Chapter 4

Deflection of Beams

CHAPTER HIGHLIGHTS

- *Deflection of Beams*
- *Deformation of Beam Under Transverse Loading*
- *Equation of the Elastic Curve*
- *Double Integration Method*
- *Macaulay's Method*
- *Moment Area Methods*
- *First Moment Area Theorem*
- *Second Moment Area Theorem*
- *Energy Based Methods-castigliano's Theorems*
- *Deflection of a Beam Due to Bending Moments*

Deflection of Beams

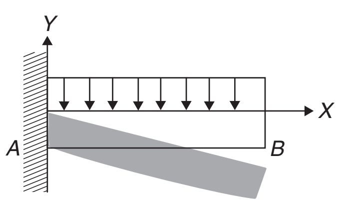
When a beam is loaded with concentrated or distributed loads, the axis of beam deflects. The deflection should be within permissible limits to prevent misalignment, to maintain dimensional accuracy, etc. Therefore, while designing, not only the strength but also the deflection is an important factor to be considered.

Of particular interest is the determination of the maximum deflection of a beam under a given loading, since the design specifications of a beam will generally include a maximum allowable value for its deflection.

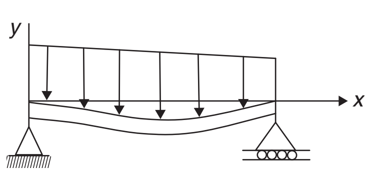
A prismatic beam subjected to pure bending is bent into an arc of circle, and that within the elastic range, the curvature of the neutral surface can be expressed as

$$\frac{1}{R} = \frac{M}{El}$$

where 'M' is the bending moment, 'E' the modulus of elasticity and 'I' the moment of inertia of the cross-section about its neutral axis.



1. **Cantilever beam**



2. **Simply supported beam:** To determine the slope and deflection of the beam at any given point, we first derive the following second order linear differential equation, which governs the elastic curve characterizing the shape of the deformed beam.

$$\frac{d^2y}{dx^2} = \frac{M(x)}{El}$$

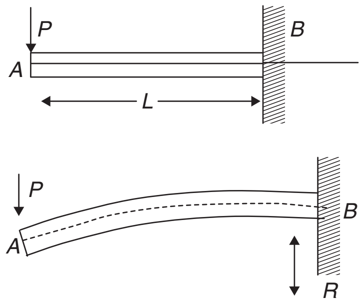
If the bending moment can be represented for all values of 'x' by a single function $M(x)$ as in the case of the beams and loadings, shown in above figures, the slope $\theta = \frac{dy}{dx}$ and the deflection 'y' at any point of the beam may be obtained through two successive integrations. The two constants of integration introduced in the process will be determined from the boundary conditions indicated in the figure.

However, if different analytical functions are required to represent the bending moment in various portions of the beam, different differential equations will also be required, leading to different functions defining the elastic curve in various portions of the beam.

Deformation of Beam under Transverse Loading

$$\frac{1}{R} = \frac{M(x)}{El}$$

Consider a cantilever beam 'AB' of length L, subjected to a concentrated load 'P' at its free end 'A' as shown in the following figure:



We have

$$M(x) = -P \times x$$

$$\text{and } \frac{1}{R} = \frac{-Px}{El},$$

which shows that the curvature of the neutral surface varies linearly with x, form zero at 'A', where R_A itself is infinite, to $\frac{-PL}{El}$ at B, where $R_B = \frac{El}{PL}$, R_A and R_B being the radius of curvature at A and B, respectively.

Equation of the Elastic Curve

We first recall from elementary calculus that the curvature of a plane curve at a point $Q(x, y)$ of the curve can be expressed as

$$\frac{1}{R} = \frac{\dfrac{d^2 y}{dx^2}}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}$$

where $\frac{dy}{dx}$ and $\frac{d^2 y}{dx^2}$ are the first and second derivatives of the function $y(x)$ represented by that curve. But, in the case of the elastic curve of a beam, the slope $\frac{dy}{dx}$ is very small, and its square is negligible compared to unity. Therefore,

$$\frac{1}{R} = \frac{d^2 y}{dx^2}$$

$$\frac{d^2 y}{dx^2} = \frac{M(x)}{El}$$

The equation obtained is a second order linear differential equation. It is the governing differential equation for the elastic curve.

This is the differential equation for deflection

Here, y = deflection

$$\theta = \frac{dy}{dx} = \text{slope}$$

$$M = EI\frac{d^2 y}{dx^2} = \text{moment}$$

It is also to be noted that shear force

$$F = -\frac{dM}{dx} = -EI\frac{d^3 y}{dx^3}$$

$$\text{Load intensity } q = \frac{dF}{dx} = -E\frac{d^4 y}{dx^4}$$

The product EI is known as flexural rigidity. In the case of a prismatic beam it is taken as constant.

Double Integration Method

Taking x from one end (usually from left end) and with sagging moment as positive

$$EI\frac{d^2 y}{dx^2} = M$$

$$EI\frac{dy}{dx} = \int_0^x M dx + C_1$$

$$EI\frac{d^2 y}{dx} = \int_0^x \int_0^x M dx + C_1 + C_2$$

The constants C_1 and C_2 are found out by applying boundary conditions.

