COORDINATE GEOMETRY

12

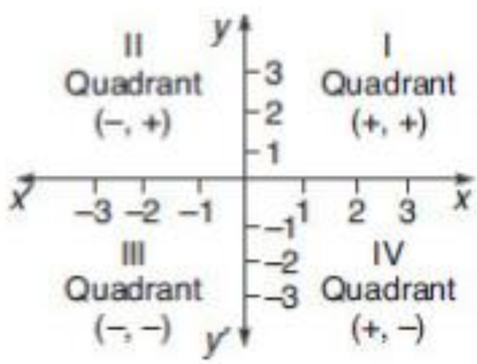
From the CAT point of view, Coordinate Geometry by itself is not a very significant chapter. Basically, applied questions are asked in the form of tabular representation or regarding the shape of the structure formed. However, it is advised to go through the basics and important formulae to have a feel-good effect as also to be prepared for surprises, if any, in the examination. Logical questions might be asked based on the formulae and concepts contained in this chapter. Besides, the student will have an improved understanding of the graphical representation of functions if he/she has gone through coordinate geometry.

The students who face any problems in this chapter can stop after solving LOD II and can skip LOD III.

CARTESIAN COORDINATE SYSTEM

Rectangular Coordinate Axes

Let *X'OX* and *Y'OY* be two mutually perpendicular lines through any point *O* in the plane of the paper. Point *O* is known as the origin. The line *X'OX* is called the x-axis or axis of *x*; the line *Y'OY* is known as the y-axis or axis of *y*; and the two lines taken together are called the coordinate axes or the axes of coordinates.



Any point can be represented on the plane described by the coordinate axes by specifying its x and y coordinates. The x coordinate of the point is also known as the abscissa while the y coordinate is also known as the ordinate.

1. Distance Formula: If two points p and Q are such that they are represented by the points (x_1, y_1) and (x_2, y_2) on the x-y plane (Cartesian plane), then the distance between the points p and Q = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Illustration

Problem 1: Find the distance between the points (5, 2) and (3, 4).

Solution : Distance = $\sqrt{(5-3)^2 + (2-4)^2}$

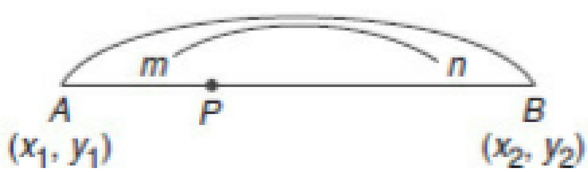
$$\left[\text{Using the formula } \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right]$$

$= 2\sqrt{2}$ units

2. Section Formula: If any point (x, y) divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally,

then $x = (mx_2 + nx_1)/(m + n)$

$y = (my_2 + ny_1)/(m + n)$ (See figure)



If any point (x, y) divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m: n$ externally,

then $x = (mx_2 - nx_1)/(m - n)$

$y = (my_2 - ny_1)/(m - n)$

Illustration

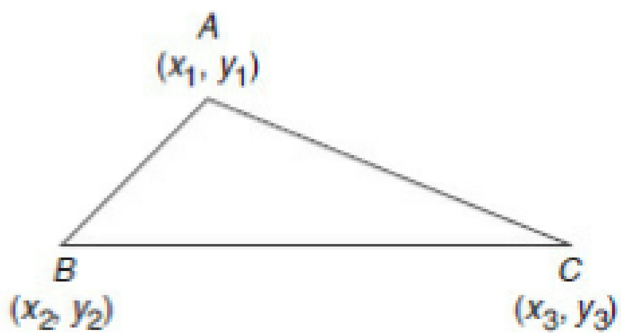
Problem 2: Find the point which divides the line segment joining (2, 5) and (1, 2) in the ratio 2 : 1 internally.

Solution: $X = (2.1 + 1.2) / (1 + 2) = 4/3$

$Y = (2.2 + 1.5) / (2 + 1) = 9/3 = 3$

3. Area of a Triangle: The area of a triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$$\left[\frac{\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}}{2}\right]$$



> ***Note:*** Since the area cannot be negative, we have to take the modulus value given by the above equation.

Corollary: If one of the vertices of the triangle is at the origin and the other two vertices are $A(x_1, y_1)$, $B(x_2, y_2)$, then the area of triangle is $\left|\frac{(x_1 y_2 - x_2 y_1)}{2}\right|$.

Illustration

Problem 3: Find the area of the triangle (0, 4), (3, 6) and (–8, –2).

Solution: Area of triangle = |1/2 {0 (6– (–2)) + 3 ((–2) –4) + (–8) (4–6)}|

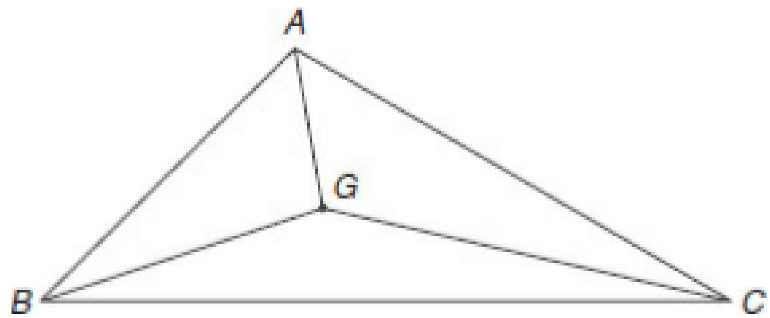
= |1/2 {(0) + 3 (–6) + (–8) (–2)}|

= |1/2 (–2)| = |–1| = 1 square unit

4. Center of gravity or centroid of a triangle: The centroid of a triangle is the point of intersection of its medians (the line joining the vertex to the middle point of the opposite side). Centroid divides the medians in the ratio 2 : 1. In other words, the *CG* or the centroid can be viewed as a point at which the whole weight of the triangle is concentrated.

Formula: If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the coordinates of the vertices of a triangle, then the coordinates of the centroid G of that triangle are

$$x = (x_1 + x_2 + x_3)/3 \text{ and } y = (y_1 + y_2 + y_3)/3$$



Illustration

Problem 4: Find the centroid of the triangle whose vertices are (5, 3), (4, 6) and (8, 2).

Solution: X coordinate = (5 + 4 + 8)/3 = 17/3

Y coordinate = (3 + 6 + 2)/3 = 11/3

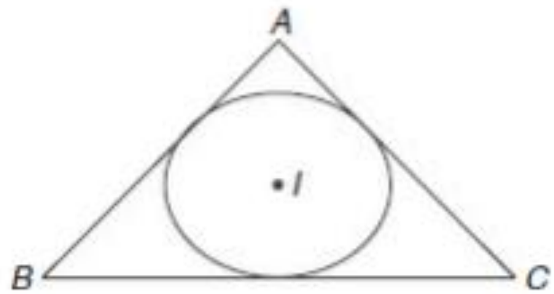
5. In-center of a triangle: The center of the circle that touches the sides of a triangle is called its in-center. In other words, if the three sides of the triangle are tangential to the circle then the center of that circle represents the in-center of the triangle.

The in-center is also the point of intersection of the internal bisectors of the angles of the triangle. The distance of the in-center from the sides of the triangle is the same and this distance is called the in-radius of the triangle.

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the coordinates of the vertices of a triangle, then the coordinates of its in-center are

$$x = \frac{(ax_1 + bx_2 + cx_3)}{(a + b + c)} \quad \text{and} \quad y = \frac{(ay_1 + by_2 + cy_3)}{(a + b + c)}$$

where, $BC = a$, $AB = c$ and $AC = b$.

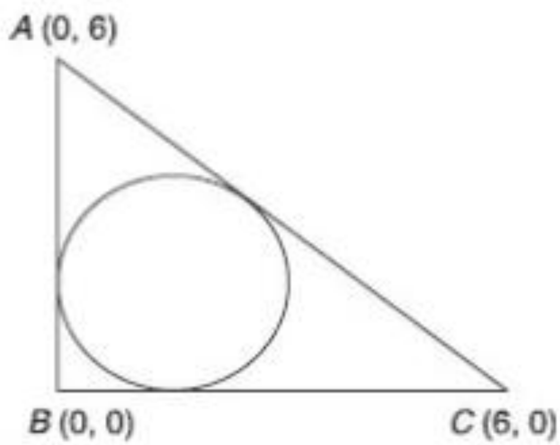


Illustration

Problem 5: Find the in-center of the right-angled isosceles triangle having one vertex at the origin and having the other two vertices at (6, 0) and (0, 6).

Solution: Obviously, the length of the two sides AB and BC of the triangle is 6 units and the length of the third side is $(6^2 + 6^2)^{1/2}$.

Hence, $a = c = 6$, $b = 6\sqrt{2}$.



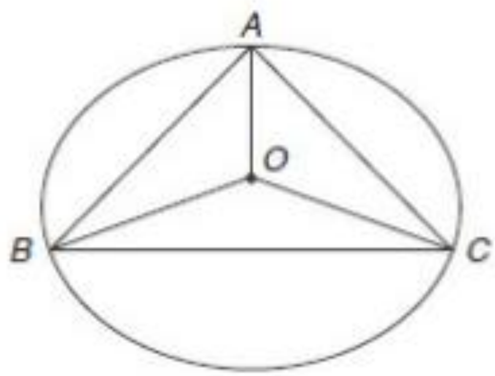
In-center will be at

$$\frac{(6.0 + 6\sqrt{2}.0 + 6.6)}{(6 + 6 + 6\sqrt{2})}, \frac{(6.6 + 6\sqrt{2}.0 + 6.0)}{(6 + 6 + 6\sqrt{2})}$$

$$= \frac{36}{12 + 6\sqrt{2}}, \frac{36}{12 + 6\sqrt{2}}$$

6. Circumcenter of a triangle The point of intersection of the perpendicular bisectors of the sides of a triangle is called its circumcenter. It is equidistant from the vertices of the triangle. It is also known as the center of the circle which passes through the three vertices of a triangle (or the center of the circle that circumscribes the triangle.)

Let ABC be a triangle. If O is the circumcenter of the triangle ABC, then $OA = OB = OC$ and each of these three represent the circum radius.



Illustration

Problem 6: What will be the circumcenter of a triangle whose sides are $3x - y + 3 = 0$, $3x + 4y + 3 = 0$ and $x + 3y + 11 = 0$?

Solution: Let ABC be the triangle whose sides AB, BC and CA have the equations $3x - y + 3 = 0$, $3x + 4y + 3 = 0$ and $x + 3y + 11 = 0$ respectively.

Solving the equations, we get the points A, B and C as $(-2, -3)$, $(-1, 0)$ and $(7, -6)$ respectively.

The equation of a line perpendicular to BC is $4x - 3y + k = 0$.

[For students unaware of this formula, read the section on straight lines later in the chapter.]

This will pass through $(3, -3)$, the mid-point of BC, if $12 + 9 + k = 0$ fi $k = -21$.

putting $k_1 = -21$ in $4x - 3y + k = 0$, we get $4x - 3y - 21 = 0$ (1)

as the equation of the perpendicular bisector of BC.

Again, the equation of a line perpendicular to CA is $3x - y + k_1 = 0$.

This will pass through $(5/2, -9/2)$, the mid-point of AC if

$15/2 + 9/2 + k_1 = 0$ fi $k_1 = -12$

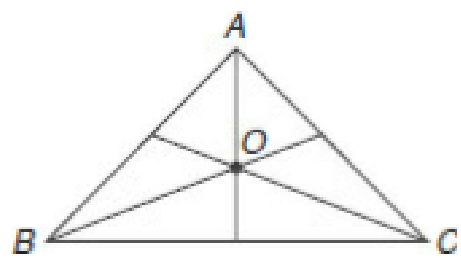
putting $k_1 = -12$ in $3x - y + k_1 = 0$, we get $3x - y - 12 = 0$ (2)

as the perpendicular bisector of AC.

Solving (1) and (2), we get $x = 3, y = -3$.

Hence, the coordinates of the circumcenter of ΔABC are $(3, -3)$.

7. Orthocenter of a triangle: The orthocenter of a triangle is the point of intersection of the perpendiculars drawn from the vertices to the opposite sides of the triangle.



Illustration

Problem 7: Find the orthocenter of the triangle whose sides have the equations $y = 15$, $3x = 4y$, and $5x + 12y = 0$.

Solution: Let ABC be the triangle whose sides BC, CA and AB have the equations $y = 15$, $3x = 4y$, and $5x + 12y = 0$ respectively.

Solving these equations pairwise, we get coordinates of A, B and C as $(0, 0)$, $(-36, 15)$ and $(20, 15)$, respectively.

AD is a line passing through *A* (0, 0) and perpendicular to $y = 15$.

So, equation of *AD* is $x = 0$.

The equation of any line perpendicular to $3x - 4y = 0$ is represented by $4x + 3y + k = 0$.

This line will pass through (–36, 15) if $-144 + 45 + k = 0$ fi $k = 99$.

So the equation of *BE* is $4x + 3y + 99 = 0$.

Solving the equations of *AD* and *BE*, we get $x = 0$, $y = -33$.

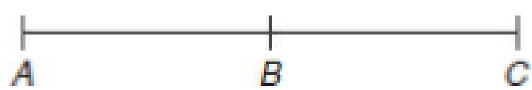
Hence, the coordinates of the orthocenter are (0, –33).

8. Collinearity of three points: Three given points *A*, *B* and *C* are said to be collinear, that is, lie on the same straight line, if any of the following conditions occur:

(i) Area of triangle formed by these three points is zero

(ii) Slope of *AB* = Slope of *AC*

(iii) Any one of the three points (say *C*) lies on the straight line joining the other two points (here *A* and *B*).



Illustration

Problem 8: Select the right option the points $(-a, -b)$, (0, 0) and (a, b) are

(a) collinear

(b) vertices of square

(c) vertices of a rectangle

(d) None of these

Solution: We can use either of the three methods to check whether the points are collinear.

But the most convenient one is (ii) in this case.

Let A, B, C are the points whose coordinates are $(-a, -b)$, $(0, 0)$ and (a, b).

Slope of $BC = b/a$

Slope of $AB = b/a$

So, the straight line made by points A, B and C is collinear.

Hence, (a) is the answer.

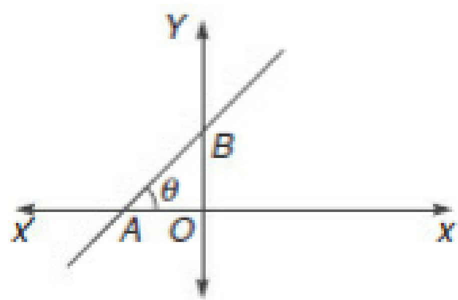
[If you have not understood this here, you are requested to read the following section on straight lines and their slopes and then re-read this solution]

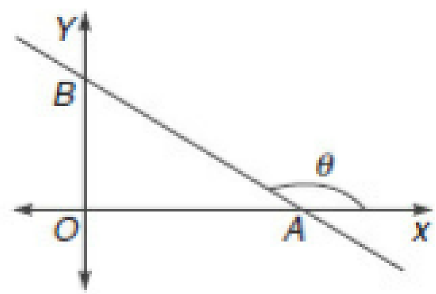
Alternative: Draw the points on paper assuming the paper to be a graph paper. This will give you an indication regarding the nature of points. In the above question, point (a, b) is in first quadrant for $a > 0, b > 0$ and point $(-a, -b)$ is directly opposite to the point (a, b) in the third quadrant with the third point $(0, 0)$ in the middle of the straight line joining the points A and B.

You can check this by assuming any value for 'a' and 'b'.

Also, you can use this method for solving any problem involving points and diagrams made by those points. However, you should be fast enough to trace the points on paper. A little practice of tracing points might help you.

9. *Slope of a line:* The slope of a line joining two points A (x_1, y_1) and B (x_2, y_2) is denoted by m and is given by $m = (y_2 - y_1)/(x_2 - x_1) = \tan q$, where q is the angle that the line makes with the positive direction of x-axis. This angle q is taken positive when it is measured in the anti-clockwise direction from the positive direction of the axis of x.





Illustration

Problem 9: Find the equation of a straight line passing through $(2, -3)$ and having a slope of 1 unit.

Solution: Here slope = 1

And point given is $(2, -3)$.

So, we will use point-slope formula for finding the equation of straight line. This formula is given by:

$(y - y_1) = m\,(x - x_1)$

So, equation of the line will be $y - (-3) = 1\,(x - 2)$

$\Rightarrow y + 3 = x - 2$

$\Rightarrow y - x + 5 = 0$

10. Different Forms of the Equations of a Straight Line:

(a) General Form: The general form of the equation of a straight line is $ax + by + c = 0$.

first degree equation in x and y, where a, b and c are real constants and a, b are not simultaneously equal to zero.

In this equation, slope of the line is given by $\frac{-a}{b}$.

The general form is also given by $y = mx + c$; where m is the slope and c is, the intercept on y-axis.

In this equation, slope of the line is given by m.

(b) Line parallel to the X-axis: The equation of a straight line parallel to the x-axis and at a distance b from it, is given by $y = b$.

Obviously, the equation of the x-axis is $y = 0$.

(c) Line parallel to Y-axis: The equation of a straight line parallel to the y-axis and at a distance a from it, is given by $x = a$.

Obviously, the equation of y-axis is $x = 0$.

(d) Slope Intercept Form: The equation of a straight line passing through the point A (x_1, y_1) and having a slope m is given by

$(y - y_1) = m\,(x - x_1)$

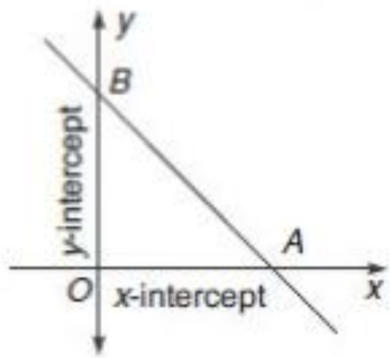
(e) Two points Form: The equation of a straight line passing through two points $A\,(x_1, y_1)$ and $B\,(x_2, y_2)$ is given by

$$(y - y1) = \frac{(y_2 - y_1)\,(x - x_1)}{(x_2 - x_1)}$$

Its slope = $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

(f) Intercept Form: The equation of a straight line making intercepts a and b on the axes of x and y respectively is given by

$$x/a + y/b = 1$$



If a straight line cuts x-axis at A and the y-axis at B then OA and OB are known as the intercepts of the line on x-axis and y-axis respectively.

11. Perpendicularity and Parallelism:

Condition for two lines to be parallel: Two lines are said to be parallel, if their slopes are equal.

For this to happen, ratio of coefficient of x and y in both the lines should be equal.

In a general form, this can be stated as: line parallel to $ax + by + c = 0$ is $ax + by + k = 0$

or $dx + ey + k = 0$ if $a/d = b/e$ where k is a constant.

Illustration

Problem 10: Which of the lines represented by the following equations are parallel to each other?

1. $x + 2y = 5$
2. $2x - 4y = 6$
3. $x - 2y = 4$
4. $2x + 6y = 8$

(a) 1 and 2

(b) 2 and 4

(c) 2 and 3

(d) 1 and 4

Solution: Go through the options and check which of the two lines given will satisfy the criteria for two lines to be parallel. It will be obvious that option (c) is correct, that is, the line $2x - 4y = 6$ is parallel to the line $x - 2y = 4$.

Problem 11: Find the equation of a straight line parallel to the straight line $3x + 4y = 7$ and passing through the point $(3, -3)$.

Solution: Equation of the line parallel to $3x + 4y = 7$ will be of the form $3x + 4y = k$.

This line passes through $(3, -3)$, so this point will satisfy the equation of straight line $3x + 4y = k$. So, $3.3 + 4.(-3) = k \Rightarrow k = -3$.

Hence, equation of the required straight line will be $3x + 4y + 3 = 0$.

Condition for two lines to be perpendicular: Two lines are said to be perpendicular if product of the slopes of the lines is equal to -1.

For this to happen, the product of the coefficients of x + the product of the coefficients of y should be equal to zero.

Illustration

Problem 12: Which of the following two lines are perpendicular?

1. $x + 2y = 5$

2. $2x - 4y = 6$

3. $2x + 3y = 4$

4. $2x - y = 4$

(a) 1 and 2

(b) 2 and 4

(c) 2 and 3

(d) 1 and 4

Solution: Check the equations to get option (d) as the correct answer.

Problem 13: Find the equation of a straight line perpendicular to the straight line $3x + 4y = 7$ and passing through the point $(3, -3)$.

Solution: Equation of the line perpendicular to $3x + 4y = 7$ will be of the form $4x - 3y = K$.

This line passes through $(3, -3)$, so this point will satisfy the equation of straight line $4x - 3y = K$. So, $4.3 - 3.-3$ fi $K = 21$.

Hence, equation of required straight line will be $4x - 3y = 21$.

12. Length of Perpendicular or Distance of a Point from a line: The length of perpendicular from a given point (x_1, y_1) to a line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Corollary:

(a) Distance between two parallel lines.

If two lines are parallel, the distance between them will always be the same.

When two straight lines are parallel whose equations are $ax + by + c = 0$ and $ax + by + c_1 = 0$, then the distance between them is given by $\frac{|c - c_1|}{\sqrt{a^2 + b^2}}$.

(b) The length of the perpendicular from the origin to the line $ax + by + c = 0$ is given by $\frac{|c|}{\sqrt{a^2 + b^2}}$.

Illustration

Problem 14: Two sides of a square lie on the lines $x + y = 2$ and $x + y = -2$. Find the area of the square formed in this way.

Solution: Obviously, the difference between the parallel lines will be the side of the square.

To convert it into the form of finding the distance of a point from a line, we will have to find out a point at which any one of these two lines cut the axes and then we will draw a perpendicular from that point to the other line, and this distance will be the side of the square.

To find the point at which the equation of the line $x + y = 2$ cut the axes, we will put once $x = 0$ and then again $y = 0$.

When $x = 0$, $y = 2$, so the coordinates of the point where it cuts y-axis is (0, 2).

Now the point is (0, 2), and the equation of line on which perpendicular is to be drawn is $x + y = -2$.

