:

Solution:

$$\text{Let } p(x) = ax^2 + bx + c$$

$$\therefore p(0) = 1 \Rightarrow c = 1$$

$$\text{Also, } p(1) = 4 \text{ \& } p(-1) = 6$$

$$\Rightarrow a + b + 1 = 4 \text{ \& } a - b + 1 = 6$$

$$\Rightarrow a + b = 3 \text{ \& } a - b = 5$$

$$\Rightarrow a = 4 \text{ \& } b = -1$$

$$p(x) = 4x^2 - x + 1$$

$$p(b) = 16 - 2 + 1 = 15$$

$$p(-2) = 16 + 2 + 1 = 19$$

Question241

A value of θ for which $\frac{2 + 3i \sin \theta}{1 - 2i \sin \theta}$ is purely imaginary, is:
[2016]

Options:

A. $\sin^{-1} \left(\frac{\sqrt{3}}{4} \right)$

B. $\sin^{-1} \left(\frac{1}{\sqrt{3}} \right)$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: B

Solution:

Solution:

Rationalizing the given expression $\frac{(2 + 3i \sin \theta)(1 + 2i \sin \theta)}{1 + 4\sin^2 \theta}$

For the given expression to be purely imaginary, real part of the above expression should be equal to zero.

$$\Rightarrow \frac{2 - 6\sin^2 \theta}{1 + 4\sin^2 \theta} = 0 \Rightarrow \sin^2 \theta = \frac{1}{3}$$

$$\Rightarrow \sin \theta = \pm \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

Question242

The point represented by $2 + i$ in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by :
[Online April 9, 2016]

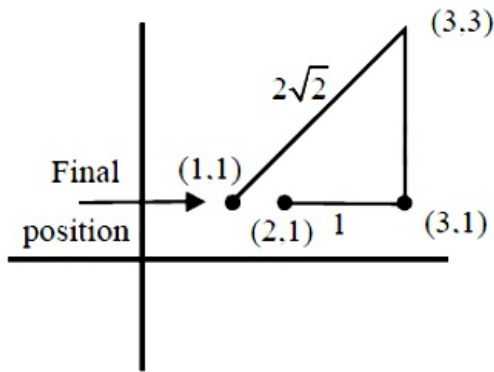
Options:

- A. $1 + i$
- B. $2 + 2i$
- C. $-2 - 2i$
- D. $-1 - i$

Answer: A

Solution:

Solution:



So new position is at the point $1 + i$

Question243

The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is:
[2016]

Options:

- A. 6
- B. 5
- C. 3
- D. -4

Answer: C

Solution:

$$(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$$

Case I

$x^2 - 5x + 5 = 1$ and $x^2 + 4x - 60$ can be any real number $\Rightarrow x = 1, 4$

Case II

$x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60$ has to be an even number
 $\Rightarrow x = 2, 3$

where 3 is rejected because for $x = 3$,

$x^2 + 4x - 60$ is odd

Case III

$x^2 - 5x + 5$ can be any real number and

$$\begin{aligned}
 x^2 + 4x - 60 &= 0 \\
 \Rightarrow x &= -10, 6 \\
 \Rightarrow \text{Sum of all values of } x \\
 &= -10 + 6 + 2 + 1 + 4 = 3
 \end{aligned}$$

Question244

If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1$, $\left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to:
[Online April 10, 2016]

Options:

- A. $\frac{3}{4}$
- B. $\frac{1}{2}$
- C. $2\sqrt{2}$
- D. 2

Answer: A

Solution:

Solution:

$$\begin{aligned}
 \sqrt{2x+1} - \sqrt{2x-1} &= 1 \\
 \Rightarrow 2x+1 + 2x-1 - 2\sqrt{4x^2-1} &= 1 \\
 \Rightarrow 4x-1 &= 2\sqrt{4x^2-1} \\
 \Rightarrow 16x^2-8x+1 &= 16x^2-4 \\
 \Rightarrow 8x &= 5 \\
 \Rightarrow x &= \frac{5}{8} \text{ which satisfies equation (i)} \\
 \text{So, } \sqrt{4x^2-1} &= \frac{3}{4}
 \end{aligned}$$

Question245

If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from -1 , then $|b|$ is equal to :
[Online April 9, 2016]

Options:

- A. 2
- B. 3
- C. $\sqrt{3}$
- D. $\sqrt{2}$

Answer: C

Solution:

Solution:

$$x^2 + bx - 1 = 0 \text{ common root}$$

$$x^2 + x + b = 0$$

$$x = \frac{b+1}{b-1}$$

$$\text{Put } x = \frac{b+1}{b-1} \text{ in equation}$$

$$\left(\frac{b+1}{b-1}\right)^2 + \left(\frac{b+1}{b-1}\right) + b = 0$$

$$\text{Put } x = \frac{b+1}{b-1} \text{ in equation}$$

$$\left(\frac{b+1}{b-1}\right)^2 + \left(\frac{b+1}{b-1}\right) + b = 0$$

$$(b+1)^2 + (b+1)(b-1) + b(b-1)^2 = 0$$

$$b^2 + 1 + 2b + b^2 - 1 + b(b^2 - 2b + 1) = 0$$

$$2b^2 + 2b + b^3 - 2b^2 + b = 0$$

$$b^3 + 3b = 0$$

$$b(b^2 + 3) = 0$$

$$b^2 = -3$$

$$b = \pm\sqrt{3}i$$

$$|b| = \sqrt{3}$$

Question 246

If z is a non-real complex number, then the minimum value of $\frac{1 \operatorname{Im} z^5}{(\operatorname{Im} z)^5}$ is :
[Online April 11, 2015]

Options:

A. -1

B. -4

C. -2

D. -5

Answer: B

Solution:

Solution:

$$\text{Let } z = re^{i\theta}$$

$$\text{Consider } \frac{\operatorname{Im} z^5}{(\operatorname{Im} z)^5} = \frac{r^5(\sin 5\theta)}{r^5(\sin \theta)^5}$$

$$(\because e^{i\theta} = \cos \theta + i \sin \theta)$$

$$= \frac{\sin 5\theta}{\sin^5 \theta} = \frac{16\sin^5 \theta - 20\sin^3 \theta + 5\sin \theta}{\sin^5 \theta}$$

$$= \frac{16\sin^5 \theta}{\sin^5 \theta} - \frac{20\sin^3 \theta}{\sin^5 \theta} + \frac{5\sin \theta}{\sin^5 \theta}$$

$$= 5\operatorname{cosec}^4 \theta - 20\operatorname{cosec}^2 \theta + 16$$

$$\text{minimum value of } \frac{\operatorname{Im} z^5}{(\operatorname{Im} z)^5} \text{ is } -4$$

Question 247

A complex number z is said to be unimodular if $|z| = 1$.

Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - \bar{z}_1 z_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a:
[2015]

Options:

- A. circle of radius 2.
- B. circle of radius $\sqrt{2}$.
- C. straight line parallel to x -axis
- D. straight line parallel to y-axis.

Answer: A

Solution:

Solution:

$$\begin{aligned} \left| \frac{z_1 - 2z_2}{2 - \bar{z}_1 z_2} \right| &= 1 \\ \Rightarrow |z_1 - 2z_2|^2 &= |2 - \bar{z}_1 z_2|^2 \\ \Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) &= (2 - \bar{z}_1 z_2)(2 - z_1 \bar{z}_2) \\ \Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) &= (2 - \bar{z}_1 z_2)(2 - z_1 \bar{z}_2) \\ \Rightarrow (z_1 \bar{z}_1) - 2z_1 \bar{z}_2 - 2\bar{z}_1 z_2 + 4z_2 \bar{z}_2 &= 4 - 2\bar{z}_1 z_2 - 2z_1 \bar{z}_2 + z_1 \bar{z}_1 z_2 \bar{z}_2 \\ \Rightarrow |z_1|^2 + 4|z_2|^2 &= 4 + |z_1|^2 |z_2|^2 \\ \Rightarrow |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2 |z_2|^2 &= 0 \\ (|z_1|^2 - 4)(1 - |z_2|^2) &= 0 \\ \because |z_2| &\neq 1 \\ \therefore |z_1|^2 &= 4 \\ \Rightarrow |z_1| &= 2 \\ \Rightarrow \text{Point } z_1 &\text{ lies on circle of radius 2} \end{aligned}$$

Question 248

Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to:
[2015]

Options:

- A. 3
- B. -3
- C. 6

D. -6

Answer: A

Solution:

Solution:

$$\alpha, \beta = \frac{6 \pm \sqrt{36+8}}{2} = 3 \pm \sqrt{11}$$

$$\alpha = 3 + \sqrt{11}, \beta = 3 - \sqrt{11}$$

$$\begin{aligned} \therefore a_n &= (3 + \sqrt{11})^n - (3 - \sqrt{11})^n \frac{a_{10} - 2a_8}{2a_9} \\ &= \frac{(3 + \sqrt{11})^{10} - (3 - \sqrt{11})^{10} - 2(3 + \sqrt{11})^8 + 2(3 - \sqrt{11})^8}{2[(3 + \sqrt{11})^9 - (3 - \sqrt{11})^9]} \\ &= \frac{(3 + \sqrt{11})^8[(3 + \sqrt{11})^2 - 2] + (3 - \sqrt{11})^8[2 - (3 - \sqrt{11})^2]}{2[(3 + \sqrt{11})^9 - (3 - \sqrt{11})^9]} \\ &= \frac{(3 + \sqrt{11})^8(9 + 11 + 6\sqrt{11} - 2) + (3 - \sqrt{11})^8(2 - 9 - 11 + 6\sqrt{11})}{2[(3 + \sqrt{11})^9 - (3 - \sqrt{11})^9]} \\ &= \frac{6(3 + \sqrt{11})^9 - 6(3 - \sqrt{11})^9}{2[(3 + \sqrt{11})^9 - (3 - \sqrt{11})^9]} = \frac{6}{2} = 3 \end{aligned}$$

