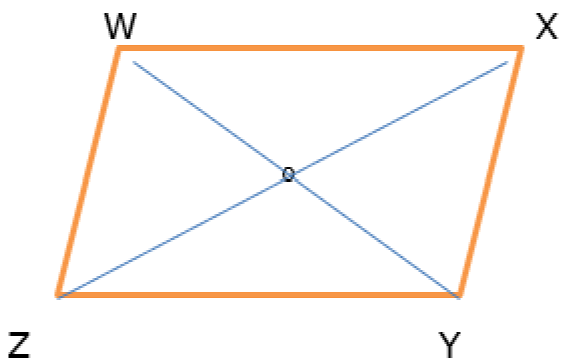
Quadrilaterals

Practice set 5.1

Q. 1. Diagonals of a parallelogram WXYZ intersect each other at point O. If ∠ XYZ = 135° then what is the measure of ∠XWZ and ∠YZW? If l(OY)= 5 cm then l(WY)=?

Answer :



Given ZX and WY are the diagonals of the parallelogram

∠ XYZ = 135° ⇒ ∠ XWZ = 135° as the opposite angels of a parallelogram are congruent.

∠YZW + ∠ XWZ = 180° as the adjacent angels of the parallelogram are supplementary.

⇒ ∠YZW = 180° - 135° = 45°

Length of OY = 5 cm then length of WY = WO + OY = 5+5 = 10 cm

(diagonals of the parallelogram bisect each other. So, O is midpoint of WY)

Q. 2. In a parallelogram ABCD, If ∠A = (3x +12)°, ∠B = (2x -32) ° then find the value of x and then find the measures of ∠C and ∠D.

Answer :

∠A = (3x +12)°

∠B = (2x -32) °

∠A + ∠B = 180° (supplementary angles of the ||gram)

(3x +12) + (2x -32) = 180

5x – 20 = 180°
5x = 200°

∴ x= 40°

∠A = (3 × 40) +12 = 120 + 12
= 132 ⇒ ∠C = 132° (opposite ∠s are congruent)

Similarly, ∠B = 2× 40 – 32

= 80 - 32°

= 48°

⇒ ∠D = 48°(opposite ∠s are congruent)

Q. 3. Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides.

Answer : perimeter of parallelogram = 150cm

Let the one side of parallelogram be x cm then

Acc. To the given condition

Other side is (x+25) cm

Perimeter of parallelogram = 2(a+b)

150 = 2(x+x+25)

150 = 2(2x+25)

$$\frac{75 - 25}{2} = x \Rightarrow 25$$

One side is 25cm and the other side is 50cm.

Q. 4. If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.

Answer : Given that the ratio of measures of two adjacent angles of a parallelogram= 1 : 2

If one ∠ is x other would be 180 – x as the adjacent ∠s of a parallelogram are supplementary.

$$\frac{x}{180 - x} = \frac{1}{2} \Rightarrow 2x = 180 - x \Rightarrow x = 60\,°$$

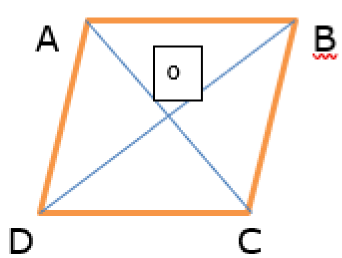
Other ∠ is 120° .

The measure of all the angles are 60 °, 120 °, 60 ° and 120 ° where 60 ° and 120 ° are adjacent ∠s and 60 ° and 60 ° are congruent opposite angles.

Q. 5. Diagonals of a parallelogram intersect each other at point O. If AO = 5, BO = 12 and AB = 13 then show that □ABCD is a rhombus.

Answer :

The figure is given below:



Given AO =5, BO = 12 and AB = 13

In Δ AOB, $AO^2 + BO^2 = AB^2$

∵ $5^2 + 12^2 = 13^2$

$25^2 + 144^2 = 169^2$

so by the Pythagoras theorem

Δ AOB is right angled at ∠ AOB.

But ∠ AOB + ∠ AOD forms a linear pair so the given parallelogram is rhombus whose diagonal bisects each other at 90°.