Some Boundary Conditions

1. At simply supported/roller ends $y = 0$
2. At fixed ends $y = 0$

$$\frac{dy}{dx} = 0$$

3. At point of symmetry $\frac{dy}{dx} = 0$

Some General Cases

1. Cantilever subjected to moment at free end
   At $x = 0$,

$$\frac{dy}{dx} = \frac{ML}{EI}$$

$$y = -\frac{ML^2}{2EI}$$

2. Cantilever subjected to concentrated load at free end
   At $x = 0$,

$$\frac{dy}{dx} = \frac{wL^2}{2EI}$$

$$y = -\frac{wL^3}{3EI}$$

3. Cantilever subjected uniform load w/unit length
   At $x = 0$,

$$\frac{dy}{dx} = \frac{wL^3}{6EI}$$

$$y = \frac{-wL^4}{8EI}$$

4. Cantilever subjected uniformly varying load, zero at free end to w/unit length at fixed end
   At $x = 0$,

$$\frac{dy}{dx} = \frac{wL^3}{24EI}$$

$$y = \frac{-wL^4}{30EI}$$

5. Simply supported beam with central concentrated load
   Deflection at centre

$$y_c = \frac{wL^3}{48EI}$$

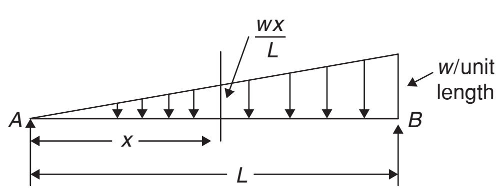
$$\text{Slope at end } \theta = \frac{-wL^3}{16EI}$$

6. Simply supported beam with uniform load w/unit length

$$y_c = \frac{5}{384}\frac{wL^4}{EI}$$

$$\theta_A = \frac{-wL^2}{24EI}$$

7. Simply supported beam with uniformly varying load, zero at end A and w/unit length at end B.



$$EI_y = -\frac{7wL^3x}{360} + \frac{wLx^3}{36} - \frac{wx^5}{120L}$$

$$EI\frac{dy}{dx} = -\frac{7wL^3}{360} + \frac{wLx^2}{12} - \frac{wx^4}{24L}$$

$\frac{dy}{dx} = 0$ at the point of maximum deflection y_{max}.

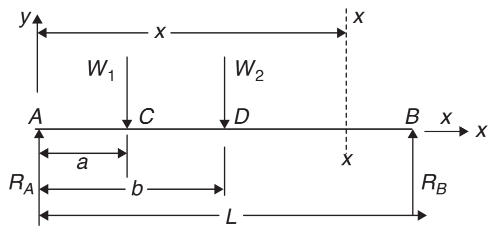
This occurs at $x = 0.5193$ L

$$Y_{max} = -0.006523\frac{wL^4}{EI}.$$

Macaulay's Method

Macaulay's method is a simplified version of double integration method. It gives a continuous expression for bending moment applicable for all portions of the beam. The constants of integration determined by using boundary conditions are also applicable for all portions of the beam.

For example, consider the case of a beam with concentrated loads as shown in the figure.



Here the expression for moment at a distance x from A is

$$M_x = EI\frac{d^2y}{dx} = R_Ax - W_1(x-a) - W_2(x-b)$$

The same expression can be used for other portions also, if we ignore the quantities $(x - a)$, etc. becoming negative.

Integrating the expression, we get $EI\frac{dy}{dx}$

$$= C_1 + R_A\frac{x^2}{2} - W_1\frac{(x-a)^2}{2} - W_2\frac{(x-b)^2}{2}$$

$$EIy = C_2 + C_1x + R_A\frac{x^3}{6} - W_1\frac{(x-a)^3}{6} - W_2\frac{(x-b)^3}{6}$$

Applying boundary condition, values of C_1 and C_2 are found out.

Now, the above expression can be used for finding out slope and deflection of any portion of the beam. If term $(x - a)$, etc. become negative they are ignored.

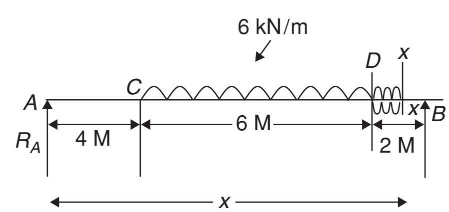
In the case of uniformly distributed loads, it is extended up to the section xx and an equal and opposite uniformly distributed load is applied to nullify it.

Solved Examples

Example 1: A 12 m long beam simply supported at ends is loaded as shown in the figure. (A uniformly distributed load of 6 kN/m acts over a length of $CD = 6$ m)

Determine the slopes at A and B.

($E = 2000$ N/cm^2 and $I = 5000$ cm^4)



Solution: Taking moments, i.e.,

$$R_A \times 12 - 6 \times 6 \times (3 + 2) = 0$$
$$\Rightarrow R_A \times 12 = 180$$
$$\Rightarrow R_A = 15 \text{ kN}$$

That is, $R_B \times 12 - 6 \times 6 \times (3 + 4) = 0$

$\Rightarrow R_B \times 12 = 252$

$\Rightarrow R_B = 21$ kN

Now, consider a section at xx in the portion DB at a distance x from A.

Extend the uniformly distributed load to xx and an equal and opposite uniformly distributed load from D to section xx. Bending moment at any section xx

$$M_x = R_A(x) - \frac{W(x-4)^2}{2} + \frac{W(x-10)^2}{2}$$

That is, $$EI\frac{d^2y}{dx^2} = 15x - \frac{6(x-4)^2}{2} + \frac{6(x-10)^2}{2}$$

Integrating

$$EI\frac{dy}{dx} = C_1 + \frac{15x^2}{2} - \frac{3(x-4)^3}{3} + \frac{3(x-10)^3}{3}$$

$$\Rightarrow EIy = C_2 + C_1x + \frac{15x^3}{6} - \frac{(x-4)^4}{4} + \frac{(x-10)^4}{4}$$

Applying boundary conditions and omitting negative terms

$$C_2 = 0 \text{ and } C_1 = -275$$

Equation for slope,

$$EI\frac{dy}{dx} = -275 + 7.5x^2 - (x-4)^3 + (x-10)^3$$

At A, $x = 0$

Slope $\frac{dy}{dx} = \theta_A$

$EI = 1000$ kN m^2

$\therefore EI\,\theta_A = -275 + 7.5 \times 0 = -275$

$\theta_A = -15.76°$.