Question249

If the two roots of the equation, $(a - 1)(x^4 + x^2 + 1) + (a + 1)(x^2 + x + 1)^2 = 0$ are real and distinct, then the set of all values of 'a' is

[Online April 11, 2015]

:

Options:

A. $\left(0, \frac{1}{2}\right)$

B. $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

C. $\left(-\frac{1}{2}, 0\right)$

D. $(-\infty, -2) \cup (2, \infty)$

Answer: B

Solution:

Solution:

$$\begin{aligned} (a - 1)(x^4 + x^2 + 1) + (a + 1)(x^2 + x + 1)^2 &= 0 \\ \Rightarrow (a - 1)(x^2 + x + 1)(x^2 - x + 1) + (a + 1)(x^2 + x + 1)^2 &= 0 \\ \Rightarrow (x^2 + x + 1)[(a - 1)(x^2 - x + 1) + (a + 1)(x^2 + x + 1)] &= 0 \\ \Rightarrow (x^2 + x + 1)(ax^2 + x + a) &= 0 \end{aligned}$$

For roots to be distinct and real, $a \neq 0$ and $1 - 4a^2 > 0$

$$\Rightarrow a \neq 0 \text{ and } a^2 < \frac{1}{4}$$

$$\Rightarrow a \in \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$$

Question250

If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in \mathbb{R}$, then the real root of this equation:
[Online April 10, 2015]

Options:

- A. exists and is equal to $-\frac{1}{2}$.
- B. exists and is equal to $\frac{1}{2}$
- C. exists and is equal to 1 .
- D. does not exist.

Answer: B

Solution:

Solution:

$\alpha = 2 + 3i$; $\beta = 2 - 3i$, $\gamma = ?$

$\alpha\beta\gamma = \frac{13}{2}$ [since product of roots = $\frac{d}{a}$]

$\Rightarrow (4 + 9)\gamma = \frac{13}{2} \Rightarrow \gamma = \frac{1}{2}$

Question251

If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left| z + \frac{1}{2} \right|$:
[2014]

Options:

- A. is strictly greater than $\frac{5}{2}$
- B. is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$
- C. is equal to $\frac{5}{2}$
- D. lie in the interval (1,2)

Answer: D

Solution:

Solution:

We know minimum value of $|Z_1 + Z_2|$ is

$|Z_1| - |Z_2|$. Thus minimum value of $\left|Z + \frac{1}{2}\right|$ is $|Z| - \frac{1}{2}$

$$\leq \left|Z + \frac{1}{2}\right| \leq |Z| + \frac{1}{2}$$

Since, $|Z| \geq 2$ therefore

$$2 - \frac{1}{2} < \left|Z + \frac{1}{2}\right| < 2 + \frac{1}{2}$$

$$\Rightarrow \frac{3}{2} < \left|Z + \frac{1}{2}\right| < \frac{5}{2}$$

Question252

For all complex numbers z of the form $1 + i\alpha$, $\alpha \in \mathbb{R}$, if $z^2 = x + iy$, then [Online April 19, 2014]

Options:

A. $y^2 - 4x + 2 = 0$

B. $y^2 + 4x - 4 = 0$

C. $y^2 - 4x - 4 = 0$

D. $y^2 + 4x + 2 = 0$

Answer: B

Solution:

Solution:

Let $z = 1 + i\alpha$, $\alpha \in \mathbb{R}$

$$z^2 = (1 + i\alpha)(1 + i\alpha)$$

$$x + iy = (1 + 2i\alpha - \alpha^2)$$

On comparing real and imaginary parts, we get

$$x = 1 - \alpha^2, y = 2\alpha$$

Now, consider option (b), which is

$$y^2 + 4x - 4 = 0$$

$$\text{LHS : } y^2 + 4x - 4 = (2\alpha)^2 + 4(1 - \alpha^2) - 4$$

$$= 4\alpha^2 + 4 - 4\alpha^2 - 4$$

$$= 0 = \text{R.H.S.}$$

$$\text{Hence, } y^2 + 4x - 4 = 0$$

Question253

Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number.

Then $z + \frac{1}{z}$ is:

[Online April 12, 2014]

Options:

A. zero

B. any non-zero real number other than 1 .

- C. any non-zero real number.
- D. a purely imaginary number.

Answer: C

Solution:

Solution:

Let $z = x + iy$

$\frac{z-i}{z+i}$ is purely imaginary means its real part is zero.

$$\frac{x+iy-i}{x+iy+i} = \frac{x+i(y-1)}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$$

$$= \frac{x^2 - 2ix(y+1) + xi(y-1) + y^2 - 1}{x^2 + (y+1)^2}$$

$$= \frac{x^2 + y^2 - 1}{x^2 + (y+1)^2} - \frac{2xi}{x^2 + (y+1)^2}$$

for pure imaginary, we have

$$\frac{x^2 + y^2 - 1}{x^2 + (y+1)^2} = 0$$

$$\Rightarrow x^2 + y^2 = 1$$

$$\Rightarrow (x+iy)(x-iy) = 1$$

$$\Rightarrow x+iy = \frac{1}{x-iy} = z$$

$$\text{and } \frac{1}{z} = x-iy$$

$$z + \frac{1}{z} = (x+iy) + (x-iy) = 2x$$

$$\left(z + \frac{1}{z}\right) \text{ is any non-zero real number}$$

Question 254

If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, then

$\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ equals:

[Online April 11, 2014]

Options:

A. 0

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. π

Answer: A

Solution:

Solution:

$$\text{Consider } \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$$

$$= \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$$

$$= (\arg(z_1) + \arg(z_2)) - (\arg(z_3) + \arg(z_4))$$

$$\text{given } \begin{pmatrix} z_2 = \bar{z}_1 \\ z_4 = \bar{z}_3 \end{pmatrix}$$

$$= (\arg(z_1) + \arg(\bar{z}_1)) - (\arg(z_3) + \arg(\bar{z}_3))$$

$$\left\{ \begin{array}{l} \text{also } (\arg(\bar{z}_1) = -\arg(z_1)) \\ \arg(\bar{z}_3) = -\arg(z_3) \end{array} \right\}$$

$$= (\arg(z_1) - \arg(z_1)) - (\arg(z_3) - \arg(z_3))$$

$$= 0 - 0 = 0$$

Question 255

Let w ($w \neq 0$) be a complex number. Then the set of all complex number z satisfying the equation $w - \bar{w}z = k(1 - z)$, for some real number k , is

[Online April 9, 2014]

Options:

A. $\{z: |z| = 1\}$

B. $\{z: z = \bar{z}\}$

C. $\{z: z \neq 1\}$

D. $\{z: |z| = 1, z \neq 1\}$

Answer: D

Solution:

Solution:

Consider the equation

$$w - \bar{w}z = k(1 - z), k \in \mathbb{R}$$

Clearly $z \neq 1$ and $\frac{w - \bar{w}z}{1 - z}$ is purely real

$$\therefore \overline{\frac{w - \bar{w}z}{1 - z}} = \frac{w - \bar{w}z}{1 - z}$$

$$\Rightarrow \frac{\bar{w} - w\bar{z}}{1 - \bar{z}} = \frac{w - \bar{w}z}{1 - z}$$

$$\Rightarrow \bar{w} - \bar{w}z - \bar{w}z + w\bar{z} = w - \bar{w}z - \bar{w}z + \bar{w}\bar{z}$$

$$\Rightarrow \bar{w} + w|z|^2 = w + \bar{w}|z|^2$$

$$\Rightarrow (w - \bar{w})(|z|^2) = w - \bar{w}$$

$$\Rightarrow |z|^2 = 1 \quad (\because w \neq 0)$$

$$\Rightarrow |z| = 1 \text{ and } z \neq 1$$

$$\therefore \text{The required set is } \{z: |z| = 1, z \neq 1\}$$

Question 256

If $a \in \mathbb{R}$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval:

[2014]

Options:

- A. $(-2, -1)$
- B. $(-\infty, -2) \cup (2, \infty)$
- C. $(-1, 0) \cup (0, 1)$
- D. $(1, 2)$

Answer: C

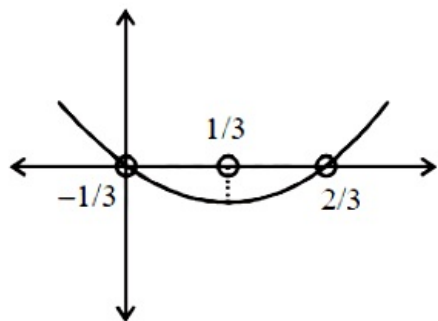
Solution:

Solution:

$$\text{Consider } -3(x - [x])^2 + 2[x - [x]] + a^2 = 0$$
$$\Rightarrow 3\{x\}^2 - 2\{x\} - a^2 = 0 \quad (\because x - [x] = \{x\})$$

$$\Rightarrow 3\left(\{x\}^2 - \frac{2}{3}\{x\}\right) = a^2, a \neq 0$$

$$\Rightarrow a^2 = 3\{x\}\left(\{x\} - \frac{2}{3}\right)$$



Now, $\{x\} \in (0, 1)$ and $-\frac{2}{3} \leq a^2 < 1$ (by graph)

Since, x is not an integer

$$\therefore a \in (-1, 1) - \{0\}$$

$$\Rightarrow a \in (-1, 0) \cup (0, 1)$$

Question257

The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has;
[Online April 19, 2014]

Options:

- A. no solution
- B. exactly one solution
- C. exactly two solution
- D. exactly four solution

Answer: A

Solution:

Solution:

$$\text{Consider } \sqrt{3x^2 + x + 5} = x - 3$$

Squaring both the sides, we get

$$3x^2 + x + 5 = (x - 3)^2$$

$$\Rightarrow 3x^2 + x + 5 = x^2 + 9 - 6x$$

$$\Rightarrow 2x^2 + 7x - 4 = 0$$

$$\Rightarrow 2x^2 + 8x - x - 4 = 0$$

$$\Rightarrow 2x(x + 4) - 1(x + 4) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = -4$$

$$\text{For } x = \frac{1}{2} \text{ and } x = -4$$

$$\text{L.H.S.} \neq \text{R.H.S. of equation, } \sqrt{3x^2 + x + 5} = x - 3$$

Also, for every $x \in \mathbb{R}$, LHS $\neq$ RHS of the given equation.

