Polynomials

IIT Foundation Material

SECTION - I Straight Objective Type

This section contains multiple choice questions. Each question has 4 choices (a),

(b), (c), (d), out of which ONLY ONE is correct. Choose the correct option.

1. If $(x-a)^2$ be a factor of $x^3 + px + q$ then (a) $p = 2a^2, q = 3a^2$ (b) $p = -3a^2, q = 2a^3$ (c) $p = 3a^2, q = 2a^3$ (d) None

2. The homogeneous function of the second degree in x and y having 2x - y as a factor, taking the value 2 when x = y = 1 and vanishing when x = -1 and y = 1 is

- (a) $2x^2 + xy y^2$ (b) $3x^2 - 2xy + y^2$ (c) $x^2 + xy - 2y^2$ (d) None
- **3.** The factor of the polynomial expressions $15-x-16x^2$ are (a) (3x+5) and (2x+3) (b) (5-3x) and (2x+3)

(c) (3-2x) and (3x+5) (d) None

4. The remainder when $7x^3 + 6x^2 - x + 1$ is divided by x - 2 is (a) 79 (b) 29 (c) 97 (d) None

5. If
$$f(x) = x^2$$
, $g(x) = x^3$ then the value of $\frac{f(b) - f(a)}{g(b) - g(a)}$

(a)
$$\frac{a-b}{a^2-ab+b^2}$$
 (b) $\frac{a+b}{a^2+ab+b^2}$

(c)
$$\frac{a+b}{a^2-ab+b^2}$$
 (d) None

6. The value of P for which
$$x-4$$
 may be a factor of $5x^3 - (P+4)x^2 - Px - (P+4)$ is
(a) 2 (b) 12 (c) 8 (d) None

7.	If $x - \frac{1}{x} = 7$ then the value of the $x^3 - \frac{1}{x^3}$ is				
	(a) 333	(b) 243	(c) 364	(d) None	
8.	If $x^2 - 3x + 2$ is a factor of the of <i>a</i> and <i>b</i> are (a) $a = -5, b = 4$		expression $x^4 + ax^2 + b$, then the value (b) $a = 4, b = -5$		
	(c) $a = 5, b = -4$		(d) None		
9.	If $a+b+c = (1-a)(1-b)(1-b)(1-b)$	(+6,bc+ca+ab) = (-c) is	=11, abc=6, th	hen the value of	
	(a) 1	(b) – 1	(c) 0	(d) None	
10.	If $x = \frac{a}{b+c}$, $y =$ is	$=\frac{b}{c+a}, z=\frac{c}{a+b}$, then the value	of $xy + yz + zx + 2xyz$	
	(a) 1	(b) 2	(c) 3	(d) None	
11.	One of the facto	ors of $8x^3 + 125y$	3		
	(a) $2x + 5y$	(b) $2x - 5y$	(c) $x - 2y$	(d) None	
12.	The remainder by $x-1$ is	when the polyno	mial $P(x) = x^4$ –	$-3x^2+2x+1$ is divided	
	(a) 2	(b) 1	(c) – 1	(d) 3	
13.	If the polynomi remainder wher (a) 2	ials $ax^3 + 4x^2 + 3$ in divided $x - 3$ b (b) 1	$3x - 4$ and $x^3 - 4$ y then the value of (c) - 1	4x+a leave the same of a is (d) 3	
14.	If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $x - 1$ and $x + 1$, the remainder are respectively 5 and 19. Then the remainder when $f(x)$ is divided by $x - 2$ is (a) 2 (b) 7 (c) 8 (d) 10				
15.	One of the factor (a) $x-3$	ors of the polynom $(b) x - 2$	nial $x^3 - 3x^2 + 4x^3$ (c) $x - 1$	x - 12 is (d) $x - 4$	