At B, $x = 12$ m

$EI\,\theta_B = -275 + 7.5 \times 12^2 - 8^3 + 2^3$

$\theta_B = 17.25°$

Example 2: In the above problem find maximum deflection.

Solution: Maximum deflection occurs at $\frac{dy}{dx} = 0$.

Assume that it occurs in the portion CD. The term $(x - 10)$ will be negative.

$\therefore$ From the slope equation,

$0 = -275 + 7.5x^2 - (x - 4)^3$ for zero slope

Let $f(x) = -275 + 7.5x^2 - (x - 4)^3$

At $x = 5$, $f(x) = -275 + 7.5 \times 25 - (1)^3$

$= -88.5$

By trial $x \simeq 6.2$ for $f(x) = 0$

This is the point at which maximum deflection occurs.

Deflection equation is

$$El_Y = 0 - 275x + 2.5x^3 - \frac{(x-4)^4}{4}$$

$\therefore$ When $x = 6.2$

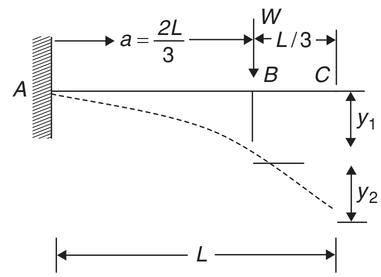
$$EIy_{max} = -1115.04$$

$$Y_{max} = \frac{-1115.04}{1000} = -1.115 \text{ m}$$

$= 1115$ mm downward

Example 3: A cantilever beam of length L is subjected to a concentrated load W at a distance $\frac{L}{3}$ from the free end. Find the deflection of the free end.

Solution:



Deflection at $B = \frac{Wa^3}{3EI} = y_1$

Since there is no load in the portion BC,

Slope $\left(\frac{dy}{dx}\right)_B$ is maintained throughout the portion slope at

$B = \frac{Wa^2}{2EI}$

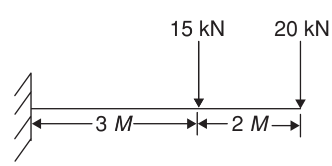
Deflection at $C = y_1 + y_2 = \frac{W_a^3}{3EI}, \frac{W_a^2}{2EI} \times \frac{L}{3}$

$$= \frac{Wa^2}{3EI}\left(a + \frac{L}{2}\right)$$

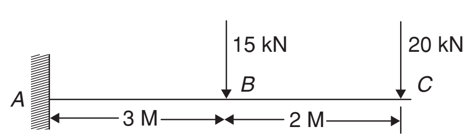
putting $a = \frac{2L}{3}$, $C = \frac{14}{81}\frac{WL^3}{EI}$

Example 4: Find the deflection at the end of the cantilever beam shown in the figure.

Take $EI = 4 \times 10^4$ kNm^2



Solution:



Equation for deflection at end $= \dfrac{-W\ell^3}{3EI}$

Equation for slope $= \dfrac{W\ell^2}{2EI}$

Total deflection = Deflection due to 20 kN + Deflection due to 15 kN

$$= \frac{20\times 5^3}{3EI} + \frac{W_B\ell^3}{3EI} + BC\frac{W_B\ell^2}{2EI}$$

$$= \frac{20\times 5^3}{3EI} + \frac{15\times 3^3}{3EI} + \frac{2\times 15\times 3^2}{2EI}$$

$$= \frac{1}{EI}(833.33+135+135)$$

$$= \frac{1103.33}{4\times 10^4} = 0.0276 \text{ m}$$

$$= 27.6 \text{ mm}$$

Moment Area Methods

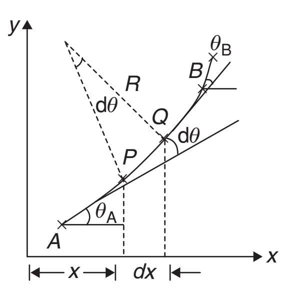
Change in slope and deflection between two points on a beam can be found out using moment area theorems (or Mohr's theorems)

We have seen that

$$\frac{1}{R} = \frac{d^2y}{dx^2} = \frac{M}{EI}$$

where M_x is the bending moment and EI is the flexural rigidity.

First Moment Area Theorem



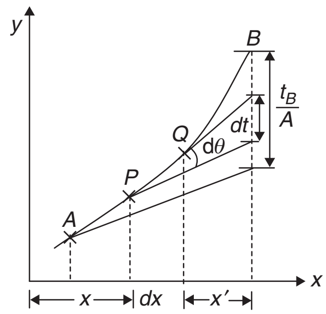
The figure shows elastic curve AB of an initially straight beam (exaggerated).