Q. 6. In the figure 5.12, □PQRS and □ABCR are two parallelograms. If ∠P = 110°then find the measures of all angles of □ABCR.

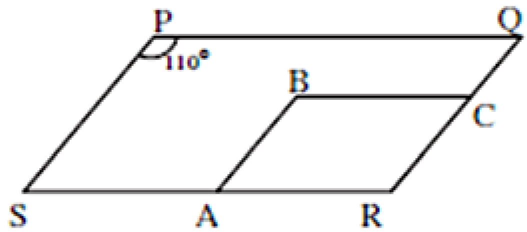


Fig. 5.12

Answer : given PQRS and ABCR are two ∥gram.

∠P = 110° ⇒ ∠R = 110°

(opposite ∠s of parallelogram are congruent)

Now if , ∠R = 110° ⇒ ∠ B = 110°

∠B + ∠A = 180°

(adjacent ∠s of a parallelogram are supplementary)

⇒ ∠A = 70° ⇒ ∠C = 70°

(opposite ∠s of parallelogram are congruent)

Q. 7. In figure 5.13 □ABCD is a parallelogram. Point E is on the ray AB such that BE = AB then prove that line ED bisects seg BC at point F.

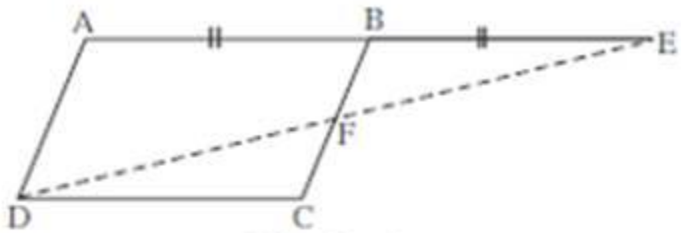


Fig. 5.13

Answer : Given, □ABCD is a parallelogram

And BE = AB

But AB = DC (opposite sides of the parallelogram are equal and parallel)

⇒ DC = BE

In Δ BEF and ∠DCF

∠DFC = ∠BFE (vertically opposite angles)

∠DFC = ∠ BFE (alternate ∠s on the transversal BC with AB and DC as ∥)

And BE = AB (given)

Δ BEF ≅ ∠DCF (by AAS criterion)

⇒ BF =FC (corresponding parts of the congruent triangles)

⇒ F is mid-point of the line BC. Hence proved.

Practice set 5.2

Q. 1. In figure 5.22, □ABCD is a parallelogram, P and Q are midpoints of side AB and DC respectively, then prove □APCQ is a parallelogram.

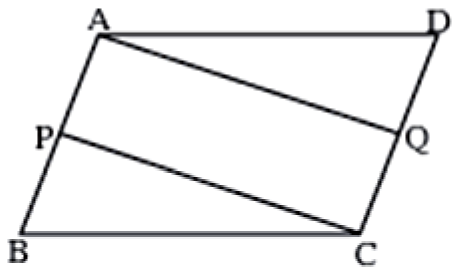


Fig. 5.22

Answer : Given AB ∥ to DC and AB = DC as ABCD is ∥gram.

⇒ AP ∥CQ (parts of ∥ sides are ∥) & 1/2 AB = 1/2 DC

⇒ AP = QC (P and Q are midpoint of AB and DC respectively)

⇒ AP = PB and DQ = QC

Hence APCQ is a parallelogram as the pair of opposite sides is = and ∥.

Q. 2. Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.

Answer : Opposite angle property of parallelogram says that the opposite angles of a parallelogram are congruent.

Given a rectangle which had at least one angle as 90°.

If ∠ A is 90° and AD = BC (opposite sides of rectangle are ∥ and =)

AB is transversal

⇒ ∠ A + ∠B = 180 (angles on the same side of transversal is 180°)

But ∠B + ∠C is 180 (AD ∥ BC, opposite sides of rectangle)

⇒ ∠ A = ∠C = 90°

Since opposite ∠s are equal this rectangle is a parallelogram too.

Q. 3. In figure 5.23, G is the point of concurrence of medians of ΔDEF. Take point H on ray DG such that D-G-H and DG = GH, then prove that □GEHF is a parallelogram.