So, distance = $\frac{|1.0 + 1.2 + 2|}{\sqrt{1^2 + 1^2}} = \frac{4}{\sqrt{2}}$

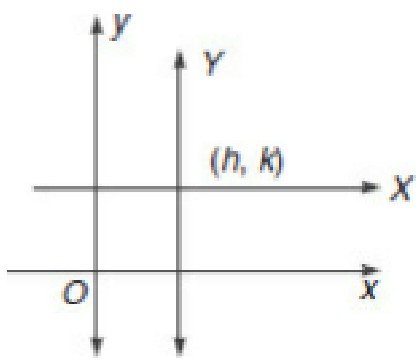
$\therefore$ Area = $\frac{16}{2} = 8$

Alternatively: Draw the points on the paper and you will get the length of diagonal as 4 units; so, length of side will be $2\sqrt{2}$ and, therefore, the area will be 8 sq units.

Alternatively: you can also use the formula for the distance between two parallel lines as

$$\frac{|2+2|}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}}$$

13. Change of axes: If origin (0, 0) is shifted to (h, k) then the coordinates of the point (x, y) referred to the old axes and (X, Y) referred to the new axes can be related with the relation $x = X + h$ and $y = Y + k$.



Illustration

Problem 15: If origin (0, 0) is shifted to (5, 2), what will be the coordinates of the point in the new axis which was represented by (1, 2) in the old axis?

Solution: Let (X, Y) be the coordinates of the point in the new axis.

Then, $1 = X + 5 \therefore X = -4$

$2 = Y + 2 \therefore Y = 0$

So, the new coordinates of the point will be (–4, 0).

14. Point of intersection of two lines: Point of intersection of two lines can be obtained by solving the equations as simultaneous equations.

An Important Result

If all the three vertices of a triangle have integral coordinates, then that triangle cannot be an equilateraltriangle.

LEVEL OF DIFFICULTY (I)

1. Find the distance between the points (3, 4) and (8, –6).

   (a) $\sqrt{5}$

   (b) $5\sqrt{5}$

   (c) $2\sqrt{5}$

   (d) $4\sqrt{5}$

2. Find the distance between the points (5, 2) and (0, 0).

   (a) $\sqrt{27}$

   (b) $\sqrt{21}$

   (c) $\sqrt{29}$

   (d) $\sqrt{31}$

3. Find the value of π if the distance between the points $(8, \pi)$ and (4, 3) is 5.

   (a) 6

   (b) 0

   (c) Both (a) and (b)

   (d) None of these

4. Find the value of c if the distance between the point (c, 4) and the origin is 5 units.

   (a) 3

(b) –3

(c) Both (a) and (b)

(d) none of these

5. Find the mid-point of the line segment made by joining the points (3, 2) and (6, 4).

(a) $\left(\frac{9}{2}, 3\right)$

(b) $\left(\frac{-3}{2}, -1\right)$

(c) $\left(\frac{9}{2}, -\frac{3}{2}\right)$

(d) $\left(\frac{3}{-1}\right)$

6. If the origin is the mid-point of the line segment joined by the points (2, 3) and (x, y), find the value of (x, y).

(a) (2, 3)

(b) (–2, 3)

(c) (–2, –3)

(d) (2, –3)

7. Find the points that divide the line segment joining (2, 5) and (–1, 2) in the ratio 2: 1 internally.

(a) (1, 2)

(b) (–3, 2)

(c) (3, 1)

(d) (0, 3)

8. In what ratio does the x-axis divide the line segment joining the points (2, –3) and (5, 6)?

(a) 2 : 1

(b) 1 : 2

(c) 3 : 4

(d) 2 : 3

9. How many squares are possible if two of the vertices of a quadrilateral are (1, 0) and (2, 0)?

(a) 1

(b) 2

(c) 3

(d) 4

10. If the point R (1, –2) divides externally the line segment joining p (2, 5) and Q in the ratio 3 : 4, what will be the coordinates of Q?

(a) (–3, 6)

(b) (2, –4)

(c) (7/3, 28/3)

(d) (1, 2)

11. Find the coordinates of the points that trisect the line segment joining (1, –2) and (–3, 4).

(a) $\left(\frac{-1}{3}, 0\right)$

(b) $\left(\frac{-5}{3}, 2\right)$

(c) Both (a) and (b)

(d) None of these

12. Find the coordinates of the point that divides the line segment joining the points (6, 3) and (–4, 5) in the ratio 3: 2 internally.

(a) $\left(0, \frac{-21}{5}\right)$

(b) $\left(0, \frac{21}{5}\right)$

(c) $\left(\frac{11}{2}, \frac{14}{3}\right)$

(d) $\left(\frac{-11}{2}, \frac{-14}{3}\right)$

13. In Question 12 , find the coordinates of the point if it divides the points externally.

(a) (24, –9)

(b) (3, –5)

(c) (–24, 9)

(d) (5, –3)

14. In what ratio is the line segment joining (–1, 3) and (4, –7) divided at the point (2, –3)?

(a) 3 : 2

(b) 2 : 3

(c) 3 : 5

(d) 5 : 3

15. In question 14, find the nature of division.

(a) Internal

(b) External

(c) Cannot be said

(d) All of these

16. In what ratio is the line segment made by the points (7, 3) and (–4, 5) divided by the y-axis?

(a) 2 : 3

(b) 4 : 7

(c) 3 : 5

(d) 7 : 4

17. What is the nature of the division in the above question?

(a) External

(b) Internal

(c) Cannot be said

(d) All of these

18. If the coordinates of the mid-point of the line segment joining the points (2, 1) and (1, –3) is (x, y) then the relation between x and y can be best described by

(a) $3x + 2y = 5$

(b) $6x + y = 8$

(c) $5x - 2y = 4$

(d) $2x - 5y = 4$

19. Points (6, 8), (3, 7), (–2, –2) and (1, –1) are joined to form a quadrilateral. What will be this structure?

(a) Rhombus

(b) parallelogram

(c) Square

(d) Rectangle

20. Points *s* (4, –1), (6, 0), (7, 2) and (5, 1) are joined to be a vertex of a quadrilateral. What will be the structure?

(a) Rhombus

(b) Parallelogram

(c) Square

(d) Rectangle

21. What will be the centroid of a triangle whose vertices are (2, 4), (6, 4) and (2, 0)?

(a) $\left(\frac{7}{2}, \frac{5}{2}\right)$

(b) (3, 5)

(c) $\left(\frac{10}{3}, \frac{8}{3}\right)$

(d) (1, 4)

22. The distance between the lines $4x + 3y = 11$ and $8x + 6y = 15$ is

(a) 4

(b) $\frac{7}{10}$

(c) $\frac{5}{7}$

(d) 26

23. If the mid-point of the line joining (3, 4) and (π, 7) is (x, y) and $2x + 2y + 1 = 0$, then what will be the value of π?

(a) 15

(b) $\frac{-17}{2}$

(c) –15

(d) $\frac{17}{2}$

24. Find the third vertex of the triangle whose two vertices are (–3, 1) and (0, –2) and the centroid is the origin.

(a) (2, 3)

(b) $\frac{-4}{3}, \frac{14}{3}$

(c) (3, 1)

(d) (6, 4)

25. Find the area of the triangle whose vertices are (1, 3), (–7, 6) and (5, –1).

(a) 20

(b) 10

(c) 18

(d) 24

26. Find the area of the triangle whose vertices are $(a, b + c)$, $(a, b - c)$ and $(-a, c)$.

(a) $2ac$

(b) $2bc$

(c) $b(a + c)$

(d) $c(a - b)$

27. The number of lines that are parallel to $2x + 6y + 7 = 0$ and have an intercept of length 10 between the coordinate axes is

(a) 0

(b) 1

(c) 2

(d) infinite

28. Which of the following three points represent a straight line?

(a) $\left(\frac{-1}{2}, 3\right)$, $(-5, 6)$ and $(-8, 8)$

(b) $\left(\frac{-1}{2}, 3\right)$, $(5, 6)$ and $(-8, 8)$

(c) $\left(\frac{1}{2}, 3\right)$, $(-5, 6)$ and $(-8, 8)$

(d) $\left(\frac{-1}{2}, 3\right)$, $\left(\frac{5}{6}\right)$ and $(8, 8)$

29. Which of the following will be the equation of a straight line that is parallel to the y-axis at a distance 11 units from it?

(a) $x = +11, x = -11$

(b) $y = 11, y = -11$

(c) $y = 0$

(d) None of these

30. Which of the following will be the equation of a straight line parallel to the y-axis at a distance of 9 units to the left?

(a) $x = -9$

(b) $x = 9$

(c) $y = 9$

(d) $y = -9$

31. What can be said about the equation of the straight line $x = 7$?

(a) It is the equation of a straight line at a distance of 7 units towards the right of the y-axis.

(b) It is the equation of a straight line at a distance of 7 units towards the left of the y-axis.

(c) It is the equation of a straight line at a distance of 7 units below the x-axis.

(d) It is the equation of a straight line at a distance of 7 units above the x-axis.

32. What can be said about the equation of the straight line $y = -8$?

(a) It is the equation of a straight line at a distance of 8 units below the x-axis.

(b) It is the equation of a straight line at a distance of 8 units above the x-axis.

(c) It is the equation of a straight line at a distance of 8 units towards the right of the y-axis.

(d) It is the equation of a straight line at a distance of 8 units towards the left of the y-axis.

33. Which of the following straight lines passes through the origin?

(a) $x + y = 4$

(b) $x2 + y2 = -6$

(c) $x + y = 5$

(d) $x = 4y$

34. What will be the point of intersection of the equation of lines $2x + 5y = 6$ and $3x + 4y = 7$?

(a) $\left(\frac{11}{7}, \frac{4}{7}\right)$

(b) $\left(\frac{-11}{7}, 4\right)$

(c) $\left(3, \frac{-2}{7}\right)$

(d) $\left(4, \frac{-2}{5}\right)$

35. If P (6, 7), Q (2, 3) and R (4, –2) be the vertices of a triangle, then which of the following is not a point contained in this triangle?

(a) (4, 3)

(b) (3, 3)

(c) (4, 2)

(d) (6, 1)

36. What will be the reflection of the point (4, 5) in the second quadrant?

(a) (– 4, – 5)

(b) (– 4, 5)

(c) (4, – 5)

(d) None of these

37. What will be the reflection of the point (4, 5) in the third quadrant?

(a) (– 4, – 5)

(b) (– 4, 5)

(c) (4, – 5)

(d) None of these

38. What will be the reflection of the point (4, 5) in the fourth quadrant?

(a) (– 4, – 5)

(b) (– 4, 5)

(c) (4, –5)

(d) None of these

39. If the origin gets shifted to (2, 2), then what will be the new coordinates of the point (4, –2)?

(a) (– 2, 4)

(b) (2, 4)

(c) (4, 2)

(d) (2, – 4)

40. What will be the length of the perpendicular drawn from the point (4, 5), upon the straight line $3x + 4y = 10$?

(a) $\frac{12}{5}$

(b) $\frac{32}{5}$

(c) $\frac{22}{5}$

(d) $\frac{42}{5}$

LEVEL OF DIFFICULTY (II)

1. Find the area of the quadrilateral, the coordinates of whose angular points taken in order are (1, 1), (3, 4), (5, –2) and (4, –7).

(a) 20.5

(b) 41

(c) 82

(d) 61.5

2. Find the area of the quadrilateral, the coordinates of whose angular points taken in order are (–1, 6), (–3, –9), (5, –8) and (3, 9).

(a) 48

(b) 96

(c) 192

(d) 72

3. Two vertices of a triangle are (5, –1) and (–2, 3). If the orthocenter of the triangle is the origin, what will be the coordinates of the third point?

(a) (4, 7)

(b) (– 4, 7)

(c) (– 4, –7)

(d) (4, –7)

4. Find the equation of the straight line passing through the origin and the point of intersection of the lines $x/a + y/b = 1$ and $x/b + y/a = 1$.

(a) $y = x$

(b) $y = -x$

(c) $y = 2x$

(d) $y = -2x$

5. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1, 1). Which of the following may be an equation which represents any of the other three straight lines?

(a) $7x - 4y = 3$

(b) $7x - 4y + 3 = 0$

(c) $y + 1 = 0$

(d) $4x + 7y = 3$

6. The points $(\pi-1, \pi + 2)$, $(\pi, \pi + 1)$, $(\pi + 1, \pi)$ are collinear for

(a) $\pi = 0$

(b) $\pi = 1$

(c) $\pi = -1/2$

(d) any value of π

7. The straight line joining (1, 2) and (2, –2) is perpendicular to the line joining (8, 2) and (4, π). What will be the value of π?

(a) –1

(b) 1

(c) 3

(d) None of these

8. What will be the length of the perpendicular drawn from the point (–3, –4) to the straight line $12(x + 6) = 5(y - 2)$?

(a) $5\left(\frac{4}{13}\right)$

(b) $5\left(\frac{1}{13}\right)$

(c) $3\left(\frac{2}{13}\right)$

(d) $3\left(\frac{1}{13}\right)$

9. The area of the triangle with vertices at $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$ is

(a) 0

(b) $a + b + c$

(c) $a2 + b2 + c2$

(d) 1

10. Find the distance between the two parallel straight lines $y = mx + c$ and $y = mx + d$. [Assume $c > d$]

(a) $\left(\frac{(c-d)}{(1+m^2)^{\frac{1}{2}}}\right)$

(b) $\left(\frac{(d-c)}{(1+m^2)^{\frac{1}{2}}}\right)$

(c) $\left(\frac{d}{(1+m^2)^{\frac{1}{2}}}\right)$

(d) $\left(\frac{-d}{(1+m)^{\frac{1}{2}}}\right)$

11. What will be the equation of the straight line that passes through the intersection of the straight lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ and is perpendicular to the straight line $3x - 4y = 5$?

(a) $8x + 6y = \frac{32}{7}$

(b) $4x + 3y = \frac{84}{17}$

(c) $4x + 3y = \frac{62}{17}$

(d) $8x + 6y = \frac{58}{17}$

12. In question 11, find the equation of the straight line if it is parallel to the straight line $3x + 4y = 5$.

(a) $12x + 16y = \frac{58}{17}$

(b) $3x + 4y = \frac{58}{17}$

(c) $6x + 8y = \frac{58}{17}$

(d) None of these

13. The orthocenter of the triangle formed by the points (0, 0), (8, 0) and (4, 6) is

(a) $\left(4, \frac{8}{3}\right)$

(b) (3, 4)

(c) (4, 3)

(d) $\left(3, \frac{5}{2}\right)$

14. The area of a triangle is 5 square units; two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. what will be the third vertex?

(a) $\left(\frac{5}{3}, \frac{13}{3}\right)$

(b) $\left(\frac{7}{2}, \frac{13}{2}\right)$

(c) (3, 4)

(d) (1, 2)

15. The equations of two equal sides *AB* and *AC* of an isosceles triangle *ABC* are $x + y = 5$ and $7x - y = 3$ respectively. What will be the equation of the side *BC* if area of triangle *ABC* is 5 square units?

(a) $x + 3y - 1 = 0$

(b) $x - 3y + 1 = 0$

(c) $2x - y = 5$

(d) $x + 2y = 5$

16. Three vertices of a rhombus, taken in order are (2, –1), (3, 4) and (–2, 3). Find the fourth vertex.

(a) (3, 2)

(b) (–3, –2)

(c) (–3, 2)

(d) (3, –2)

17. Four vertices of a parallelogram taken in order are (–3, –1), (a, b), (3, 3) and (4, 3). What will be the ratio of a to b?

(a) 4 : 1

(b) 1 : 2

(c) 1 : 3

(d) 3 : 1

18. What will be the new equation of straight line $3x + 4y = 6$, if the origin gets shifted to (3, – 4)?

(a) $3x + 4y = 5$

(b) $4x – 3y = 4$

(c) $3x + 4y + 1 = 0$

(d) $3x + 4y – 13 = 0$

19. What will be the value of p if the equation of straight line $2x + 5y = 4$ gets changed to $2x + 5y = p$ after shifting the origin at (3, 3)?

(a) 16

(b) –17

(c) 12

(d) 10

20. A line passing through the points (a, 2a) and (–2, 3) is perpendicular to the line $4x + 3y + 5 = 0$. Find the value of a.

(a) –14/3

(b) 18/5

(c) 14/3

(d) –18/5

ANSWER KEY

Level of Difficulty (I)

1. (b)
2. (c)
3. (c)
4. (c)
5. (a)
6. (c)
7. (d)
8. (b)
9. (c)
10. (c)
11. (c)
12. (b)

13. (c)
14. (a)
15. (a)
16. (d)
17. (b)
18. (b)
19. (b)
20. (a)
21. (c)
22. (b)
23. (c)
24. (c)
25. (b)
26. (a)
27. (c)
28. (a)
29. (a)
30. (a)
31. (a)
32. (a)
33. (d)
34. (a)
35. (d)
36. (b)
37. (a)
38. (c)
39. (d)

40. (c)

Level of Difficulty (II)

1. (a)
2. (b)
3. (c)
4. (a)
5. (a)
6. (d)
7. (b)
8. (b)
9. (a)
10. (a)
11. (c)
12. (d)
13. (a)
14. (b)
15. (d)
16. (b)
17. (a)
18. (c)
19. (b)
20. (b)

Hints

Level of Difficulty (II)

1. Use the area of a triangle formula for the two parts of the quadrilateral separately and then add them.

4. Find the point of intersection of the lines by solving the simultaneous equations and then use the two-point formula of a straight line.

   Alternative: After finding out the point of intersection, use options to check.

6. For three points to be collinear,

   (i) Either the slope of any two of the three points should be equal to the slope of any other two points. OR

   (ii) The area of the triangle formed by the three points should be equal to zero.
   Solve using options.

7. Form the equation of the straight lines and then use the options.

10. point of intersection of $y = mx + c$ with x-axis is $(-c/m, 0)$.

    Now use the formula for the distance of a point to a straight line.

11. Find the point of intersection of the lines and then put the coordinate of this point into the equation $4x + 3y = K$, which is perpendicular to the equation of straight line $3x - 4y = 5$, to find out K.

13. Orthocenter is the point of intersection of altitudes of a triangle and centroid divides the straight line formed by joining circumcenter and the orthocenter in the ratio 2: 1.

    Let the vertices of the triangle be O (0, 0), A (8, 0) and B (4, 6).

The equation of an altitude through O and perpendicular to AB is $y = 2/3x$ and similarly the equation of an altitude through A and perpendicular to OB is $2x + 3y = 16$. Now find the point of intersection of these two straight lines.

14. Use the options.

    Alternative: Draw the points in the Cartesian co-ordinate system and then use the simple geometry formula to calculate the point using the options.

15. Draw the points and then check with the options.

    Alternative: Find out the point of intersection with the help of options and then use the formula for area of ?.

17. Sum of x and y co-ordinates of opposite vertices in a parallelogram are same.

19. If the origin gets changed to (h, k) from $(0, 0)$ then

    Old x co-ordinate = New x co-ordinate + h

    Old y co-ordinate = New y co-ordinate + k

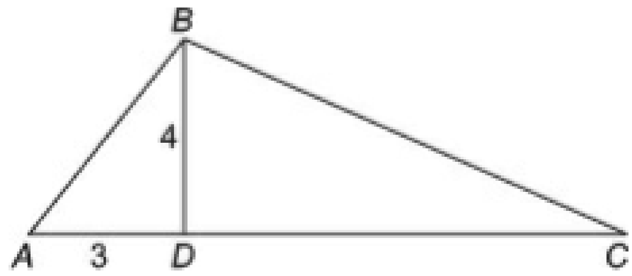
20. Equation of any straight line perpendicular to the line $4x + 3y + 5 = 0$ will be of the form of $3x – 4y = k$, where k is any constant.

    Now form the equation of the straight line with the given two points and then equate.

REVIEW CAT SCAN

REVIEW CAT Scan 1

1. In the figure given below, if angle $ABC = 90$, and BD is perpendicular to AC, and $BD = 4$ cm and $AD = 3$ cm, what will be the length of BC?



(a) 13

(b) 20/3

(c) 16/3

(d) 9

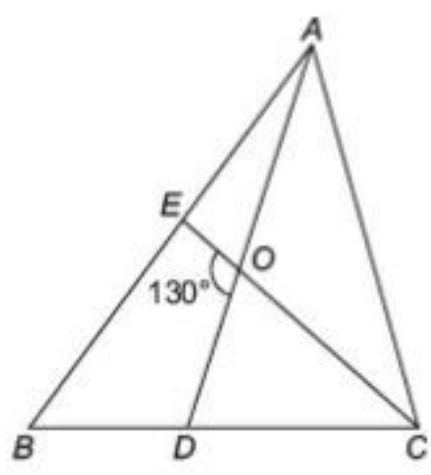
2. In the figure below, the measure of an angle formed by the bisectors of two angles in a triangle ABC is 130. Find the measure of an angle B.

(a) 40

(b) 45

(c) 50

(d) 80



3. the perimeter of a right triangle is 36 and the sum of the square of its sides is 450. The area of the right triangle is

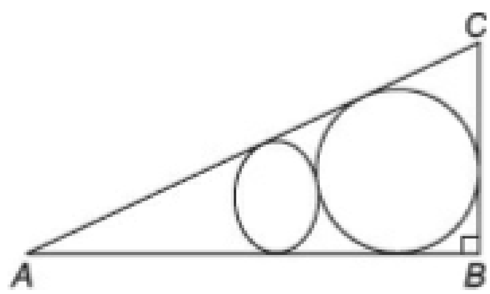
(a) 42

(b) 54

(c) 62

(d) 100

4. The circles are tangent to one another and each circle is tangent to the sides of the right triangle ABC with right angle ABC. If the larger circle has radius 12 and the smaller circle has radius 3, what is the area of the triangle?



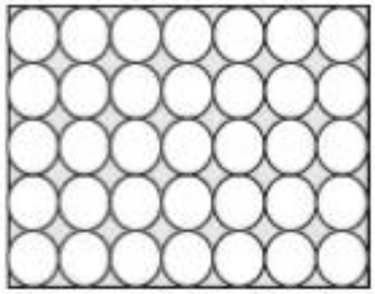
(a) 420

(b) 620

(c) 540

(d) 486

5. All the circles are tangent to one another / or the sides of the rectangle. All circles have radius 1. What is the area of the shaded region to the nearest whole unit, i.e. the region outside all the circles but inside the rectangle?