$\therefore$ Given equation has no solution.

Question258

**The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is:
[Online April 12, 2014]**

Options:

A. 2

B. -2

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: C

Solution:

Solution:

$$x^2 + |2x - 3| - 4 = 0$$

$$|2x - 3| = \begin{cases} (2x - 3) & \text{if } x > \frac{3}{2} \\ -(2x - 3) & \text{if } x < \frac{3}{2} \end{cases}$$

$$\text{for } x > \frac{3}{2}, \quad x^2 + 2x - 3 - 4 = 0$$

$$x^2 + 2x - 7 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 28}}{2} = \frac{-2 \pm 4\sqrt{2}}{2} = -1 \pm 2\sqrt{2}$$

$$\text{Here } x = 2\sqrt{2} - 1 \quad \left\{ 2\sqrt{2} - 1 < \frac{3}{2} \right\}$$

$$\text{for } x < \frac{3}{2}$$

$$x^2 - 2x + 3 - 4 = 0$$

$$\Rightarrow x^2 - 2x - 1 = 0$$

$$\Rightarrow x = \frac{2 \pm \sqrt{4 + 4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

$$\text{Here } x = 1 - \sqrt{2} \quad \left\{ (1 - \sqrt{2}) < \frac{3}{2} \right\}$$

$$\text{Sum of roots : } (2\sqrt{2} - 1) + (1 - \sqrt{2}) = \sqrt{2}$$

Question259

If α and β are roots of the equation, $x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$ for some k , and $\alpha^2 + \beta^2 = 66$, then $\alpha^3 + \beta^3$ is equal to:
[Online April 11, 2014]

Options:

- A. $248\sqrt{2}$
- B. $280\sqrt{2}$
- C. $-32\sqrt{2}$
- D. $-280\sqrt{2}$

Answer: D

Solution:

Solution:

$$x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$$

$$\text{or, } x^2 - 4\sqrt{2}kx + 2k^4 - 1 = 0$$

$$\alpha + \beta = 4\sqrt{2}k \text{ and } \alpha \cdot \beta = 2k^4 - 1$$

Squaring both sides, we get

$$(\alpha + \beta)^2 = (4\sqrt{2}k)^2 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 32k^2$$

$$66 + 2\alpha\beta = 32k^2$$

$$66 + 2(2k^4 - 1) = 32k^2$$

$$66 + 4k^4 - 2 = 32k^2 \Rightarrow 4k^4 - 32k^2 + 64 = 0$$

$$\text{or, } k^4 - 8k^2 + 16 = 0 \Rightarrow (k^2)^2 - 8k^2 + 16 = 0$$

$$\Rightarrow (k^2 - 4)(k^2 - 4) = 0 \Rightarrow k^2 = 4, k^2 = 4$$

$$\Rightarrow k = \pm 2$$

$$\text{Now, } \alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$$

$$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}k)[66 - (2k^4 - 1)]$$

Putting $k = -2$, ($k = +2$ cannot be taken because it does not satisfy the above equation)

$$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}(-2))[66 - 2(-2)^4 - 1]$$

$$\alpha^3 + \beta^3 = (-8\sqrt{2})(66 - 32 + 1) = (-8\sqrt{2})$$

$$\therefore \alpha^3 + \beta^3 = -280\sqrt{2}$$

Question260

If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation,

$ax^2 + bx + 1 = 0$ ($a \neq 0, a, b, \in \mathbb{R}$), then the equation,

$x(x + b^3) + (a^3 - 3abx) = 0$ as roots :

[Online April 9, 2014]

Options:

- A. $\alpha^{3/2}$ and $\beta^{3/2}$
- B. $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$
- C. $\sqrt{\alpha\beta}$ and $\alpha\beta$
- D. $\alpha^{-\frac{3}{2}}$ and β^{-3} ?

Answer: A

Solution:

Solution:

Let $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ be the roots of $ax^2 + bx + 1 = 0$

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \left(\frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} \right) = -\frac{b}{a}$$

$$\frac{1}{\sqrt{\alpha}\sqrt{\beta}} = \frac{1}{a} \Rightarrow a = \sqrt{\alpha\beta}$$

$$b = -(\sqrt{\alpha} + \sqrt{\beta})$$

$$x(x + b^3) + (a^3 - 3abx) = 0$$

$$\Rightarrow x^2 + (b^3 - 3ab)x + a^3 = 0$$

Putting values of a and b, we get

$$x^2 + [(-\sqrt{\alpha} + \sqrt{\beta})^3 + 3(\sqrt{\alpha\beta})(\sqrt{\alpha} + \sqrt{\beta})] + (\alpha\beta)^{3/2} = 0$$

$$\Rightarrow x^2 - [\alpha^{3/2} + \beta^{3/2} + 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta}) - 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta})]x + (\alpha\beta)^{3/2} = 0$$

$$\Rightarrow x^2 - (\alpha^{3/2} + \beta^{3/2})x + \alpha^{3/2}\beta^{3/2} = 0$$

Roots of this equation are $\alpha^{3/2}, \beta^{3/2}$

Question261

If non-zero real numbers **b** and **c** are such that $\min f(x) > \max g(x)$,

where $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ ($x \in \mathbb{R}$) then $\left| \frac{c}{b} \right|$

lies in the interval:

[Online April 19, 2014]

Options:

A. $\left(0, \frac{1}{2} \right)$

B. $\left[\frac{1}{2}, \frac{1}{\sqrt{2}} \right)$

C. $\left[\frac{1}{\sqrt{2}}, \sqrt{2} \right]$

D. $(\sqrt{2}, \infty)$

Answer: D

Solution:

Solution:

We have

$$f(x) = x^2 + 2bx + 2c^2$$

$$\text{and } g(x) = -x^2 - 2cx + b^2, (x \in \mathbb{R})$$

$$\Rightarrow f(x) = (x + b)^2 + 2c^2 - b^2$$

$$\text{and } g(x) = -(x + c)^2 + b^2 + c^2$$

$$\text{Now, } f_{\min} = 2c^2 - b^2 \text{ and } g_{\max} = b^2 + c^2$$

$$\text{Given : } \min f(x) > \max g(x)$$

$$\Rightarrow 2c^2 - b^2 > b^2 + c^2$$

$$\Rightarrow c^2 > 2b^2$$

$$\Rightarrow |c| > |b| \sqrt{2}$$

$$\Rightarrow \frac{|c|}{|b|} > \sqrt{2} \Rightarrow \left| \frac{c}{b} \right| > \sqrt{2}$$

$$\Rightarrow \left| \frac{c}{b} \right| \in (\sqrt{2}, \infty)$$

Question262

If equations $ax^2 + bx + c = 0$ ($a, b, c \in \mathbb{R}, a \neq 0$) and $2x^2 + 3x + 4 = 0$ have a common root, then $a : b : c$ equals:
[Online April 9, 2014]

Options:

A. 1: 2: 3

B. 2: 3: 4

C. 4: 3: 2

D. 3: 2: 1

Answer: B

Solution:

Solution:

Let α, β be the common roots of both the equations.

For first equation $ax^2 + bx + c = 0$

we have

$$\alpha + \beta = \frac{-b}{a} \dots (i)$$

$$\alpha \cdot \beta = \frac{c}{a} \dots (ii)$$

For second equation $2x^2 + 3x + 4 = 0$

we have

$$\alpha + \beta = \frac{-3}{2} \dots (iii)$$

$$\alpha \cdot \beta = \frac{2}{1} \dots (iv)$$

Now, from (i) & (iii) & from (ii) & (iv)

$$\frac{-b}{a} = \frac{-3}{2} \quad \frac{c}{a} = \frac{2}{1}$$

$$\frac{b}{a} = \frac{3/2}{1}$$

Therefore on comparing we get $a = 1, b = \frac{3}{2}$ & $c = 2$

putting these values in first equation, we get

$$x^2 + \frac{3}{2}x + 2 = 0 \text{ or } 2x^2 + 3x + 4 = 0$$

from this, we get $a = 2, b = 3; c = 4$

or $a : b : c = 2 : 3 : 4$

Question263

If z is a complex number of unit modulus and argument θ , then
 $\arg \left(\frac{1+z}{1+\bar{z}} \right)$ equals:
[2013]

Options:

- A. $-\theta$
- B. $\frac{\pi}{2} - \theta$
- C. θ
- D. $\pi - \theta$

Answer: C**Solution:****Solution:**Given $|z| = 1$, $\arg z = \theta$

$$\Rightarrow \bar{z} = \frac{1}{z}$$

$$\therefore \arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg\left(\frac{1+z}{1+\frac{1}{z}}\right) = \arg(z) = \theta$$

Question 264

Let z satisfy $|z| = 1$ and $z = 1 - \bar{z}$.**Statement 1 : z is a real number.****Statement 2: Principal argument of z is $\frac{\pi}{3}$** **[Online April 25, 2013]****Options:**

- A. Statement 1 is true Statement 2 is true; Statement 2 is a correct explanation for Statement 1 .
- B. Statement 1 is false; Statement 2 is true
- C. Statement 1 is true, Statement 2 is false.
- D. Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1 .

