CENTRE OF MASS

- Point at which weight of the body is concentrated is called 'centre of gravity'.
- Point at which entire mass of the body appears to be concentrated is called 'centre of mass'.
- Centre of mass undergoes translatory motion, when a constant resultant force acts on the body and it represents the motion of the entire body.
- Centre of mass obeys newtons laws of motion At the centre of mass actual mass needn't be present.
- Internal forces can't produce acceleration in centre of mass.
- Location of centre of mass is independent of coordinate system.
- In uniform gravitational fields and for small bodies centre of mass and centre of gravity coincide.
- Coordinates of C.M. of a system of particles m(w, w, z) = m(w, w, z)

$$m_1(x_1y_1z_1), m_2(x_2y_2z_2)$$
.....

$$\mathbf{x}_{\rm cm} = \frac{\mathbf{m}_1 \mathbf{x}_1 + \mathbf{m}_2 \mathbf{x}_2 + \dots}{\mathbf{m}_1 + \mathbf{m}_2 + \dots}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \dots}{m_1 + m_2 + \dots}$$

$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + \dots}{m_1 + m_2 + \dots}$$

In vector notation

$$\bar{r}_{cm} = \frac{\sum \bar{m_1 r_1}}{m_1}$$
 where $r_1 = x_1 \hat{i} + y_1 \hat{i} + z_1 \hat{k}$

• Velocity of centre of mass is

$$\mathbf{v}_{CM} = \frac{\mathbf{m}_1 \mathbf{v}_1 + \mathbf{m}_2 \mathbf{v}_2 + \dots}{\mathbf{m}_1 + \mathbf{m}_2 + \dots}$$
$$= \frac{total \text{ momentum}}{\mathbf{m}_1 + \mathbf{m}_2 + \dots}$$

total mass

• Acceleration of centre of mass is

$$a_{CM} = \frac{m_1 a_1 + m_2 a_2 + \dots}{m_1 + m_2 + \dots} = \frac{net \ force}{total \ mass}$$

• To a system of particles $m_1(x_1y_1)$, $m_2(x_2y_2)$ another particle of mass m_3 is added so that centre of mass shifts to the origin then coordinates of third particle are

$$x_3 = -\frac{(m_1x_1 + m_2x_2)}{m_3}; y_3 = -\frac{(m_1y_1 + m_2y_2)}{m_3}$$

- To a circular disc of radius R_1 another of radius R_2 and of the same material is added then shift
 - in the CM is

$$\mathbf{x} = \frac{R_2^2 (R_1 + R_2)}{R_1^2 + R_2^2}$$

From a disc of radius R a disc of radius r is removed from one end. Then shift in its centre

of mass is
$$x = \frac{r^2 (R - r)}{R^2 - r^2}$$

• Out of a uniform solid sphere of radius R, if a sphere of radius r is removed, then the shift in

centre of mass is
$$\frac{r^3(R-r)}{(R^3-r^3)}$$

In case of two bodies, the ratio of distance of centre of mass from the bodies is in the inverse ratio of their masses. If m_1 and m_2 are masses of two bodies separated by a distance 'd' then

$$\mathbf{x}_1 = \frac{m_2 d}{m_1 + m_2}, \ \mathbf{x}_2 = \frac{m_1 d}{m_1 + m_2}$$
$$\mathbf{x}_1, \mathbf{x}_2 \text{ are the distances of CM, from } \mathbf{m}_1, \mathbf{m}_2$$

CONCEPTUAL QUESTIONS

- Centre of mass is a

 point 2.mass 3.vector 4.scalar
- 2. Centre of mass does not depend on
 - 1. masses of particles
 - 2. position of particles
 - 3. internal forces 4. external forces
- 3. The moon moves around the earth in circular orbit and earth moves around the sun in an elliptical orbit. The Centre of mass of earth-moon system moves around the sun in
 - 1. a circular orbit 2. an elliptical orbit
 - 3. hyperbolic path 4. parabolic path
- 4. A cricket ball is thrown upwards by a spin bowler

1. the path of the particle on the surface of the ball is a parabola

2. the path of the Centre of mass of the ball is a parabola

3. the path of a particle on the surface is complex one

4. 2 and 3 are correct

- 5. When a force is applied on a body, Newton's second law is applicable to
 - 1. Centre of mass 2. any part of the body
 - 3. centre of gravity 4. None
- 6. Centre of mass of the earth-moon system lies
 1. on the surface of earth
 - 2. on the surface of the moon
 - 3. within the earth
 - 4. at the mid-point of the line joining their centres
- 7. A wooden sphere and a copper sphere of same radius are kept in contact with each other. Their centre of mass will be
 - 1. at their point of contact
 - 2. within the wooden sphere
 - 3. within the copper sphere
 - 4. outside the spheres
- 8. There are five bodies with their centre of mass lying on x-axis. Then their combined centre of mass
 - 1. lies only y-axis 2. lies on z-axis
 - 3. lies on x-axis 4. may be anywhere
- 9. Three identical spheres each of radius r are placed touching each other on a horizontal table. The centre of mass of the system is located.

1. at one of the centres of the spheres

2. at the mid point joining the medians of any two spheres

3. at the point of intersection of the medians of the triangle formed by the centres of the spheres 4. at the mid point of a median of the triangle formed by the centres of the two spheres.

- 10. Two balls are thrown at the same time in air, while they are in air, the acceleration of their centre of mass
 - 1. depends on masses of the balls
 - 2. depends on the direction of motion of the balls
 - 3. depends on speeds of the balls

4. is equal to acceleration due to gravity

11. A cylinder is completely filled with water. If $\frac{1}{4}$ th of the vielume of water leads out, its control of mass

the volume of waterleaks out, its centre of mass1. moves up2. moves down

3. does not change 4.moves towards one wall

- 12. The Na⁺ and Cl⁻ ions which are initially at certain distance apart start moving towards each other due to mutual attraction. As they are moving nearer, their centre of mass
 - shifts nearer to Na⁺ 2. shifts nearer to Cl⁻
 remains in the same position 4. none
- 13. Consider a two particle system with the particles having masses m₁ and m₂. If the first particle is pushed towards the centre of mass

through a distance 'd', the distance by which the second one should be pushed towards the centre of mass to keep the centre of mass at the same position is

1. d 2.
$$\frac{m_1 d}{m_2}$$
 3. $\frac{m_2 d}{m_1}$ 4. $\frac{m_2 d}{m_1 + m_2}$

		<u>KEY</u>		
1.1	2.3	3.4	4.4	5.1
6.3	7.3	8.3	9.3	10.4
11.2	12.3	13.4		

LEVEL - I

- 1. 10gm., 30gm and 60gm are at 20cm, 40cm and 80cm on light metre scale. The position of the centre of mass of the system is
 - 1. 0 2. 62cm 3. 74 cm 4. 100cm
- 2. A system consists of two masses connected by a massless rod that lies along x-axis. The distance of centre of mass from O is