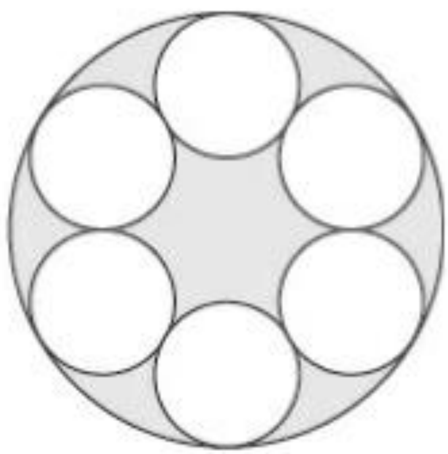


(a) 27
(b) 28

(c) 29

(d) 30

6. Given below are six congruent circles drawn internally tangent to a circle of a radius 21; each smaller circle is also tangent to each of its adjacent circles. Find the shaded area between the circle and the six smaller circles.



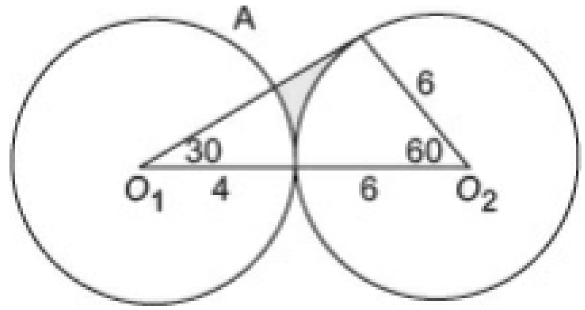
(a) 136π

(b) 196π

(c) 180π

(d) 147π

7. In the figure below, O_1 and O_2 are centers of the circles O_1A is the circle centers at O_2.



Find the area of the shaded region.

(a) 22 – 7π

(b) 24 – 7π

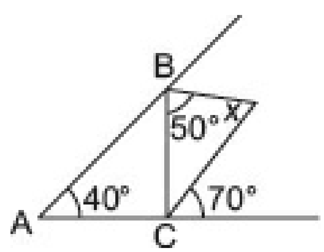
(c) 18 – 9π

(d) 24 – π 22/3

REVIEW CAT Scan 2

1. In the figure, which of the following is correct?

Given: $AB = BC$



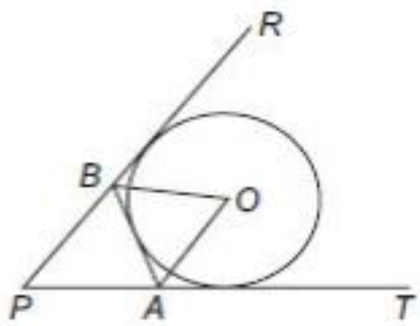
(a) $x = 60$

(b) $x = 70$

(c) $x = 10$

(d) $x = 120$

2. Triangle PAB is formed by three tangents to circle O and angle $APB = 40$; then the angle BOA equals



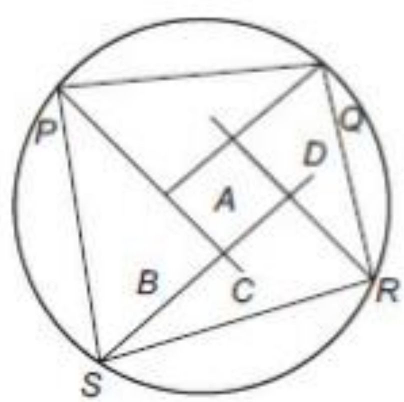
(a) 70

(b) 55

(c) 60

(d) 50

3. $PQRS$ is a cyclic quadrilateral. The angle bisector of angle P, Q, R and S intersects at A, B, C and D as shown in the figure below. Then these four points form a quadrilateral $ABCD$, where $ABCD$ is a



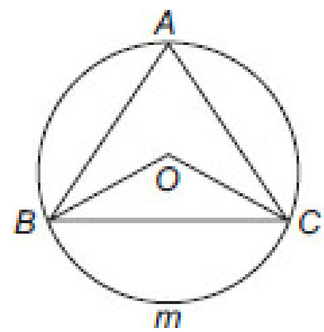
(a) Rectangle

(b) Square

(c) Rhombus

(d) Cyclic quadrilateral

4. In the given figure, O is the center of the circle; angle $BOC = m°$, angle $BAC = n°$, then which of the following is correct?



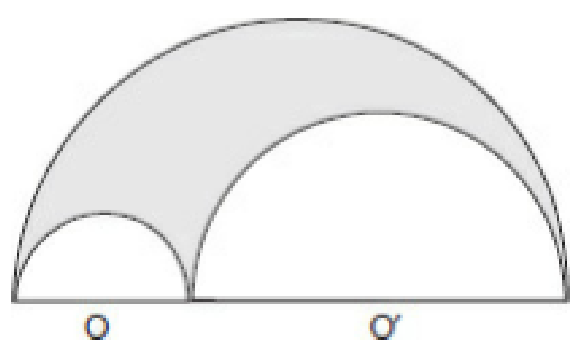
(a) $m + n = 90$

(b) $m + n = 180$

(c) $2m + n = 180$

(d) $m + 2n = 180$

5. Find the area of shaded portion given that the circles with centers O and O' are 6 cm and 18 cm in diameter respectively and ACB is a semi circle.



(a) 54 p cm2

(b) 27 p cm2

(c) 36 p cm2

(d) 18 p cm2

6. There are two spheres and one cube. The cube is inside the bigger sphere and the smaller sphere is inside the cube. Find the ratio of surface areas of the bigger sphere to the smaller sphere.

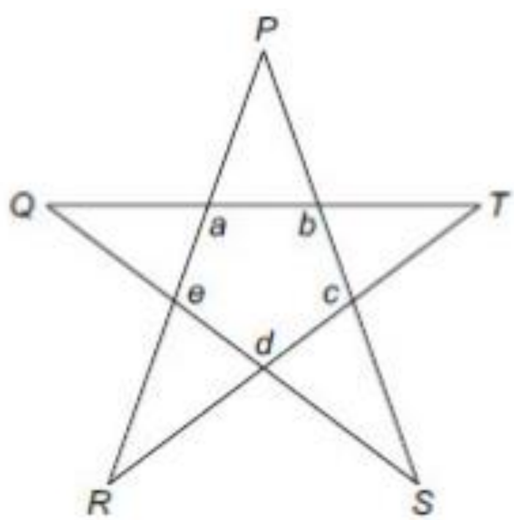
(a) 3 : 1

(b) 2 : 1

(c) 4 : 1

(d) 2 : 1

7. In the adjoining figure, a star is shown. What is the sum of the angles *P*, *Q*, *R*, *S* and *T*?



(a) 240

(b) 180

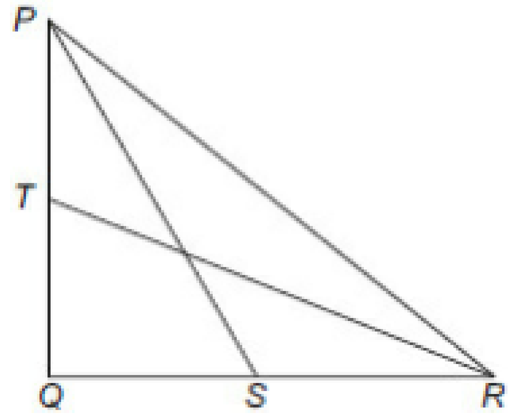
(c) 120

(d) Cannot be determined

REVIEW CAT Scan 3

1. In the figure given below *PS* and *RT* are the medians each measuring 4 cm.

Triangle *PQR* is right-angled at *Q*. What is the area of the triangle *PQR*?



(a) 5.2

(b) 6.4

(c) 6.2

(d) 7.2

2. The area of the largest triangle that can be inscribed in a semi-circle whose radius is

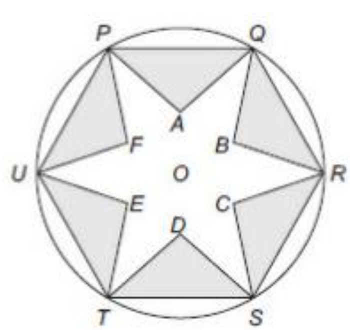
(a) 2*R*2

(b) 3*R*2

(c) *R*2

(d) 3*R*2/2

3. *O* is the center of a circle having radius (*OP*) = *r*. *PQRSTU* is a regular hexagon and *PAQBRCSDTEUFP* is a regular six pointed star.



Find the perimeter of hexagon *PQRSTU*.

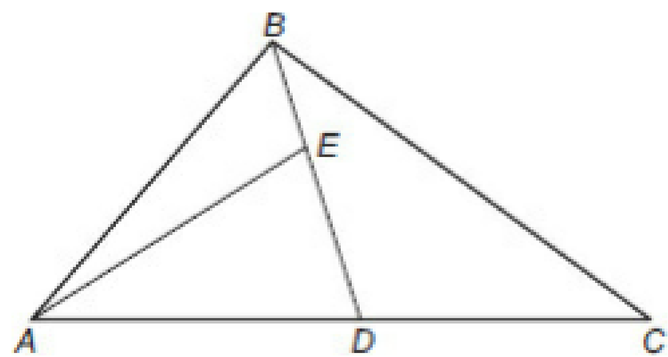
(a) $12r$

(b) $9r$

(c) $6r$

(d) $8r$

4. In the given figure, *ABC* is a triangle in which $AD = 3CD$ and *E* lies on *BD*, $DE = 2BE$. What is the ratio of area of triangle *ABE* and area of triangle *ABC*?



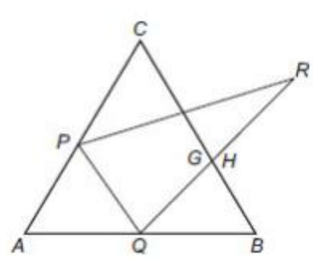
(a) 1/12

(b) 1/8

(c) 1/6

(d) 1/10

5. In the given figure, *P* and *Q* are the mid points of *AC* and *AB*. Also, $PG = GR$ and $HQ = HR$. What is the ratio of area of triangle *PQR*: area of triangle *ABC*?



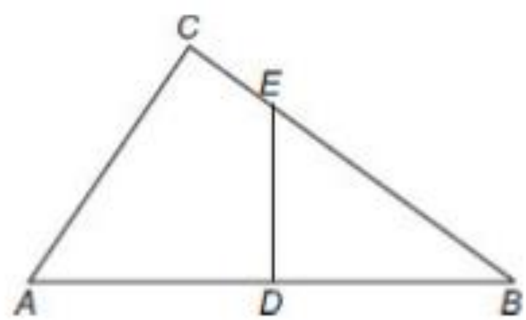
(a) 1/2

(b) 2/3

(c) 3/5

(d) 1/3

6. In the given figure, it is given that angle $C = 90$, $AD = DB$, DE is perpendicular to $AB = 20$, and $AC = 12$. The area of quadrilateral $ADEC$ is



(a) 37(1/2)

(b) 75

(c) 48

(d) 58(1/2)

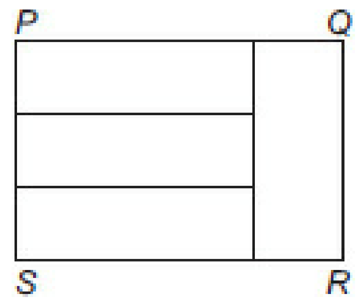
7. Rectangle $PQRS$ contains four congruent rectangles. If the smaller dimension of one of the small rectangles is 4 units, what is the area of rectangle $PQRS$ in square units?

(a) 144

(b) 172

(c) 156

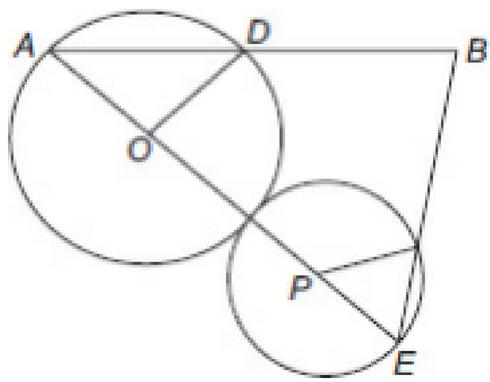
(d) 192



REVIEW CAT Scan 4

1. If the radii of the circles with centers *O* and *p*, as shown below are 4 and 2 units respectively, find the area of triangle *ABC*.

   Given: angle *DOA* = angle *EPC* = 90



   (a) 36

   (b) 62

   (c) 18

   (d) 48

2. A cube of side 16 cm is painted red on all the faces and then cut into smaller cubes, each of side 4 cm. What is the total number of smaller cubes having none of their faces painted?

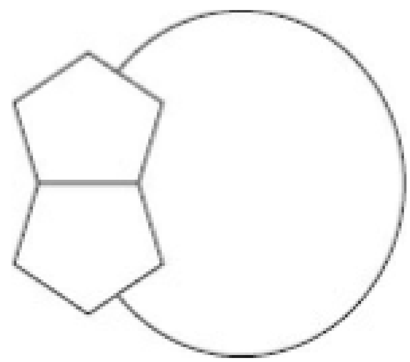
   (a) 16

(b) 8

(c) 12

(d) 24

3. Identical regular pentagons are placed together side by side to form a ring in the manner shown. The diagram shows the first two pentagons. How many are needed to make a full ring?



(a) 9

(b) 10

(c) 11

(d) 12

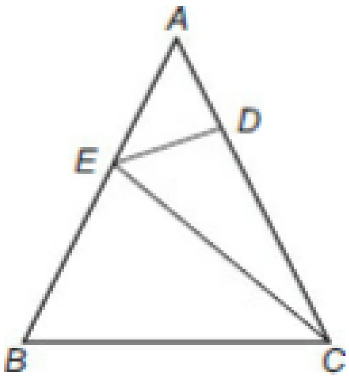
4. Find angle *EBC* + angle *ECB* from the given figure, given *ADE* is an equilateral triangle and angle *DCE* = 20°.

(a) 160

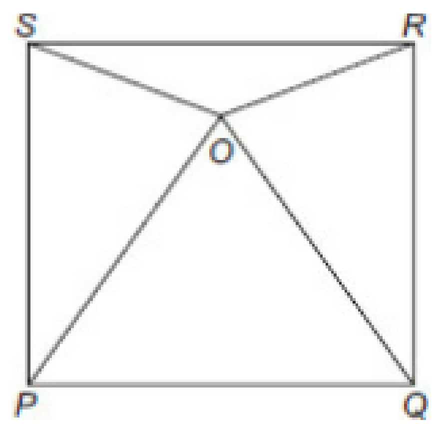
(b) 140

(c) 100

(d) 120



5. *PQRS* is a square and POQ is an equilateral triangle. What is the value of angle *SOR*?

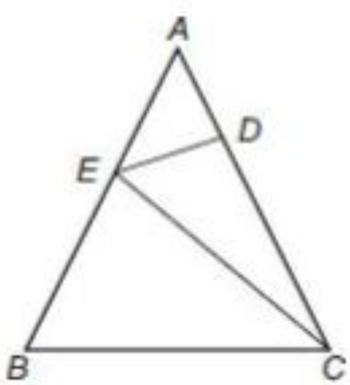


(a) 160

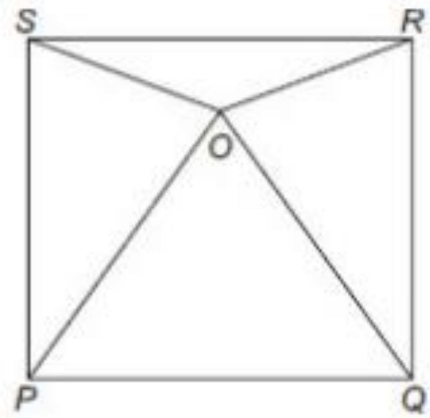
(b) 140

(c) 100

(d) 120



5. *PQRS* is a square and POQ is an equilateral triangle. What is the value of angle *SOR*?



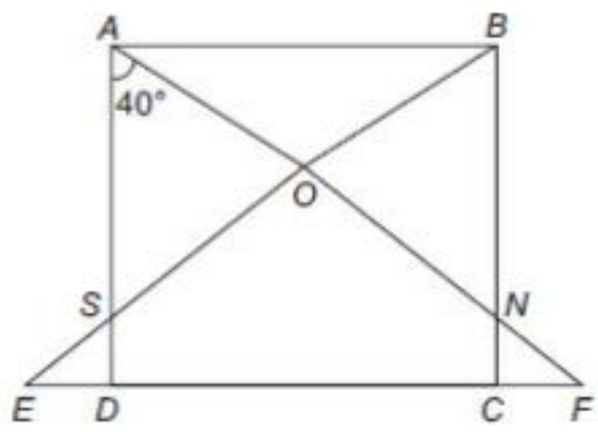
(a) 150

(b) 120

(c) 125

(d) 100

6. In the following figure *ABCD* is a square, angle *DAO* = 40, then find angle *BNO*.



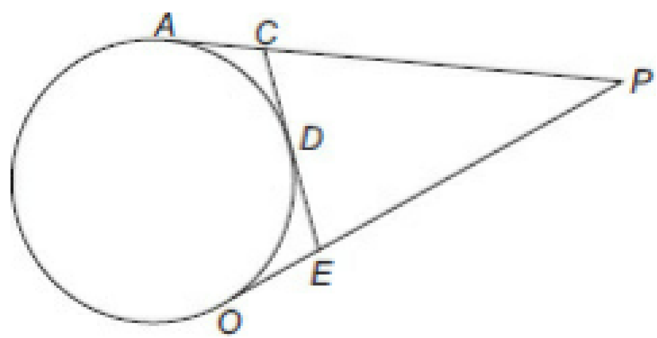
(a) 50

(b) 60

(c) 30

(d) 40

7. From an external pointp, tangents PA and PB are drawn to a circle with center O. if CO is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of ΔCPD.



(a) 21

(b) 28

(c) 24

(d) 25

REVIEW CAT Scan 5

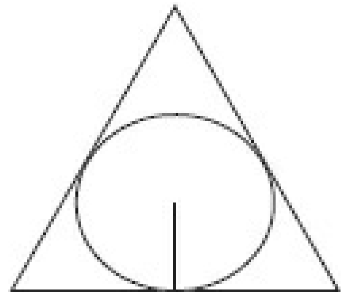
1. A circle is inscribed in an equilateral triangle; the radius of the circle is 2 cm. find the area of triangle.

(a) $12\sqrt{3}$

(b) $15\sqrt{3}$

(c) $12\sqrt{3}$

(d) $18\sqrt{3}$



2. If interior angle of a regular polygon is 168°, then find the number of sides in that polygon.

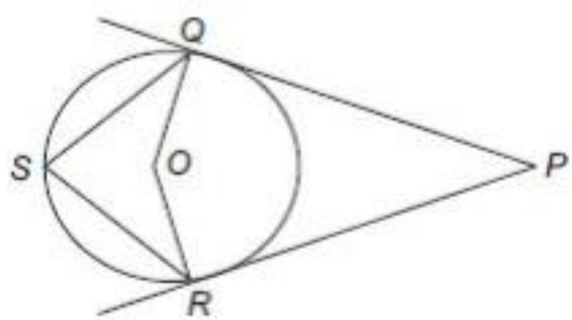
   (a) 10

   (b) 20

   (c) 30

   (d) 25

3. In the given figure, *PQ* and *PR* are two tangents to the circle, whose center is *O*. If angle *QPR* = 40°, find angle *QSR*.



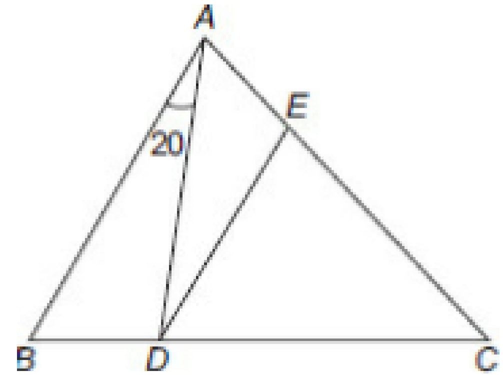
   (a) 60

   (b) 70

   (c) 80

   (d) 50

4. In the figure *AB* || *DE*, angle *BAD* = 20° and angle *DAE* = 30°, and *DE* = *EC*. Then –*ECD* =?



(a) 60

(b) 65

(c) 75

(d) 70

5. There is an equilateral triangle of side 32 cm. The mid-points of the sides are joined to form another triangle, whose mid-points are again joined to form still another triangle. This process is continued for 'n' number of times. The sum of the perimeters of all the triangles is 180 cm. find the value of n.

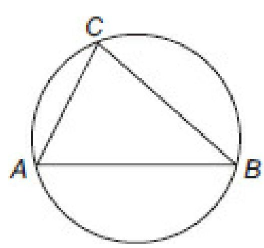
(a) 4

(b) 5

(c) 8

(d) 3

6. There is a circle of diameter AB and radius 26 cm. If chord CA is 10 cm long, find the ratio of area of triangle ABC to the remaining area of circle.



(a) 0.60

(b) 0.30

(c) 0.29

(d) 0.52

7. Ram Singh has a rectangular plot of land of dimensions 30 m * 40 m. He wants to construct a unique swimming pool which is in the shape of an equilateral triangle. Find the area of the largest swimming pool which he can have?