Answer: B**Solution:****Solution:**Let $z = x + iy$, $\bar{z} = x - iy$ Now, $z = 1 - \bar{z}$

$$\Rightarrow x + iy = 1 - (x - iy)$$

$$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

$$\text{Now, } |z| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$$

$$\Rightarrow y = \pm \frac{\sqrt{3}}{2}$$

Now, $\tan \theta = \frac{y}{x}$ (θ is the argument)

$$= \frac{\sqrt{3}}{2} \div \frac{1}{2} \text{ (+ ve since only principal argument)}$$

$$= \sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

Hence, z is not a real number

So, statement- 1 is false and 2 is true.

Question265

Let $a = \operatorname{Im} \left(\frac{1+z^2}{2iz} \right)$, where z is any non-zero complex number.

The set $A = \{ a : |z| = 1 \text{ and } z \neq \pm 1 \}$ is equal to:

[Online April 23, 2013]

Options:

A. (-1,1)

B. [-1,1]

C. [0,1)

D. (-1,0]

Answer: A

Solution:

Solution:

$$\text{Let } z = x + iy \Rightarrow z^2 = x^2 - y^2 + 2ixy$$

$$\text{Now, } \frac{1+z^2}{2iz} = \frac{1+x^2-y^2+2ixy}{2i(x+iy)} = \frac{(x^2-y^2+1)+2ixy}{2ix-2y}$$

$$= \frac{(x^2-y^2+1)+2ixy}{-2y+2ix} \times \frac{-2y-2ix}{-2y-2ix}$$

$$= \frac{y(x^2+y^2-1)+x(x^2+y^2+1)i}{2(x^2+y^2)}$$

$$a = \frac{x(x^2+y^2+1)}{2(x^2+y^2)}$$

$$\text{Since, } |z| = 1 \Rightarrow \sqrt{x^2+y^2} = 1 \Rightarrow x^2+y^2 = 1$$

$$\therefore a = \frac{x(1+1)}{2 \times 1} = x$$

$$\text{Also } z \neq 1 \Rightarrow x+iy \neq 1$$

$$\therefore A = (-1, 1)$$

Question266

If $Z_1 \neq 0$ and Z_2 be two complex numbers such that $\frac{Z_2}{Z_1}$ is a purely

imaginary number, then $\left| \frac{2Z_1+3Z_2}{2Z_1-3Z_2} \right|$ is equal to:

[Online April 9, 2013]

Options:

A. 2

B. 5

C. 3

D. 1

Answer: D

Solution:

Solution:

Let $z_1 = 1 + i$ and $z_2 = 1 - i$

$$\frac{z_2}{z_1} = \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = -i$$

$$\frac{2z_1 + 3z_2}{2z_1 - 3z_2} = \frac{2 + 3\left(\frac{z_2}{z_1}\right)}{2 - 3\left(\frac{z_2}{z_1}\right)} = \frac{2 - 3i}{2 + 3i}$$

$$\begin{aligned} \left| \frac{2z_1 + 3z_2}{2z_1 - 3z_2} \right| &= \left| \frac{2 - 3i}{2 + 3i} \right| = \left| \frac{2 - 3i}{2 + 3i} \right| \left[\because \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \right] \\ &= \frac{\sqrt{4+9}}{\sqrt{4+9}} = 1 \end{aligned}$$

Question267

If p and q are non-zero real numbers and $\alpha^3 + \beta^3 = -p$, $\alpha\beta = q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}$, $\frac{\beta^2}{\alpha}$ is:

[Online April 25, 2013]

Options:

A. $px^2 - qx + p^2 = 0$

B. $qx^2 + px + q^2 = 0$

C. $px^2 + qx + p^2 = 0$

D. $qx^2 - px + q^2 = 0$

Answer: B

Solution:

Solution:

Given $\alpha^3 + \beta^3 = -p$ and $\alpha\beta = q$

Let $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ be the root of required quadratic equation.

$$\text{So, } \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{-p}{q}$$

$$\text{and } \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = q$$

Hence, required quadratic equation is

$$x^2 - \left(\frac{-p}{q} \right) x + q = 0$$

$$\Rightarrow x^2 + \frac{p}{q}x + q = 0 \Rightarrow qx^2 + px + q^2 = 0$$

Question268

If α and β are roots of the equation $x^2 + px + \frac{3p}{4} = 0$ such that $|\alpha - \beta| = \sqrt{10}$, then p belongs to the set :
[Online April 22, 2013]

Options:

A. $\{2, -5\}$

B. $\{-3, 2\}$

C. $\{-2, 5\}$

D. $\{3, -5\}$

Answer: C

Solution:

Solution:

Given quadratic eqn. is

$$x^2 + px + \frac{3p}{4} = 0$$

$$\text{So, } \alpha + \beta = -p, \alpha\beta = \frac{3p}{4}$$

$$\text{Now, given } |\alpha - \beta| = \sqrt{10}$$

$$\Rightarrow \alpha - \beta = \pm\sqrt{10}$$

$$\Rightarrow (\alpha - \beta)^2 = 10 \Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 10$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 10$$

$$\Rightarrow p^2 - 4 \times \frac{3p}{4} = 10 \Rightarrow p^2 - 3p - 10 = 0$$

$$\Rightarrow p = -2, 5 \Rightarrow p \in \{-2, 5\}$$

Question269

If a complex number z satisfies the equation $z + \sqrt{2} |z + 1| + i = 0$, then $|z|$ is equal to :
[Online April 22, 2013]

Options:

A. 2

B. $\sqrt{3}$

C. $\sqrt{5}$

D. 1

Answer: C

Solution:

Solution:

Given equation is

$$z + \sqrt{2} |z + 1| + i = 0$$

put $z = x + iy$ in the given equation.

$$(x + iy) + \sqrt{2} |x + iy + 1| + i = 0$$

$$\Rightarrow x + iy + \sqrt{2} [\sqrt{(x+1)^2 + y^2}] + i = 0$$

Now, equating real and imaginary part, we get

$$x + \sqrt{2} \sqrt{(x+1)^2 + y^2} = 0 \text{ and}$$

$$y + 1 = 0 \Rightarrow y = -1$$

$$\Rightarrow x + \sqrt{2} \sqrt{(x+1)^2 + (-1)^2} = 0 \quad (\because y = -1)$$

$$\Rightarrow \sqrt{2} \sqrt{(x+1)^2 + 1} = -x$$

$$\Rightarrow 2[(x+1)^2 + 1] = x^2$$

$$\Rightarrow x^2 + 4x + 4 = 0$$

$$\Rightarrow x = -2$$

$$\text{Thus, } z = -2 + i(-1) \Rightarrow |z| = \sqrt{5}$$

Question270

If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$ $a, b, c \in \mathbb{R}$, have a common root, then $a : b : c$ is [2013]

Options:

A. 1: 2: 3

B. 3: 2: 1

C. 1: 3: 2

D. 3: 1: 2

Answer: A

Solution:

Solution:

Given equations are

$$x^2 + 2x + 3 = 0 \quad \dots (i)$$

$$ax^2 + bx + c = 0 \quad \dots (ii)$$

Roots of equation (i) are imaginary roots in order pair.

According to the question (ii) will also have both roots same as (i).

$$\text{Thus } \frac{a}{1} = \frac{b}{2} = \frac{c}{3} = \lambda \text{ (say)}$$

$$\Rightarrow a = \lambda, b = 2\lambda, c = 3\lambda$$

Hence, required ratio is 1: 2: 3

Question271

The least integral value α of x such that $\frac{x-5}{x^2+5x-14} > 0$ satisfies:

[Online April 23, 2013]

Options:

A. $\alpha^2 + 3\alpha - 4 = 0$

B. $\alpha^2 - 5\alpha + 4 = 0$

C. $\alpha^2 - 7\alpha + 6 = 0$

D. $\alpha^2 + 5\alpha - 6 = 0$

Answer: A

Solution:

Solution:

$$\frac{x-5}{x^2+5x-14} > 0 \Rightarrow x^2+5x-14 < x-5$$

$$\Rightarrow x^2+4x-9 < 0$$

$$\Rightarrow \alpha = -5, -4, -3, -2, -1, 0, 1$$

$\alpha = -5$ does not satisfy any of the options

$\alpha = -4$ satisfy the option (a) $\alpha^2 + 3\alpha - 4 = 0$

Question272

The values of ' a ' for which one root of the equation $x^2 - (a+1)x + a^2 + a - 8 = 0$ exceeds 2 and the other is lesser than 2 , are given by :

[Online April 9, 2013]

Options:

A. $3 < a < 10$

B. $a \geq 10$

C. $-2 < a < 3$

D. $a \leq -2$

Answer: C

Solution:

Solution:

$$x^2 - (a+1)x + a^2 + a - 8 = 0$$

Since roots are different, therefore $D > 0$

$$\Rightarrow (a+1)^2 - 4(a^2 + a - 8) > 0$$

$$\Rightarrow (a-3)(3a+1) < 0$$

There are two cases arises

Case I. $a-3 > 0$ and $3a+1 < 0$

$$\Rightarrow a > 3 \text{ and } a < -\frac{11}{3}$$

Hence, no solution in this case

Case II : $a-3 < 0$ and $3a+11 > 0$

$$\Rightarrow a < 3 \text{ and } a > -\frac{11}{3}$$

$$\therefore -\frac{11}{3} < a < 3 \Rightarrow -2 < a < 3$$

Question273

$|z_1 + z_2|^2 + |z_1 - z_2|^2$ is equal to
[Online May 26, 2012]

Options:

- A. $2(|z_1| + |z_2|)$
- B. $2(|z_1|^2 + |z_2|^2)$
- C. $|z_1| |z_2|$
- D. $|z_1|^2 + |z_2|^2$

Answer: B

Solution:

Solution:

$$\begin{aligned}
 & |z_1 + z_2|^2 + |z_1 - z_2|^2 \\
 &= |z_1|^2 + |z_2|^2 + 2|z_1||z_2| + |z_1|^2 + |z_2|^2 - 2|z_1||z_2| \\
 &= 2|z_1|^2 + 2|z_2|^2 = 2[|z_1|^2 + |z_2|^2]
 \end{aligned}$$

Question274

Let Z and W be complex numbers such that $|Z| = |W|$, and $\arg Z$ denotes the principal argument of Z .

Statement 1: If $\arg Z + \arg W = \pi$, then $Z = -\overline{W}$

Statement 2: $|Z| = |W|$, implies $\arg Z - \arg \overline{W} = \pi$

[Online May 19, 2012]

Options:

- A. Statement 1 is true, Statement 2 is false.
- B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
- C. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
- D. Statement 1 is false, Statement 2 is true.

Answer: A

Solution:

Solution:

Let $|Z| = |W| = r \Rightarrow Z = re^{i\theta}, W = re^{i\varphi}$

where $\theta + \varphi = \pi$

$\therefore \overline{W} = re^{-i\varphi}$

Now, $\overline{Z} = re^{i(\pi - \varphi)} = re^{i\pi} \times e^{-i\varphi} = -re^{-i\varphi}$
 $= -\overline{W}$

Thus, statement- 1 is true but statement- 2 is false.

Question275

Let Z_1 and Z_2 be any two complex number. Statement

1: $|Z_1 - Z_2| \geq |Z_1| - |Z_2|$

Statement 2: $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$

[Online May 7, 2012]

Options:

- A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.
- B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.
- C. Statement 1 is true, Statement 2 is false.
- D. Statement 1 is false, Statement 2 is true.

Answer: B

Solution:**Solution:**

Statement -1 and 2 both are true. It is fundamental property. But Statement -2 is not correct explanation for Statement -1.

Question276

If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies:

[2012]

Options:

- A. either on the real axis or on a circle passing through the origin.
- B. on a circle with centre at the origin
- C. either on the real axis or on a circle not passing through the origin.
- D. on the imaginary axis.

Answer: A

Solution:

Solution:

$$\frac{z^2}{z-1} = \frac{\bar{z}^2}{\bar{z}-1} \left[\because \left(\frac{z_1}{z_2} \right) = \frac{\bar{z}_1}{\bar{z}_2} \right]$$

$$\Rightarrow z\bar{z}z - z^2 = z \cdot \bar{z} \cdot \bar{z} - \bar{z}^2$$

$$\Rightarrow |z|^2 \cdot z - z^2 = |z|^2 \cdot \bar{z} - \bar{z}^2$$

$$\Rightarrow |z|^2 (z - \bar{z}) - (z - \bar{z})(z + \bar{z}) = 0$$

$$\Rightarrow (z - \bar{z})(|z|^2 - (z + \bar{z})) = 0$$

$$\text{Either } z - \bar{z} = 0 \text{ or } |z|^2 - (z + \bar{z}) = 0$$

$$\text{Either } z = \bar{z} \Rightarrow \text{real axis}$$

$$\text{or } |z|^2 = z + \bar{z} \Rightarrow z\bar{z} - z - \bar{z} = 0$$

represents a circle passing through origin.

Question277

Let $p, q, r \in \mathbb{R}$ and $r > p > 0$. If the quadratic equation $px^2 + qx + r = 0$ has two complex roots α and β , then $|\alpha| + |\beta|$ is
[Online May 19, 2012]

Options:

- A. equal to 1
- B. less than 2 but not equal to 1
- C. greater than 2
- D. equal to 2

Answer: C

Solution:

Solution:

Given quadratic equation is $px^2 + qx + r = 0$

$$D = q^2 - 4pr$$

Since α and β are two complex root

$$\therefore \beta = \bar{\alpha} \Rightarrow |\beta| = |\bar{\alpha}| \Rightarrow |\beta| = |\alpha| \quad (\because |\bar{\alpha}| = |\alpha|)$$

Consider

$$|\alpha| + |\beta| = |\alpha| + |\alpha| \quad (\because |\beta| = |\alpha|)$$

$$= 2|\alpha| > 2 \quad (\because |\alpha| > 1)$$

Hence, $|\alpha| + |\beta|$ is greater than 2

Question278

If the sum of the square of the roots of the equation $x^2 - (\sin \alpha - 2)x - (1 + \sin \alpha) = 0$ is least, then α is equal to
[Online May 12, 2012]

Options:

- A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

Solution:

Solution:

Given equation is

$$x^2 - (\sin \alpha - 2)x - (1 + \sin \alpha) = 0$$

Let x_1 and x_2 be two roots of quadratic equation

$$\therefore x_1 + x_2 = \sin \alpha - 2 \text{ and } x_1 x_2 = -(1 + \sin \alpha)$$

$$(x_1 + x_2)^2 = (\sin \alpha - 2)^2 = \sin^2 \alpha + 4 - 4 \sin \alpha$$

$$\Rightarrow x_1^2 + x_2^2 = \sin^2 \alpha + 4 - 4 \sin \alpha - 2x_1 x_2$$

$$= \sin^2 \alpha + 4 - 4 \sin \alpha + 2(1 + \sin \alpha)$$

$$= \sin^2 \alpha - 2 \sin \alpha + 6$$

Now, By putting

$$\alpha = \frac{\pi}{6}, \alpha = \frac{\pi}{4}, \alpha = \frac{\pi}{3} \text{ and } \alpha = \frac{\pi}{2} \text{ in (i) one by one}$$

We get least value of $x_1^2 + x_2^2$ at $\frac{\pi}{2}$

$$\text{Hence, } \alpha = \frac{\pi}{2}$$

Question 279

The value of k for which the equation $(k - 2)x^2 + 8x + k + 4 = 0$ has both roots real, distinct and negative is

[Online May 7, 2012]

Options:

A. 6

B. 3

C. 4

D. 1

Answer: B

Solution:

Solution:

$$(k - 2)x^2 + 8x + k + 4 = 0$$

If real roots then,

$$8^2 - 4(k - 2)(k + 4) > 0$$

$$\Rightarrow k^2 + 2k - 8 < 16$$

$$\Rightarrow k^2 + 6k - 4k - 24 < 0$$

$$\Rightarrow (k + 6)(k - 4) < 0$$

$$\Rightarrow -6 < k < 4$$

If both roots are negative

then $\alpha\beta$ is + ve

$$\Rightarrow \frac{k+4}{k-2} > 0 \Rightarrow k > -4$$

$$\text{Also, } \frac{k-2}{k+4} > 0 \Rightarrow k > 2$$

Roots are real so $-6 < k < 4$

So, 6 and 4 are not correct.

Since, $k > 2$, so 1 is also not correct value of k .

$$\therefore k = 3$$

Question280

If $\omega(\neq 1)$ is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals
[2011]

Options:

- A. (1,1)
- B. (1,0)
- C. (-1,1)
- D. (0,1)

Answer: A

Solution:

Solution:

$$(1 + \omega)^7 = A + B\omega$$

$$(-\omega^2)^7 = A + B\omega (\because \omega^{14} = \omega^{12} \cdot \omega^2 = \omega^2)$$

$$-\omega^2 = A + B\omega$$

$$1 + \omega = A + B\omega$$

$$\Rightarrow A = 1, B = 1$$

Question281

Let for $a \neq a_1 \neq 0$ $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$.