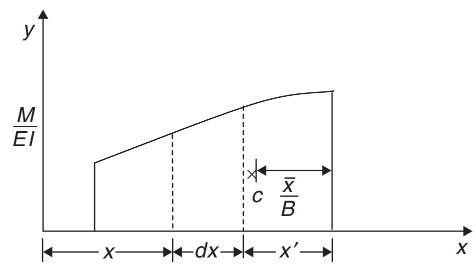
For the infinitesimally small distance dx, from the geometry of the figure we can see that $dx = Rd\theta$ or $d\theta = \dfrac{dx}{R} = \dfrac{M}{EI}dx$.

On integration over the segment AB, $\int_A^B d\theta = \int_A^B \dfrac{M}{EI}dx$ or change in slope,

$\theta_B - \theta_A =$ Area of $\dfrac{M}{EI}$ diagram between A and B. This is the first moment area theorem.

Second Moment Area Theorem





Let $t_{B/A}$ be the vertical distance of point B from the tangent to the elastic curve at point A. This distance is termed as the **tangential deviation** of B with respect to A.

$t_{B/A} = \int_A^B dt$ where dt is the vertical distance at B corresponding to the tangents at P and Q subtending angle $d\theta$. From the geometry we can see that

$$dt = x'd\theta$$

For infinitesimal values of dt and $d\theta$

$$\therefore \quad t_{B/A} = \int_A^B dt = \int_A^B x'd\theta = \int_A^B \frac{M}{EI}x'dx$$

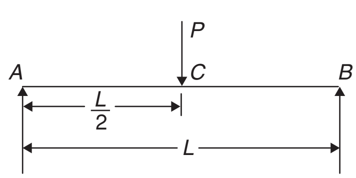
The right hand side of the above equation represents the first moment of area of the $\dfrac{M}{EI}$ diagram and is equal to

(The area of $\dfrac{M}{EI}$ diagram between A and B) × (Horizontal distance of centroid of the area from B)

$$\text{or } t_{B/A} = \text{area of } \frac{M}{EI} \text{ diagram}\Big]_B^A \cdot \frac{\bar{x}}{B}$$

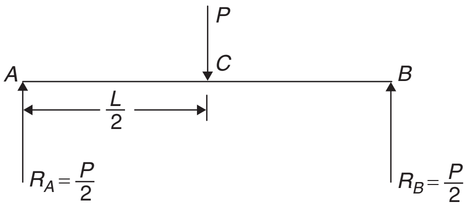
where $\dfrac{\bar{x}}{B}$ is the distance of centroid of area from B. It is to be noted here that $t_{B/A}$ need not be equal to $t_{A/B}$.

Example 5:

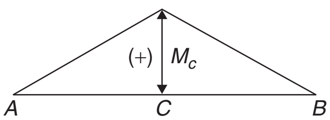


For the simply supported beam loaded as shown in the above figure, determine slope at B and deflection at C by area moment method. (Flexural rigidity = EI)

Solution:



From symmetry, reaction $R_A = R_B = \dfrac{P}{2}$



Bending moment diagram

Bending moment at centre (C)

$$= \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$$

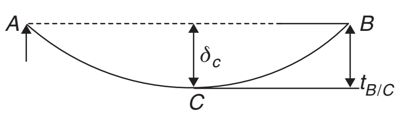
Since tangent at C of the deflection curve is horizontal, slope at B,

θ_B = Area of $\dfrac{M}{EI}$ diagram between B and C

$$= \frac{PL}{4} \times \frac{L}{2} \times \frac{1}{2} \times \frac{1}{EI}$$

$$= \frac{PL^2}{16EI}$$

Deflection at C



From geometry of elastic curve, deflection at C,

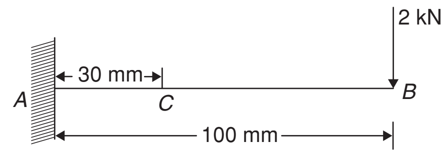
δ_c = tan gential deviation $t_{B/C}$

= area of $\dfrac{M}{EI}$ diagram between B and C × distance of centroid of the area

$$= \frac{PL^2}{16EI} \times \frac{2}{3} \times \frac{L}{2}$$

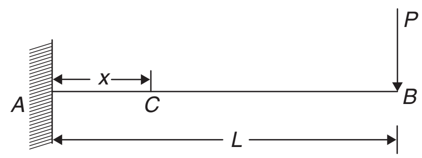
$$= \frac{PL^3}{48EI}.$$

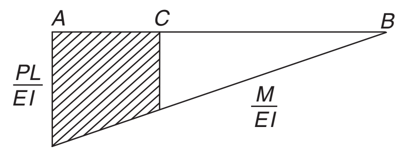
Example 6:



A cantilever beam ACB is loaded as shown in the above figure. If the flexural rigidity is 22 Nm^2 determine the slope at point C using moment area method.

Solution:





$EI = 22$ Nm^2

Slope at C,

θ_c = Area of $\dfrac{M}{EI}$ diagram between A and C

$$= -\left[\frac{PL}{EI} \times \frac{L}{2} - \frac{PL}{EI} \times \frac{(L-x)}{L} \times \frac{(L-x)}{2}\right]$$

$$= -\frac{P}{2EI}[L^2 - (L-x)^2]$$

$$= -\frac{Px}{2EI}[2L - x]$$

$$= \frac{-2000 \times 0.03}{2 \times 22}[2 \times 0.1 - 0.03]$$

$= -0.2318$ radian

∴ The magnitude of the slope is 0.2318 radian.

Energy-based Methods–Castigliano's Theorems

A structure subjected to external forces undergoes deformation. The deformation work involves two parts

1. The external work supplying mechanical energy to the system
2. The internal work in the form of strain energy resisting the external forces.