(a) $300\sqrt{3}$ sq cm

(b) $225\sqrt{3}$ sq cm

(c) 300 sq cm

(d) 225√sq cm

ANSWER KEY

REVIEW CAT Scan 1

1. (b)
2. (d)
3. (b)
4. (d)
5. (d)
6. (d)
7. (d)

REVIEW CAT Scan 2

1. (d)
2. (a)
3. (d)
4. (a)
5. (b)
6. (a)
7. (b)

REVIEW CAT Scan 3

1. (b)
2. (c)
3. (c)
4. (a)
5. (a)
6. (d)
7. (d)

REVIEW CAT Scan 4

1. (a)
2. (b)
3. (b)
4. (c)
5. (a)
6. (d)
7. (b)

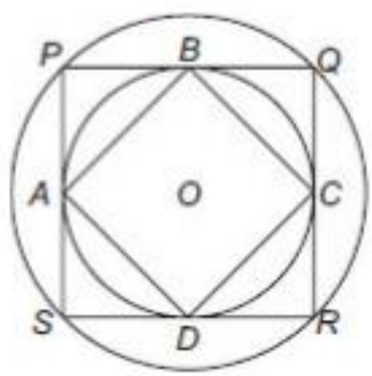
REVIEW CAT Scan 5

1. (a)
2. (c)
3. (b)
4. (b)
5. (a)
6. (c)
7. (a)

TASTE OF THE EXAMS-BLOCK IV

CAT

1. The figure below shows two concentric circles with centre *O*. *PQRS* is a square inscribed in the outer circle. It also circumscribes the inner circle, touching it at points *B*, *C*, *D* and *A*. What is the ratio of the perimeter of the outer circle to that of polygon *ABCD*? **(CAT 1999)**



(a) $\frac{\pi}{4}$

(b) $\frac{3\pi}{2}$

(c) $\frac{\pi}{2}$

(d) p

2. There is a circle of radius 1 cm. Each member of a sequence of regular polygons $S_1(n)$, n = 4, 5, 6, ..., where n is the number of sides of the polygon, is circumscribing the circle: and each member of the sequence of regular polygons $S_2(n)$, n = 4, 5, 6, ... where n is the number of sides of the polygon, is inscribed in the circle. Let $L_1(n)$ and $L_2(n)$ denote the perimeters of the corresponding polygons of $S_1(n)$ and $S_2(n)$, then $\frac{\{L_1(13) + 2\pi\}}{L_2(17)}$ is **(CAT 1999)**

(a) greater than $\frac{\pi}{4}$ and less than 1

(b) greater than 2

(c) greater than 1 and less than 2

(d) less than $\frac{\pi}{4}$

3. There is a square field of side 500 m length each. It has a compound wall along its perimeter. At one of its corners, a triangular area of the field is to be cordoned off by erecting a straight-line fence. The compound wall and the fence will form its borders. If the length of the fence is 100 m, what is the maximum area that can be cordoned off? **(CAT 1999)**

(a) 2,500 sq m

(b) 10,000 sq m

(c) 5,000 sq m

(d) 20,000 sq m

5. Consider a circle with unit radius. There are seven adjacent sectors, $S1$, $S2$, $S3$, ..., $S7$, in the circle such that their total area is 1/8 of the area of the circle. Further, the area of the jth sector is twice that of the $(j-1)$th sector, for $j = 2, ..., 7$. What is the angle, in radians, subtended by the arc of $S1$ at the centre of the circle? **(CAT 2000)**

(a) π/508

(b) π/2040

(c) π/1016

(d) π/1524

6. If a, b and c are the sides of a triangle, and $a^2 + b^2 + c^2 = bc + ca + ab$, then the triangle is **(CAT 2000)**

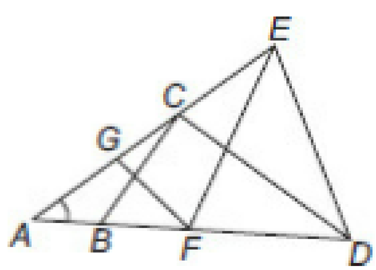
(a) equilateral

(b) isosceles

(c) right-angled

(d) obtuse-angled

7. In the figure, $AB = BC = CD = DE = EF = FG = GA$. Then –$DAE$ is approximately **(CAT 2000)**



(a) 15°

(b) 20°

(c) 30°

(d) 25°

8. A square, whose side is 2 m, has its corners cut away so as to form an octagon with all sides equal. Then the length of each side of the octagon, in metres, is **(CAT 2001)**

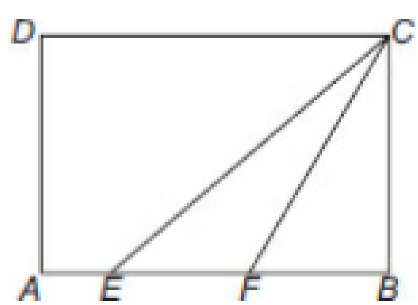
(a) $\frac{\sqrt{2}}{\sqrt{2}+1}$

(b) $\frac{2}{\sqrt{2}+1}$

(c) $\frac{2}{\sqrt{2}-1}$

(d) $\frac{\sqrt{2}}{\sqrt{2}-1}$

9. In the diagram, *ABCD* is a rectangle with *AE* = *EF* = *FB*. What is the ratio of the areas of Δ*CEF* and that of the rectangle? **(CAT 2001)**



(a) 1/6

(b) 1/8

(c) 1/9

(d) None of these

11. Euclid has a triangle in mind. Its longest side has length 20 and another of its sides has length 10. Its area is 80. What is the exact length of its third side? **(CAT 2001)**

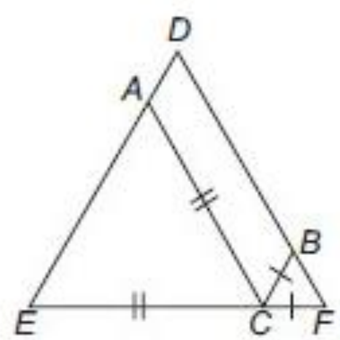
(a) $\sqrt{260}$

(b) $\sqrt{250}$

(c) $\sqrt{240}$

(d) $\sqrt{270}$

12. In DDEF shown below, points *A*, *B* and *C* are taken on *DE*, *DF* and *EF* respectively such that *EC* = *AC* and *CF* = *BC*. If –*D* = 400, then –*ACB* = **(CAT 2001)**



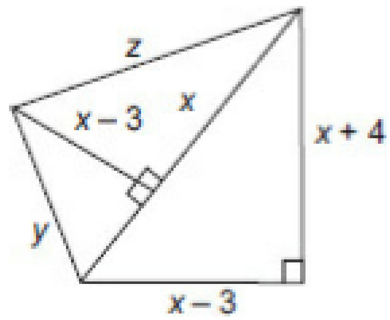
(a) 140°

(b) 70°

(c) 100°

(d) None of these

13. Based on the figure below, what is the value of *x*, if *y* = 10? **(CAT 2001)**



(a) 10

(b) 11

(c) 12

(d) None of these

14. A rectangular pool of 20 m wide and 60 m long is surrounded by a walk-way of uniform width. If the total area of the walkway is 516 m2, how wide, in metres, is the walkway? **(CAT 2001)**

(a) 4.3 m

(b) 3 m

(c) 5 m

(d) 3.5 m

15. In D*ABC*, the internal bisector of –*A* meets *BC* at *D*. If *AB* = 4, *AC* = 3 and –*A* = 600, then the length of *AD* is **(CAT 2002)**

(a) $2\sqrt{3}$

(b) $\frac{12\sqrt{3}}{7}$

(c) $\frac{15\sqrt{3}}{8}$

(d) $\frac{6\sqrt{3}}{7}$

16. The length of the common chord of two circles of radii 15 cm and 20 cm, whose centres are 25 cm apart, is **(CAT 2002)**

(a) 24 cm

(b) 25 cm

(c) 15 cm

(d) 20 cm

17. Four horses are tethered at four corners of a square plot of side 14 m so that the adjacent horses can just reach one another. There is a small circular pond of area 20 m^2 at the centre. Find the ungrazed area. **(CAT 2002)**

(a) 22 m^2

(b) 42 m^2

(c) 84 m^2

(d) 168 m^2

18. In the figure given below, *ABCD* is a rectangle. The area of the isosceles right triangle *ABE* = 7 cm^2; *EC* = 3(*BE*). The area of *ABCD* (in cm^2) is **(CAT 2002)**

(a) 21 cm^2

(b) 28 cm^2

(c) 42 cm2

(d) 56 cm2

19. The area of the triangle whose vertices are (a, a), $(a + 1, a + 1)$ and $(a + 2, a)$ is **(CAT 2002)**

(a) $a3$

(b) 1

(c) $2a$

(d) 21/2

20. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is **(CAT 2002)**

(a) 1/2

(b) 2/3

(c) 1/4

(d) 3/4

21. Neeraj has agreed to mow a lawn, which is a 20 m × 40 m rectangle. He mows it with 1 m wide strip. If Neeraj starts at one corner and mows around the lawn toward the centre, about how many times would he go round before he has mowed half the lawn? **(CAT 2002)**

(a) 2.5

(b) 3.5

(c) 3.8

(d) 4

22. A piece of string is 40 cm long. It is cut into three pieces. The longest piece is three times as long as the middle-sized and the shortest piece is 23 cm shorter than the longest piece. Find the length of the shortest piece. (**CAT 2002)**

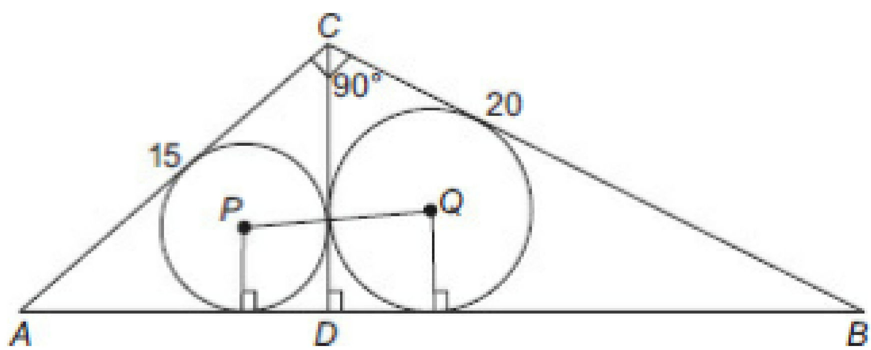
(a) 27

(b) 5

(c) 4

(d) 9

23. In the figure, *ACB* is a right-angled triangle. *CD* is the altitude. Circles are inscribed within the D*ACD* and D*BCD*. *P* and *Q* are the centers of the circles. The distance *PQ* is **(CAT 2002)**



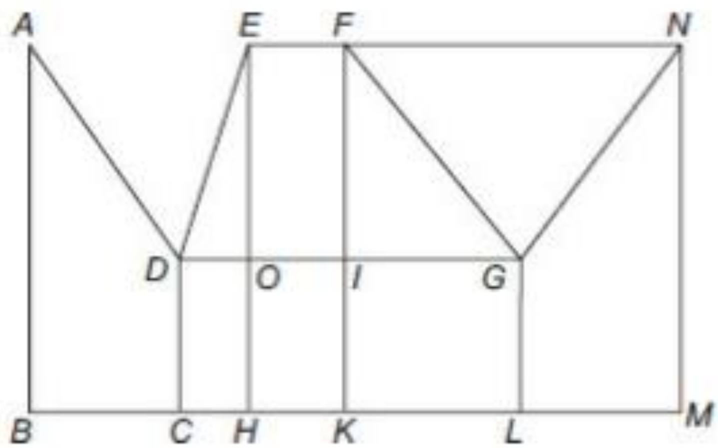
(a) 5

(b) $\sqrt{50}$

(c) 7

(d) 8

Directions for Questions 24 and 25: Answer the questions based on the following diagram.



In the above diagram, –*ABC* = 90° = –*DCH* = –*DOE* = –*EHK* = –*FKL* = –*GLM* = –*LMN*

AB = *BC* = 2*CH* = 2*CD* = *EH* = *FK* = 2*HK* = 4*KL* = 2*LM* = *MN* **(CAT 2002)**

24. The magnitude of –*FGO* =

(a) 30°

(b) 45°

(c) 60°

(d) None of these

25. What is the ratio of the areas of the two quadrilaterals *ABCD* to *DEFG*?

(a) 1 : 2

(b) 2 : 1

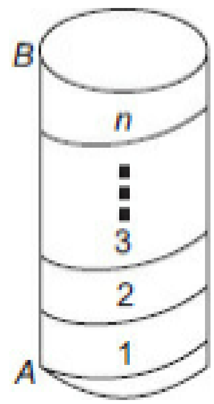
(c) 12 : 7

(d) None of these

Directions for Questions 26 to 28: Answer the questions on the basis of the information given below.

Consider a cylinder of height h cm and radius $r = 2/\pi$ cm as shown in the figure (not drawn to scale). A string of a certain length, when wound on its cylindrical surface, starting at point A and ending at point B, gives a maximum of n turns (in other words, the string's length is the minimum length required to wind n turns). **(CAT 2003)**

26. What is the vertical spacing between the two consecutive turns?



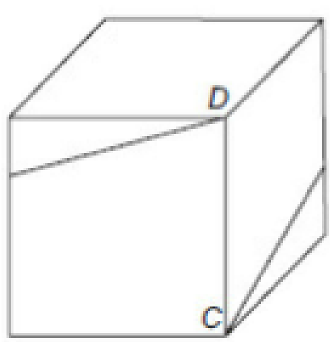
(a) $\frac{h}{n}$ cm

(b) $\frac{h}{\sqrt{n}}$ cm

(c) $\frac{h}{n^2}$ cm

(d) Cannot be determined

27. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale). The length of the string is



(a) $\sqrt{2}n$ cm

(b) $\sqrt{17}n$ cm

(c) n cm

(d) $\sqrt{13}n$ cm

28. In the set-up of the previous two questions, how is h related to n?

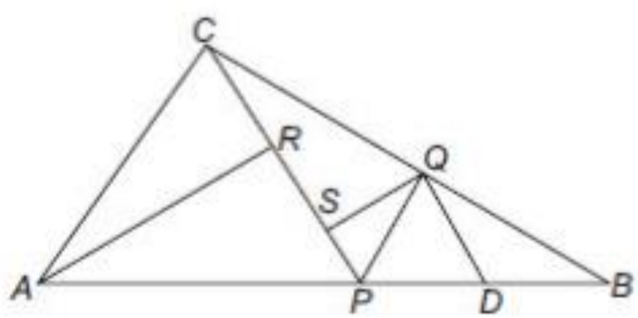
(a) $h = \sqrt{2}n$

(b) $h = \sqrt{17}n$

(c) $h = n$

(d) $h = \sqrt{13}n$

29. In the figure (not drawn to scale) given below, P is a point on AB such that $AP : PB = 4 : 3$. PQ is parallel to AC and QD is parallel to CP. In DARC, –ARC = 90°, and in DPQS, –PSQ = 90°. The length of QS is 6 cm. What is the ratio of $AP : PD$? **(CAT 2003)**



(a) 10:3

(b) 2:1

(c) 7:3

(d) 8:3

30. A car is being driven, in a straight line and at a uniform speed, towards the base of a vertical tower. The top of the tower is observed from the car and, in the process, it takes 10 min for the angle of elevation to change from 45° to 60°. After how much more time will this car reach the base of the tower? **(CAT 2003)**

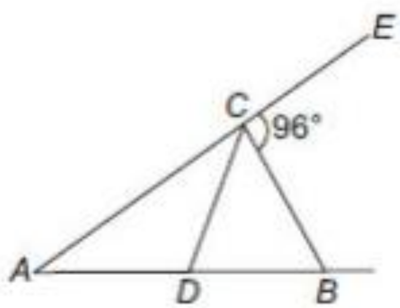
(a) $5(\sqrt{3}+1)$

(b) $6(\sqrt{3}+\sqrt{2})$

(c) $7(\sqrt{3}-1)$

(d) $8(\sqrt{3}-2)$

31. In the figure (not drawn to scale) given below, if $AD = CD = BC$ and –BCE = 96°, how much is the value of DBC? **(CAT 2003)**



(a) 32°

(b) 84°

(c) 64°

(d) Cannot be determined

32. Consider two different cloth-cutting processes. In the first one, n circular cloth pieces are cut from a square cloth piece of side a in the following

steps: the original square of side a is divided into n smaller squares, not necessarily of the same size, then a circle of maximum possible area is cut from each of the smaller squares. In the second process, only one circle of maximum possible area is cut from the square of side a and the process ends there. The cloth pieces remaining after cutting the circles are scrapped in both the processes. The ratio of the total area of scrap cloth generated in the former to that in the latter is **(CAT 2003)**

(a) 1 : 1

(b) $\sqrt{2}:1$

(c) $\frac{n(4-\pi)}{4n-\pi}$

(d) $\frac{4n-\pi}{n(4-\pi)}$

33. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3, \ldots$ be the areas and P_1, $P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$, respectively, then the ratio $\frac{P_1+P_2+P_3+\ldots}{A_1+A_2+A_3+\ldots}$ equals **(CAT 2003)**

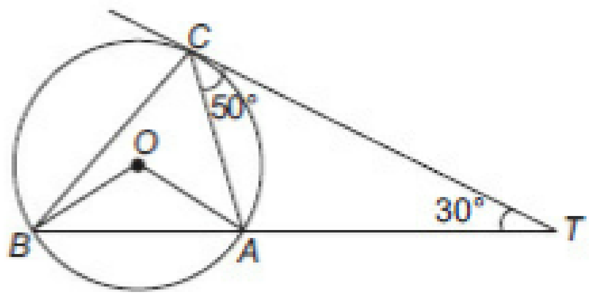
(a) $\frac{2(1+\sqrt{2})}{a}$

(b) $\frac{2(2-\sqrt{2})}{a}$

(c) $\frac{2(2+\sqrt{2})}{a}$

(d) $\frac{2(1+2\sqrt{2})}{a}$

34. In the figure given below (not drawn to scale), *A*, *B* and *C* are three points on a circle with centre *O*. The chord *BA* is extended to a point *T* such that *CT* becomes a tangent to the circle at point *C*. If –*ATC* = 30° and –*ACT* = 50°, then the angle –*BOA* is **(CAT 2003)**



(a) 100°

(b) 150°

(c) 80°

(d) Cannot be determined

35. Let *ABCDEF* be a regular hexagon. What is the ratio of the area of the Δ*ACE* to that of the hexagon *ABCDEF*? **(CAT 2003)**

(a) 1/3

(b) 1/2

(c) 2/3

(d) 5/6

36. A piece of paper is in the shape of a right-angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was 35% reduction in the length of the hypotenuse of the triangle. If the

area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle? **(CAT 2003)**

(a) 16.665

(b) 16.565

(c) 15.465

(d) 14.365

37. A square tin sheet of side 12 inches is converted into a box with open top in the following steps. The sheet is placed horizontally. Then, equal-sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If x is an integer, then what value of x maximizes the volume of the box? **(CAT 2003)**

(a) 3

(b) 4

(c) 1

(d) 2

38. A rectangular sheet of paper, when halved by folding it at the midpoint of its longer side, results in a rectangle, whose longer and shorter sides are in the same proportion as the longer and shorter sides of the original rectangle. If the shorter side of the original rectangle is 2, what is the area of the smaller rectangle? **(CAT 2004)**

(a) $4\sqrt{2}$

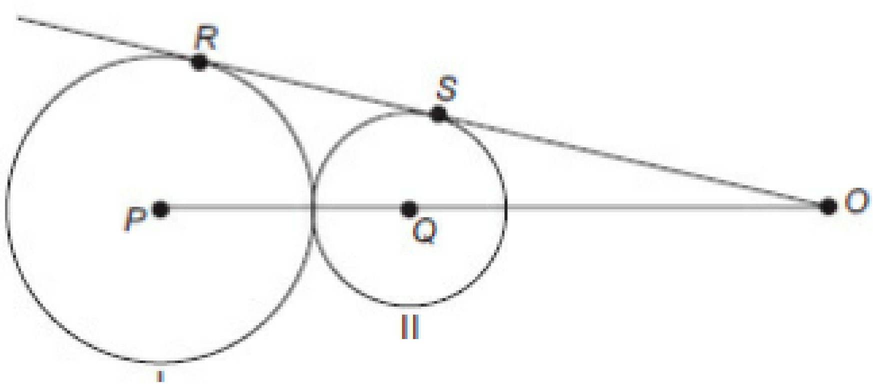
(b) $2\sqrt{2}$

(c) $\sqrt{2}$

(d) None of these

Directions for Questions 39 to 41: Answer the questions on the basis of the information given below:

In the adjoining figure I and II, are circles with centers at *P* and *Q* respectively. The two circles touch each other and have common tangent that touches them at points *R* and *S* respectively. This common tangent meets the line joining *P* and *Q* at *O*. The diameters of I and II are in the ratio 4 : 3. It is also known that the length of *PO* is 28 cm. **(CAT 2004)**



39. What is the ratio of the length of *PQ* to that of *QO*?

(a) 1 : 4

(b) 1 : 3

(c) 3 : 8

(d) 3 : 4

40. What is the radius of the circle II?

(a) 2 cm

(b) 3 cm

(c) 4 cm

(d) 5 cm

41. The length of *SO* is

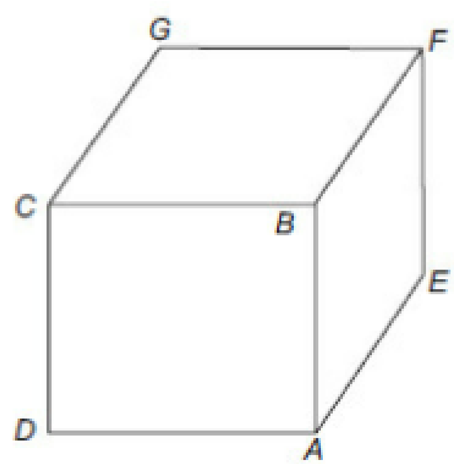
(a) $8\sqrt{3}$ cm

(b) $10\sqrt{3}$ cm

(c) $12\sqrt{3}$ cm

(d) $14\sqrt{3}$ cm

42. If the lengths of diagonals *DF*, *AG* and *CE* of the cube shown in the adjoining figure are equal to the three sides of a triangle, then the radius of the circle circumscribing that triangle will be **(CAT 2004)**



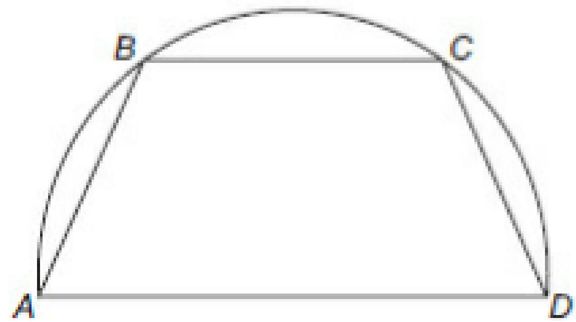
(a) Equal to the side of cube

(b) $\sqrt{3}$ times the side the cube

(c) $\frac{1}{\sqrt{3}}$ times the side of the cube

(d) impossible to find from the given information.