If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(b)$ is :
[2011 RS]

Options:

- A. 3
- B. 9
- C. 6
- D. 18

Answer: D

Solution:

Solution:

$p(x) = 0$
 $\Rightarrow f(x) = g(x)$
 $\Rightarrow ax^2 + bx + c = a_1x^2 + b_1x + c_1$
 $\Rightarrow (a - a_1)x^2 + (b - b_1)x + (c - c_1) = 0$
It has only one solution, $x = -1$
 $\Rightarrow b - b_1 = a - a_1 + c - c_1 \dots (i)$
Sum of roots $\frac{-(b - b_1)}{(a - a_1)} = -1 - 1$
 $\Rightarrow \frac{b - b_1}{2(a - a_1)} = 1 \dots\dots(ii)$
 $\Rightarrow b - b_1 = 2(a - a_1)$
Now $p(-2) = 2$
 $\Rightarrow f(-2) - g(-2) = 2$
 $\Rightarrow 4a - 2b + c - 4a_1 + 2b_1 - c_1 = 2$
 $\Rightarrow 4(a - a_1) - 2(b - b_1) + (c - c_1) = 2 \dots (iii)$
From equations, (i), (ii) and (iii)
 $a - a_1 = c - c_1 = \frac{1}{2}(b - b_1) = 2$
Now, $p(2) = f(2) - g(2)$
 $= 4(a - a_1) + 2(b - b_1) + (c - c_1)$
 $= 8 + 8 + 2 = 18$

Question 282

Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3) . Rahul made a mistake in writing down coefficient of x to get roots (3,2) . The correct roots of equation are : [2011 RS]

Options:

- A. 6,1
- B. 4,3
- C. -6,-1
- D. -4,-3

Answer: A

Solution:

Solution:

Let the correct equation be
 $ax^2 + bx + c = 0$
Now, Sachin's equation
 $ax^2 + bx + c' = 0$
Given that, roots found by Sachin's are 4 and 3
 $\Rightarrow -\frac{b}{a} = 7 \dots\dots(i)$
Rahul's equation, $ax^2 + b'x + c = 0$
Given that roots found by Rahul's are 3 and 2

$$\Rightarrow \frac{c}{a} = 6 \dots\dots(ii)$$

From (i) and (ii), roots of the correct equation

$$x^2 - 7x + 6 = 0 \text{ are } 6 \text{ and } 1$$

Question283

Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line $\operatorname{Re} z = 1$ then it is necessary that : [2011]

Options:

A. $\beta \in (-1, 0)$

B. $|\beta| = 1$

C. $\beta \in (1, \infty)$

D. $\beta \in (0, 1)$

Answer: C

Solution:

Solution:

Since both the roots of given quadratic equation lie in the line $\operatorname{Re} z = 1$ i.e., $x = 1$, hence real part of both the roots are 1

Let both roots be $1 + i\alpha$ and $1 - i\alpha$

Product of the roots, $1 + \alpha^2 = \beta$

$$\therefore \alpha^2 + 1 \geq 1$$

$$\therefore \beta \geq 1 \Rightarrow \beta \in (1, \infty)$$

Question284

The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals [2010]

Options:

A. 1

B. 2

C. ∞

D. 0

Answer: A

Solution:

Solution:

Let $z = x + iy$

$|z - 1| = |z + 1| \Rightarrow (x - 1)^2 + y^2 = (x + 1)^2 + y^2$
 $\Rightarrow x = 0 \Rightarrow \operatorname{Re} z = 0$
 $|z - 1| = |z - i| \Rightarrow (x - 1)^2 + y^2 = x^2 + (y - 1)^2$
 $\Rightarrow x = y$
 $|z + 1| = |z - i| \Rightarrow (x + 1)^2 + y^2 = x^2 + (y - 1)^2$
 $\Rightarrow x = -y$
 Only (0,0) will satisfy all conditions.
 $\Rightarrow$ Number of complex number $z = 1$

Question 285

If α and β are the roots of the equation $x^2 - x + 1 = 0$, then
 $\alpha^{2009} + \beta^{2009} =$
[2010]

Options:

- A. -1
- B. 1
- C. 2
- D. -2

Answer: B

Solution:

Solution:

$$x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1 - 4}}{2}$$

$$x = \frac{1 \pm \sqrt{3}i}{2}$$

$$\alpha = \frac{1}{2} + i \frac{\sqrt{3}}{2} = -\omega^2$$

$$\beta = \frac{1}{2} - i \frac{\sqrt{3}}{2} = -\omega$$

$$\alpha^{2009} + \beta^{2009} = (-\omega^2)^{2009} + (-\omega)^{2009}$$

$$= -\omega^2 - \omega = 1$$

Question 286

If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all
 real values of x , the expression $3b^2x^2 + 6bcx + 2c^2$ is :
[2009]

Options:

- A. less than $4ab$
- B. greater than $-4ab$
- C. less than $-4ab$
- D. greater than $4ab$

Answer: B

Solution:

Solution:

Given that roots of the equation

$bx^2 + cx + a = 0$ are imaginary

$\therefore c^2 - 4ab < 0$

Let $y = 3b^2x^2 + 6bcx + 2c^2$

$\Rightarrow 3b^2x^2 + 6bcx + 2c^2 - y = 0$

As x is real, $D \geq 0$

$\Rightarrow 36b^2c^2 - 12b^2(2c^2 - y) \geq 0$

$\Rightarrow 12b^2(3c^2 - 2c^2 + y) \geq 0$ [$\because b^2 \geq 0$]

$\Rightarrow c^2 + y \geq 0 \Rightarrow y \geq -c^2$

But from eqn. (i), $c^2 < 4ab$ or $-c^2 > -4ab$

$\therefore$ we get $y \geq -c^2 > -4ab$

$\Rightarrow y > -4ab$

Question 287

If $z - \frac{4}{z} = 2$, then the maximum value of $|z|$ is equal to :
[2009]

Options:

A. $\sqrt{5} + 1$

B. 2

C. $2 + \sqrt{2}$

D. $\sqrt{3} + 1$

Answer: A

Solution:

Solution:

Given that $\left| z - \frac{4}{z} \right| = 2$

$|z| = \left| z - \frac{4}{z} + \frac{4}{z} \right| \leq \left| z - \frac{4}{z} \right| + \frac{4}{|z|}$

$\Rightarrow |z| \leq 2 + \frac{4}{|z|}$

$\Rightarrow |z|^2 - 2|z| - 4 \leq 0$

$\Rightarrow \left(|z| - \frac{2 + \sqrt{20}}{2} \right) \left(|z| - \frac{2 - \sqrt{20}}{2} \right) \leq 0$

$\Rightarrow (|z| - (1 + \sqrt{5}))(|z| - (1 - \sqrt{5})) \leq 0$

$\frac{+}{-\infty} - \frac{+}{\infty}$

$(1 - \sqrt{5})(1 + \sqrt{5})$

$\Rightarrow (-\sqrt{5} + 1) \leq |z| \leq (\sqrt{5} + 1)$

$\Rightarrow |z|_{\max} = \sqrt{5} + 1$

Question288

The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is [2009]

Options:

- A. 1
- B. 4
- C. 3
- D. 2

Answer: D

Solution:

Solution:

Let the roots of equation $x^2 - 6x + a = 0$ be α and 4β and that of the equation $x^2 - cx + 6 = 0$ be α and 3β . Then
 $\alpha + 4\beta = 6$... (i) $4\alpha\beta = a$... (ii)
and $\alpha + 3\beta = c$... (iii) $3\alpha\beta = 6$... (iv)
 $\Rightarrow a = 8$ (from (ii) and (iv))
 $\therefore$ The equation becomes $x^2 - 6x + 8 = 0$
 $\Rightarrow (x - 2)(x - 4) = 0$
 $\Rightarrow$ roots are 2 and 4
 $\Rightarrow \alpha = 2, \beta = 1 \therefore$ Common root is 2

Question289

The conjugate of a complex number is $\frac{1}{i-1}$ then that complex number is [2008]

Options:

- A. $\frac{-1}{i-1}$
- B. $\frac{1}{i+1}$
- C. $\frac{-1}{i+1}$
- D. $\frac{1}{i-1}$

Answer: C

Solution:

Solution:

$$\left(\frac{1}{i-1}\right) = \frac{1}{(i-1)} = \frac{1}{-i-1} = \frac{-1}{i+1}$$

Question290

If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is [2007]

Options:

- A. 6
- B. 0
- C. 4
- D. 10

Answer: A

Solution:

Solution:

$$|z + 1| = |z + 4 - 3| \leq |z + 4| + |-3| \leq |3| + |-3|$$

$$\Rightarrow |z + 1| \leq 6 \Rightarrow |z + 1|_{\max} = 6$$

Question291

If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is [2007]

Options:

- A. $(3, \infty)$
- B. $(-\infty, -3)$
- C. $(-3, 3)$
- D. $(-3, \infty)$

Answer: C

Solution:

Solution:

Let α and β are roots of the equation

$$x^2 + ax + 1 = 0$$

$$\alpha + \beta = -a \text{ and } \alpha\beta = 1$$

Given that $|\alpha - \beta| < \sqrt{5}$

$$\Rightarrow \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} < \sqrt{5}$$

$$(\because (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta)$$

$$\Rightarrow \sqrt{a^2 - 4} < \sqrt{5} \Rightarrow a^2 - 4 < 5$$

$$\Rightarrow a^2 - 9 < 0 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$$

$$\Rightarrow a \in (-3, 3)$$

Question292

All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 lie in the interval [2006]

Options:

A. $-2 < m < 0$

B. $m > 3$

C. $-1 < m < 3$

D. $1 < m < 4$

Answer: C

Solution:

Solution:

Given equation is $x^2 - 2mx + m^2 - 1 = 0$

$$\Rightarrow (x - m)^2 - 1 = 0$$

$$\Rightarrow (x - m + 1)(x - m - 1) = 0$$

$$\Rightarrow x = m - 1, m + 1$$

$$m - 1 > -2 \text{ and } m + 1 < 4$$

$$\Rightarrow m > -1 \text{ and } m < 3 \Rightarrow -1 < m < 3$$

Question293

If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$ respectively, then the value of $2 + q - p$ is [2006]

Options:

A. 2

B. 3

C. 0

D. 1

Answer: B

Solution:

Solution:

Given that $x^2 + px + q = 0$

Sum of roots = $\tan 30^\circ + \tan 15^\circ = -p$

Product of roots = $\tan 30^\circ \cdot \tan 15^\circ = q$

$$\tan 45^\circ = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \cdot \tan 15^\circ} \Rightarrow \frac{-p}{1 - q} = 1$$

$$\Rightarrow -p = 1 - q \Rightarrow q - p = 1$$

$$\therefore 2 + q - p = 3$$

Question294

If $z^2 + z + 1 = 0$, where z is complex number, then the value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \dots + \left(z^6 + \frac{1}{z^6}\right)^2$ is

[2006]

Options:

A. 18

B. 54

C. 6

D. 12

Answer: D

Solution:

Solution:

$$z^2 + z + 1 = 0 \Rightarrow z = \omega \text{ or } \omega^2$$

$$\text{So, } z + \frac{1}{z} = \omega + \omega^2 = -1$$

$$\left[\because \frac{1}{z} = \omega^2 \text{ and } 1 + \omega + \omega^2 = 0 \right]$$

$$z^2 + \frac{1}{z^2} = \omega^2 + \omega = -1$$

$$[\because \omega^3 = 1]$$

$$z^3 + \frac{1}{z^3} = \omega^3 + \omega^3 = 2$$

$$z^4 + \frac{1}{z^4} = -1, z^5 + \frac{1}{z^5} = -1$$

$$\text{and } z^6 + \frac{1}{z^6} = 2$$

$$\therefore \text{The given sum} = 1 + 1 + 4 + 1 + 1 + 4 = 12$$

Question295

If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is

[2006]

Options:

A. $\frac{1}{4}$

B. 41

C. 1

D. $\frac{17}{7}$

Answer: B

Solution:

Solution:

$$y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$$

$$3x^2(y - 1) + 9x(y - 1) + 7y - 17 = 0$$

$D \geq 0 \because x$ is real

$$81(y - 1)^2 - 4 \times 3(y - 1)(7y - 17) \geq 0$$

$$\Rightarrow (y - 1)(y - 41) \leq 0 \Rightarrow 1 \leq y \leq 41$$

$\therefore$ Max value of y is 41

Question296

If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on

[2005]

Options:

A. an ellipse

B. a circle

C. a straight line

D. a parabola

Answer: C

Solution:

Solution:

$$\text{Given that } w = \frac{z}{z - \frac{1}{3}i}$$

$$\Rightarrow |w| = \frac{|z|}{\left|z - \frac{1}{3}i\right|} = 1 \quad \left[\because \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \right]$$

$$\Rightarrow |z| = \left|z - \frac{1}{3}i\right|$$

$\Rightarrow$ distance of z from origin and point $\left(0, \frac{1}{3}\right)$ is same hence z lies on bisector of the line joining points $(0,0)$ and $(0, 1/3)$

Hence z lies on a straight line.

Question297

If z_1 and z_2 are two non- zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to [2005]

Options:

- A. $\frac{\pi}{2}$
- B. $-\pi$
- C. 0
- D. $-\frac{\pi}{2}$

Answer: C

Solution:

Solution:

$|z_1 + z_2| = |z_1| + |z_2| \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.

Question298

If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x - 1)^3 + 8 = 0$, are [2005]

Options:

- A. $-1, -1 + 2\omega, -1 - 2\omega^2$
- B. $-1, -1, -1$
- C. $-1, 1 - 2\omega, 1 - 2\omega^2$
- D. $-1, 1 + 2\omega, 1 + 2\omega^2$

Answer: C

Solution:

Solution:

$\because (x - 1)^3 + 8 = 0 \Rightarrow (x - 1) = (-2)^{1/3}$
 $\Rightarrow x - 1 = -2$ or -2ω or $-2\omega^2$
or $x = -1$ or $1 - 2\omega$ or $1 - 2\omega^2$

Question299

In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $-\tan\left(\frac{Q}{2}\right)$ are the roots of

**$ax^2 + bx + c = 0$, $a \neq 0$ then
[2005]**

Options:

A. $a = b + c$

B. $c = a + b$

C. $b = c$

D. $b = a + c$

Answer: B

Solution:

Solution:

$\tan\left(\frac{P}{2}\right), \tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$

$$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = -\frac{b}{a}$$

$$\tan\left(\frac{P}{2}\right) \cdot \tan\left(\frac{Q}{2}\right) = \frac{c}{a}$$

$$\frac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right) \tan\left(\frac{Q}{2}\right)} = \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$$

$$\left[\because P + Q = \frac{\pi}{2} \right]$$

$$\Rightarrow \frac{-\frac{b}{a}}{1 - \frac{c}{a}} = 1 \Rightarrow -\frac{b}{a} = \frac{a - c}{a}$$

$$\Rightarrow -b = a - c \Rightarrow c = a + b$$

Question300

**If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals
[2005]**

Options:

A. -2

B. 3

C. 2

D. 1

Answer: D

Solution:

Solution:

Let $\alpha, \alpha + 1$ be roots

Then $\alpha + \alpha + 1 = b =$ sum of roots
 $\alpha(\alpha + 1) = c =$ product of roots
 $\therefore b^2 - 4c = (2\alpha + 1)^2 - 4\alpha(\alpha + 1) = 1$

Question301

If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval [2005]

Options:

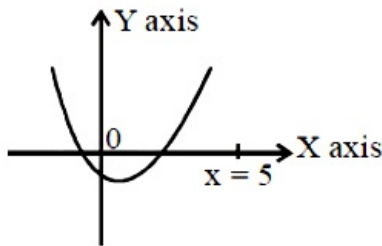
- A. (5,6]
- B. (6, ∞)
- C. $(-\infty, 4)$
- D. [4,5]

Answer: C

Solution:

Solution:

Given that both roots of quadratic equation are less than 5 then (i)



Discriminant ≥ 0

$$4k^2 - 4(k^2 + k - 5) \geq 0$$

$$4k^2 - 4k^2 - 4k + 20 \geq 0$$

$$4k \leq 20 \Rightarrow k \leq 5$$

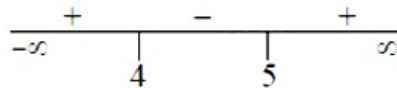
(ii) $p(5) > 0$

$$\Rightarrow f(5) > 0; 25 - 10k + k^2 + k - 5 > 0$$

$$\Rightarrow k^2 - 9k + 20 > 0$$

$$\Rightarrow k(k - 4) - 5(k - 4) > 0$$

$$\Rightarrow (k - 5)(k - 4) > 0$$



$$(iii) \frac{\text{Sum of roots}}{2} < 5$$

$$\Rightarrow -\frac{b}{2a} = \frac{2k}{2} < 5$$

$$\Rightarrow k < 5$$

The intersection of (i), (ii) & (iii) gives

$$k \in (-\infty, 4)$$

Question302

The value of a for which the sum of the squares of the roots of the

equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is [2005]

Options:

A. 1

B. 0

C. 3

D. 2

Answer: A

Solution:

Solution:

Given equation is $x^2 - (a - 2)x - a - 1 = 0$

$\Rightarrow \alpha + \beta = a - 2; \alpha\beta = -(a + 1)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= a^2 - 2a + 6 = (a - 1)^2 + 5$

For min. value of $\alpha^2 + \beta^2$, $a - 1 = 0$

$\Rightarrow a = 1$

Question303

If $z = x - iy$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q} \right) / (p^2 + q^2)$ is equal to [2004]

Options:

A. -2

B. -1

C. 2

D. 1

Answer: A

Solution:

Solution:

Given that $z^{\frac{1}{3}} = p + iq$

$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p + iq)$

$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

Comparing both side, we get

$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2 \dots\dots\dots (i)$

and $y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2 \dots\dots\dots (ii)$

Adding (i) and (ii), we get

$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 \therefore \left(\frac{x}{p} + \frac{y}{q} \right) / (p^2 + q^2) = -2$

Question304

Let z and w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then $\arg z$ equals
[2004]

Options:

A. $\frac{5\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Solution:

Given that $\arg zw = \pi$

$$\Rightarrow \arg z + \arg w = \pi$$

$$z + iw = 0 \Rightarrow \bar{z} = -i\bar{w}$$

Replace i by $-i$, we get

$$\therefore \bar{z} = i\bar{w} \Rightarrow \arg z = \frac{\pi}{2} + \arg w$$

$$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z \quad (\text{from (i)})$$

$$\arg z = \frac{3\pi}{4}$$

Question305

If $|z^2 - 1| = |z|^2 + 1$, then z lies on
[2004]

Options:

A. an ellipse

B. the imaginary axis

C. a circle

D. the real axis

Answer: B

Solution:

Solution:

Given that $|z^2 - 1| = |z|^2 + 1 \Rightarrow z^2 - 1 = (\bar{z}z + 1)^2$
 $[\because |z|^2 = z\bar{z}]$
 $\Rightarrow (z^2 - 1)(\bar{z}^2 - 1) = (\bar{z} + 1)^2 (\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2)$
 $\Rightarrow z^2 \bar{z}^2 - z^2 - \bar{z}^2 + 1 = z^2 \bar{z}^2 + 2z\bar{z} + 1$
 $\Rightarrow z^2 + 2z\bar{z} + \bar{z}^2 = 0$
 $\Rightarrow (z + \bar{z})^2 = 0 \Rightarrow z = -\bar{z}$
 $\Rightarrow z$ is purely imaginary

Question306

If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is [2004]

Options:

- A. 4
- B. 12
- C. 3
- D. $\frac{49}{4}$

Answer: D

Solution:

Solution:

Given that 4 is a root of $x^2 + px + 12 = 0$
 $\Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$
 Now, the equation $x^2 + px + q = 0$ has equal roots.
 $\therefore D = 0$
 $\Rightarrow p^2 - 4q = 0 \Rightarrow q = \frac{p^2}{4} = \frac{49}{4}$

Question307

If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its root are [2004]

Options:

- A. -1,2
- B. -1,1
- C. 0,-1
- D. 0,1

Answer: C

Solution:

Solution:

Let the second root be α .