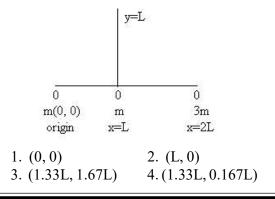
$$m_1 = 0.4 \text{ kg}$$
 $m_2 = 0.6 \text{ kg}$
O X X X X
 $x_1 = 2m$ $x_2 = 7m$

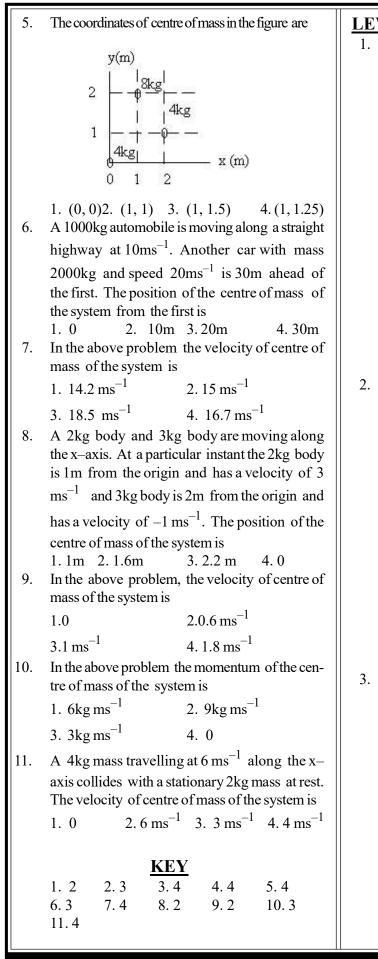
$$3 \text{ kg} = 4 \text{ kg} = 0 \text{ kg}$$

 $0 = 0 = 0$
 $0.3 \text{m} = 1 \text{m}$

1. 0.21m 2.0.35m 3.0.42m 4.0.48 m

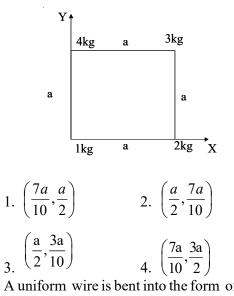
4. Coordinates of centre of mass of the following system are



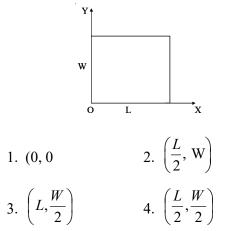


<u>LEVEL – II</u>

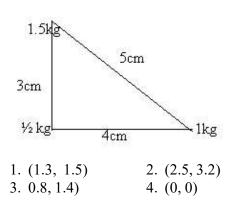
1. Four bodies of masses 1, 2, 3, 4 kg respectively are placed at the comers of a square of side 'a'. Coordinates of centre of mass are

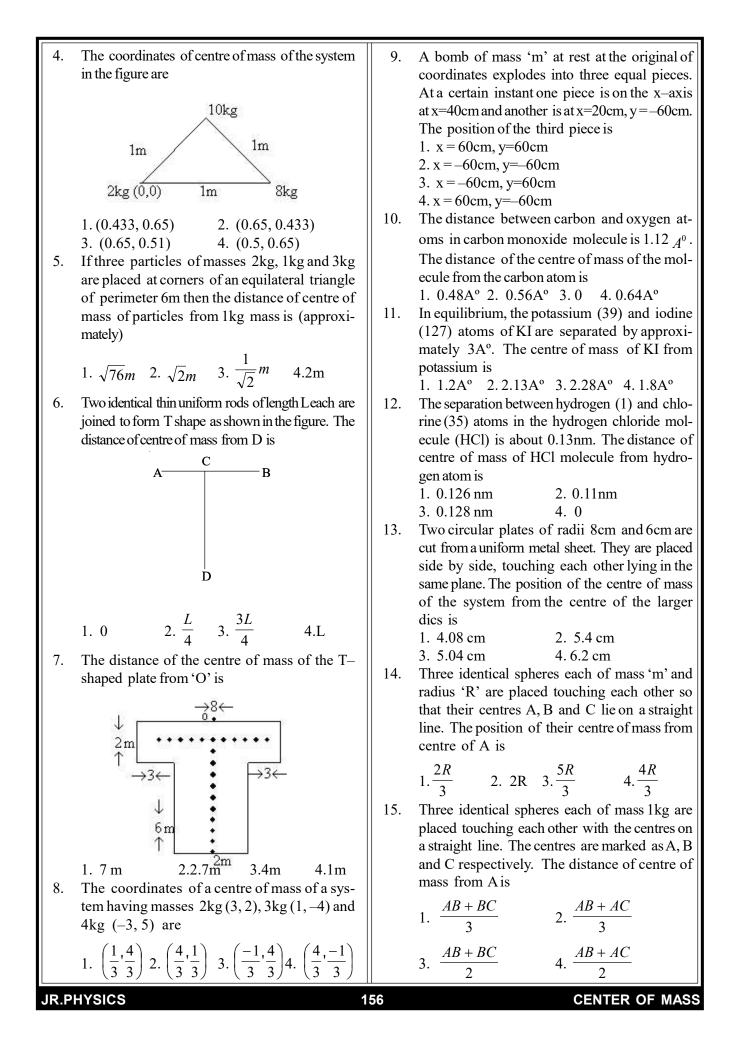


2. A uniform wire is bent into the form of a rectangle of length L and width W. The coordinates of its centre of mass are



3. The coordinates of centre of mass of the system in the figure are





17. Two charged particles of masses m and 2m are placed at a distance d apart on a smooth horizontal table. The position of centre of mass from lighter one is

1. 0 2.
$$\frac{d}{3}$$
 3. $\frac{2d}{3}$ 4. d

18. Two charged particles of masses m and 2m are placed at a distance d apart on a smooth horizontal table. Because of their mutual attraction, they move towards each other and collide. Where will the collision occur from lighter one?

1. 0 2.
$$\frac{d}{3}$$
 3. $\frac{2d}{3}$ 4. d

19. Two boats of masses 400kg and 500kg are connected by a long rope. A boy of mass 50kg in the first boat pulls other boat by means of a rope with a force 950N. With what acceleration does the centre of mass of the system of boats move?