43. On a semicircle with diameter AD, chord BC is parallel to the diameter. Further, each of the chords AB and CD has length 2, while AD has length 8. What is the length of BC? **(CAT 2004)**



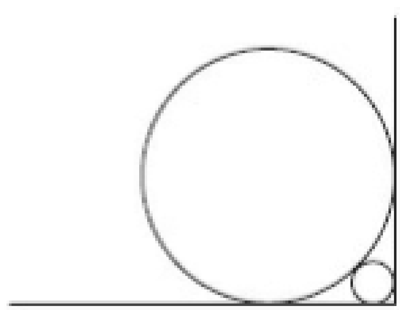
(a) 7.5

(b) 7

(c) 7.75

(d) None of these

44. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the adjoining figure. What is the radius of the smaller circle? **(CAT 2004)**



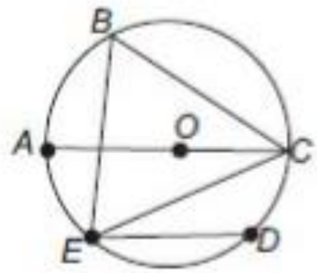
(a) $3-2\sqrt{2}$

(b) $4-2\sqrt{2}$

(c) $7-4\sqrt{2}$

(d) $6-4\sqrt{2}$

45. In the given figure, chord *ED* is parallel to the diameter *AC* of the circle. If –*CBE* = 65°, then what is the value of –*DEC*? **(CAT 2004)**



(a) 35°

(b) 55°

(c) 45°

(d) 25°

46. Two identical circles intersect so that their centers, and the points at which they intersect, form a square of side 1 cm. The area in sq. cm of the portion that is common to the two circles is

(a) π/4

(b) (π/2) – 1

(c) π/5

(d) $\sqrt{2}-1$ **(CAT 2005)**

47. A jogging park has two identical circular tracks touching each other, and a rectangular track enclosing the two circles. The edges of the rectangles are tangential to the circles. Two friends, *A* and *B*, start jogging simultaneously from the point where one of the circular tracks touches the smaller side of the rectangular track. *A* jogs along the rectangular track,

while B jogs along the two circular tracks in a figure of eight. Approximately, how much faster than A does B have to run, so that they take the same time to return to their starting point? **(CAT 2005)**

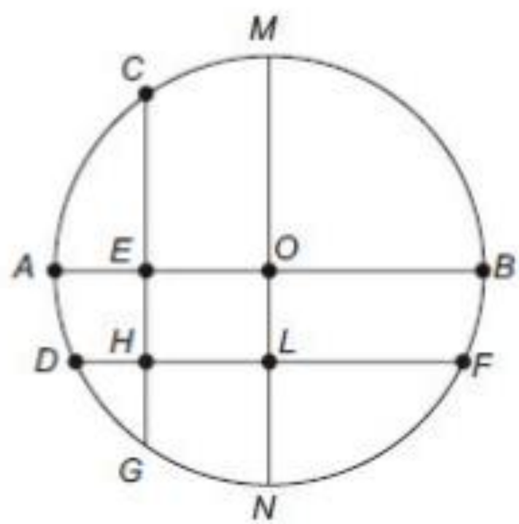
(a) 3.88%

(b) 4.22%

(c) 4.44%

(d) 4.72%

48. In the following figure, the diameter of the circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition, CG is perpendicular to AB such that $AE : EB = 1 : 2$, and DF is perpendicular to MN such that $NL : LM = 1 : 2$. The length of DH in cm is **(CAT 2005)**



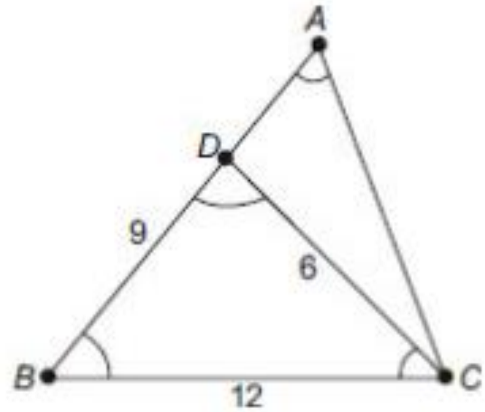
(a) $2\sqrt{2}-1$

(b) $\frac{2\sqrt{2}-1}{2}$

(c) $\frac{3\sqrt{2}-1}{2}$

(d) $\frac{2\sqrt{2}-1}{3}$

49. Consider the triangle ABC shown in the following figure where $BC = 12$ cm, $DB = 9$ cm, $CD = 6$ cm and $–BCD = –BAC$ **(CAT 2005)**



What is the ratio of the perimeter of $\angle ADC$ to that of the $\angle BDC$?

(a) 7/9

(b) 8/9

(c) 6/9

(d) 5/9

50. P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. What is the perimeter of the quadrilateral $PQSR$? **(CAT 2005)**

(a) $2r(1+\sqrt{3})$

(b) $2r(2+\sqrt{3})$

(c) $r(1+\sqrt{5})$

(d) $2r+\sqrt{3}$

51. A rectangular floor is fully covered with square tiles of identical size. The tiles on the edges are white and the tiles in the interior are red. The number of white tiles is the same as the number of red tiles. A possible value

of the number of tiles along one edge of the floor is **(CAT 2005)**

(a) 10

(b) 12

(c) 14

(d) 16

52. A semi-circle is drawn with *AB* as its diameter. From *C*, a point on *AB*, a line perpendicular to *AB* is drawn meeting the circumference of the semi-circle at *D*. Given that *AC* = 2 cm and *CD* = 6 cm, the area of the semi-circle (in sq. cm) will be: **(CAT 2006)**

(a) 32π

(b) 50π

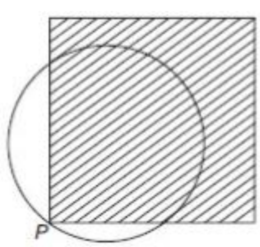
(c) 40.5π

(d) 81π

(e) undeterminable

Directions for Questions 53 and 54: Answer questions on the basis of the information given below:

A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminium of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner *P* of the square sheet and the diameter of the hole originating at *P* is in line with a diagonal of the square. **(CAT 2006)**



53. The proportion of the sheet area that remains after punching is:

(a) $\frac{\pi + 2}{8}$

(b) $\frac{6 - \pi}{8}$

(c) $\frac{4 - \pi}{4}$

(d) $\frac{\pi - 2}{4}$

(e) $\frac{14 - 3\pi}{6}$

54. Find the area of the part of the circle (round punch) falling outside the square sheet.

(a) $\frac{\pi}{4}$

(b) $\frac{\pi - 1}{2}$

(c) $\frac{\pi - 1}{4}$

(d) $\frac{\pi - 2}{2}$

(e) $\frac{\pi - 2}{4}$

55. An equilateral triangle *BPC* is drawn inside a square *ABCD*. What is the value of the angle *APD* in degrees? **(CAT 2006)**

(a) 75

(b) 90

(c) 120

(d) 135

(e) 150

56. In a triangle ABC, the lengths of the sides AB and AC equal 17.5 cm and 9 cm respectively. Let D be a point on the line segment BC such that AD is perpendicular to BC. If $AD = 3$ cm, then what is the radius (in cm) of the circle circumscribing the triangle ABC? **(CAT 2008)**

(a) 17.05

(b) 27.85

(c) 22.45

(d) 32.25

(e) 26.25

57. Consider obtuse-angled triangles with sides 8 cm, 15 cm and x cm. If x is an integer then how many such triangles exist? **(CAT 2008)**

(a) 5

(b) 21

(c) 10

(d) 15

(e) 14

58. Consider a square $ABCD$ with midpoints E, F, G, H of AB, BC, CD and DA respectively. Let L denote the line passing through F and H. Consider points P and Q, on L and inside $ABCD$ such that the angles APD and BQC both equal 120°. What is the ratio of the area of $ABQCDP$ to the remaining area inside $ABCD$? **(CAT 2008)**

(a) $\frac{4\sqrt{2}}{3}$

(b) $2+\sqrt{3}$

(c) $\frac{10-3\sqrt{3}}{9}$

(d) $1+\frac{1}{\sqrt{3}}$

(e) $2\sqrt{3}-1$

59. Two circles, both of radii 1 cm, intersect such that the circumference of each one passes through the centre of the other. What is the area (in sq. cm.) of the intersecting region? **(CAT 2008)**

(a) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

(b) $\frac{2\pi}{3}+\frac{\sqrt{3}}{2}$

(c) $\frac{4\pi}{3}-\frac{\sqrt{3}}{2}$

(d) $\frac{4\pi}{3}+\frac{\sqrt{3}}{2}$

(e) $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$

60. Consider a right circular cone of base radius 4 cm and height 10 cm. A cylinder is to be placed inside the cone with one of the flat surfaces resting on the base of the cone. Find the largest possible total surface area (in sq. cm) of the cylinder. **(CAT 2008)**

(a) $100\pi/3$
(b) $80\pi/3$

(c) $120\pi/3$

(d) $130\pi/9$

(e) $110\pi/7$

61. From a triangle *ABC* with sides of lengths 40 ft, 25 ft and 35 ft, a triangular portion *GBC* is cut off where *G* is the centroid of *ABC*. The area, in sq.ft, of the remaining portion of triangle *ABC* is **(CAT 2017)**

(a) $225\sqrt{3}$

(b) $500/\sqrt{3}$

(c) $275/\sqrt{3}$

(d) $250/\sqrt{3}$

62. Let *ABC* be a right-angled isosceles triangle with hypotenuse *BC*. Let *BQC* be a semi-circle, away from *A*, with diameter *BC*. Let *BPC* be an arc of a circle centered at *A* and lying between *BC* and *BQC*. If *AB* has length 6 cm then the area, in sq cm, of the region enclosed by *BPC* and *BQC* is **(CAT 2017)**

(a) $9\pi - 18$

(b) 18

(c) 9π

(d) 9

63. A solid metallic cube is melted to form five solid cubes whose volumes are in the ratio 1:1:8:27:27. The percentage by which the sum of the surface areas of these five cubes exceeds the surface area of the original cube is nearest to **(CAT 2017)**

(a) 10

(b) 50

(c) 60

(d) 20

64. A ball of diameter 4 cm is kept on top of a hollow cylinder standing vertically. The height of the cylinder is 3 cm, while its volume is 9π cm3. Then the vertical distance, in cm, of the topmost point of the ball from the base of the cylinder is **(CAT 2017)**

65. Let *ABC* be a right-angled triangle with *BC* as the hypotenuse. Lengths of *AB* and *AC* are 15 km and 20 km, respectively. The minimum possible time, in minutes, required to reach the hypotenuse from *A* at a speed of 30 km per hour is **(CAT 2017)**

66. The shortest distance of the point $\left(\frac{1}{2}, 1\right)$ from the curve $y = |x - 1| + |x + 1|$ is **(CAT 2017)**

(a) 1

(b) 0

(c) $\sqrt{2}$

(d) $\sqrt{3}/2$

67. Let *ABCDEF* be a regular hexagon with each side of length 1 cm. The area (in sq cm) of a square with *AC* as one side is **(CAT 2017)**

(a) $3\sqrt{2}$

(b) 3

(c) 4

(d) $\sqrt{3}$

68. The base of a vertical pillar with uniform cross section is a trapezium whose parallel sides are of lengths 10 cm and 20 cm while the other two sides are of equal length. The perpendicular distance between the parallel sides of the trapezium is 12 cm. If the height of the pillar is 20 cm, then the total area, in square cm, of all six surfaces of the pillar is **(CAT 2017)**

(a) 1300

(b) 1340

(c) 1480

(d) 1520

69. If three sides of a rectangular park have a total length 400 ft, then the area of the park is maximum when the length (in ft) of its longer side is **(CAT 2017)**

70. *ABCD* is a quadrilateral inscribed in a circle with centre *O*. If –*COD* = 120 degrees and –*BAC* = 30 degrees, then the value of –*BCD* (in degrees) is **(CAT 2017)**

71. Let *P* be an interior point of a right-angled isosceles triangle *ABC* with hypotenuse *AB*. If the perpendicular distance of *P* from each of *AB*, *BC*, and *CA* is $4(\sqrt{2} - 1)$ cm, then the area, in sq cm, of the triangle *ABC* is **(CAT 2017)**

72. The points (2, 5) and (6, 3) are two end points of a diagonal of a rectangle. If the other diagonal has the equation $y = 3x + c$, then c is **(CAT 2017)**

(a) –5

(b) –6

(c) –7

(d) –8

73. A right circular cone, of height 12 ft, stands on its base which has diameter 8 ft. The tip of the cone is cut off with a plane which is parallel to the base and 9 ft from the base. With π = 22/7, the volume, in cubic ft, of the remaining part of the cone is

(CAT 2018)

74. In a circle, two parallel chords on the same side of a diameter have lengths 4 cm and 6 cm. If the distance between these chords is 1 cm, than the radius of the circle, in cm, is **(CAT 2018)**

(a) $\sqrt{11}$

(b) $\sqrt{13}$

(c) $\sqrt{12}$

(d) $\sqrt{14}$

75. Points *E*, *F*, *G*, and *H* lie on the sides *AB*, *BC*, *CD*, and *DA*, respectively, of a square *ABCD*. If *EFGH* is also a square whose area is 62.5% of that of *ABCD* and *CG* is longer than *EB*, then the ratio of length of *EB* to that of *CG* is **(CAT 2018)**

(a) 2 : 5

(b) 4 : 9

(c) 3 : 8

(d) 1 : 3

76. In a circle with centre O and radius 1 cm, an arc AB makes an angle 60 degrees at O. Let R be the region bounded by the radii OA, OB and the arc AB. If C and D are two points on OA and OB, respectively, such that $OC = OD$ and the area of triangle OCD is half that of R, then the length of OC, in cm, is **(CAT 2018)**

(a) $\left(\frac{\pi}{4}\right)$

(b) $\left(\frac{\pi}{6}\right)^{\frac{1}{2}}$

(c) $\left(\frac{\pi}{4\sqrt{3}}\right)^{\frac{1}{2}}$

(d) $\left(\frac{\pi}{3\sqrt{3}}\right)^{\frac{1}{2}}$

77. Given an equilateral triangle T_1 with side 24 cm, a second triangle T_2 is formed by joining the midpoints of the sides of T_1. Then a third triangle T_3 is formed by joining the midpoints of the sides of T_2. If this process of forming triangles is continued, the sum of the areas, in sq cm, of infinitely many such triangles T_1, T_2, T_3,... will be **(CAT 2018)**

(a) $192\sqrt{3}$

(b) $164\sqrt{3}$

(c) $248\sqrt{3}$

(d) $188\sqrt{3}$

78. Let $ABCD$ be a rectangle inscribed in a circle of radius 13 cm. Which one of the following pairs can represent, in cm, the possible length and breadth of $ABCD$? **(CAT 2018)**

(a) 25, 10

(b) 24, 12

(c) 25, 9

(d) 24, 10

79. In a parallelogram *ABCD* of area 72 sq cm, the sides *CD* and *AD* have lengths 9 cm and 16 cm, respectively. Let *P* be a point on *CD* such that *AP* is perpendicular to *CD*. Then the area, in sq cm, of triangle *APD* is **(CAT 2018)**

(a) $18\sqrt{3}$

(b) $24\sqrt{3}$

(c) $32\sqrt{3}$

(d) $12\sqrt{3}$

80. From a rectangle *ABCD* of area 768 sq cm, a semicircular part with diameter *AB* and area 72π sq. cm is removed. The perimeter of the leftover portion, in cm, is **(CAT 2018)**

(a) 82 + 24π

(b) 88 + 12π

(c) 80 + 16π

(d) 86 + 8π

81. On a triangle *ABC*, a circle with diameter *BC* is drawn, intersecting *AB* and *AC* at points *P* and *Q*, respectively. If the lengths of *AB*, *AC*, and *CP* are 30 cm, 25 cm, and 20 cm respectively, then the length of *BQ*, in cm, is **(CAT 2018)**

82. A chord of length 5 cm subtends an angle of 60° at the centre of a circle. The length, in cm, of a chord that subtends an angle of 120° at the centre of the same circle is **(CAT 2018)**

(a) $6\sqrt{2}$

(b) 8

(c) 2π

(d) $5\sqrt{3}$

83. A triangle *ABC* has area 32 sq units and its side *BC*, of length 8 units, lies on the line $x = 4$. Then the shortest possible distance between *A* and the point (0, 0) is **(CAT 2018)**

(a) $4\sqrt{2}$ units

(b) 8 units

(c) 4 units

(d) $2\sqrt{2}$ units

84. A parallelogram *ABCD* has area 48 sq cm. If the length of *CD* is 8 cm and that of *AD* is *s* cm, then which one of the following is necessarily true? **(CAT 2018)**

(a) $5 \le s \le 7$

(b) $s \le 6$

(c) $s \ge 6$

(d) $s \ne 6$

85. The area of a rectangle and the square of its perimeter are in the ratio 1 : 25. Then the lengths of the shorter and longer sides of the rectangle are in the ratio **(CAT 2018)**

(a) 1 : 3

(b) 3 : 8

(c) 2 : 9

(d) 1 : 4

86. With rectangular axes of coordinates, the number of paths from (1, 1) to (8, 10) via (4, 6), where each step from any point (x, y) is either to $(x, y + 1)$ or to $(x + 1, y)$, is **(CAT 2019)**

87. Let T be the triangle formed by the straight line $3x + 5y - 45 = 0$ and the coordinate axes. Let the circumcircle of T have radius of length L, measured in the same unit as the coordinate axes. Then, the integer closest to L is **(CAT 2019)**

88. If the rectangular faces of a brick have their diagonals in the ratio 3: $2\sqrt{3} : \sqrt{15}$, then the ratio of the length of the shortest edge of the brick to that of its longest edge is **(CAT 2019)**

(a) $\sqrt{3} : 2$

(b) $1 : \sqrt{3}$

(c) $2 : \sqrt{5}$

(d) $\sqrt{2} : \sqrt{3}$

89. In a circle of radius 11 cm, *CD* is a diameter and *AB* is a chord of length 20.5 cm. If *AB* and *CD* intersect at a point *E* inside the circle and *CE* has length 7 cm, then the difference of the lengths of *BE* and *AE*, in cm, is **(CAT 2019)**

(a) 1.5

(b) 3.5

(c) 0.5

(d) 2.5

90. *AB* is a diameter of a circle of radius 5 cm. Let *P* and *Q* be two points on the circle so that the length of *PB* is 6 cm, and the length of *AP* is twice that of *AQ*. Then the length, in cm, of *QB* is nearest to **(CAT 2019)**

(a) 8.5

(b) 9.3

(c) 9.1

(d) 7.8

91. Let *S* be the set of all points (x, y) in the x-y plane such $|x| + |y| \leq$ and $|x| \geq$. Then, the area, in square units, of the region represented by *S* equals [TITA] **(CAT 2019)**

92. Corners are cut off from an equilateral triangle *T* to produce a regular hexagon *H*. Then, the ratio of the area of *H* to the area of *T* is **(CAT 2019)**

(a) 5 : 6

(b) 3 : 4

(c) 2 : 3

(d) 4 : 5

93. In a triangle ABC, medians AD and BE are perpendicular to each other, and have lengths 12 cm and 9 cm, respectively. Then, the area of triangle ABC, in sq cm, is **(CAT 2019)**

(a) 80

(b) 68

(c) 72

(d) 78

94. Two circles, each of radius 4 cm, touch externally. Each of these two circles is touched externally by a third circle. If these three circles have a common tangent, then the radius of the third circle, in cm, is **(CAT 2019)**

(a) $\frac{\neq}{3}$

(b) 1

(c) $\frac{1}{\sqrt{2}}$

(d) $\sqrt{2}$

95. Let ABC be a right angled triangle with hypotenuse BC of length 20cm. If AP is perpendicular on BC, then the maximum possible length of AP, in cm, is **(CAT 2019)**

(a) 10

(b) $8\sqrt{2}$

(c) $6\sqrt{2}$

(d) 5

96. A man makes complete use of 405 cc of iron, 783 cc of aluminium, and 351 cc of copper to make a number of solid right circular cylinders of each type of metal. These cylinders have the same volume and each of these has radius 3 cm. If the total number of cylinders are to be kept at a minimum, then the total surface area of all these cylinders, in sq cm, is **(CAT 2019)**

(a) $1026(1 + \pi)$

(b) 8464π

(c) 928π

(d) $1044(4 + \pi)$

97. The base of a regular pyramid is a square and each of the other four sides is an equilateral triangle, length of each side being 20 cm. The vertical height of the pyramid, in cm, is **(CAT 2019)**

(a) $\sqrt{2}$

(b) $\sqrt{3}$

(c) 12

(d) $\sqrt{5}$

98. Let A and B be two regular polygons having a and b sides, respectively. If $b = 2a$ and each interior angle of B is 3/2 times each interior angle of A, then each interior angle, in degrees, of a regular polygon with $a + b$ sides is **(CAT 2019)**

ANSWER KEY

1. (c)

2. (b)
3. (a)
4. (a)
5. (a)
6. (a)
7. (d)
8. (b)
9. (a)
10. (d)
11. (a)
12. (c)
13. (b)
14. (b)
15. (b)
16. (a)
17. (a)
18. (d)
19. (b)
20. (d)
21. (c)
22. (c)
23. (b)
24. (d)
25. (c)
26. (a)
27. (b)

28. (c)

29. (c)

30. (a)

31. (c)

32. (a)

33. (c)

34. (a)

35. (b)

36. (d)

37. (d)

38. (b)

39. (b)

40. (b)

41. (c)

42. (a)

43. (b)

44. (d)

45. (d)

46. (b)

47. (d)

48. (b)

49. (a)

50. (a)

51. (b)

52. (b)

53. (b)

54. (d)

55. (e)

56. (e)

57. (c)

58. (e)

59. (e)

60. (a)

61. (b)

62. (b)

63. (b)

64. 6

65. 24

66. (a)

67. (b)

68. (c)

69. 200

70. 90

71. 16

72. (d)

73. 198 cm3

74. (b)

75. (d)

76. (d)

77. (a)

78. (d)

79. (c)

80. (b)

\

81. 24 cm

82. (d)

83. (c)

84. (c)

85. (d)

86. 3920

87. 9

88. (b)

89. (c)

90. (c)

91. 2 units

92. (c)

93. (c)

94. (b)

95. (a)

96. (a)

97. (a)

98. 150 degrees

Solutions

1. Let the radius of the outer circle be *r*.

   ABCD is a square and length of its side would also be '*r*'.