16. The value of K, if x + 3 is as factor of
$$3x^2 + K$$
 is
(a) -2 (b) 4 (c) 11 (d) 8
17. The value of a so that polynomial $x^3 + 10x^2 + ax + 6$ is exactly divisible
by x - 1 as well as x - 2 is
(a) -37 (b) 26 (c) 45 (d) 14
18. If $2x^3 + ax^2 + 11x + a + 3$ is exactly divisible by $2x - 1$ then the value of
a is
(a) 7 (b) -7 (c) 6 (d) 5
19. If both x - 2 and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$ then
(a) $p = r$ (b) $p = \frac{1}{2}$ (c) $p = -r$ (d) None
20. If $x^2 - 1$ is a factor $ax^4 + bx^3 + cx^2 + dx + e$ then
(a) $a + c = b + d$ (b) $a + c + e = b + d$
(c) $a = b, c = d$ (d) None
21. The remainder when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by
 $g(x) = 3x - 1$ is
(a) $\frac{3}{2}$ (b) 1 (c) $\frac{-107}{27}$ (d) 4
22. The polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ are divided by $x + 2$.
If the remainder in each case is the same, then the value of a is ?
(a) 1 (b) $\frac{5}{9}$ (c) $\frac{3}{4}$ (d) $\frac{7}{5}$
23. Let R_1 and R_2 be the remainders when the polynomials
 $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and
 $x - 2$ respectively. If $2R_1 + R_2 = 6$, then the value of a is
(a) 1 (b) 2 (c) 3 (d) 4
24. Common factor for the polynomials $x^{10} - 1$ and $x^{11} - 1$ is
(a) $x - 1$ (b) $x + 1$ (c) $x - 2$ (d) $x - 4$

25 .	The value of K if $x+3$ is factor of $3x^2 + kx + 6$ is				
	(a) 10	(b) 11	(c) 8	(d) 9	
26.	For what value $2x-1$	es of a is $2x^3$	$+ax^{2}+11x+a+$	3 exactly divisible by	
	(a) – 7	(b) 4	(c) 5	(d) 8	
27.	One of the factors of the polynomial $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2)$ is				
	(a) $a-b$	(b) $b - a$	(c) <i>b</i> −1	(d) <i>a</i> -1	
28.	If $x + a$ is a fact (a) 1	tor of $x^3 + ax^2 - (b) 0$	2x+a+4, then $a = (c) 2$	the value of a is (d) 4	
	(0) 1	(2) 0	(0) 2		
29.	$x + a$ is a factor of $x^n + a^n$ (a) $n \in N$		(b) for any odd positive integer		
	(c) for any even positive integer (d) None				
30.	One of the factors of $(x-b)^5 + (b-a)^5$ is				
	(a) <i>a</i> – <i>b</i>	(b) $x - b$	(c) $x - a$	(d) $a - x$	

SECTION - II Assertion - Reason Questions

This section contains certain number of questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT - 2 (Reason). Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct. Choose the correct option.

31. STATEMENT-1 : $x^2 + 2xy + 4y^2$ is one of the factors of $8x^3 + 125y^3$ **because**

STATEMENT-2 : $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a

correct explanation for Statement-1

- (c) Statement-1 is True, Statement-2 is False
- (d) Statement-1 is False, Statement-2 is True
- **32.** STATEMENT-1 : $\frac{x^2}{y^2} + 2 + \frac{y^2}{x^2} = \left(\frac{x}{y} + \frac{y}{x}\right)^2$

because

STATEMENT-2 : $(a+b)^2 = a^2 + b^2 + 2ab$

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

33. STATEMENT-1 : x- 3 is a factor of the polynomial $x^3 - 3x^2 + 4x - 12$ **because**

STATEMENT-2 : x - a is a factor of the polynomial P(x) then P(a) = 0

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(IB) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

34. STATEMENT-1 : x + 1 and 2x - 3 are factors of $2x^3 - 9x^2 + x + 12$ **because**

STATEMENT-2: a-b,b-c,c-a are the factors of a

 $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$

(a) Statement-1 is True, Statement-2 is True; Statement-2 18 a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

35. STATEMENT-1

 $x^{4} + 2x^{3} - 13x^{2} - 14x + 24 = (x-1)(x+2)(x-3)(x+4)$

because

STATEMENT-2: If sum of the coefficients of all the terms of a polynomial is zero then x - 1 is a factor