1. 0 2.
$$1 \text{ m/s}^2$$
 3. 2 m/s^2 4. 0.5 m/s²

4. 2a

20. In a system of two identical particles, one of the particles is at rest and the other has an acceleration a, then the centre of mass has an acceleration

- 21. A boy of mass 50kg is standing at one end of a boat of length 9m and mass 400kg. He runs to the other end. The distance through which the centre of mass of the boat boy system moves is
 1. 0
 2. 1m
 3. 2m
 4. 3m
- 22. A boy of mass 50kg is standing at one end of a boat of length 9m and mass 400kg. He runs to the other end. The distance through which the boat moves back is
 - 1. 0 2. 1m 3. 2m 4. 3m
- 23. A boat of mass 60kg is floating in still water. A boy of mass 20kg walks from one end to the other end. If the length of the boat is 3m, the distance through which the boat moves is
 1. 1m 2.0.75m 3.0.5m 4.0.9m

- 24. Two particles A and B of masses 2kg and 3kg are at rest separated by 10m. If they move towards each other under a mutual force of attraction, the distance of the point from the initial position of A where they meet is
 1.5m
 2.4m
 3.6m
 4.8m
- 25. Two particles having masses m and 2m are travelling along x-axis on a smooth surface with velocities u_1 and u_2 collide. If their velocities after collision are v_1 and v_2 , then the ratio velocities of their centre of mass before and after collision is

- 26. Two particles of masses 4kg and 6kg are at rest separated by 20m. If they move towards each other under mutual force of attraction, the position of the point where they meet is 1. 12m from 4kg body
 - 2. 12m from 6kg body
 - 3. 8m from 4kg body
 - 4. 10m from 4kg body

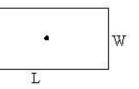
KEY

1.2	2.4	3.1	4.2	5.2
6.3	7.2	8.3	9.3	10.4
11.3	12.1	13.3	14.2	15.2
16.2	17.3	18.3	19.1	20.2
21.1	22.2	23.2	24.3	25.3
26.1				

1. The coordinates of masses are 1 (0,0), 2(a, 0) 3(a, a) and 4 (0, a).

$$\mathbf{X}_{cm} = \frac{\sum m_i \mathbf{X}_i}{\sum m_i}; \mathbf{Y}_{cm} = \frac{\sum m_i \mathbf{y}_i}{\sum m_i}$$

2. Coordinate of centre of mass are $\left(\frac{L}{2}, \frac{W}{2}\right)$



3. The coordinates are $\frac{1}{2}(0,0)$, 1(4,0) and 1.5

0,3)
$$X_{cm} = \frac{\sum m_i x_i}{\sum m_i}; Y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$$

4. See problem 3.

5. Take 1kg mass at the origin, and calculate the coordinates of c.m. and find the distance

c.m. is located at a distance $\frac{L}{A}$ from c 6. distance from $D = \frac{L}{4} + \frac{L}{2} = \frac{3L}{4}$ 7. Moment are equal about c.m., and mass is proportional to area $\therefore m_1 x_1 = m_2 x_2$ $8 \times 2x = 6 \times 2(4-x)$ (or) $x = \frac{12}{7}$:. **c.m.** from $O = \frac{12}{7} + 1 = \frac{19}{7} m$ $X_{cm} = \frac{\sum m_i x_i}{\sum m_i}; Y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$ 8. $\vec{r_1} = 40 \vec{i}$ 9. $\vec{r}_{2} = 20 \vec{i} - 60 \vec{j}; \qquad \vec{r}_{3} = ?$ $\overrightarrow{r_1} + \overrightarrow{r_2} + \overrightarrow{r_3} = 0$ $\overrightarrow{60} \overrightarrow{i} - \overrightarrow{60} \overrightarrow{j} + \overrightarrow{r_3} = 0$ (or) $\overrightarrow{r_3} = -\overrightarrow{60} \overrightarrow{i} + \overrightarrow{60} \overrightarrow{j}$ \therefore x = -60cm; y = 60cm 10. $X_c = \frac{\mathbf{m}_o}{\mathbf{m}_c + \mathbf{m}_o} x_w$ 11. See problem 10 12. See problem 10 13. $\mathbf{x} = \frac{r_2^2}{r_1^2 + r_2^2} (\mathbf{r}_1 + \mathbf{r}_2)$ 14. $X = \frac{AB + AC}{3} = \frac{2R + 4R}{3} = \frac{6R}{3} = 2R$ 15. See problem 16. $\vec{r_{cm}} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$ 17. See problem 10 18. Collision occurs at c.m. 19. Position of c.m. remains stationary $a_{\rm cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$ 20. 21. **c.m.** of the system remains stationary 22. (400 + 50)d = 50x923. See problem 22

- 24. Equal to the distance of **c.m.**
- 25. The velocity of **c.m.** of particles in a closed system does not depend upon individual velocities of particles in a collision

26.
$$x = x = \frac{6}{4+6}x^{20}$$
 $\left(x = \frac{m_2}{m_1 + m_2} \cdot AB\right)$

1. A circular disc of radius 20cm is cut from one edge of a larger circular disc of radius 50cm. The shift of centre of mass is

1. 5.7 cm 2. (-5.7 cm) 3. 3.2 cm 4. -3.2 cm

- 2. A circular plate has a uniform thickness and has a diameter 56cm. A circular disc of diameter 42cm is removed from one edge of the plate. The distance of the centre of mass of the remaining portion from the centre of mass of the plate is
- 1. 18 cm
 2. 9cm
 3. 27cm
 4. 4.5 cm
 A uniform disc of radius R is put over another uniform disc of radius 2R of same thickness and density. The peripheries of the two discs touch each other. The position of their centre of mass is

1. at $\frac{R}{3}$ from the centre of the bigger disc towards the centre of the smaller disc

2. at $\frac{R}{5}$ from the centre of the bigger disc to-

wards the centre of the smaller disc

3. at $\frac{2R}{5}$ from the centre of the bigger disc

towards the centre of the smaller disc 4. None

4. From a sphere of radius 1m, a sphere of radius 0.5m is removed from the edge. The position of centre of mass of the remaining portion from centre of the sphere is

1.
$$\frac{13}{6}m$$
 2. $\frac{16}{13}m$ 3. $\frac{14}{13}m$ 4. $\frac{1}{14}m$

5. From a uniform circular plate of radius R, a circular plate of diameter R is removed such

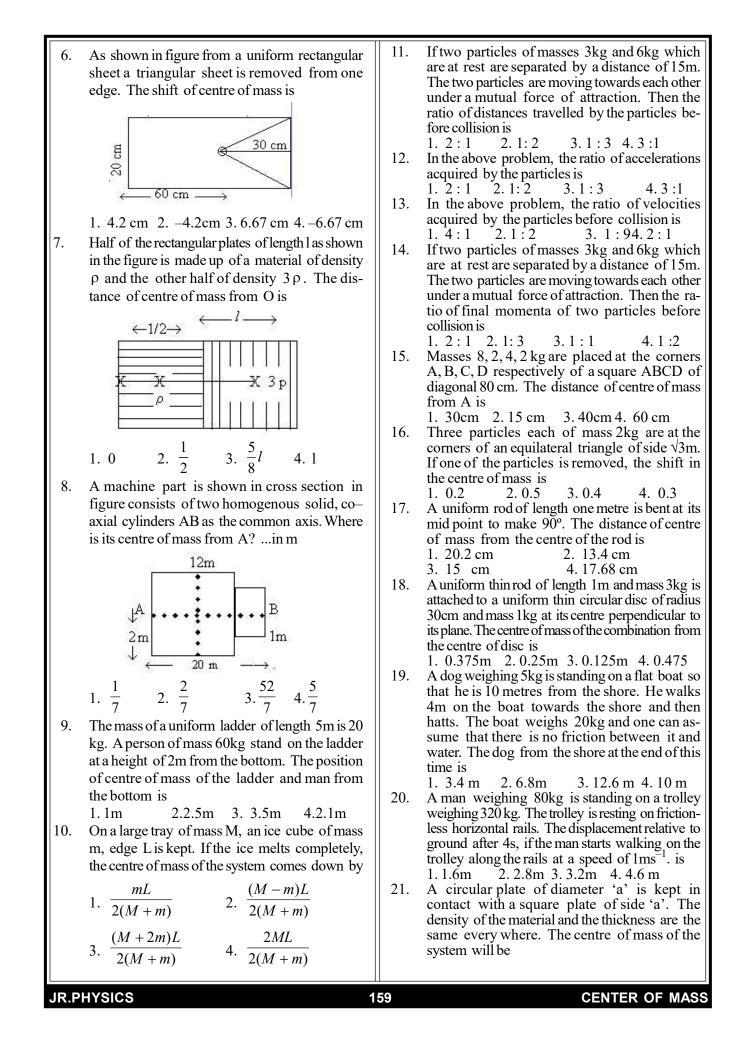
that the centre of the hole is at a distance $\frac{R}{3}$