   Perimeter of *ABCD* = 4*r*

   Hence, the required ratio = $\frac{2\pi r}{4r} = \frac{\pi}{2}$

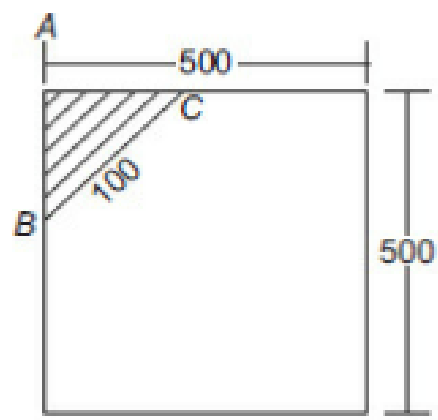
2. The perimeter of any polygon circumscribed about a circle is always greater than the circumference of the circle and the perimeter of any polygon inscribed in a circle is always less than the circumference of the

polygon inscribed in a circle is always less than the circumference of the circle.

Hence, $L_1(13) > 2\pi . 1$ and $L2(17) < 2\pi$

So $\{L_1(13) + 2\pi\} > 4p$ and hence $\frac{\{L1(13) + 2\pi\}}{L2(17)}$ will be greater than 2.

3. In this case, the area of the triangle would be maximum when it is an isosceles triangle.

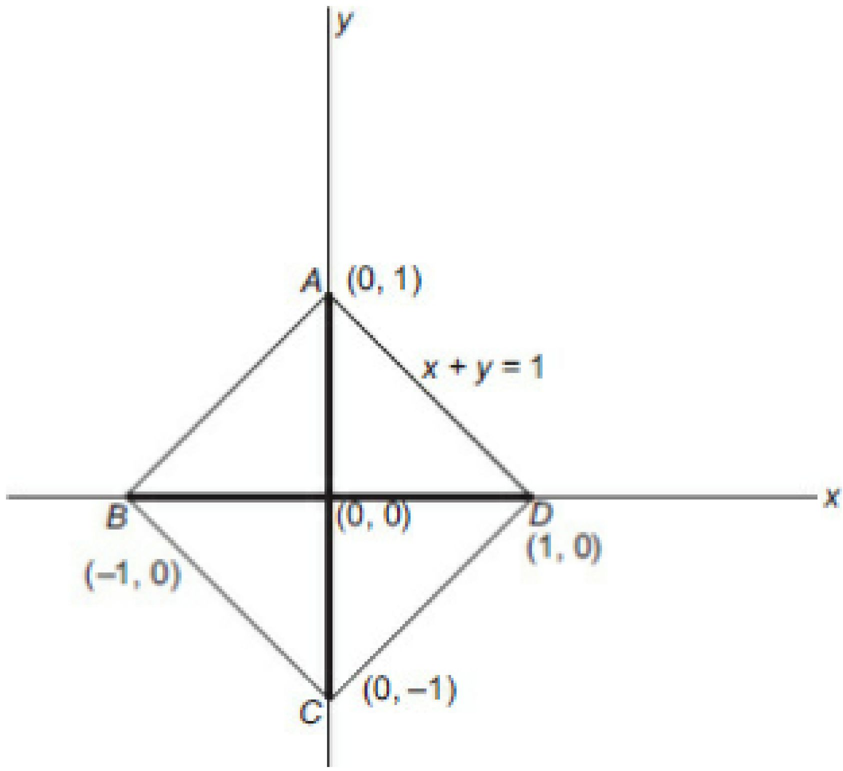


Length of $AC = \frac{100}{\sqrt{2}}$.

Area of the triangular region ABC

$$= \frac{1}{2} \times \frac{100}{\sqrt{2}} \times \frac{100}{\sqrt{2}} = 2{,}500 \text{ sq.m.}$$

4.



Equation of line BC is $\frac{X}{-1} + \frac{Y}{-1} = 1$ OR $x + y = -1$.

5. Let the area of sector S_1 be A units. Then the area of the corresponding sectors shall be $2A$, $4A$, $8A$, $16A$, $32A$ and $64A$ (since every successive sector has an angle that is twice the previous one). Hence, the total area then shall be $127A$ units. This is 1/8 of the total area of the circle. Hence, the total area of the circle will be $127A \times 8 = 1016A$ units. Since, the total angle at the center of a circle is 2p, the sector S_1should account for $\frac{1}{1016}$ of this angle. Hence, angle of sector S_1 is $\frac{\pi}{508}$.

6. $a2 + b2 + c2 = bc + ca + ab$

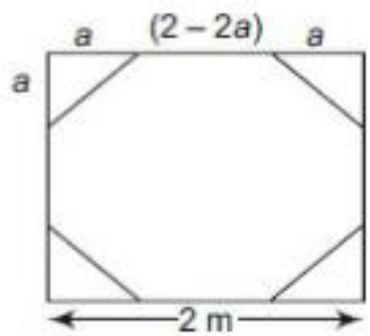
Now we can solve the problem by assuming values of a, b, c and substitute $a = b = c = 1$, we get that $RHS = LHS$. So, option (a) is correct. Even by looking at the equation directly, we can easily make out that the only condition for the equation to be true would be when $a = b = c$.

7. In the figure in triangle ACB, let Angle CAB = Angle BCA = x; then Angle $CBD = 2x$ (exterior angle). Then in triangle BCD angle CBD = angle $CDB = 2x$. Then Angle $BCD = 180 - 4x$ and Angle $DCE = 3x$. (Because the three angles at the point C should add up to 180). In triangle CDE then, angle $DCE = 3x$ = angle CED. Then angle $CDE = 180 - 6x \rightarrow$ Angle $FDE = 180 - 4x$.

Also, $AGF = 180 - 2x$ (since angle AGF = angle $AFG = x$). Then angle $FGE = 2x$ in triangle FGE, Angle FGE = Angle $FEG = 2x$.

Finally in triangle, DEF, Angle EFD = Angle $FDE = 180 - 4x$ and the third angle of the triangle $FED = x$. Thus, $180 - 4x + 180 - 4x + x = 180 \rightarrow 7x = 180 \rightarrow x = 25°$ approximately.

8. Let the length of the edge cut at each corner be 'a' m. After cutting away the corners we'll find a regular octagon. Each side of the regular octagon was $2 - 2a$.



$\therefore \sqrt{a^2 + a^2} = 2 - 2a \Rightarrow a\sqrt{2} = 2 - 2a$

$\Rightarrow a\sqrt{2} \times (1 + \sqrt{2}) = 2 \Rightarrow a = \frac{\sqrt{2}}{\sqrt{2}+1}$. Then, side of the octagon = $2 - 2a = \frac{2}{\sqrt{2}+1}$.

Option (b) is correct.

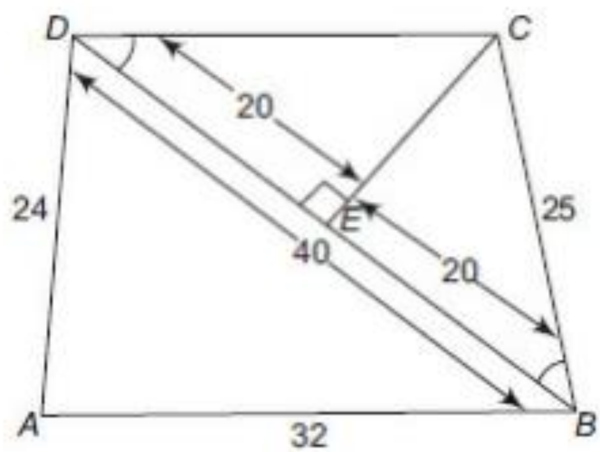
9. Let $AB = p$, $BC = q$

Area of $\Delta CEF = \frac{1}{2} \times \frac{p}{3} \times q = \frac{pq}{6}$

Area of $ABCD = pq$

Required ratio $= \frac{pq}{6} : pq = \frac{1}{6}$

10. $BD = \sqrt{32^2 + 24^2} = 40.$



Draw CE ^ BD. As ΔBCD is isosceles triangle so $DE = BE$.

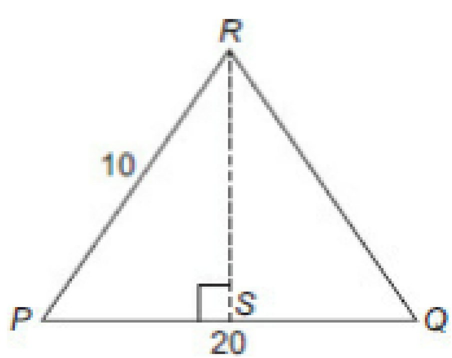
$$CE = \sqrt{25^2 - 20^2} = 15$$

So area of $\Delta ADB = \frac{1}{2} \times 32 \times 24 = 384$ sq.m

Area of $\Delta BCD = 2 \times \frac{1}{2} \times 15 \times 20 = 300$ sq.m

Hence, area of $ABCD = 384 + 300 = 684$ sq.m

11. Let's assume PQ be the longest side of 20 unit and another side AC is 10 unit. Hence RS ^ PQ.



Since area of $\Delta PQR = 80 = \frac{1}{2} PQ \times RS$

So $RS = \frac{80 \times 2}{20} = 8$. In DPSR; $PS = \sqrt{10^2 - 8^2} = 6$

Hence $SQ = 20 - 6 = 14$

So $QR = \sqrt{14^2 + 8^2} = \sqrt{196 + 64} = \sqrt{260}$ units

12. Let $\angle CAE = \angle CEA = x$, $\angle CBF = \angle CFB = y$

$\angle ACE = 180 - 2x$, $\angle BCF = 180 - 2y$

In $\angle DEF$: $x + y + 40° = 180°$

So $x + y = 140°$

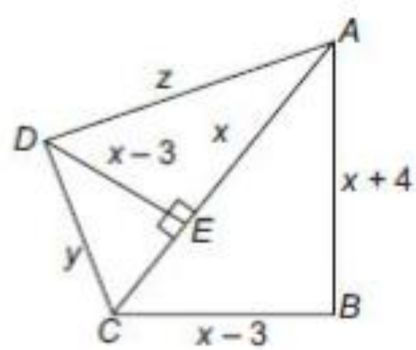
So $\angle ACB + \angle ACE + \angle BCF = 180°$

$\angle ACB = 180° - \angle ACE - \angle BCF$

$= 180° - (180° - 2x) - (180° - 2y)$

$= 2(x + y) - 180° = 2 \times 140 - 180 = 100°$

13. We can solve this problem just by checking the options.

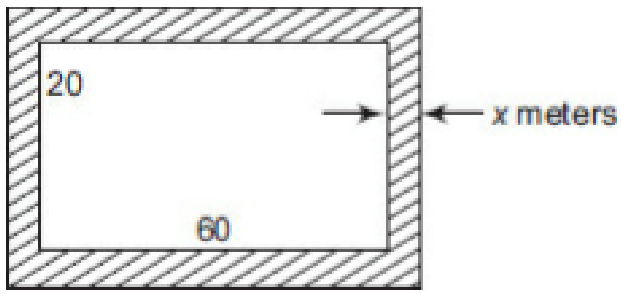


If we put $x = 11$, then in ΔABC by applying Pythagoras theorem, we get $AC = 17$ cm. and $CE = 17 - x = 17 - 11 = 6$ cm

ΔCED is a right angle triangle. For CE = 6 cm and DE = 8 cm, y = 10 cm

So for x = 11, all the conditions given in the question are satisfied.

14. Let width of the path be x meters.



According to the question:

Area of the path = 516 m^2

$\Rightarrow (60 + 2x)(20 + 2x) - 60 \times 20 = 516\ m^2$

$\Rightarrow 1200 + 120x + 40x + 4x^2 - 1200 = 516\ m^2$

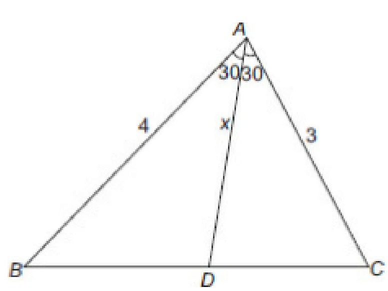
$\Rightarrow 4x^2 + 160x - 516 = 0 \Rightarrow x^2 + 40x - 129 = 0$

By solving we get x = 3.

Alternately, you could try to put the values of the width of the walkway from the options to check in which case do we get the value of the area of the walkway equaling 516.

For Option (b) 3 m we get total area = $60 \times 3 \times 2 + 20 \times 3 \times 2 + 3 \times 3 \times 4$ = 516 m^2.

15.



Let $AD = x$

Then Area of ΔABC = Area of ΔABD + Area of ΔACD

$\frac{1}{2} \times 4 \times 3 \times \sin 60 = \frac{1}{2} \times 4 \times x \times \sin 30 + \frac{1}{2} \times 3 \times x \times \sin 30$

$3\sqrt{3} = \frac{x + 3x}{4}$

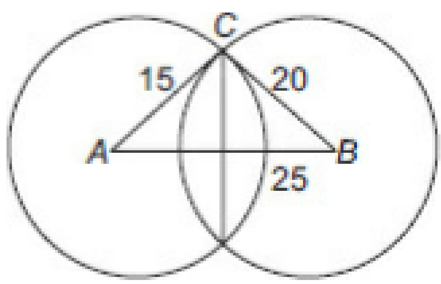
Solving for x, we get $x = \frac{12\sqrt{3}}{7}$

Option (b) is the correct answer.

16. Let the length of the chord be 'a' cm.

Area of $\Delta ABC = \frac{1}{2}(15 \times 20) = \frac{1}{2} \times 25 \times \frac{a}{2}$

$\Rightarrow a = 24$ cm



Alternately, you can think of this using Pythagoras theorem, by trying to split the length 25 of AB into two parts, through trial and error, you would arrive at a break up of 9 and 16, that would lead to two right triangles with sides: 15, 12 and 9 & 12, 16 and 20 respectively. Hence, the length of the chord would be 2 × 12 = 24.

17. Total area of the square plot = 14 × 14 = 196 m2

Total area available for grazing = 196 – 20 = 176 m2

Grazed area = $\frac{\pi \times r^2}{4} \times 4 = \pi r^2 = 22 \times 7 (r = 7)$

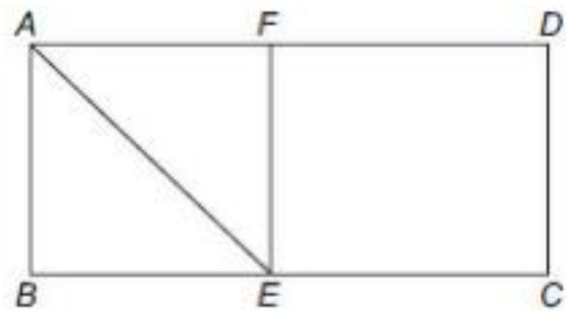
alphalist-ind">= 154 m^2

Ungrazed area = 176 – 154 = 22 m^2

18. Area of DABE = 7 cm^2

Area of $ABEF$ = 2 × Area of DABE = 14 cm^2

Area of $\angle ABCD$ = 4 × Area of $ABEF$ = 4 × 14 = 56 cm^2



19. Area of the triangle =

Let $a = 0$

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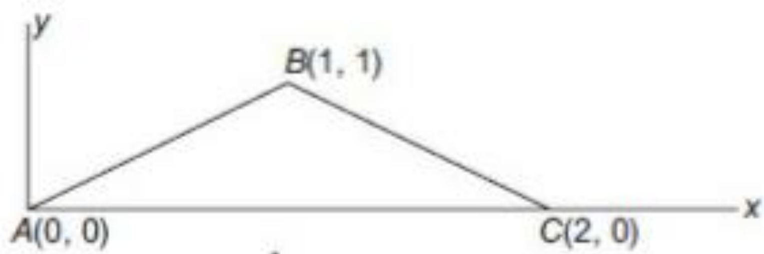
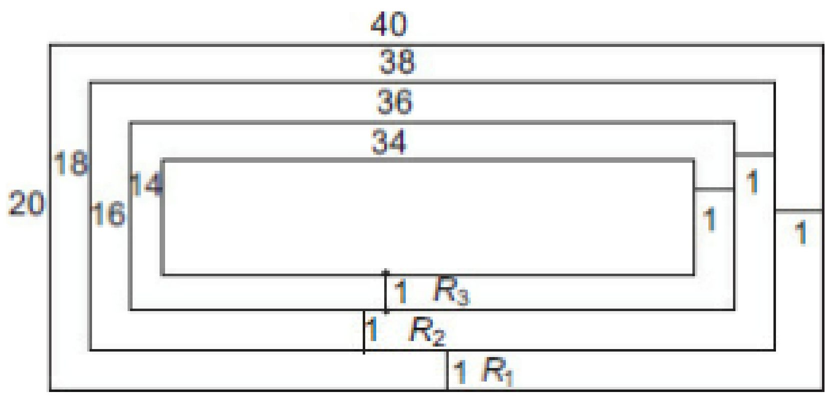


Hence, area = $\frac{1}{2}(2)(1) = 1$. By putting $a = 0$, we get only Option (b) is correct.

20. Think in terms of Pythagoras triplets here. The Pythagoras triple 3, 4, 5 would fulfill the given conditions of the problem, since if instead of walking along the adjacent sides (3 + 4), if he walks along the diagonal (5) he saves a distance (7 – 5 = 2) that is equal to half the length of the longer side.

21.



In the first round he would do 20 + 20 + 38 + 38 = 116 meters. The second round would be 18 + 18 + 36 + 36 = 108, the third round would be 100. (Note that the values of the area of the lawn mowed in every round would be an *AP*. So you need to calculate the total value only for the first two rounds and the remaining values you can get by considering the *AP*). Thus, in the fourth round he would do a total of 92 but to complete half the lawn he would need to do an additional of only 76 (400 – 116 – 108 – 100). This can be achieved in approximately 0.8 of the fourth round.

22. Let the middle sized piece = a

Largest piece = $3a$

Shortest = $3a - 23$,

$3a + a + (3a - 23) = 40$

$a = 9$

The shortest piece = $3a = 3(9) - 23 = 4$.

23. $AB = \sqrt{(15)^2 + (20)^2} = 25$

$\frac{1}{CD^2} = \frac{1}{15^2} + \frac{1}{20^2} = \frac{625}{(300)^2}$

$\frac{1}{CD} = \frac{25}{300}$

CD = 12 cm

$AD = \sqrt{15^2 - 12^2} = 9$ cm

$BD = \sqrt{20^2 - 12^2} = 16$ cm

Area of $\Delta ACD = \frac{1}{2} \times 12 \times 9 = 54$

$s = \frac{1}{2}(15 + 12 + 9) = 18$

$r_1 = \frac{\text{Area}}{s} \Rightarrow r_1 = 3$

Area of $\Delta BCD = \frac{1}{2} \times 16 \times 12 = 96$

$EB = a\sqrt{2}$

Construct PS, to intersect the perpendicular from Q to the base AB at S.

In DPQS, $PS = r_1 + r_2 = 7$ cm

$QS = r_2 - r_1 = 1$ cm

Hence, $PQ = \sqrt{50}$ cm

24. If If $\tan\theta = \frac{FI}{IG} = \frac{\frac{AB}{2}}{\frac{AB}{4}} = 2$

Option (d) is correct.

25. If you assume *AB* as 4, the area of the trapezium *ABCD* = 12, while the area of *DEFG* = 7. Hence, the ratio would be 12 : 7.

Alternately, you could solve this using the formulae as follows:

Area of quadrilateral *ABCD*

$$= = \frac{1}{2} \times \frac{AB}{2} \times AB + \frac{AB}{2} \times AB = \frac{3AB^2}{4}$$

Area of quadrilateral *DEFG*

$$= \frac{1}{2} \times \frac{AB}{2} \times \frac{AB}{2} + \frac{AB}{2} \times \frac{AB}{2} + \frac{1}{2} \times \frac{AB}{4} \times \frac{AB}{2}$$

$$= \frac{7AB^2}{16}$$

Hence, the required ratio = $\frac{3AB^2}{4} : \frac{7AB^2}{16}$ = 12 : 7.

26. The whole height of the cylinder is divided into '*n*' equal parts.

So the vertical spacing between two consecutive turns = $\frac{h}{n}$.

27. If we unfold the lateral surfaces of the cube. It becomes a rectangle of sides 4*n* & *n*, and the string becomes the diagonal. Hence the required length = $\sqrt{(4n)^2 + n^2} = n\sqrt{17}$ cm.

28. If we unfold the lateral surfaces of the cylinder, it becomes a rectangle of sides – 2π*r* & h; and the length of the string equals to the diagonal. Diagonal = 16(*n*)2 + h2 = 17(*n*2) or *h* = *n*

29. In Δ*BAC*, *AC* || *PQ*

$$\therefore \quad \frac{CQ}{QB} = \frac{AP}{PB} = \frac{4}{3}$$

In ΔBPC, $PC \parallel DQ$

$\therefore \frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$

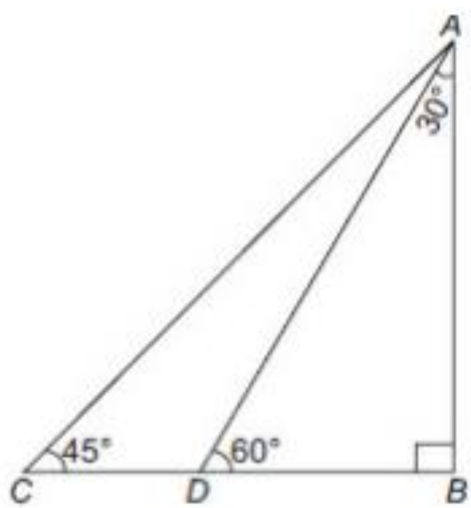
If $\frac{PD}{DB} = \frac{4}{3}$

$\therefore PD = \frac{4}{7} PB$

$$\frac{AP}{PD} = \frac{AP}{\frac{4}{7}PB} = \frac{7}{4} \times \frac{AP}{PB} = \frac{7}{4} \times \frac{4}{3} = \frac{7}{3}.$$

Option (c) is correct.