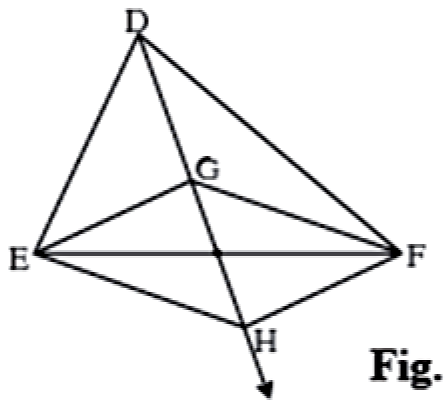
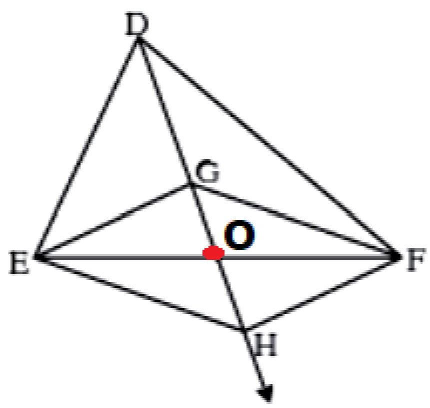


Fig. 5.23

Answer : Given G is the point of concurrence of medians of Δ DEF so the medians are divided in the ratio of 2:1 at the point of concurrence. Let O be the point of intersection of GH AND EF.

The figure is shown below:



⇒ DG = 2 GO

But DG = GH

⇒ 2 GO = GH

Also DO is the median for side EF.

⇒ EO = OF

Since the two diagonals bisects each other

⇒ GEHF is a ∥gram.

Q. 4. Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle. (Figure 5.24)

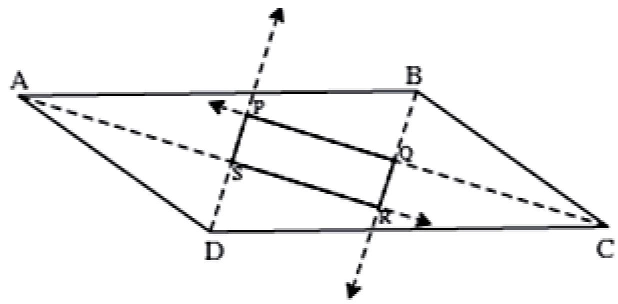


Fig. 5.24

Answer : Given ABCD is a parallelogram

AR bisects ∠BAD, DP bisects ∠ADC , CP bisects ∠BCD and BR bisects ∠CBA

∠BAD + ∠ABC = 180° (adjacent ∠s of parallelogram are supplementary)

But 1/2 ∠BAD = ∠BAR

1/2 ∠ABC = ∠RBA

∠BAR + ∠RBA = 1/2 × 180° = 90°

⇒ Δ ARB is right angled at ∠ R since its acute interior angles are complementary.

Similarly Δ DPC is right angled at ∠ P and

Also in Δ COB , ∠BOC = 90° ⇒ ∠POR = 90° (vertically opposite angles)

Similarly in ΔADS , ∠ASD = 90° = ∠PSR (vertically opposite angles)

Since vertically opposite angles are equal and measures 90° the quadrilateral is a rectangle.

Q. 5. **In figure 5.25, if points P, Q, R, S are on the sides of parallelogram such that AP = BQ = CR = DS then prove that □PQRS is a parallelogram.**

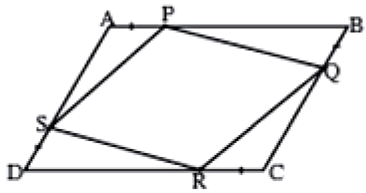


Fig. 5.25

Answer : Given ABCD is a parallelogram so

AD = BC and AD ∥BC

and DC = AB and DC ∥ AB

also AP = BQ = CR = DS

⇒ AS = CQ and PB = DR

in ΔAPS and Δ CRQ

∠ A = ∠C (opposite ∠s of a parallelogram are congruent)

AS = CQ

AP = CR

ΔAPS ≅ Δ CRQ(SAS congruence rule)

⇒ PS = RQ (c.p.c.t.)

Similarly PQ= SR

Since both the pair of opposite sides are equal

PQRS is ∥gram.