from original centre. Shift in centre of mass is

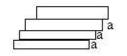
$$\frac{R}{6}$$
 2. $\frac{R}{14}$ 3. $\frac{R}{9}$ 4. $\frac{R}{3}$

JR.PHYSICS

1.



- 1. at the point of contact of the bodies
- 2. inside the circular plate
- 3. inside the square plate 4. outside the system
- 22. Four identical planks each of lengths 'L' are arranged one above the other over a table as shown. Each projects a distance 'a' beyond the edge of the one that is below it. What is the maximum possible value of 'a' for the system to be in equilibrium without tripping forward?



1. L/5 2. L/4 3. L/3 4. L

23. Two masses 'm₁' and 'm₂' (m₁>m₂) are connected to the ends of a light inextensible string which passes over the surface of a smooth fixed pulley. If the system is released from rest, the acceleration of the centre of mass of the system will be (g=acceleration due to gravity)

1.
$$\frac{g(m_1 - m_2)}{(m_1 + m_2)}$$
 2. $\frac{g(m_1 - m_2)^2}{(m_1 + m_2)^2}$

3.
$$\frac{g(m_1 + m_2)}{(m_1 - m_2)}$$
 4. $\frac{g(m_1 + m_2)}{(m_1 - m_2)}$

24. Three identical particles each of mass 0.1kg are arranged at three corners of a square of side $\sqrt{2}$ m. The distance of the centre of mass from the fourth corner is

1. 2/3 m 2. 4/3 m 3. 1m 4.8/3m

- 25. ABCD is a square of side "L". Two masses each of 1kg are placed at the corners C and D. Where should a mass of 2kg be placed, so that the centre of mass of the system coincides with the centre of the square?1. at the corner A 2. at the corner B
 - 3. at the mid point of AB
- 4. at the mid point of AC
 26. Two identical particles each of mass 'm' are projected from points A and B on the ground with same initial speed 'u' making an angle 'θ' as shown in the figure, such that their trajecto-

velocity of the centre of mass is



1. $u \cos \theta$ 2. $2u \cos \theta 3$. $u \sin \theta$ 4. $2u \sin \theta$

ries are in the same vertical plane. The initial

27. Figure shows a square plate of uniform thickness and side length $\sqrt{2}$ m. One fourth of the plate is removed as indicated. The distance of centre of mass of the remaining portion from the centre of the original square plate is



$$1. \ 1/3m \qquad 2. \ 1/2m \qquad 3.1/6m \qquad 4. \ 1/8m$$

- 28. Two spheres made of copper have radii 'r' and '2r'. They are arranged to touch each other. The distance of the centre of mass of the system from the point of contact of the two spheres is 1. 3r/2 2. 5r/3 3. 8r/3 4. 2r
- 29. Two particles of masses "p" and "q" (p>q) are separated by a distance "d". The shift in the centre of mass when the two particles are interchanged is

$$\begin{array}{rl} 1. \ d(p+q) \, / \, (p-q) & 2. \ d(p-q) \, / \, (p+q) \\ 3. \ d \ p / (p-q) & 4. \ d \ q / \, (p-q) \end{array}$$

30. A force of 15N is acting on a particle of mass 3kg towards north and a force of 20N is acting on another particle of mass 2kg towards east. The acceleration of the centre of mass of the two particle system is

1. 7 m/s^2 2. 5 m/s^2 3.1 m/s² 4.Zero

- 31. Six identical particles each of mass ½ kg are arranged at the corners of a regular hexagon of side length ½m. If one of the particles is removed, the shift in the centre of mass is
 1. 1/10m 2. 1/12m 3.1/14m 4.1/8m
- 32. Two particles of masses 1kg and 2kg are separated by a distance of 60cm. If the 1kg particle moves towards the 2kg particle over a distance of 15cm, the shift in the centre of mass will be
 1. 5cm 2. 10cm 3. 7.5 cm 4. 15cm
- 33. A uniform hemisphere has radius 'R'. The centre of mass lies on the line perpendicular to its base. The height of the centre of mass above the centre of the base is

1. 3 R/8 2. R/2 3.5R/8 4. 3R/4

34. Two bodies of masses (M+m) and (M-m) are connected to the ends of a light inextensible string and the string is laid over a smooth fixed pulley. The system is released from rest. The velocity of the centre of mass of the two body system 't' seconds after the release is

$$1.gt \frac{m}{M} = 2.2gt \frac{m^2}{M^2} \ 3.gt \frac{m^2}{M^2} \ 4.$$
 Zero

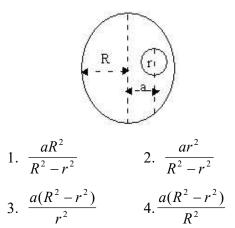
CENTER OF MASS

35. Two bodies of masses ' m_1 ' and ' m_2 ' are moving with velocity ' v_1 ' and ' v_2 ' respectively in the same direction. The total momentum of the system in the frame of reference attached to the centre of mass is (v is relative velocity between the masses)

1.
$$\frac{m_1 m_2 v}{m_1 - m_2}$$

2. $\frac{2 m_1 m_2 v}{m_1 + m_2}$
3. zero
4. $\frac{4 m_1 m_2 v}{m_1 + m_2}$

36. A circular hole of radius 'r' is made in a disk of radius 'R' and of uniform thickness at a distance 'a' from the centre of the disk. The distance of the new centre of mass from the original centre of mass is



- 37. A smooth string passes over a fixed light pulley and masses 1kg and 2kg are tied to the two ends of the string. The system is held at rest and then released. The centre of mass of the system 1. remains in the same position
 - 2. moves down 3. moves up
 - 4. first moves up and then moves down
- 38. Four identical particles each of mass 1kg are arranged at the corners of a square of side length $2\sqrt{2}$ m. If one of the particles is removed, find the shift in the centre of masses? 1. 8/3 m 2. 4/3m 3. 2/3 m 4. 2m
- 39. Two particles of masses 0.2kg and 0.3kg are initially at a distance 1.5m apart. The 0.2 kg mass is moved towards the 0.3kg mass over a distance of 30cm. By how much distance the 0.3kg mass is to be moved towards the 0.2kg mass, if the centre of mass of the system is to remain at the original position?
 1. 0.6 2. 0.72 3. 0.12 4. 0.36

- 40. Two particles of masses 2kg and 3kg are at a distance of 5m apart. If half of the mass of the lighter particle is removed from it and added to the heavier mass, the shift in the centre of mass 1.3 m
 2.4m
 3.1m
 4.'0'
- 41. Four identical particles each of mass 'm' are arranged at the corners of a square of side length "*l*". If the masses of the particles at the end of a side are doubled, the shift in the centre of mass of the system?