30. Let $AB = H$



$\therefore \tan 45° = \frac{AB}{BC} \rightarrow AB = BC$

$\therefore \tan 60° = \frac{AB}{BD} \therefore \sqrt{3} = \frac{AB}{BD}, BD = \frac{AB}{\sqrt{3}}$

$$\begin{aligned} CD &= BC - BD \\ \therefore \quad &= AB - \frac{AB}{\sqrt{3}} \end{aligned}$$

As time for traveling CD, i.e. $AB - \frac{AB}{\sqrt{3}}$ is 10 min.

$\therefore$ Time required for traveling BD = $\dfrac{\frac{AB}{\sqrt{3}}}{AB-\frac{AB}{\sqrt{3}}} \times 10$

$= \dfrac{1}{\sqrt{3}-1} \times 10$

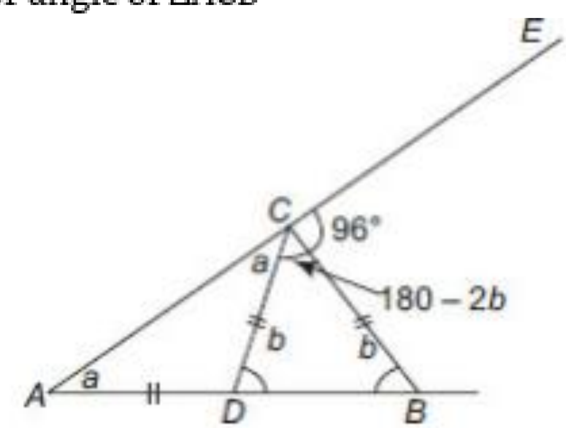
$= \dfrac{10}{\sqrt{3}-1}$

$= \left(\dfrac{10}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$= \dfrac{10(\sqrt{3}+1)}{2}$

$= 5(\sqrt{3}+1)$ minutes

31. $\angle ECB$ is exterior angle of ΔACB



$\angle A + \angle B = \angle ECB = 96°$ (Sum of interior opposite angles).

i.e. $a + b = 96°$ (1)

Also $a + (180 - 2b) + 96° = 180°$

$\therefore a - 2b + 96° = 0$

$\therefore a - 2b = -96°$ (2)

Solving (1) and (2)

$b = 64°$ and $a = 32°$

$\therefore \angle DBC = b = 64°$

32. Let the side of square be a.

Area of the square = a^2

Therefore, area of the largest circle which can be cut from the square = $\pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$

Area scrapped = $a^2 - \frac{\pi}{4}a^2 = a^2\left(1-\frac{\pi}{4}\right)$

$$\therefore \frac{\text{Area scrapped}}{\text{Area of square}} = \frac{a^2\left(1-\frac{\pi}{4}\right)}{a^2} = 1-\frac{\pi}{4}$$

As this ratio does not depend on the value of the side of the square.

Therefore, whether we cut a circle from smaller square or larger square, scrapped area will be a fixed percentage of square. Therefore, the required ratio will be 1 : 1.

33. Let $P_1 = P$ and $A_1 = A$. Then according to the question:

$$\frac{P_1 + P_2 + P_3 + \ldots}{A_1 + A_2 + A_3 + \ldots} = \frac{P + \frac{P}{\sqrt{2}} + \cdots \infty}{A + \frac{A}{2} + \cdots \infty} = \frac{\frac{P}{1-\frac{1}{\sqrt{2}}}}{2A}$$

$$= \frac{P\sqrt{2}}{\sqrt{2}-1} \times \frac{1}{2A}$$

$$= \left(\frac{\sqrt{2}P(\sqrt{2}+1)}{2A}\right)$$

$$= \frac{\sqrt{2} \times 4a(\sqrt{2}+1)}{2 \times a^2} \qquad (\text{since, } P = 4a,\ A = a^2)$$

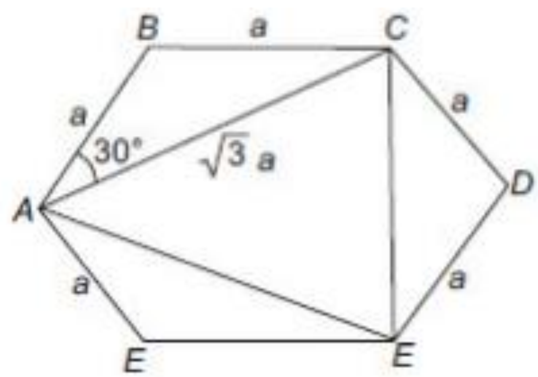
$$= \frac{\sqrt{2} \times 2(\sqrt{2}+1)}{a} = \frac{2(2+\sqrt{2})}{a}$$

34. We can see from the triangle CAT that, $\angle CAT = 100$. We can also see that $\angle OCA = 40$. Hence, $\angle OAC = 40$. Thus, $\angle OAB = \angle OBA = 40°$.

Thus, $\angle BOA = 180 - 40 - 40 = 100$.

35. Let the side of the hexagon $ABCDEF$ be 'a'

$\frac{a}{\sin 30°} = \frac{AC}{\sin 120°}$ or $AC = a\sqrt{3}$. So ΔACE is equilateral triangle with side $\sqrt{3}$.



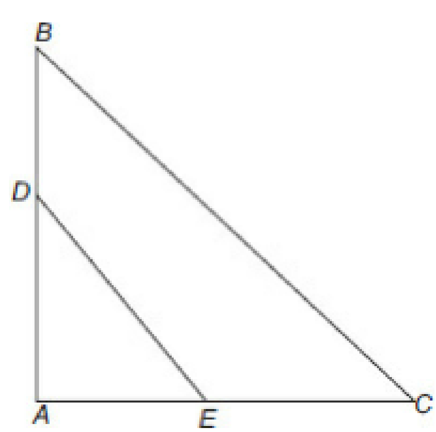
Area of hexagon = $\frac{\sqrt{3}}{4}a2 \times 6$

Area as $\angle ACE = \frac{\sqrt{3}}{4}(\sqrt{3}a)^2$

Therefore, the required ratio

$$= \frac{\sqrt{3}}{4}(\sqrt{3}a)^2 : \frac{\sqrt{3}}{4}a^2 \times 6 = 1:2$$

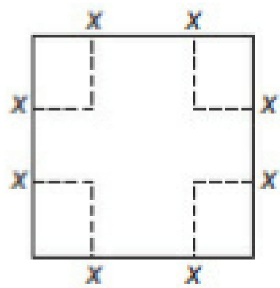
36.



DDAE & DBAC are similar triangles and the ratio of the area of the similar triangles is the ratio of the square of the corresponding sides of the triangle.

Hence, the area of the smaller triangle = 34 × 0.65 × 0.65 = 14.365

37. The length and breadth of the base would be $(12 - 2x)$ and $(12 - 2x)$ respectively. The height of the box would be 'x'. Thus, the total volume would be $(12 - 2x)(12 - 2x)x$. Checking the options, it can be seen that the volume is maximized for $x = 2$.



38. Let the length of the longer side be 'l' then according to the question:

$$\frac{l}{2} = \frac{2}{\frac{l}{2}} \Rightarrow \frac{l}{2} = \frac{4}{l} \text{ or } l = 2\sqrt{2}$$

Area of smaller rectangle = $\frac{l}{2} \times 2 = 2\sqrt{2}$ sq. units

39. In ΔPRO, $PR \parallel QS$. Then,

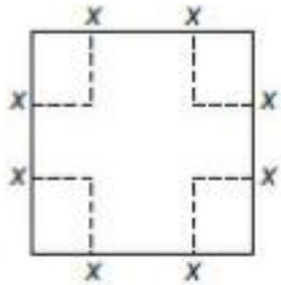
$$\frac{OP}{OQ} = \frac{PR}{QS} = \frac{4}{3}$$

$$\frac{28}{OQ} = \frac{4}{3}$$

$$OQ = 21\text{cm}$$

$$PQ = OP - OQ = 28 - 21 = 7 \text{ cm}$$

$$\frac{PQ}{OQ}=\frac{7}{21}=\frac{1}{3}$$



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$$\frac{28}{OQ}=\frac{4}{3}$$

$$OQ = 21\text{cm}$$

$$PQ = OP - OQ = 28 - 21 = 7 \text{ cm}$$

$$\frac{PQ}{OQ}=\frac{7}{21}=\frac{1}{3}$$

40. $PQ = 7 = PR + QS$

$$\frac{PR}{QS}=\frac{4}{3}$$

$$\Rightarrow \quad QS = 3 \text{ cm}$$

41. ΔOSQ is a right angle triangle.

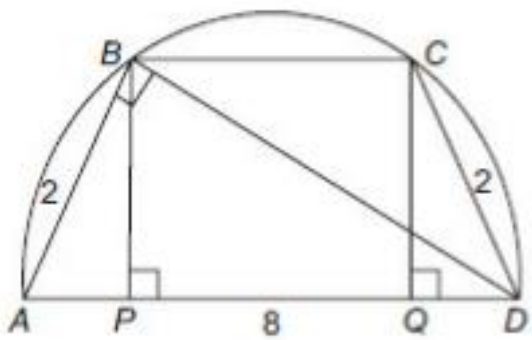
$$SO = \sqrt{OQ^2 - QS^2}$$
$$= \sqrt{21^2 - 3^2} = 12\sqrt{3} = \text{cm}$$

42. Let the length of the cube is '*a*'.

Length of side of triangle = $a\sqrt{3}$

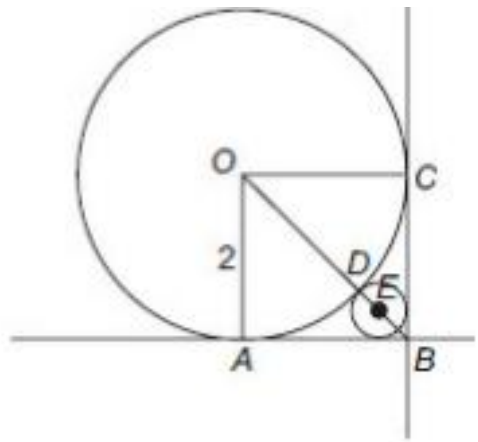
Radius of circumcircle of the triangle = $\frac{\text{side}}{\sqrt{3}} = a\sqrt{3}/\sqrt{3} = a$. Hence, option (a) is correct.

43.



$$\frac{1}{2} \times AB \times BD = \frac{1}{2} \times AD \times BP$$
$$2\sqrt{8^2 - 2^2} = 8 \times BP$$
$$BP = \frac{\sqrt{15}}{2}$$
$$AP = DQ = \sqrt{2^2 - \left(\frac{\sqrt{15}}{2}\right)^2} = \sqrt{4 - \frac{15}{4}} = \frac{1}{2}$$
$$BC = PQ = 8 - \left(\frac{1}{2} + \frac{1}{2}\right) = 7$$

44. Let the radius of smaller circle $ED = a$. Let E be the center of the smaller circle.



$\therefore EB = a\sqrt{2}$

$\therefore OB = EB + ED + OD$

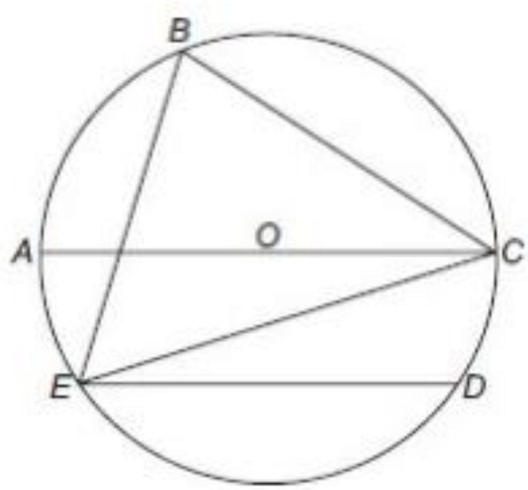
$= a\sqrt{2} + a + 2$

Also $OB = 2\sqrt{2}$

$\Rightarrow a\sqrt{2} + a + 2 = 2\sqrt{2}$

$\Rightarrow a = 6 - 4\sqrt{2}$

45.

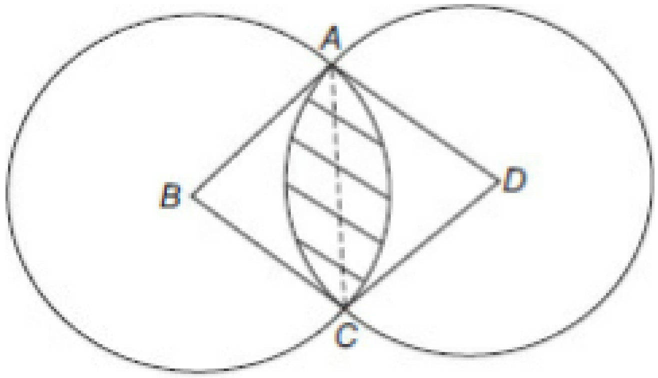


$\angle EAC = \angle CBE$ are the same (as they are in the same arc).

$\angle CEA = 90°$, (it is the angle subtended by the semi-circle)

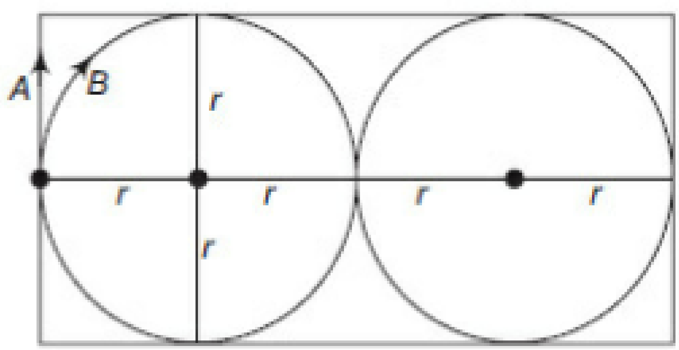
In ΔAEC, $\angle ECA = 180° - 65° - 90° = 25°$. As $AC \parallel ED$ then $\angle DEC = \angle ACE = 25°$.

46. Common area = 2 × (area of sector ADC – area of ΔADC)



$$= 2 \times \left(\frac{90°\pi}{360°} \times 1^2 - \frac{1}{2} \times 1 \times 1 \right) = \frac{\pi}{2} - 1$$

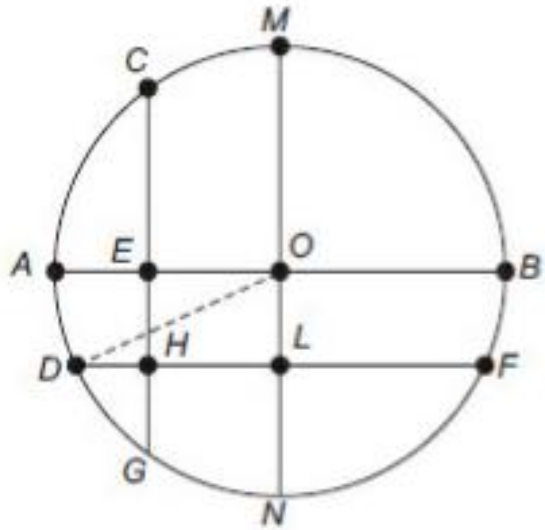
47. Distance covered by $A = 2r + 2r + 4r + 4r = 12r$



Distance covered by B = $2\pi r + 2\pi r = 4\pi r$. The percentage difference in speed should be such that B should be able to cover the extra distance in the same time. Since, time is constant, the percentage difference of speed would be equal to the percentage difference of distance. This, would be given by: $\frac{4\pi r - 12r}{12r} \times 100 = 4.72\%$ approximately.

48. In order to find DH, we can think of finding DL and subtracting HL from it.

As $NL : LM = 1 : 2$, $NL = 1$ cm, $ML = 2$ cm. Similarly $AE = 1$ cm, $BE = 2$ cm



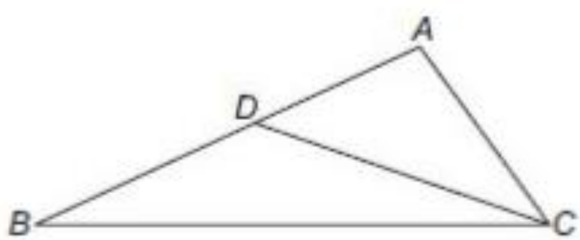
Construct OD, $OD = 1.5$ cm

$OL = ON - LN = 1.5 - 1 = 0.5$ cm

In ΔODL: $DL = \sqrt{OD^2 - OL^2} = \sqrt{1.5^2 - 0.5^2} = \sqrt{2}$ cm

$\Rightarrow DH = DL - HL = \sqrt{2} - \frac{1}{2} = \frac{2\sqrt{2} - 1}{2}$. Option (b).

49. Ratio of perimeters of two similar triangles is the ratio of corresponding sides of the triangles.



In ΔBDC and ΔBCA, $\angle B$ is common and $\angle BCD = \angle BAC$. Therefore, DBDC ~ DBCA.

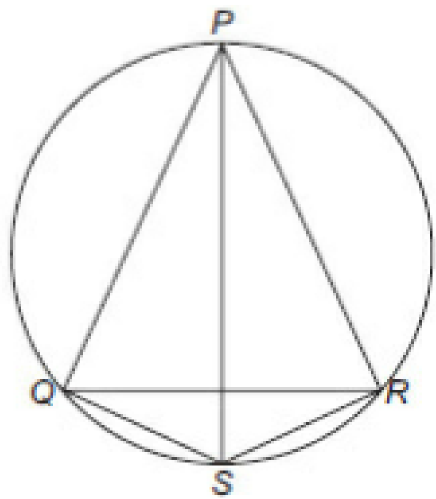
$$\frac{BD}{BC} = \frac{DC}{AC} = \frac{BC}{AB} \Rightarrow \frac{9}{12} = \frac{6}{AC} = \frac{12}{AB}$$

$AC = 8$ cm, $AB = 16$ cm.

$DA = 16 - 9 = 7$ cm

Perimeter of ΔDAC: Perimeter of $\Delta BDC = (6 + 7 + 8) : (9 + 6 + 12) = 21 : 27$ $= 7 : 9$.

50.



From the figure, it is clear that the triangle PRS is a 30 – 60 – 90 triangle with $\angle RPS$ as 30 and $\angle RSP$ as 60. Since $PS = 2r$, thus, $SR = SQ = r$ and $PR = PQ = r\sqrt{3}$. Hence, correct option is (a).

51. You would need to solve this using options. Suppose, there are 10 tiles along an edge of the rectangle – then all these edge tiles would be white. So, the number of white tiles would be $10 + 10 + x + x$ (where x is the number of unique tiles on the other edge of the rectangle).

With $x = 1$, the number of white tiles = 22, and the number of total tiles = $10 \times 3 = 30$. This cannot be the answer, as the number of red tiles would only be 8, which is less than required.

Next, take x as 2. In this case, White tiles = 24 and total tiles = $10 \times 4 = 40$. This cannot be the answer, as the number of red tiles would only be 16, which is less than required.

Next, take x as 3. In this case, White tiles = 26 and total tiles = 10 × 5 = 50. This cannot be the answer as the number of red tiles would only be 24, which is less than required.

Next, take x as 4. In this case, White tiles = 28 and total tiles = 10 × 6 = 60. This cannot be the answer, as the number of red tiles would be 32 which is more than required. Thus, we can reject 10 as the answer, as increasing x would only increase the number of red tiles further.

We, then check for option (b), whereby there could be 12 tiles on an edge of the rectangle.

Again, the number of white tiles would be 12 + 12 + x + x.

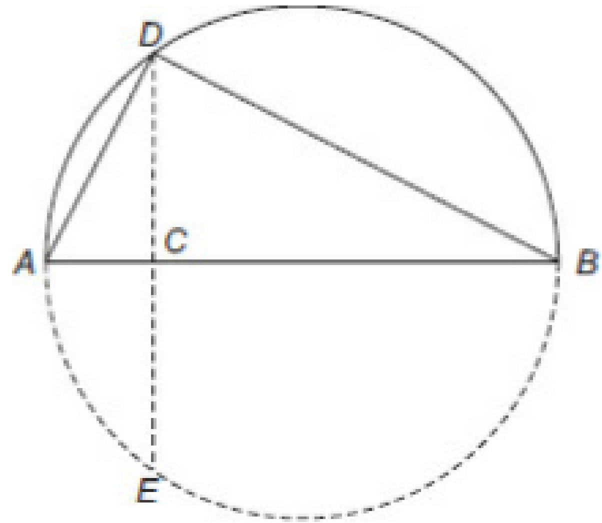
For x = 1, white tiles = 26 and total tiles = 12 × 3 = 36 tiles. This cannot be the answer, as the number of red tiles would be 10 only, which is less than required.

For x = 2, white tiles = 28 and total tiles 12 × 4 = 48 tiles. This cannot be the answer as the number of red tiles would be 20 only, which is less than required.

For x = 3, white tiles = 30 and total tiles 12 × 5 = 60 tiles. In this case, we see that the number of red tiles would also be equal to 30. Thus, 12 tiles along an edge is possible as an answer. Thus, the

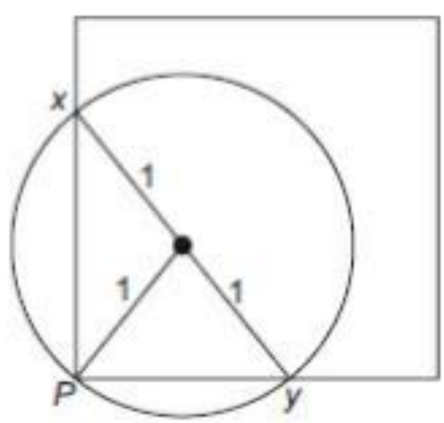
(b) option is correct.