If P represents the force and x represents the displacement corresponding to the point of application of the force, both the external and internal works are functions of x and P.

Therefore strain energy $U = U(x, P)$

The incremental energy stored is given by $\Delta U = P \,.\, \Delta x$

When $\Delta U \to 0$,

$$\boxed{P_i = \frac{\partial u}{\partial x_i}} \qquad (1)$$

From the above, it can be stated that the unknown forces acting at any point in a structure is the partial derivative of the internal energy associated with the structure with respect to the displacements at that point. This is the **first theorem of Castigliano.**

Now, the complimentary energy stored in the system is

$$U_c = \int_P x \cdot \Delta P$$

When $\Delta P \to 0,\ x_i = \dfrac{\partial U_c}{\partial P}$ (2)

This means that the displacement is obtained by taking partial derivative of the internal energy with respect to loads that are applied in the direction of displacement.

Above is the second theorem of Castigliano. This is used for finding displacement at specific points in a structure.

Deflection of a Beam Due to Bending Moments

Bending moment M is a function of load P and distance L, i.e., $M = f(P, L)$.

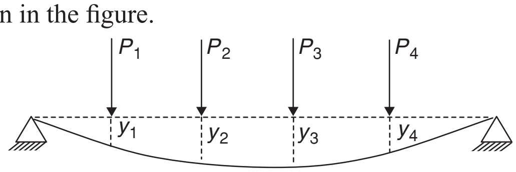
Strain energy due to bending moment is given by

$$U = \int_0^L \frac{M^2 dx}{2EI}$$

So deflection or displacement is obtained by applying Castigliano's second theorem

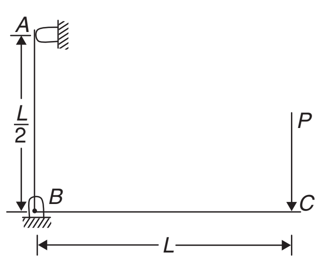
That is, $x_i = \dfrac{\partial u}{\partial P}$

Suppose point loads P_1, P_2, P_3, etc. are acting on a beam as shown in the figure.



The displacement below any of the points is obtained by differentiating total energy stored in the beam with respect to the corresponding point load. Thus, $y_1 = \dfrac{\partial u}{dP_1}$, $y_2 = \dfrac{\partial u}{\partial P_2}$, etc.

Example 7:



A frame ABC with 2 arms is hinged at B and loaded at free end C as shown in the figure. Each arm of the frame has a flexural rigidity EI.

The vertical deflection at C is

(A) $\dfrac{5P^2L^3}{12EI}$

(B) $\dfrac{3P^2L^3}{4EI}$

(C) $\dfrac{P^2L^3}{2EI}$

(D) $\dfrac{5P^2L^3}{8EI}$

Solution:

Bending moment in BC at a distance x from C

$$M_x = -Px$$

Strain energy in BC

$$U_{BC} = \int_O^L \frac{(-Px)^2}{2EI} dx$$

$$= \frac{P^2}{2EI}\left[\frac{x^3}{3}\right]_O^L$$

$$= \frac{P^2L^3}{6EI}$$

Bending moment in AB at a distance y from B

$$M_y = PL$$

Strain energy in AB

$$U_{AB} = \int_O^{\frac{L}{2}} \frac{(PL)^2}{2EI} dy$$

$$= \frac{(PL)^2}{2EI}[y]_O^{\frac{L}{2}}$$

$$= \frac{P^2L^2}{2EI}\cdot\frac{L}{2}$$

$$= \frac{P^2L^3}{4EI}$$

Total strain energy in the frame due to the applied load P.

$$U = U_{AB} + U_{BC}$$

$$= \frac{P^2L^3}{4EI} + \frac{P^2L^3}{6EI}$$

$$= \frac{3P^2L^3 + 2P^2L^3}{12EI}$$

$$= \frac{5P^2L^3}{12EI}$$

EXERCISES

Practice Problems I

Direction for questions 1 to 15: Select the correct alternative from the given choices.

Direction for questions 1 and 2: A cantilever beam of 5 m length carries a uniformly distributed load of 15 N/m over its entire length. The beam is fixed at one end and supported by a prop in the other end.

1. The reaction at the supported end is
(A) 30.376 N (B) 26.212 N
(C) 32.215 N (D) 28.125 N

2. The position of the maximum deflection from the supported end is
(A) 1.88 m (B) 2.11 m
(C) 2.56 m (D) 1.91 m

Direction for questions 3 and 4: A simply supported beam with a span of 6 m carries a point load of 25 kN at 4 m from the left support. Given $I_{xx} = 55\times10^{-6}$ m^4 and $E = 200$ GN/m^2

3. The deflection under the load will be
(A) 8.08 mm (B) 5.09 mm
(C) 8.08 cm (D) 5.09 cm

4. The position of the maximum deflection occurs at
(A) 4.03 m (B) 2.46 m
(C) 3.63 m (D) 3.27 m

5. A cantilever of 4 m span is loaded with a point load of 20 kN/m at a distance of 1 m from the free end. The downward deflection of the cantilever at the free end is [Take $E = 2 \times 10^5$ N/mm^2 and $I = 2 \times 10^8$ mm^4]
(A) 5.25 mm (B) 6.23 mm
(C) 6.75 mm (D) 5.78 mm

6. A girder of uniform section and constant depth is freely supported over a span of 3 m. If it is centrally loaded with a load of 4 kN then the end slopes are [Take I = 800 cm^4 and $E = 20\times10^6$ N/cm^2]
(A) –0.081° and +0.081°
(B) –0.042° and +0.042°
(C) –0.033° and +0.033°
(D) +0.033° and –0.033°