1.
$$\frac{l}{4\sqrt{2}}$$
 2. $\frac{l}{6\sqrt{2}}$ 3. $\frac{l}{\sqrt{2}}$ 4. $\frac{l}{5\sqrt{2}}$

42. Six identical particles each of mass 'm' are arranged at the corners of a regular hexagon of side length "L". If the mass of one of the particles is doubled, the shift in the centre of mass?

1. L 2.
$$6L/7$$
 3. $L/7$ 4. $L/\sqrt{3}$

43. A bomb shell of mass 3kg is moving with a velocity of 10m/s on the ground. Suddenly it explodes in to two pieces of masses 1kg and 2kg. The 2kg mass continues to move in the same direction with a velocity of 8 m/s. The velocity of the centre of mass of the bomb after the explosion?

1. 8 m/s 2. 16 m/s 3. 10 m/s 4. 2 m/s

44. Three particles each of mass 'm' are arranged at the corners of an equilateral triangle of side "L". If one of the masses is doubled, the shift in the centre of mass of the system.

1.
$$\frac{L}{\sqrt{3}}$$
 2. $\frac{L}{4\sqrt{3}}$ 3. $\frac{\sqrt{3}L}{4}$ 4. $\frac{L}{2\sqrt{3}}$

45. Four identical particles each of mass "m" are arranged at the corners of a square of side length "L". If the masses of the particles at the end of a diagonal are doubled the shift in the centre of mass of the system?

1.
$$\frac{L}{\sqrt{2}}$$
 2. $\frac{L}{2\sqrt{2}}$ 3. '0' 4. $\frac{L}{3}$

46. Four identical particles each of mass "m" are arranged at the corners of a square of side of side length "L". If one of the masses is doubled, the shift in the centre of mass of the system. w.r.t. diagonally opposite mass

1.
$$\frac{L}{\sqrt{2}}$$
 2. $\frac{3\sqrt{2}L}{5}$ 3. $\frac{1}{4\sqrt{2}}$ 4. $\frac{1}{5\sqrt{2}}$

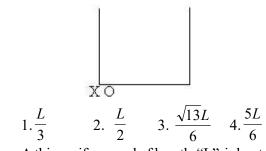
47. Three identical particles each of mass "m" are arranged at the corners of an equilateral triangle of side length "L". If two of the particles are replaced by particles of double the mass, the shift in the centre of mass of the system is

$$1.\frac{L}{2\sqrt{3}}$$
 $2.\frac{L}{5\sqrt{3}}$ $3.\frac{L}{3\sqrt{3}}$ $4.\frac{L}{4\sqrt{3}}$

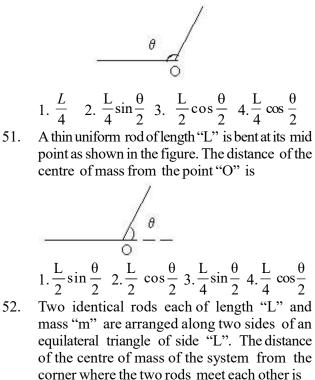
48. Six identical particles each of mass "m" are arranged at the corners of a regular hexagon of side length "L". If the masses of any two adjacent particles are doubled, the shift in the centre of mass is

1. L / 8 2. $\sqrt{3L/8}$ 3. $\sqrt{3L/16}$ 4. $\sqrt{3L/4}$

49. Three identical rods each of length "L" and mass "m" are joined to form a U-shape as shown in the figure. The distance of the centre of mass of the system from the corner indicated?



50. A thin uniform rod of length "L" is bent at its mid point as shown in the figure. The distance of the centre of mass from the point "O" is



1.
$$\frac{L}{4}$$
 2. $\sqrt{3}L$ 3. $\frac{\sqrt{3}L}{4}$ 4. $\frac{\sqrt{3}L}{2}$

53. Three identical balls each of mass "m" are at the vertices of an equilateral triangle of side length "L". If the balls move under the influence of the mutual gravitational forces, the distance each ball moves before meeting the other two is

1.
$$\frac{L}{2}$$
 2. $\frac{L}{\sqrt{3}}$ 3. $\frac{2L}{\sqrt{3}}$ 4. L

- 54. A bomb is initially at rest on the ground and suddenly explodes in to two fragments of equal masses. One of the fragments is found moving north with a velocity of 20 m/s. The velocity and acceleration of the centre of mass of the bomb after the explosion are
- 1. 20, 0 2. 20, g 3. 0, g 4. 0, 0
 55. Two particles of mass "p" and "q" (p>q) are at a certain distance apart. On interchanging the position of the masses, the centre of mass of the system is found to shift through a distance "d". The distance between the two particles is

1.
$$\frac{d(p-q)}{q}$$
2.
$$\frac{d(p-q)}{p}$$
3.
$$\frac{d(p+q)}{p-q}$$
4.
$$\frac{d(p-q)}{p+q}$$

56. A uniform square sheet has a side length of 2R. A circular sheet of maximum possible area is removed from one of the quadrants of the square sheet. The centre of mass of the remaining portion from the centre of the original sheet is at a distance of

1.
$$\frac{\Pi R}{\sqrt{2}[16 - \Pi]}$$
 2. $\frac{\Pi}{[16 - \Pi]}$
3. $\frac{R}{\Pi[16 - \Pi]}$ 4. $\frac{R\Pi}{16 - \Pi}$

57. A uniform circular sheet has a radius r. A square sheet of diagonal length equal to radius of the plate is removed from one end of it. The distance of the centre of mass of the reminder plate from the centre of the original sheet is

1.
$$\frac{r}{2(2\Pi - 1)}$$
 2. $\frac{r}{[\Pi - 2]}$
3. $\frac{2r}{\Pi - 3}$ 4. $\frac{r}{2(\Pi - 1)}$

58. A shell in flight explodes into n equal fragments k of the fragments reach the ground earlier than the other fragments. The acceleration of their centre of mass subsequently will be