Practice set 5.3

Q. 1. Diagonals of a rectangle ABCD intersect at point O. If AC = 8 cm then find BO and if ∠CAD = 35° then find ∠ACB.

Answer : The diagonals of a rectangle are congruent to each other and bisects each other at the point of intersection so since AC = 8 cm

⇒ BD = 8 cm and

O is point of intersection so DO = OB = AO = OC = 4 cm

∠CAD = 35 ° given

⇒ ∠ACB = 35 °

(since AB ∥ DC and AC is transversal ∴ ∠CAD and ∠ACB are pair of alternate interior angle.)

Q. 2. In a rhombus PQRS if PQ = 7.5 then find QR. If ∠QPS = 75° then find the measure of ∠PQR and ∠SRQ.

Answer : Given quadrilateral is a rhombus.

⇒ all the sides are congruent /equal

⇒ PQ = QR = 7.5

Also ∠QPS = 75° (given)

⇒∠QPS = 75° (opposite angles are congruent)

But ∠QPS + ∠PQR = 180° (adjacent angles are supplementary)

⇒ ∠PQR = 105°

∴ ∠SRQ = 105° (opposite angles)

Q. 3

Diagonals of a square IJKL intersects at point M, Find the measures of ∠IMJ,∠JIK and ∠LJK.

Answer : The given quadrilateral is a square

⇒ all the angles are 90°

∴ ∠JIK = 90°

Since the diagonals are ⊥ to each other ∠IMJ = 90°

Since the diagonals os a square are bisectors of the angles also

∠LJK = ∠IJL = 1/2 × 90° = 45°

Q. 4. Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.

Answer : Let the diagonal AC = 20cm and BD = 21

$AB^2 = BO^2 + AO^2$

$AB^2 = (10.5)^2 + (10)^2$

(the diagonals of a rhombus bisect each other at 90°)

$AB^2 = 110.25 + 100$

AB = $\sqrt{210.25}$ = 14.5cm (side of the rhombus)

Perimeter = 4a = 14. 5 × 4 = 58cm

Q. 5. State with reasons whether the following statements are 'true' or 'false'.

(i) Every parallelogram is a rhombus.
(ii) Every rhombus is a rectangle.
(iii) Every rectangle is a parallelogram.
(iv) Every square is a rectangle.
(v) Every square is a rhombus.
(vi)Every parallelogram is a rectangle.

Answer : (i) False.

Explanation: Every Parallelogram cannot be the rhombus as the diagonals of a rhombus bisects each other at 90° but this is not the same with every parallelogram. Hence the statement if false.

(ii) False.

Explanation: In a rhombus all the sides are congruent but in a rectangle opposite sides are equal and parallel. Hence the given statement is false.

(iii) True.

Explanation: The statement is true as in a rectangle opposite angles and adjacent angles all are 90°. And for any quadrilateral to be parallelogram the opposites angles should be congruent.

(iv) True.

<u>Explanation:</u> Every square is a rectangle as all the angles of the square at $90°$, diagonal bisects each other and are congruent , pair of opposite sides are equal and parallel . Hence every square is a rectangle is true statement.

(v) True.

<u>Explanation:</u> The statement is true as all the test of properties of a rhombus are meet by square that is diagonals are perpendicular bisects each other , opposite sides are parallel to each other and the diagonals bisects the angles.

(vi) False.

<u>Explanation:</u>

Every parallelogram is a rectangle is not true as rectangle has each angle of $90°$ measure but same is not the case with every parallelogram.

Practice set 5.4

Q. 1. In □IJKL, side IJ || side KL $\angle I = 108°$ $\angle K = 53°$ then find the measures of $\angle J$and $\angle L$.

Answer : IJ ‖ KL and IL is transversal

$\angle I + \angle L = 180°$ (adjacent angles on the same side of the transversal)

$\Rightarrow \angle L = 180° – 108° = 72°$

Now again IJ ‖ KL and JK is transversal

$\angle J + \angle K= 180 °$ (adjacent angles on the same side of the transversal)

$\Rightarrow \angle K = 180° – 53° = 127°$

Q. 2. In □ABCD, side BC || side AD, side AB ≅ sided DC If $\angle A = 72°$ then find the measures of $\angle B$, and $\angle D$.