7. An I section steel girder of moment of inertia 3000 cm^4 and depth 30 cm is used as a simply supported beam for a span of 5 m. If the maximum bending stress is not to exceed 5000 N/cm^2, the slope at a point 2 m from one end under a uniformly distributed load is (Take $E = 20 \times 10^6$ N/cm^2)
(A) –0.62° (B) –0.56°
(C) –0.047° (D) –0.39°

8. A cantilever 3 m long is loaded with a concentrated load of 100 N at the free end. If the cross section is rectangular 6 cm × 14 cm deep, the slope at a distance of 1 m from the free end is ($E = 10^6$ N/cm^2)
(A) 0.22° (B) 0.19°
(C) 0.16° (D) 0.18°

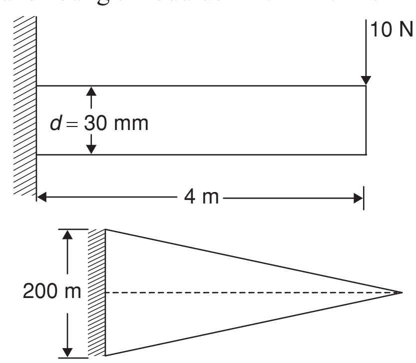
9. A cantilever of rectangular cross-section 50 mm wide and 25 mm deep and 300 cm long carries a uniformly distributed load W. The maximum value of W, if the maximum deflection is not to exceed 1.5 mm is ($E = 70 \times 10^3$ N/mm^2)
(A) 6.75×10^{-4} N/mm (B) 5.26×10^{-4} N/mm
(C) 3.01×10^{-4} N/mm (D) 8.58×10^{-4} N/mm

10. A simply supported beam of span 10 m carries a load of 10 kN at a distance of 4 m from one end. The distance of maximum deflection from same end is [Take I = 5000 cm^4 and $E = 2 \times 10^5$ N/mm^2]
(A) 4.32 m (B) 4.71 m
(C) 4.96 m (D) 5.02 m

11. A cantilever beam of span 5 m is loaded by a weight W at the free end. The deflection at the free end was 1.5 cm. The slope at the free end in radians will be
(A) 0.45 (B) 0.045
(C) 0.65 (D) 0.065

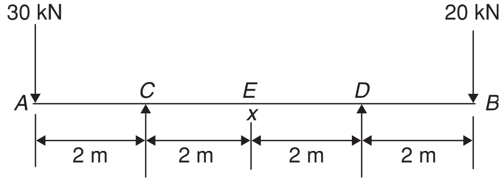
12. A cantilever beam of cross section 12 × 24 mm^2 and length 240 mm is having a load of 1 kN at the free end. A simply supported beam made of same material and having a cross section of 6 × 12 mm^2 with identical load and deflection will have a span of
(A) 200 mm (B) 240 mm
(C) 220 mm (D) 280 mm

13. A triangular-shaped cantilever beam is loaded as shown in the figure. The maximum deflection of the beam will be [Take Young's modulus = 1.2×10^5 N/mm^2



(A) 6.72 mm (B) 5.93 mm
(C) 8.16 mm (D) 7.33 mm

Direction for questions 14 and 15: For the double overhanging beam loaded as shown in the figure Young's modulus = 200 GPa and moment of inertia = 5×10^6 mm^4.



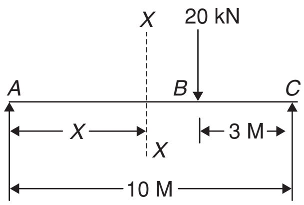
14. Deflection at point A relative to top of support is
(A) 29.33 mm up (B) 32.41 mm up
(C) 29.33 mm down (D) 32.41 mm down

15. Deflection at point E relative to top of support is
(A) 12 mm up (B) 10 mm up
(C) 12 mm down (D) 10 mm down

Practice Problems 2

Direction for questions 1 to 10: Select the correct alternative from the given choices.

1.



A simply supported beam of 10 m span is loaded as shown in the figure. The deflection at point B is $\left(E = 200 \text{ kN/mm}^2 I = 2\times10^8 \text{ mm}^4\right)$
(A) 6.87 mm (B) 9.32 mm
(C) 8.36 mm (D) 8.57 mm

Direction for questions 2 and 3: A simply supported beam is loaded by a couple M_1 as shown in figure. The beam is 3 m long and of square cross-section of 50 mm size.

2. Distance of the position of maximum deflection from left end is (given $E = 200$ GN/m^2)
(A) 3 m (B) $\sqrt{2}$ m
(C) $\sqrt{3}$ m (D) 2 m

3. If the maximum deflection is 6 mm allowable load M_1 is
(A) 1156 Nm (B) 1085 Nm
(C) 1022 Nm (D) 995 Nm

4. The deflection equation for a simply supported beam of 10 m span with a concentrated load is

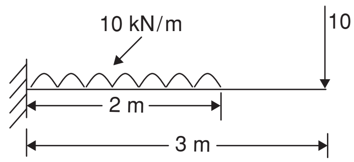
$$EIy = x^3 - \frac{5}{3}(x-4)^3 - 64x$$

The slopes at the ends of the beam are (Take EI = 10,000 kNm^2; Span = 10 m)
(A) −0.26°, 0.22° (B) −0.36°, 0.32°
(C) −0.32°, 0.41° (D) −0.28°, 0.26°