1. g 2. (n-k)g 3.
$$\frac{(n-k)g}{k}$$
 4. $\frac{(n-k)}{n}g$

JR.PHYSICS

CENTER OF MASS

<u>K E Y</u>

1.2	2.2	3.2	4.4	5.3
6.4	7.3	8.3	9.4	10.1
11.1	12.1	13.4	14.3	15.1
16.2	17.4	18.1	19.2	20.3
21.3	22.1	23.2	24.2	25.3
26.3	27.3	28.2	29.2	30.2
31.1	32.1	33.3	34.3	35.3
36.2	37.2	38.3	39.3	40.3
41.2	42.3	43.3	44.2	45.3
46.4	47.2	48.2	49.3	50.4
51.3	52.3	53.2	54.3	55.3
56.1	57.1	58.4		

HINTS

1. Shift of centre of mass
$$x = \frac{-r^2 a}{R^2 - r^2}$$

- Where r radius of removed disc R radius of original disc a distance between the centres
- 2. See problem 1

3.
$$x = \frac{r^2 a}{R^2 + r^2}$$

compare with problem 1 New position = R - x

4. Refer problem 1

6.
$$O = Xcm = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}; \text{ but } m = eAt$$

$$O = A_1 X_1 + A_2 X_2$$
 (or) $\frac{-A_2 X_2}{A_1} = \frac{A_2 X_2}{A - A_1}$

Here A's are areas, and x_2 is the second body's centre of mass

7.
$$\mathbf{x} = \frac{e_1 x_1 + e_2 x_2}{e_1 + e_2}$$

8. If C is the centre of mass, then

$$AC = \frac{m_1(AC_1) + m_2(AC_2)}{m_1 + m_2}$$

and mass = density x volume

9. Mass of ladder acts at 2.5m Mass of man acts at 2m

$$\therefore \mathbf{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

10. The initial position of **c.m.** is $x = \frac{mL}{2(m+M)}$ and it shifts by the same

11. Distance travelled = Distance from centre of mass

12.
$$\frac{S_1}{S_2} = \frac{\frac{1}{2}a_1t^2}{\frac{1}{2}a_2t^2} = \frac{a_1}{a_2} = \frac{m_2}{m_1}$$
; F = constant

13.
$$\frac{v_1}{v_2} = \frac{a_1 t}{a_2 t} = \frac{m_2}{m_1}$$
 14. $\frac{P_1}{P_2} = \frac{m_1 v_1}{m_2 v_2}$

TRUE OR FALSE TYPE QUESTIONS :

- Consider the following statements A and B and identify the correct answer.
 A. centre of mass lie inside or outside the body B. Mass must be present at centre of mass
 Both A & B are true
 A is true but B is false
 A is false but B is true
 Both A & B are false
 - 2. Consider the following two statements A and B and identify the correct answer
 - 1. The centre of mass of a system of particles depends on forces on the particles.
 - 2. In the absence of external force, the centre of mass of system moves with uniform velocity
 - 1. Both A & B are true
 - 2. A is true but B is false
 - 3. A is false but B is true
 - 4. Both A & B are false
 - 3. Choose the correct statement about the centre of mass (CM) of system of two particles.
 - a. The C.M. lies on the line joining the two particles midway between them

b. The C.M. lies on the line joining them at a point whose distance from each particle is inversely proportional to the mass of that particle c. The C.M. lies on the line joining them at a point whose distance from each particle is proportional to the square of the mass of the particle d. The C.M. is on the line joining them at a point whose distance from each particle is proportional to the square of the mass of the particle d. The C.M. is on the line joining them at a point whose distance from each particle is proportional to the square of the mass of the particle d. The C.M. is on the line joining them at a point whose distance from each particle is proportional to the mass of that particle spectrum.

1. a is correct2. b is correct3. c is correct4. d is correct

4. (Consider the two st	atements A and B and	7.	Study the fallowing			
	identify the correct an		/.	Study the following LIST – I	LIST – II		
1. A wooden sphere and a copper sphere of		9 0	entre of mass of ring	e. mass is present			
	same radius are kept in contact with each other			entre of mass of non	f. coincides with		
	the centre of mass will be with in the wooden		0.0	uniform rod	centre of gravity		
5	sphere.		c. centre of mass of small g. on the heavier si				
1	b. Three identical spl	neres each of radius R are		funnel	0		
1	placed touching each	other on horizontal table.	d.	centre of mass of a	h. at geometric centre		
		he system is located at the	uniform disc				
• · · · ·	-	the medians of the triangle	1. $a \rightarrow f; b \rightarrow e; c \rightarrow g; d \rightarrow h$				
	formed by the centres of	-	2. $a \rightarrow g; b \rightarrow h; c \rightarrow e; d \rightarrow f$				
	1. Both A & B are co		3. $a \rightarrow h; b \rightarrow g; c \rightarrow f; d \rightarrow e$				
	 A is false but B is A is true but B is factorial 		4. $a \rightarrow e; b \rightarrow f; c \rightarrow h; d \rightarrow g$				
	4. Both A & B are fa		8.	Study the following			
			0.1	LIST — I	LIST — II e. at one-fourth of		
		sees m_1 and m_2 ($m_1 > m_2$)	-	position of centre of mass of a triangle	maxi-mum height		
		a force inversely propor-		indos or a trialigio	from the base		
t	tional to the square	of the distance between	b. 1	Position of centre of	f. at the point of inter-		
t t	them. The particles an	re initially held at rest and		mass of a sphere	section of medians		
t	then released. Which c	one is correct.		Position of centre of	g. at the point of inter-		
6	a. the c.m. moves tow	vards m ₁		mass of a cone	section of diagonals		
1	b. the c.m. moves to	wards m ₂		Position of centre of	h. at the centre		
	c. the c.m. remains a	t rest		mass of square			
	d. c.m. moves at righ	t angles to the line joining		1. $a \rightarrow f; b \rightarrow h; c -$	$\rightarrow e; d \rightarrow g$		
1	m ₁ and m ₂			2. $a \rightarrow g; b \rightarrow h; c \rightarrow e; d \rightarrow f$			
	1. a is correct	2. b is correct		3. $a \rightarrow g; b \rightarrow h; c -$			
	3. c is correct	4. d is correct		4. $a \rightarrow g; b \rightarrow f; c -$			
6. Match the following:			A: Assertion	,			
	LIST – I	LIST – II		R : Reason			
	LISI – I	L151 – 11		A: Both A and F	R are true. R is correct		
a. Pos	sition of centre of mas	s e. is zero		explanation of A			
	e algebraic sum of	f. in non uniform gravi			are true. But R is not		
	nents of all the masse	e		correct explanation			
	out centre of mass			C. A is true. R is D. Both A and R			
	ntre of mas and	g. is independent of	0				
cen	tre of gravity	frame of reference	9.		ss of a body may lie where		
	incide			there is no mass	0 1 1 2 2 1		
		e h. in uniform gravita-	R : Centre of mass of a body is a point, where the whole mass of the body is supposed to be				
-	of gravity do not coincide tional field			the whole mass of the concentrated	he body is supposed to be		
	The correct match is			1.A 2.B	3. C 4. D		
	1. $a \rightarrow e; b \rightarrow g; c \rightarrow f; d \rightarrow h$ 2. $a \rightarrow g; b \rightarrow e; c \rightarrow f; d \rightarrow h$		10. A : Two particles (starting from rest) move				
2. $a \rightarrow g, b \rightarrow e, c \rightarrow f, d \rightarrow f$ 3. $a \rightarrow g; b \rightarrow e; c \rightarrow h; d \rightarrow f$				inder a mutual force of at-			
	4. $a \rightarrow h; b \rightarrow e; c \rightarrow f; d \rightarrow g$			traction. The velocit	y of centre of mass is zero		
					o not alter the state of mo-		
				tion of centre of mas			
				1.A 2.B	3. C 4. D		
JR.PH	YSICS	1	64		CENTER OF MASS		
			0 -1		OENTER OF MASS		