Answer : Given that BC ‖ AD and BC = AD (congruent)

$\Rightarrow$ the quadrilateral is a parallelogram (pair of opposite sides are equal and parallel)

$\angle A = 72°$

$\Rightarrow \angle C = 72°$ (opposite angles of parallelogram are congruent)

$\angle B = 180° – 72° = 108°$ (adjacent angles of a parallelogram are supplementary)

∠D = 108° (opposite angles of parallelogram are congruent)

Q. 3. In □ABCD, side BC < side AD (Figure 5.32) side BC || side AD and if side BE≅ side CD then prove that ∠ABC ≅ ∠DCB.

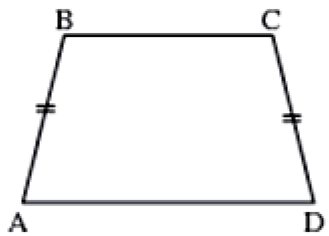


Fig. 5.32

Answer : The figure of the question is given below:

Construction: we will draw a segment ∥ to BA meeting BC in E through point D.

Given BC ∥ AD

And AB ∥ ED (construction)

⇒ AB = DE (distance between parallel lines is always same)

Hence ABDE is parallelogram

⇒ ∠ABE ≅ ∠DEC (corresponding angles on the same side of transversal)

And segBA ≅ seg DE (opposite sides of a ∥gram)

But given BA ≅ CD

So seg DE ≅ seg CD

⇒∠CED ≅ ∠DCE (∵ Δ CED is isosceles with CE = CD)

(Angle opposite to opposite sides are equal)

⇒ ∠ABC ≅ ∠DCB

Practice set 5.5

Q. 1. In figure 5.38, points X, Y, Z are the midpoints of side AB, side BC and side AC of ΔABC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find the length of XY, YZ, XZ.

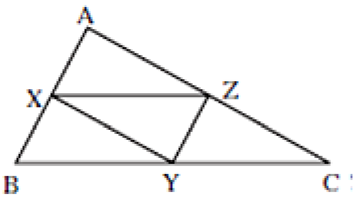


Fig. 5.38

Answer : Given X , Y and Z is the mid-point of AB, BC and AC.

Length of AB = 5 cm

So length of ZY = 1/2 × AB =1/2 × 5=2.5 cm (line joining mid-point of two sides of a triangle is parallel of the third side and is half of it)

Similarly, XZ = 1/2 × BC = 1/2 × 11= 5.5cm

Similarly, XY = 1/2 × AC = 1/2 × 9 = 4.5cm

Q. 2. In figure 5.39, □PQRS and □MNRL are rectangles. If point M is the midpoint of side PR then prove that,

i. SL = LR. Ii. LN = 1/2SQ.

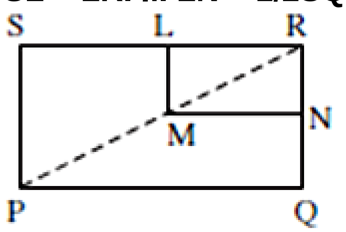


Fig. 5.39

Answer : The two rectangle PQRS and MNRL

In Δ PSR,

∠ PSR = ∠ MLR = 90°

∴ ML ∥ SP when SL is the transversal

M is the midpoint of PR (given)

By mid-point theorem a parallel line drawn from a mid-point of a side of a Δ meets at the Mid-point of the opposite side.

Hence L is the mid-point of SR

⇒ SL= LR

Similarly if we construct a line from L which is parallel to SR

This gives N is the midpoint of QR

Hence LN∥ SQ and L and N are mis points of SR and QR respectively

And LN = 1/2 SQ (mid-point theorem)

Q. 3. In figure 5.40, ΔABC is an equilateral triangle. Points F,D and E are midpoints of side AB, side BC, side AC respectively. Show that ΔEFD is an equilateral triangle.

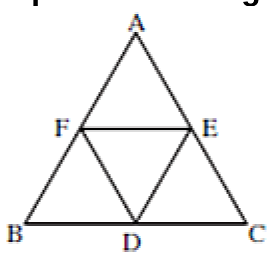


Fig. 5.40

Answer : Given F, D and E are mid-point of AB, BC and AC of the equilateral ΔABC ∴ AB =BC = AC

So by mid-point theorem

Line joining mid-points of two sides of a triangle is 1/2 of the parallel third side.