5. A cantilever beam 5 m long carries a UDL of 1.5 kN/m over its entire length. The free end of the cantilever is supported by a spring such that deflection at the free end is zero. The stiffness of the spring is
(Take $E = 2 \times 10^5$ N/mm^2; $I = 60 \times 10^6$ mm^4)
(A) 452 N/mm (B) 512 N/mm
(C) 405 N/mm (D) 384 N/mm

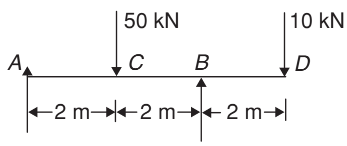
6. Half span of a simply supported beam of Length L is subjected to a uniformly distributed load of w/unit length. Deflection at the centre of the beam is,
(A) $\frac{5}{384}wL^4$ (B) $\frac{5}{768}wL^4$
(C) $\frac{7}{384}wL^4$ (D) $\frac{7}{768}wL^4$

7.



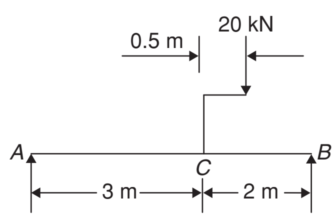
For the cantilever loaded as shown in the figure, the deflection at the end is (Flexural rigidity is given as 36,000 kNm^2)
(A) 4.325 mm (B) 4.675 mm
(C) 3.425 mm (D) 3.675 mm

8.



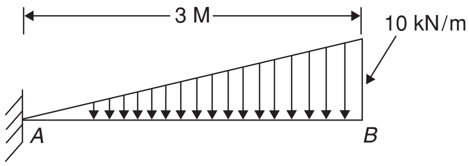
For the beam loaded as shown in the figure, with a flexural rigidity 36,000 kNm^2, the deflection at point C is, (Downward deflection +ve)
(A) −1.3 mm (B) −1.6 mm
(C) +1.3 mm (D) +1.6 mm

9.



Taking flexural rigidity 10,000 kN-m^2, the deflection of the beam shown in the figure at point C is
(A) −5.4 mm (B) −5.8 mm
(C) −4.4mm (D) −6.6 mm

10.

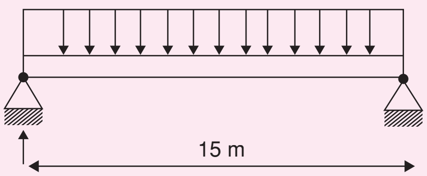


The deflection at point B for the beam shown in the figure, assuming a flexural rigidity of 10,000 kN-m^2 is

(A) 7.43 mm (B) 7.52 mm
(C) 8.43 mm (D) 8.52 mm

Previous Years' Questions

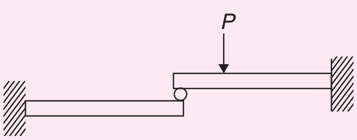
1. A steel beam of breadth 120 mm and height 750 mm is loaded as shown in the figure. Assume E_{steel} = 200 GPa.



The value of maximum deflection of the beam is **[2004]**

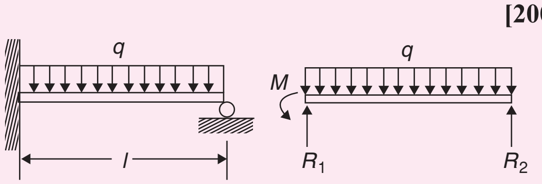
(A) 93.75 mm (B) 83.75 mm
(C) 73.75 mm (D) 63.75 mm

2. Two identical cantilever beams are supported as shown, with their free ends in contact through a rigid roller. After the load P is applied, the free ends will have **[2005]**



(A) equal deflections but not equal slopes.
(B) equal slopes but not equal deflections.
(C) equal slopes as well as equal deflections.
(D) neither equal slopes nor equal deflections.

3. A uniformly loaded propped cantilever beam and its free body diagram are shown below. The reactions are **[2007]**



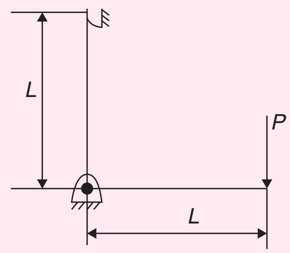
(A) $R_1 = \frac{5ql}{8}$, $R_2 = \frac{3ql}{8}$, $M = \frac{ql^2}{8}$

(B) $R_1 = \frac{3ql}{8}$, $R_2 = \frac{5ql}{8}$, $M = \frac{ql^2}{8}$

(C) $R_1 = \frac{5ql}{8}$, $R_2 = \frac{3ql}{8}$, $M = 0$

(D) $R_1 = \frac{3ql}{8}$, $R_2 = \frac{5ql}{8}$, $M = 0$

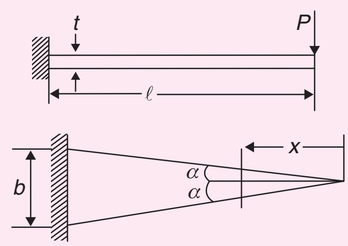
4. A frame of two arms of equal length L is shown in the adjacent figure. The flexural rigidity of each arm of the frame is EI. The vertical deflection at the point of application of load P is **[2009]**



(A) $\frac{PL^3}{3EI}$ (B) $\frac{2PL^3}{3EI}$

(C) $\frac{PL^3}{EI}$ (D) $\frac{4PL^3}{3EI}$

5. A triangular-shaped cantilever beam of uniform thickness is shown in the figure. The Young's modulus of the material of the beam is E. A concentrated load P is applied at the free end of the beam.