11	A. The control of many of a two martials system	MID	TL CODDECT TYPE OUESTIONS
11.	A : The centre of mass of a two particle system	19.	LTI CORRECT TYPE QUESTIONS Identify the correct statements from the following
	lies on the line joining them, being closer to the	19.	
	heavier particle.		a. Algebraic sum of moments of masses about centre of mass is zero
	R: The centre of mass is a point where whole mass of		b. For small bodies centre of mass coincides
	the system is supposed to be concentrated		with centre of gravity.
	1. A 2. B 3. C 4. D		c. Position of centre of mass depends on co-
12.	A : particle in the system of particles has a com-		ordinate system.
	plex motion. But its centre of mass has		d. Position of centre of mass is independent of
	translatory motion		mass distribution
	R : The centre of mass is a point where whole		1. a & b are correct 2. b & c are correct
	mass of the system is supposed to be concentrated		3. a, b & c are correct 4. All are correct
		20.	If external forces acting on a system have zero
12		20.	resultant, the centre of mass
13.	A: The centre of mass of an isolated system		a. may move b. may accelerate
	has constant velocity.		c. must not move d. must not accelerate
	R : The proton and electron attract and move		1. a & b are correct 2. b & c are correct
	towards each other. No external force is applied,		3. c & d are correct 4. a & d are correct
1	therefore their centre of mass remains at rest	21.	Pick out the wrong statements out of the follow-
	1. A 2. B 3. C 4. D		ing. In the absence of external force
14.	A : When a person walks on a stationary boat		a. the K.E. of the system of particles do not follow
1	in still water, centre of mass of person and boat		the law of conservation of kinetic energy
	system is not displaced		b. the centre of mass of the system of particles
	R : Internal force cannot alter the position of		obey the Newton's law of motion.
	centre of mass		c. the centre of mass is accelerated
	1.A 2.B 3.C 4.D		d. the centre of mass always moves along a
15	-		curved path
15.	A : When an external force is applied at the cen-		1. b & c are wrong 2. a & b are wrong
	tre of mass of a free system of particles then it		3. a, b & c are wrong 4. a, c & d are wrong
	undergoes translatory motion only	22.	Identify the correct one from the following statements.
	R : The torque of the force acting at centre of		a. the position of centre of mass in a co-ordinate
	mass about that centre of mass is zero		system does not change if a man moves from
	1.A 2.B 3.C 4.D		the one end to other end on a floating wooden
16.	A : The position of centre of mass of a body		long in still water.
	depends on its shape and size		b. When a man moves from one end to other
	R: Centre of mass of a body is a point where		end on a floating wooden log in still water, it
1	the whole mass of the body is supposed to be		moves in opposite direction c. Due to action and reaction the wooden log
1	concentrated		floating in still water moves in opposite direc-
1	1.A 2.B 3.C 4.D		tion as the man on it moves from one end to the
17.	A: A shell moving in a parabolic path explodes		other end
1	in mid air. The centre of mass of the fragments will		1. b & c are true 2. a & d are true
1	follow the same parabolic path		3. a, b & c are true 4. All are correct
	R : Explosion is due to internal forces, which	23.	A stationary bomb explodes into two parts of
1	cannot alter the state of motion of a body.		masses in the ratio $2:3$. If the heavier part
1	1. A 2. B 3. C 4. D		moves with a velocity 6ms ⁻¹ towards east,
10	-		then
18.	A : When a body dropped from a height ex-		a. the lighter part moves with velocity 9ms^{-1}
1	plodes in mid air, the pieces fly in such a way		towards west
	that their centre of mass keeps moving vertically		b. the lighter part remains at rest
1	downwards.		c. the velocity of the centre of mass of the two
1	R : Explosion occurs under internal forces only.		parts is zero after explosion.
1	External forces = 0		d. the centre of mass moves along a parabolic
	1.A 2.B 3.C 4.D		path
1			1. b & c are true 2. a & d are true
1			3. a & c are true 4. a, c & d are true
			,

24.	In which of the following cases, the centre of mass of a rod is certainly not at its centre? a. The density increases from left to right upto the centre and then decrease b. The density decreases from left to right upto the centre and then increase c. The density continuously increases from left to right d. The density continuously decreases from left to right 1. a & b are correct 2. b & c are correct 3. c & d are correct 4. a & d are correct	 28. A trolley filled with sand moves on a smooth horizontal surface with a velocity v0. A small hole is made at the base of it from which sand is leaving out vertically down at constant rate. As the sand leaves out a. the velocity of the trolley remains constant b. the velocity of the trolley increases c. the velocity of the trolley decreases d. the momentum of trolley + leaked out sand is conserved 1. a & b are correct 2. c & d are correct 3. a & c are correct 		
25.	Mass of a ring is non–uniformly distributed around its geometric centre. If R is radius of the ring, then		KEV	
	a. centre of mass does not coincide with geo-		KEY 1.2 2.3 3.4 4.2 5.3	
	metric centre		$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
	b. Position centre of mass from the geometric		11.1 12.2 13.1 14.1 15.2	
	centre will be x $(0 \le x \le R)$		16.4 17.1 18.1 19.1 20.4	
	c. centre of mass will be nearer to the greater		21.4 22.3 23.3 24.3 25.3	
	mass distribution		26. 1 27. 1 28. 4	
	d. centre of mass may lie out side the periphery			
	 only a & b are correct only b & c are correct 		QUESTION FROM PREVIOUS	
	3. a, b & c are correct 4. All the correct		EAMCET QUESTIONS	
26.	P is the centre of mass of a system of four point masses A, B, C and D which are coplanar but not collinear a. P may or may not coincide with one of the	1.	Particles of masses m, 2m, 3m nm grams are placed on the same line at distances, 1, 21, 31, nl cm from a fixed point. The distance of centre of mass of the particles from the fixed	
	point masses		point in centimeters in [2002 Engg]	
	b. P must lie within or on the edge of at least one of the triangles formed by taking A, B, C and D three at a time		1. $\frac{(2n+1)l}{3}$ 2. $\frac{l}{n+1}$	
	c. Pmust lie on a line joining two of the points A, B, C, D		3. $\frac{n(n^2+l)l}{2}$ 4. $\frac{2l}{n(n^2+l)l}$	
27.	 d. Plies out side the quadrangle ABCD 1. a & b are correct 2. All are correct 3. All are false 4.a & c are correct A disc of radius 'r' is removed from the disc of radius 'R' then 	2.	A system consists of two identical particles. One particle is at rest and the other particle has an acceleration 'a'. The centre of mass of the sys- tem has an acceleration of	
	a. the minimum shift in centre of mass is zero b. the maximum shift in centre of mass cannot be		1. 2a 2. a 3. $\frac{a}{2}$ 4. $\frac{a}{4}$	
		3.	Two particles of mass 1kg and 3kg have posi-	
	greater than $\frac{r^2}{(R+r)}$		tion vectors $2i+3j-2k$ and $-2i+3j-4k$	
	c. centre of mass must lie where mass exists		respectively. The centre of mass has a position vectors [1999 Engg]	
	d. the shift in centre of mass is $\frac{r^2}{(R+r)}$		1. $\overline{i+3}\overline{j-2k}$ 2. $-\overline{i+3}\overline{j-2k}$	
	1. a b are correct2. b,c are correct3. a, b, d are correct4. a, b, c, d are correct		3. $-\bar{i}+3\bar{j}+2\bar{k}$ 4. $-\bar{i}+3\bar{j}-2\bar{k}$	
JR.P	HYSICS	66	CENTER OF MASS	