∴ FE = 1/2 BC =

Similarly, DE = 1/2 AB

And FD = 1/2 AC

But AB =BC = AC

⇒ 1/2 AB = 1/2 BC = 1/2 AC

⇒ DE = FD = FE

Since all the sides are equal ΔDEF is a equilateral triangle.

Q. 4. In figure 5.41, seg PD is a median of ΔPQR, Point T is the midpoint of seg PD. Produced QT intersects PR at M. Show that

$$\frac{PM}{PR} = \frac{1}{3}.$$

[Hint : draw DN || QM.]

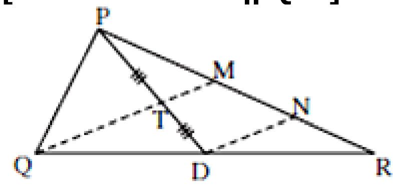


Fig. 5.41

Answer : PD is median so QD = DR (median divides the side opposite to vertex into equal halves)

T is mid-point of PD

⇒ PT = TD

In ΔPDN

T is mid-point and is ∥ to TM (by construction)

⇒TM is mid-point of PN

PM =MN……………….1

Similarly in ΔQMR

QM ∥ DN (construction)

D is mid –point of QR

⇒ MN = NR…………………..2

From 1 and 2

PM = MN = NR

Or PM = 1/3 PR

$$\Rightarrow \frac{PM}{PR} = \frac{1}{3}.\ \text{hence proved}$$

Problem set 5

Q. 1 A. Choose the correct alternative answer and fill in the blanks.

If all pairs of adjacent sides of a quadrilateral are congruent then it is called ....

A. rectangle
B. parallelogram
C. trapezium
D. rhombus

Answer : As per the properties of a rhombus:- A rhombus is a parallelogram in which adjacent sides are equal(congruent).

Q. 1 B. Choose the correct alternative answer and fill in the blanks.

If the diagonal of a square is 12√2 cm then the perimeter of square is ......

A. 24 cm
B. 24√2 cm
C. 48 cm
D. 48√2 cm

Answer : Here d= 12√2 = √2 s where s is side of square

Given diagonal = 20 cm

$$\Rightarrow s = \frac{12\sqrt{2}}{\sqrt{2}} = 12$$

Therefore, perimeter of the square is 4s = 4 x 12

= 48cm. (C)

Q. 1 C. Choose the correct alternative answer and fill in the blanks.

If opposite angles of a rhombus are (2x)° and (3x - 40)° then value of x is ....

A. 100°
B. 80°

C. 160°
D. 40°

Answer : As rhombus is a parallelogram with opposite angles equal

$\Rightarrow$ 2x = 3x -40

x= 40°

Q. 2. Adjacent sides of a rectangle are 7 cm and 24 cm. Find the length of its diagonal.

Answer : Adjacents sides are 7cm and 24 cm

In a rectangle angle between the adjacent sides is 90°

$\Rightarrow$ the diagonal is hypotenuse of right Δ

By pythagorus theorem

$Hypotenuse^2 = side^2 + side^2$

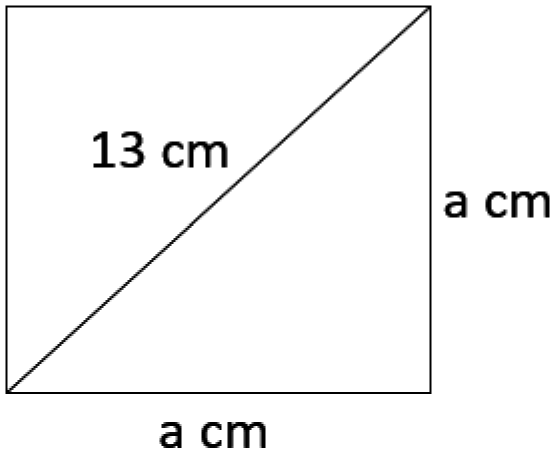
$$Hypotenuse^2 = 49 + 576 = \sqrt{625} = 25 \text{ cm}$$