The area moment of inertia about the neutral axis of a cross-section at a distance x measured from the free end is **[2011]**

(A) $\frac{bxt^3}{6\ell}$ (B) $\frac{bxt^3}{12\ell}$

(C) $\frac{bxt^3}{24\ell}$ (D) $\frac{xt^3}{12}$

6. A cantilever beam of length L is subjected to a moment M at the free end. The moment of inertia of the beam cross section about neutral axis is I and the Young's modulus is E. The magnitude of the maximum deflection is **[2012]**

(A) $\frac{ML^2}{2EI}$ (B) $\frac{ML^2}{EI}$

(C) $\frac{2ML^2}{EI}$ (D) $\frac{4ML^2}{EI}$

7. The flexural rigidity (EI) of a cantilever beam is assumed to be constant over the length of the beam shown in the figure. If a load P and bending moment PL/2 are applied at the free end of the beam then the value of the slope at the free end is **[2014]**

(A) $\frac{1PL^2}{2EI}$ (B) $\frac{PL^2}{EI}$

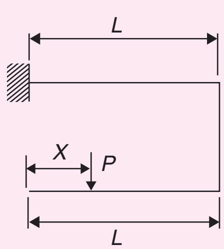
(C) $\frac{3PL^2}{2EI}$ (D) $\frac{5PL^2}{2EI}$

8. A cantilever beam of length, L, with uniform cross–section and flexural rigidity, EI, is loaded uniformly by a vertical load, w per unit length. The maximum vertical deflection of the beam is given by **[2014]**

(A) $\frac{wL^4}{8EI}$ (B) $\frac{wL^4}{16EI}$

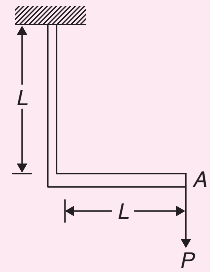
(C) $\frac{wL^4}{4EI}$ (D) $\frac{wL^4}{24EI}$

9. A force P is applied at a distance x from the end of the beam as shown in the figure. What would be the value of x so that the displacement at 'A' is equal to zero? **[2014]**



(A) $0.5L$ (B) $0.25L$
(C) $0.33L$ (D) $0.66L$

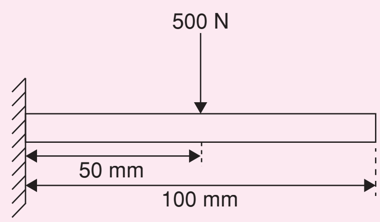
10. A frame is subjected to a load P as shown in the figure. The frame has a constant flexural rigidity EI. The effect of axial load is neglected. The deflection at point A due to the applied load P is **[2014]**



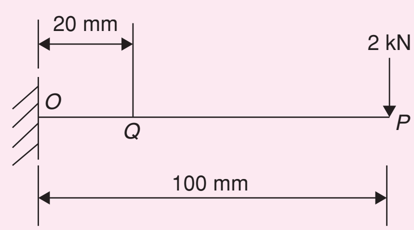
(A) $\frac{1}{3}\frac{PL^3}{EI}$ (B) $\frac{2}{3}\frac{PL^3}{EI}$

(C) $\frac{PL^3}{EI}$ (D) $\frac{4}{3}\frac{PL^3}{EI}$

11. A cantilever beam with flexural rigidity of 200 N.m^2 is loaded as shown in the figure. The deflection (in mm) at the tip of the beam is _____. **[2015]**



12. A cantilever beam with square cross-section of 6 mm side is subjected to a load of 2 kN normal to the top surface as shown in the figure. The Young's modulus of elasticity of the material of the beam is 210 GPa. The magnitude of slope (in radian) at Q(20 mm from the fixed end) is ______. **[2015]**



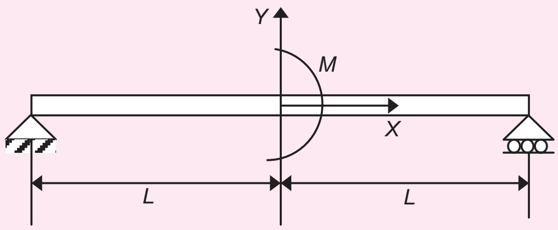
13. A cantilever beam having square cross-section of side a is subjected to an end load. If a is increased by 19%, the tip deflection decreases approximately by **[2016]**
(A) 19% (B) 29%
(C) 41% (D) 50%

Hence, the correct option is (D).

14. A simply supported beam of length $2L$ is subjected to a moment M at the mid-point $x = 0$ as shown in the figure. The deflection in the domain $0 \le x \le L$ is given by

$$W = \frac{-Mx}{12EIL}(L - x)(x + c)$$

where E is the Young's modulus, I is the area momentum of inertia and c is a constant (to be determined)



The slope at the center $x = 0$ is: **[2016]**

(A) $ML/(2EI)$
(B) $ML/(3EI)$
(C) $ML/(6EI)$
(D) $ML/(12EI)$

Answer Keys

Exercises

Practice Problems 1

1. D **2.** B **3.** A **4.** D **5.** C **6.** A **7.** C **8.** C **9.** A **10.** B
11. A **12.** B **13.** B **14.** C **15.** B

Practice Problems 2

1. D **2.** C **3.** B **4.** B **5.** D **6.** B **7.** C **8.** A **9.** C **10.** A

Previous Years' Questions

1. A **2.** A **3.** A **4.** D **5.** B **6.** A **7.** B **8.** A **9.** C **10.** D
11. 0.24 to 0.28 **12.** 0.15 to 0.17 **13.** D **14.** C