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

36. STATEMENT-1 : $3x^3 - 5x^2 + 8x + 9$ is a polynomial in x of degrees. **because**

STATEMENT-2 : The highest power of x in an algebraic expression is called the degree of the polynomial.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

- (c) Statement-1 is True, Statement-2 is False
- (d) Statement-1 is False, Statement-2 is True
- **37.** STATEMENT -1 : x+5 is a factor of x^2+6x+5 **because**
 - STATEMENT-2: A polynomial P(x), if g(x) devides P(x) exactly

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

- (c) Statement-1 is True, Statement-2 is False
- (d) Statement-1 is False, Statement-2 is True
- **38.** STATEMENT-1 : One of the factors of ax by + bx ay is a + b **because**

STATEMENT-2 : $a^2 - 2ab + b^2 = (a+b)^2$

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

:

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

39. STATEMENT-1

$$\frac{(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2)}{(a - b)^3 + (b - c)^3 + (c - a)^3} = (a + b)(b + c)(c + a)$$

because

STATEMENT-2 : One of the factors for $(x = 2y)^{3} + (2y - 3z)^{3} + (3z - x)^{3}$ is x - 2y (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

40. STATEMENT-1 : If a polynomial P(x) is divided by ax+b, the remainder is the

Because

STATEMENT-2 : (x-a)(x-b) is a factor of a polynomial P(x) if

P(a) = 0 and P(b) = 0

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is false

(d) Statement-1 is False, Statement-2 is True

SECTION - III

Linked Comprehension Type

This section contains paragraphs. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. Choose the correct option.

Paragraph for Question Nos. 41 to 43

If $ax^3 + bx^2 + x + b$ has x + 2 as a factor and leaved a remainder 4 when divides by x - 2 then

41.	The value of a is					
	(a) 1	(b) 0	(c) 2	(d) 3		
42 .	The value of b is					
	(a) 0	(b) 1	(c) 2	(d) 3		
40		(, (3, 1	2			

One of the factor of $ax^3 + bx^2 + x - 6$ is 43. (b) x-2 (c) x+3 (d) None (a) x + 1

Paragraph for Question Nos. 44 to 46

$$a^{2} + 2ab + b^{2} = (a+b)^{2}, a^{2} - 2ab + b^{2} = (a-b)^{2}$$

- a) Here the first and third terms are perfect squares.
- b) The middle term = 2 (Product of square roots of first and third term.

44. One of the factor of
$$\frac{x^2}{4y^2} - \frac{2}{3} + \frac{4y^2}{9x^2}$$
 is
(a) $\left(\frac{x}{2y} - \frac{2y}{3x}\right)^2$ (b) $\left(\frac{x}{2y} - \frac{y}{3x}\right)^2$
(c) $\left(\frac{x}{2y} + \frac{2y}{3x}\right)^2$ (d) None

45.

 $25(3x-4y)^2 - K(9x^2+16y^2) + 16(3x+4y)^2$ is a perfect square then the value of K is (b) 24 (c) 40 (d) 34 (a) 20

46. If $\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) + K$ is a perfect square, then the value of K is (a) 4 (b) 6 (c) 5 (d) 7

Paragraph for Question Nos. 47 to 49

Let f(x) be polynomial in x of degree not less than 1 and a be real number. If f(x) is divided by (x-a) then the remainder is f(a). If x-a is a factor of f(x) then f(a)=0.

- **47.** The remainder, when $f(x) = x^2 + 4x + 5$ is divided by x 5 is (a) 50 (b) 318 (c) 0 (d) 4
- **48.** The remainder, when $f(x) = x^3 + 5x 3$ is divided by 2x 1 is (a) 50 (b) 318 (c) 0 (d) 4
- **49.** The remainder when $(a-b)x^2 + (b-c)x + c a$ is divided by x-1(a) 50 (b) - 318 (c) 0 (d) 4

Paragraph for Question Nos. 50 to 52

Factorization of algebraic expression of the form $a^3 + b^3 + c^3$ when a + b + c = 0 $a^3 + b^3 + c^3 = 3abc$ if a + b + c = 0