length of the diagonal = 25cm

Q. 3. If diagonal of a square is 13 cm then find its side.

Answer : given Diagonal of the Square = 13cm

The angle between each side of the square is 90°



Using Pythagoras theorem

Hypotenuse^2 = side^2 + side^2

$$\Rightarrow 13^2 = a^2 + a^2$$

$$\Rightarrow 13^2 = 2a^2$$

$$\Rightarrow \frac{13^2}{2} = a^2$$

$$\Rightarrow \frac{13}{\sqrt{2}} = a$$

Side = $13/\sqrt{2}$ cm

Q. 4. Ratio of two adjacent sides of a parallelogram is 3 : 4, and its perimeter is 112 cm. Find the length of its each side.

Answer : In a parallelogram opposite sides are equal

Let the sides of parallelogram be x and y

$2x + 2y = 112$ and given $\frac{x}{y} = \frac{3}{4} \Rightarrow 4x = 3y$

$\Rightarrow 2\left(\frac{3y}{4}\right) + 2y = 112$

$\Rightarrow 7y = 224$

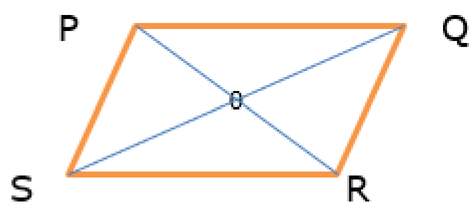
y= 32

x= 24

four sides of the parallelogram are 24cm , 32 cm, 24cm, 32cm.

Q. 5. Diagonals PR and QS of a rhombus PQRS are 20 cm and 48 cm respectively. Find the length of side PQ.

Answer : According to the properties of Rhombus diagonals of the rhombus bisects each other at 90°



In the rhombus PQRS

SO = OQ = 10 cm

PO= OR = 12cm

So in ΔPOQ

∠ POQ = 90°

⇒ PQ is hypotenuse

By Pythagoras theorem,

$10^2 + 24^2 = PQ^2$

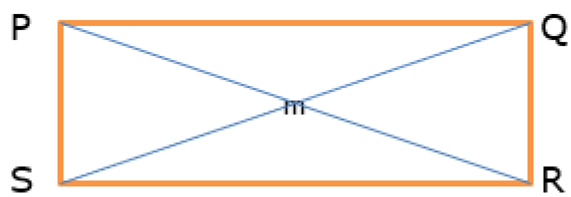
$100 + 576 = PPQ^2$

676 = PQ^2

26cm = PQ Ans

Q. 6. Diagonals of a rectangle PQRS are intersecting in point M. If ∠QMR = 50° then find the measure of ∠MPS.

Answer : The figure is given below:



Given PQRS is a rectangle

⇒ PS ∥ QR (opposite sides are equal and parallel)

QS and PR are transversal

So ∠ QMR = ∠ MPS (vertically opposite angles)

Given ∠ QMR = 50°

∴ ∠ MPS = 50°

Q. 7. In the adjacent Figure 5.42, if seg AB || seg PQ, seg AB ≅ seg PQ, seg AC || seg PR, seg AC ≅ seg PR then prove that, seg BC || seg QR and seg BC ≅ seg QR.

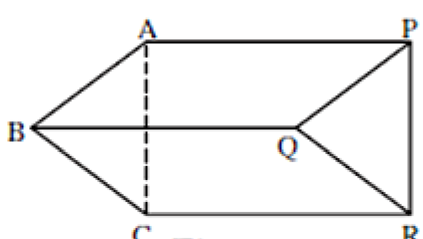


Fig. 5.42

Answer : Given

AB ∥ PQ

AB ≅ PQ (or AB = PQ)

⇒ ABPQ is a parallelogram (pair of opposite sides is equal and parallel)

⇒ AP ∥ BQ and AP ≅ BQ……………..1

Similarly given,

AC ∥ PR and AC ≅ PR

⇒ACPR is a parallelogram (pair of opposite sides is equal and parallel)

⇒ AP ∥ CR and AP ≅ CR ……………………2

From 1 and 2 we get

BQ ∥ CR and BQ ≅ CR

Hence BCRQ is a parallelogram with a pair of opposite sides equal and parallel.

Hence proved.