Then $\alpha + (1 - p) = -p \Rightarrow \alpha = -1$

Also $\alpha \cdot (1 - p) = 1 - p$

$\Rightarrow (\alpha - 1)(1 - p) = 0 \Rightarrow p = 1 [\because \alpha = -1]$

$\therefore$ Roots are $\alpha = -1$ and $1 - p = 0$

Question308

If $\left(\frac{1+i}{1-i} \right)^x = 1$ then

[2003]

Options:

- A. $x = 2n + 1$, where n is any positive integer
- B. $x = 4n$, where n is any positive integer
- C. $x = 2n$, where n is any positive integer
- D. $x = 4n + 1$, where n is any positive integer.

Answer: B

Solution:

Solution:

Given that

$$\left(\frac{1+i}{1-i} \right)^x = 1 \Rightarrow \left[\frac{(1+i)^2}{1-i^2} \right]^x = 1$$

$$\left(\frac{1+i^2+2i}{1+1} \right)^x = 1 \Rightarrow (i)^x = 1; \quad \therefore x = 4n; \quad n \in \mathbb{I}^+$$

Question309

If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $\text{Arg}(z) - \text{Arg}(\omega) = \frac{\pi}{2}$, then $\overline{z}\omega$ is equal to

[2003]

Options:

- A. -1
- B. 1
- C. $-i$
- D. i

Answer: A

Solution:

Solution:

$$\begin{aligned} |\bar{z}\omega| &= |\bar{z}| |\omega| = |z| |\omega| = |z\omega| = 1 [\because |\bar{z}| = |z|] \\ \text{Arg}(z\omega) &= \arg(\bar{z}) + \arg(\omega) \\ &= -\arg(z) + \arg \omega = -\frac{\pi}{2} \\ [\because \arg(\bar{z}) &= -\arg(z)] \\ \therefore z\omega &= -1 \end{aligned}$$

Question310

The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is [2003]

Options:

- A. 3
- B. 2
- C. 4
- D. 1

Answer: C

Solution:

Solution:

$$\begin{aligned} \text{Given that } x^2 - 3|x| + 2 = 0 &\Rightarrow |x|^2 - 3|x| + 2 = 0 \\ \Rightarrow (|x| - 2)(|x| - 1) &= 0 \\ \Rightarrow |x| = 1, 2 &\Rightarrow x = \pm 1, \pm 2 \\ \therefore \text{No. of solution} &= 4 \end{aligned}$$

Question311

The value of ' a ' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is [2003]

Options:

- A. $-\frac{1}{3}$
- B. $\frac{2}{3}$
- C. $-\frac{2}{3}$
- D. $\frac{1}{3}$

Answer: B

Solution:

Solution:

Let one roots of given equation be α

$\therefore$ Second roots be 2α then

$$\alpha + 2\alpha = 3\alpha = \frac{1 - 3a}{a^2 - 5a + 3}$$

$$\Rightarrow \alpha = \frac{1 - 3a}{3(a^2 - 5a + 3)} \dots\dots(i)$$

$$\text{and } \alpha \cdot 2\alpha = 2\alpha^2 = \frac{2}{a^2 - 5a + 3}$$

$$\therefore 2 \left[\frac{1}{9} \frac{(1 - 3a)^2}{(a^2 - 5a + 3)^2} \right] = \frac{2}{a^2 - 5a + 3}$$

[from (i)]

$$\frac{(1 - 3a)^2}{(a^2 - 5a + 3)} = 9$$

$$\Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27$$

$$\Rightarrow 39a = 26 \Rightarrow a = \frac{2}{3}$$

Question312

Let Z_1 and Z_2 be two roots of the equation $Z^2 + aZ + b = 0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then
[2003]

Options:

A. $a^2 = 4b$

B. $a^2 = b$

C. $a^2 = 2b$

D. $a^2 = 3b$

Answer: D

Solution:

Solution:

Given that $Z^2 + aZ + b = 0$;

$Z_1 + Z_2 = -a$ & $Z_1 Z_2 = b$

$0, Z_1, Z_2$ form an equilateral triangle $\therefore 0^2 + Z_1^2 + Z_2^2 = 0 \cdot Z_1 + Z_1 \cdot Z_2 + Z_2 \cdot 0$

(for an equilateral triangle,

$$Z_1^2 + Z_2^2 + Z_3^2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1)$$

$$\Rightarrow Z_1^2 + Z_2^2 = Z_1 Z_2$$

$$\Rightarrow (Z_1 + Z_2)^2 = 3Z_1 Z_2$$

$$\therefore a^2 = 3b$$

Question313

If $|z - 4| < |z - 2|$, its solution is given by
[2002]

Options:

A. $\operatorname{Re}(z) > 0$

B. $\operatorname{Re}(z) < 0$

C. $\operatorname{Re}(z) > 3$

D. $\operatorname{Re}(z) > 2$

Answer: C

Solution:

Solution:

Given that $|z - 4| < |z - 2|$

Let $z = x + iy$

$$\Rightarrow |(x - 4) + iy| < |(x - 2) + iy|$$

$$\Rightarrow (x - 4)^2 + y^2 < (x - 2)^2 + y^2$$

$$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$$

$$\Rightarrow x > 3 \Rightarrow \operatorname{Re}(z) > 3$$

Question 314

z and w are two non zero complex numbers such that $|z| = |w|$ and $\operatorname{Arg} z + \operatorname{Arg} w = \pi$ then z equals
[2002]

Options:

A. $\overline{\omega}$

B. $-\overline{\omega}$

C. ω

D. $-\omega$

Answer: B

Solution:

Solution:

Let $|z| = |\omega| = r$

$$\therefore z = re^{i\theta}, \omega = re^{i\varphi} \quad \text{where } \theta + \varphi = \pi$$

$$\therefore z = re^{i(\pi - \varphi)} = re^{i\pi} \cdot e^{-i\varphi} = -re^{-i\varphi} = -\overline{\omega}$$

$$[\because e^{i\pi} = -1 \text{ and } \overline{\omega} = re^{-i\varphi}]$$

Question 315

The locus of the centre of a circle which touches the circle $|z - z_1| = a$

and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be [2002]

Options:

- A. an ellipse
- B. a hyperbola
- C. a circle
- D. none of these

Answer: B

Solution:

Solution:

Let the circle be $|z - z_0| = r$. Then according to given conditions $|z_0 - z_1| = r + a \dots (i)$

$|z_0 - z_2| = r + b \dots (ii)$

Subtract (ii) from (i)

we get $|z_0 - z_1| - |z_0 - z_2| = a - b$.

$\therefore$ Locus of centre z_0 is $|z - z_1| - |z - z_2| = a - b$, which represents a hyperbola.

Question316

If p and q are the roots of the equation $x^2 + px + q = 0$, then [2002]

Options:

- A. $p = 1, q = -2$
- B. $p = 0, q = 1$
- C. $p = -2, q = 0$
- D. $p = -2, q = 1$

Answer: A

Solution:

Solution:

$p + q = -p \Rightarrow q = 2p$

and $pq = q \Rightarrow q(p - 1) = 0$

$\Rightarrow q = 0$ or $p = 1$

If $q = 0$, then $p = 0$

or $p = 1$, then $q = -2$.

Question317

Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$

[2002]

Options:

- A. is always positive
- B. is always negative
- C. does not exist
- D. none of these

Answer: A

Solution:

Solution:

Product of real roots $= \frac{c}{a} = \frac{9}{t^2} > 0, \forall t \in \mathbb{R}$

$\therefore$ Product of real roots is always positive.

Question318

Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then

[2002]

Options:

- A. $a + b + 4 = 0$
- B. $a + b - 4 = 0$
- C. $a - b - 4 = 0$
- D. $a - b + 4 = 0$

Answer: A

Solution:

Solution:

Let α and β are roots of the equation $x^2 + ax + b = 0$ and γ and δ be the roots of the equation $x^2 + bx + a = 0$ respectively

$\therefore \alpha + \beta = -a, \alpha\beta = b$ and $\gamma + \delta = -b, \gamma\delta = a$

Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow a^2 - 4b = b^2 - 4a$

$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$

$\Rightarrow a + b + 4 = 0 \quad (\because a \neq b)$

Question319

If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α / β and β / α as its roots is

[2002]

Options:

A. $3x^2 - 19x + 3 = 0$

B. $3x^2 + 19x - 3 = 0$

C. $3x^2 - 19x - 3 = 0$

D. $x^2 - 5x + 3 = 0$.

Answer: A

Solution:

Given that $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$;

$\Rightarrow \alpha$ & β are roots of equation, $x^2 = 5x - 3$

or $x^2 - 5x + 3 = 0$

$\therefore \alpha + \beta = 5$ and $\alpha\beta = 3$

Thus, the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$$x^2 - x \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha} \right) + \frac{\alpha\beta}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - x \left(\frac{\alpha^2 + \beta^2}{\alpha\beta} \right) + 1 = 0$$

$$\text{or } 3x^2 - 19x + 3 = 0$$