Hence a, b, c are the factors of $a^3 + b^3 + c^3$ if a + b + c = 0

50. One of the factors of
$$(x - y)^3 + (y + z)^3 + (z - x)^3$$
 is
(a) $x - y$ (b) $x + y + z$ (c) $x - y + z$ (d) None

51. One of the factor of $P^3(q-r)^3(r-p)^3 + r^3(p-q)^3$ is (a) p (b) p-q-r (c) p+q+r (d) p-q+r

52. One of the factors of
$$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$$
 is
(a) $a - b$ (b) $a + b - c$ (c) $b - c + a$ (d) $b + c - a$

Paragraph for Question Nos. 53 to 55

 $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ $=a^{3}-b^{3}-3ab(a-b)$ If $\frac{x^2 - 1}{x} = 4$ then the value of $\frac{x^6 - 1}{x^3}$ is **53**. (a) 76 (b) 27(c) 0 (d) 36 The value of $216-144x+108x^2-27x^3$ when x=3**54**. (b) 27(d) 36 (a) 76 (c) 0The value of $(6a-5b)^3 - (3a-4b)^3 - 3(3a-b)(6a-5b)(3a-4b)$ when 55. 3a - b = 0 is (a) 76 (b) 27 (c) 0(d) 36

SECTION - IV

This section contains Matrix-Match type questions. Each question contains statements given in two columns which have to be matched. Statements (a, b, c, (d) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are a-p, a-s, b-q, b-r, c-p, c-q and d-s, then the correctly bubbled 4 x 4 matrix should be as follows:



Column I a) If $a - \frac{1}{a} = 3$ the value of $a^3 - \frac{1}{a^3}$ b) If $C - \frac{1}{C} = 5$ the value of $C^3 - \frac{1}{C^3}$ and $P^3 + 3P$ c) If $x - \frac{1}{x} = P$ the value of $x^3 - \frac{1}{x^3}$ b) 140

d) If
$$\frac{a^r - 1}{a}$$
 the value $\frac{a^6}{a^3}$ s) 4

56.

Column I

Column II p) $\frac{1+2x}{2+3x}$

a)
$$\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}} = \frac{4}{3}$$

b) If
$$\frac{1}{x + \frac{1}{1 + \frac{x+1}{2-x}}} = 1$$
 then x is q) 1

c) If
$$\frac{2x}{1 + \frac{1}{1 + \frac{x}{1 - x}}} = 1$$
 then x is r) $\frac{2}{3}$

d) If
$$1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{x}}} = \frac{13}{9}$$
 then \underline{x} is s) $\frac{1}{8}$

58. Column I

Column II

a)
$$\frac{bc}{(a-b)(a-c)} + \frac{ca}{(b-c)(b-a)} + \frac{ab}{(c-a)(c-b)}$$
 p) 1

b) The value of
$$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$$
 when $x = \frac{4ab}{a+b}$ q) $\frac{1}{2}$

c) If
$$x + y + z = 15$$
, $xy + yz + zx = 75$ r) 0

then the value of $x^3 + y^3 + z^3 - 3xyz$

d) If
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a+b+c}$$
 then $\frac{1}{a^7} + \frac{1}{b^7} + \frac{1}{c^7}$ is $s = \frac{1}{(a+b+c)^7}$

59. Column I

Column II

a) The remainder when $x^3 + x^2 - 2x + 1$ is p) - 318divided by x - 3b) The remainder when $4x^3 - 3x^2 + 2x - 1$ is q) 50divided by x + 2c) The remainder when $x^2 + 4x + 5$ is divided r) - 49by x - 5d) The remainder when $x^2 + 5x - 3$ is s) 31

a) The remainder when x + 5x - 5 is

divided 2x-1

60. Find the value if Column I

Column II

a) $a^2 + 2b + ab + 2a$ p) (a+b)(a+2)

b) $2b^2 + 8ab + 4ac + bc$

- q) (2b+c)(b+4a)
- *c*) 6*pm*+9*pm*+8*pm*+12*pm*
- d) $xy(z^2+1) + z(x^2 + y^2)$

- r) (2p+31)(3m+4n)
- s) (x + yz)(xz + y)