Q. 8. In the Figure 5.43, ABCD is a trapezium. AB || DC. Points P and Q are midpoints of seg AD and seg BC respectively.

Then prove that, PQ || AB and $PQ = \frac{1}{2}(AB + DC)$.

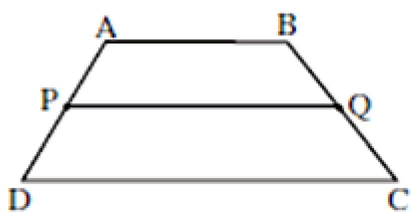


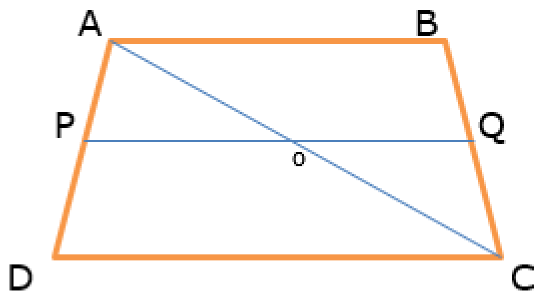
Fig. 5.43

Answer : Given AB ∥ DC

P and Q are mid points of AD and BC respectively.

Construction :- Join AC

The figure is given below:



In Δ ADC

P is mid point of AD and PQ is ‖ DC the part of PQ which is PO is also ‖ DC

By mid=point theorem

A line from the mid-point of a side of Δ parallel to third side, meets the other side in the mid-point

⇒ O is mid-point of AC

⇒ PO = 1/2 DC…………..1

Similarly in Δ ACB

Q id mid-point of BC and O is mid –point of AC

⇒ OQ‖ AB and OQ = 1/2 AB……………..2

Adding 1 and 2

PO + OQ = 1/2 (DC+ AB)

PQ = 1/2 (AB +DC)

And PQ ‖ AB

Hence proved.

Q. 9. In the adjacent figure 5.44, □ABCD is a trapezium. AB || DC. Points M and N are midpoints of diagonal AC and DB respectively then prove that MN || AB.

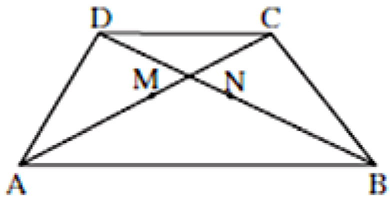


Fig. 5.44

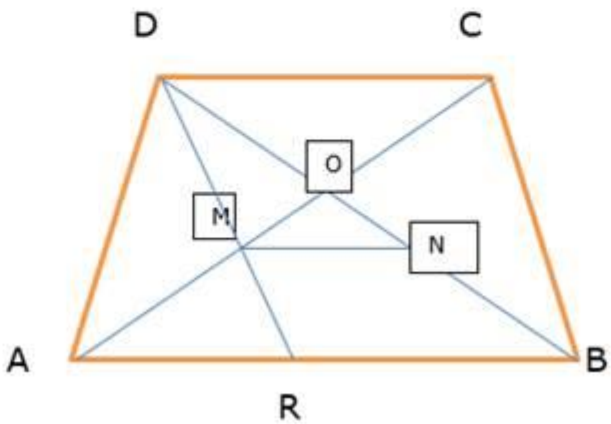
Answer : Given AB ∥ DC

M is mid-point of AC and N is mid-point of DB

Given ABCD is a trapezium with AB ∥ DC

P and Q are the mid-points of the diagonals AC and BD respectively

The figure is given below:



To Prove:- MN ∥ AB or DC and

In ΔAB

AB || CD and AC cuts them at A and C, then

∠1 = ∠2 (alternate angles)

Again, from ΔAMR and ΔDMC,

∠1 = ∠2 (alternate angles)

AM = CM (since M is the mid=point of AC)

$\angle 3 = \angle 4$ (vertically opposite angles)

From ASA congruent rule,

$\Delta AMR \cong \Delta DMC$

Then from CPCT,

AR = CD and MR = DM

Again in ΔDRB, M and N are the mid points of the sides DR and DB,

then PQ || RB

⇒ PQ || AB

⇒ PQ || AB and CD (∵ AB ∥ DC)

Hence proved.