effect of gravity explodes in mid air. The centre of [1993] mass of the fragments will 1. moves vertically upwards and then vertically downwards 2. move vertically upwards 3. move in irregular path 4. move in parabolic path, the unexploded bomb would have travelled 5. The two particles with masses 4kg and 2kg located at (1, 0, 1) and (2, 2, 0) respectively. Then the coordinates of their centre of mass is [1990] 1. $\left(\frac{4}{3}, \frac{2}{3}, \frac{2}{3}\right)$ 2. $\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$ 3. $\left(\frac{2}{3}, \frac{4}{3}, \frac{2}{3}\right)$ 4. $\left(\frac{2}{3}, \frac{2}{3}, \frac{4}{3}\right)$ 6. Four particles, each of mass 1kg, are placed at the corners of a square of side one meter in the X–Y plane. If the point of intersection of the diagonals of the square is taken as the origin, the coordinates of the centre of mass are (Med 2004) 1. (1, 1) 2. (-1, 1) 3. (1, -1) 4. (0, 0)Two particles of equal mass have velocities V_1 = 7. $4\bar{i}$ and $V_2 = 4\bar{j}$. First particle has an acceleration $\overline{a_1} = (5i+5j) \text{ ms}^{-2}$ while the acceleration of the other particle is zero. The centre of mass of the two particles moves in a path of [Eng 2004] 2. Parabola 1. Straight line 3. Circle 4. Ellipse 8. One end of a thin uniform rod of length L and mass M_1 is rivetted to the centre of a uniform circular disc of of radius 'r' and mass M2 so that both are coplanar. The centre of mass of the combination from the centre of mass of the combination from the centre of the disc is (assume that the point of attachment is at the origin) 1. $\frac{L(M_1 + M_2)}{2M_1}$ 2. $\frac{LM_1}{2(M_1 + M_2)}$ 3. $\frac{2(M_1 + M_2)}{LM_1}$ 4. $\frac{2LM_1}{(M_1 + M_2)}$ 167

A bomb travelling in a parabolic path under the

4.

9. Two objects of masses 200gm and 500gm have velocities of 10i m/s and 3i + 5j m/s respectively. The velocity of their centre of mass is [2003 E]

1. 5i-25j 2.
$$\frac{5}{7}i$$
 - 25 j 3. $5i + \frac{25}{7}j$ 4. $25i - \frac{5}{7}j$

- 10. The centre of mass of three particles of masses 1kg, 2kg, and 3kg is at (2, 2, 2). The position of the fourth mass of 4kg to be placed in the system so that the new centre of mass is at (0, 0, 0) is 2005 E 1. (-3, -3, -3)2. (-3, 3, -3)3. (2, 3, -3)4.(2,-2,3)
- Three particles each 1kg mass are placed at the 11. corners of a right angled triangle AOB, O being the origin of the co-ordinate system (OA and OB) along +ve x-direction and +ve y - direction. If the positive vector of the centre of OA = OB = 1m (in meters) 2005 M

1.
$$\frac{i+j}{3}$$
 2. $\frac{i-j}{3}$ 3. $\frac{2(i+j)}{3}$ 4. (i-j)

QUESTIONS FROM OTHER COMPETITIVE EXAMINATIONS

1. Four particles of masses m, 2m, 3m and 4m are at the verticies of a parallelogram in x-y palne with one of the adjacent angle 60° and smaller side 'a' and larger side 2a. The mass 'm' is at the origin and mass 4m on x-axis. The centre of mass of the system is [MP; PMT 1999]

1.
$$\left(\frac{3a}{2};\frac{a}{2}\right)$$

2. $\left(\frac{a}{2};\frac{3a}{2}\right)$
3. $\left(1.65a;\frac{\sqrt{3a}}{4}\right)$
4. $\left(\frac{\sqrt{3a}}{2};0.82a\right)$

- 2. Masses 8, 2, 4, 2 kg are placed at the corners A, B, C, D respectively of a square ABCD of diagonal 80cm. The distance of centre of mass from A is [BHU 1998] 1.20 cm 2.30cm 3.40cm 4.60cm
- 3. Two bodies of different masses 2kg and 4kg are moving with velocities 2m/s and 10m/s towards each other due to mutual gravitational attraction. Then the velocity of the centre of

4. 5.	The centr of mass of the system of particles does not depend on [CPMT 1993] 1. masses of the particles 2. forces on the particles 3. position of the particles 4. relative distances between the particles Two spheres of masses 2M and M are initially at rest at a distance R apart. Due to mutual force of attrction, they approach each other. When they are at separation $\frac{R}{2}$, the	9.	attract e portiona them. T then rele 1. the C 2. the C 3. the C 4. the C joining	ach other wal to the squ he particles eased C.M. moves C.M. moves C.M. remain C.M. moves m ₁ and m ₂	with a force are of the di are initially towards m towards m as at rest s at right ang	1
6.	acceleration of the centre of mass of sphereswould be[CPMT 1993]1. zero ms ⁻² 2. g ms ⁻² 3. 3g ms ⁻² 4. 12g ms ⁻² Two particles of masses 1kg and 3kg move towards eac other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two		tively p triangle centre o	laced at the of side 1 of mass of t he particle om m	e corners of 0cm. Find	f an equilateral the distance of from the corner s placed cm
7.	the relative velocity of approach of the two particles is $2m/s$, the velocity of centre of mass is 0.5m/s. When the relative velocity of approach becomes 3 m/s, the velocity of centre of mass is [IIT 1989] 1. 0.75 m/s 2. 0.5 m/s 3. 2.5 m/s 4. 0 m/s Two particles A and B initially at rest move towards each other under a mutual force of at- traction. At the instant when the speed of A is V and the speed of B is 2v, the speed of the centre of mass of the system is (IIT) 1. 0 2. v 3. 1.5v 4. 3v		1. 3 5. 1 9. 1		3.4 7.1	4. 2 8. 3