52.



In the figure, we can see that the triangles *ACD* and *ADB* are similar to each other. Thus, the ratio of the legs of *ADB* would be 1 : 3 (since that is the ratio of the legs of *ACD* given to us). Also, using Pythagoras theorem, $AD = \sqrt{40}$, then $AB = 3\sqrt{40}$. Again using the Pythagoras theorem of triangle *ABD*, we get that *AB* = 20 = diameter of the semi-circle. Hence, the area of the semi-circle = $\frac{1}{2} \times \pi \times r2 = 50\pi$.

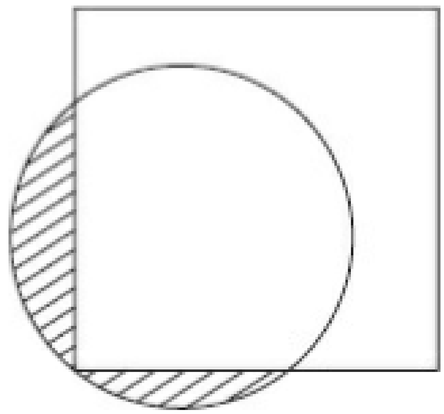
53.



The sheet area that remains after punching is = 4 – (Area of semicircle + Area of ΔPXY) = $\left(4-\left(\frac{\pi}{2}+\frac{1}{2}\times1\times2\right)\right)=\frac{6-\pi}{2}$

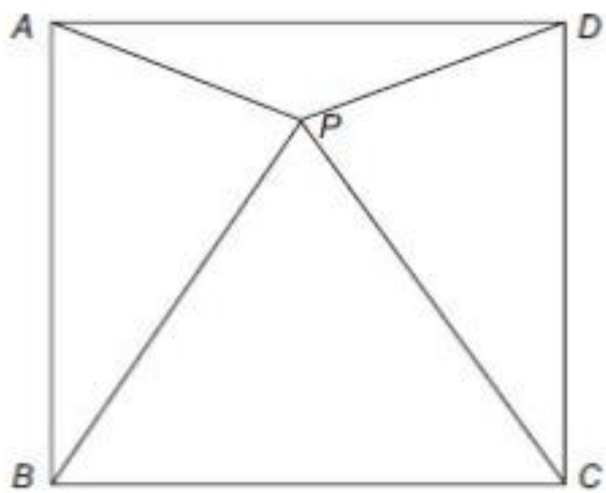
Required proportion = $\frac{6-\pi}{2}:4=\frac{6-\pi}{8}$

54. Area of the part of the circle (round punch) falling outside the square sheet = Area of the circle – (Area of semicircle + Area of ΔPXY) = π (1)2 – $\left(\frac{\pi}{2}+1\right)$



$$= \pi - \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$$

55.



In the figure, ∠BPC = 60 (given) and also, *BP* = *AB*. Hence, if you were to look at the triangle *ABP*, it is isosceles. Also, Angle *ABP* = 30 and the angles *APB* and *PAB* are equal (as they are opposite equal sides of the isosceles triangle). This means, that in the central angle *P*, *BPC* = 60 and *APB* = 75 = *CPD*.

Hence, the required angle $APD = 360 - 60 - 75 - 75 = 150$, option (e).

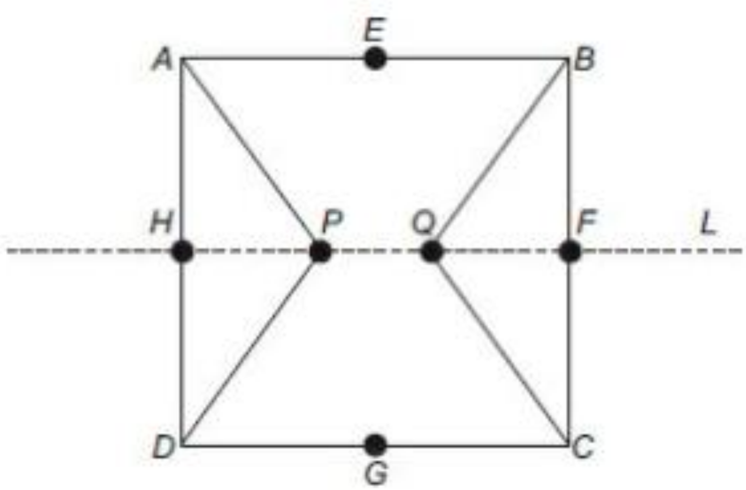
56. Radius of a circle circumscribing a triangle = circum-radius of the triangle

$$= \frac{abc}{4 \times (\text{Area of the triangle})}$$

$$= \frac{a \times b \times c}{4 \times \left(\frac{1}{2} \times b \times AD\right)} = \frac{a \times c}{2 \times AD} = \frac{17.5 \times 9}{2 \times 3} = 26.25 \text{ cm}^2$$

57. The set of all possible triangles is given by the set –{8, 15, –8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22}. But, we have to look for obtuse angled triangles only. In this context we could go through two alternate possibilities: If 15 is the largest side –in such a case, the third side should be $\sqrt{161}$ for a right angled triangle. In case, the side is greater than this value, then we will get an acute angled triangle. Hence, for this case we will get the possible values as 8, 15, 8 to 8, 15, 12. After this value uptil 8, 15, 16, we will again get an acute angled triangle (8, 15, 17 being a Pythagoras triplet, the triangle with those dimensions would be right angled). After that, 8, 15, 18 to 8, 15, 22 would all be obtuse angled triangle. Thus, there would be 10 triangles which satisfy the given requirements. We will mark Option (c) as the correct answer.

58.



Area of *ABQCDP* = Area of *ABCD* – (Area of triangle *APD* + Area of triangle *BQC*)

Given that Angle *APD* = 1200 = Angle *BQC*.

By symmetry triangles, *APD* and *BQC* are similar and have equal area.

Hence, Area of *ABQCDP* = Area of *ABCD* – 2(Area of triangle *APD*)

Now, area of triangle *APD* can be given as:

Let *DH* be '*x*',

⇒ $DH/HP = \tan 60 = \sqrt{3}$

⇒ $HP = x/\sqrt{3}$ cm

⇒ Area of triangle *APD* = $2x \times \frac{1}{2} \times \frac{x}{\sqrt{3}} = \frac{x^2}{\sqrt{3}}$ cm

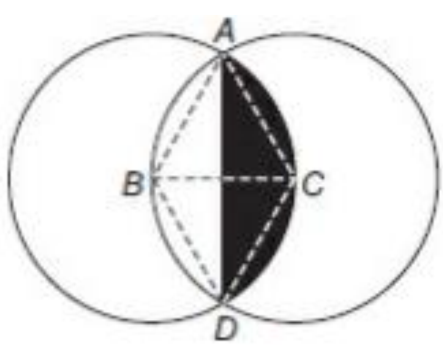
⇒ Area of *ABQCDP* = Area of *ABCD* – 2(Area of triangle *APD*)

Hence, required ratio can be given as = Area of *ABQCDP*/(2(Area of triangle *APD*))

Required ratio = $2\sqrt{3} - 1$

Hence, option (e) is the correct option.

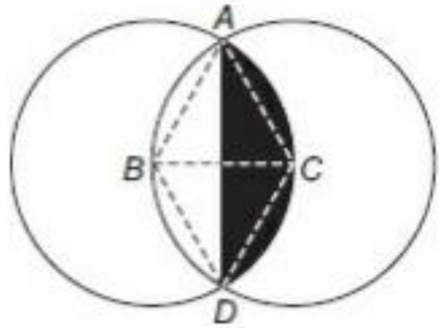
59. Both the circle have radii 1 cm then *AB* = *BC* = *CD* = *AC* = 1 cm.



Therefore, Δ*ABC* and Δ*DBC* are equilateral triangle.

Hence, $\angle ABD = \angle ABC + \angle DBC = 60° + 60° = 120°$

59. Both the circle have radii 1 cm then $AB = BC = CD = AC = 1$ cm.



Therefore, ΔABC and ΔDBC are equilateral triangle.

Hence, $\angle ABD = \angle ABC + \angle DBC = 60° + 60° = 120°$

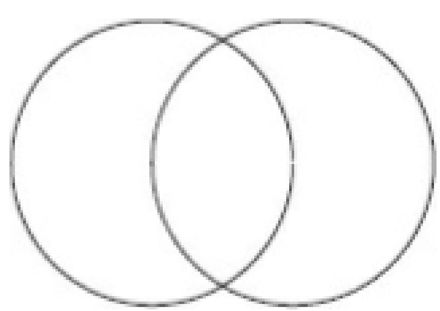
Area of the shaded portion = Area of sector $BACD$ – Area of $\Delta ABD = \frac{120}{360} \cdot 1 \cdot 1 \cdot \sin 120° = = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

Required area = 2 · Area of the shaded portion = $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)2 = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$sq.cm

Alternate Solution:

Before thinking about this problem through mensuration based processes, try thinking of it this way:

Draw a rough sketch of this situation by drawing two intersecting circles as described (both with radii 1).



A close guess estimate based on the look of the figure tells you that the required shaded area should be less than half of the area of the circle – which is 3.14 (since the radius is 1). Hence, we need an answer that is slightly lower than 1.57. At this stage, try to check the values of the options.

See what happens when you do this,

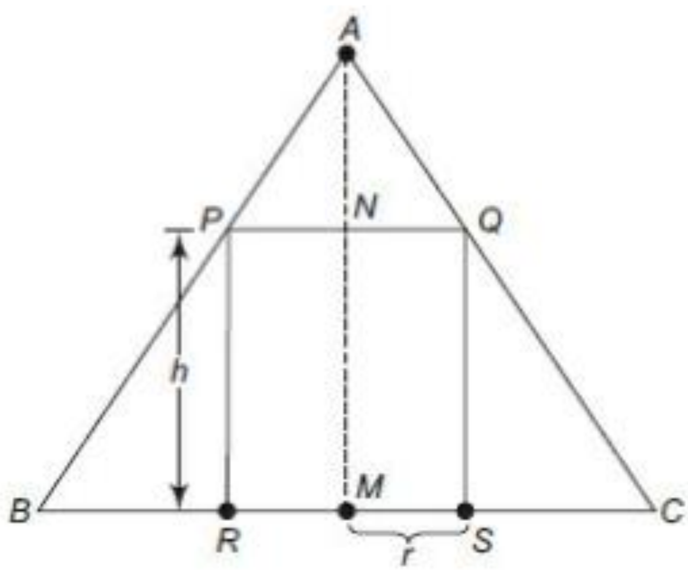
Option (a) gives a value of 0.69 (approx) which is too small.

Option (b) gives a value of 2.95 (approx) which is too large.

Option (c) gives a value of 3.33 (approx) which is too large.

Option (d) gives a value of 5.05 (approx) which is too large again. This leaves us with only option (e), whose value is 1.24 which is the only feasible value.

60. Let the height of the cylinder be 'h' cm and radius be 'r' cm.



ΔANQ is similar to ΔQSC

$$\Rightarrow \frac{AN}{NQ} = \frac{QS}{SC} \Rightarrow \frac{10-h}{r} = \frac{h}{4-r}$$

$$\Rightarrow \frac{10}{h} - 1 = \frac{r}{4-r} \Rightarrow \frac{10}{h} = \frac{4}{4-r}$$

$$\therefore h = \frac{5}{2}(4-r)$$

Surface area of the cylinder *PQRS*

$$= 2\pi[r2 + hr] = 2\pi\left[r^2 + \frac{5r}{2}(4-r)\right]$$

$$= 2\pi\left[r^2 - \frac{5}{2}r^2 + 10r\right] = 2\pi\left[10r - \frac{3}{2}r^2\right]$$

$$= 2\pi\left[-\frac{3}{2}\left(r - \frac{10}{3}\right)^2 + \frac{50}{3}\right]$$

(**Note:** This is a key step in this problem to realize that the expression $\left[10r - \frac{3}{2}r^2\right]$ can be written as a perfect square plus a constant as: $\left[-\frac{3}{2}\left(r - \frac{10}{3}\right)^2 + \frac{50}{3}\right]$.

Surface area will be maximum when $r - 10/3 = 0$ or $r = 10/3$. In this case, h = 5/3

So, largest surface area

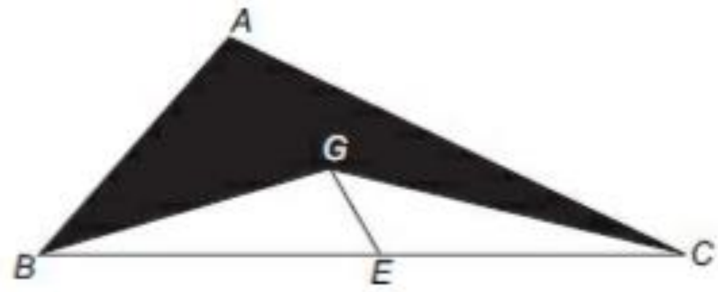
$$= 2\pi[r^2 + hr] = 2\pi\left[\frac{100}{9} + \frac{50}{9}\right] = 2\pi\left[\frac{50}{3}\right] = \frac{100\pi}{3}$$

61. Area of triangle ABC = $\sqrt{S(S-a)(S-b)(S-c)}$ where $S = \frac{40+25+35}{2} = 50$

$$= \sqrt{50(50-40)(50-35)(50-25)}$$

$$= \sqrt{50.10.15.25} = 250\sqrt{3}$$

Let *E* is the mid-point of *BC*.

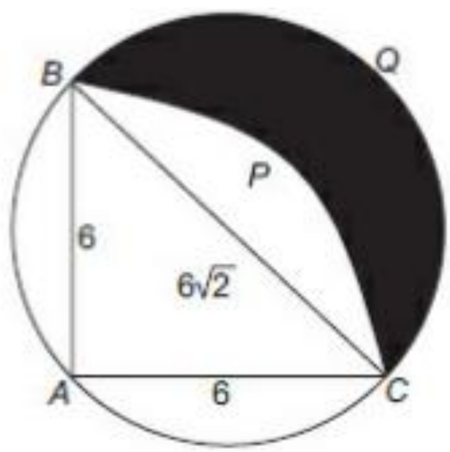


Area of ΔBGE = Area of $\Delta CGE = \frac{1}{6}$Area of ΔABC

Area of the remaining portion (after cutoff of triangle BGC)

$$= \frac{4}{6} \times 250\sqrt{3} = \frac{500}{\sqrt{3}}$$

62.



Area of arc BPC = Area of circular segment $ABPC$ – Area of ΔABC

$$= \frac{1}{4}\pi(6)^2 - \frac{1}{2} \times 6 \times 6$$

$$= 9\pi - 18$$

Area of semicircle $BQC = \frac{\pi}{2} \times r^2 = \frac{\pi \times 3\sqrt{2}^2}{2} = 9\pi$

Area of shaded portion = Area of semi circle BQC – Area of arc BPC = 9π – $(9p - 18)$ = 18 sq. cm

63. Let the side of five cubes are 1 cm, 1 cm, 2 cm, 3 cm, 3 cm respectively.

Sum of surface areas of the cubes = 6[12 + 12 + 22 + 32 + 32] = 144 sq. cm.

Volume of the five cubes = 13 + 13 + 23 + 33 + 33 = 64 cm3

Side length of the original cube = (64)1/3 = 4 cm.

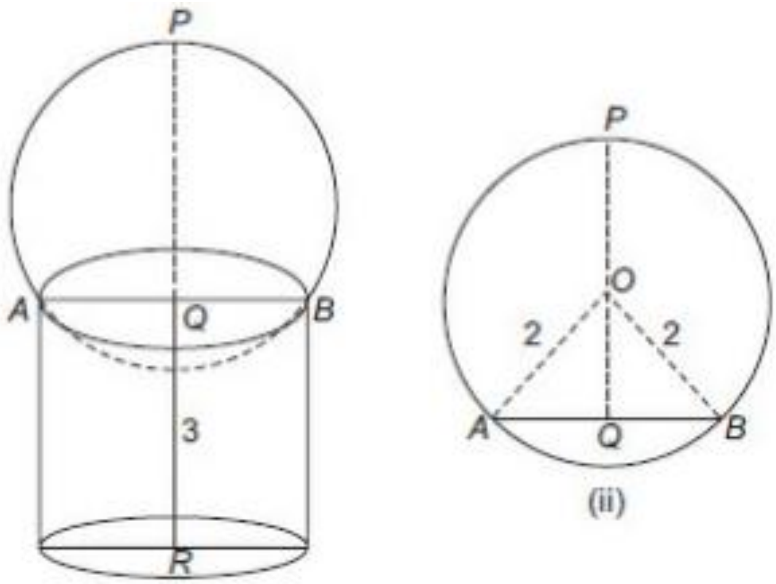
Surface area of the original cube = 6(4)2 = 96 sq. cm.

Required percentage increase from 96 to 144 sq. cm is 50%.

64. Let the radius of the cylinder is 'r' cm

$\pi r2h = V = 9\pi$ (given). Since, $h = 3$ cm (given), $r = \sqrt{3}$.

As the radius of the cylinder is not equal to the radius of the sphere so the sphere would be at the top of the cylinder as shown in the following diagram.



Distance of the top of the sphere from bottom of the cylinder = $PQ + QR = PQ + 3$

In diagram II:

In D $AOQ \rightarrow AQ = AB/2$ = Radius of the cylinder = $\sqrt{3}$.

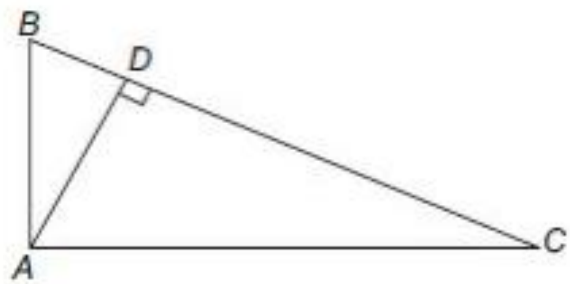
AO = Radius of the sphere = 2

$OQ = \sqrt{AO^2 - AQ^2} = 1$

Hence, $PQ = PO + OQ = 2 + 1 = 3$ cm

Thus, the height of the top of the sphere from the base of the cylinder = $PQ + QR = 3 + 3 = 6$ cm.

65.



To reach at the hypotenuse *BC* in minimum time we have to follow the shortest possible path which is *AD*, where *AD* ^ *BC*.

As we know that, $\frac{1}{AD^2} = \frac{1}{AB^2} + \frac{1}{AC^2} = \frac{1}{15^2} + \frac{1}{20^2}$

By solving the above equation we get, AD = 12 km.

Required time to reach at $BC = \frac{12}{30}$ h or $\frac{12}{30} \times 60 = 24$ min

66. $y = |x - 1| + |x + 1|$

For $x < -1$

$y = -(x - 1) - (x + 1) = -2x$

For $-1 < x < 1$

$y = -x + 1 + x + 1 = 2$

For $x > 1$

$y = x - 1 + x + 1 = 2x.$

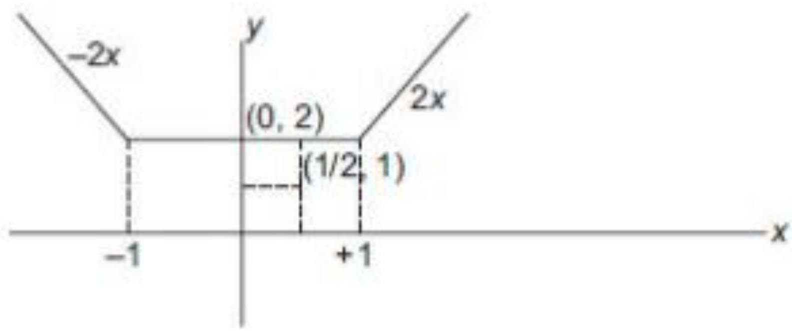
For $-1 < x < 1$

$y = -x + 1 + x + 1 = 2$

For $x > 1$

$y = x - 1 + x + 1 = 2x.$

Thus, the graph of the function, $y = |x - 1| + |x + 1|$ can be drawn as follows:

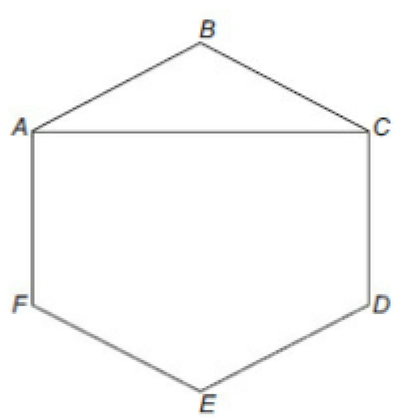


From the above curve it is clear that the shortest distance between (1/2, 1) and the curve y would be if we move vertically from the point $\left(\frac{1}{2}, 1\right)$ to the height of 2. This distance would be = 2 – 1 = 1.

67. First of all we need to find the length of AC.

In DABC:

$\angle BCA = \angle BAC = 120° - \angle CAF = 120° - 90° = 30°$



Apply sine rule in triangle *ABC*:

$$\frac{AB}{\sin 30°} = \frac{AC}{\sin 120°}$$

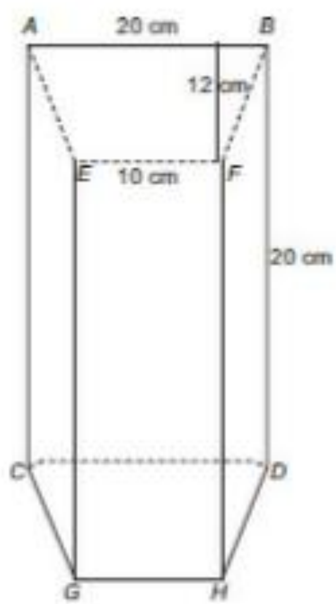
$$\frac{1}{\sin 30°} = \frac{AC}{\sin 120°}$$

$$AC = \frac{1}{\sin 30°} \cdot \sin 120° = \sqrt{3} \text{ cm}$$

$$\left(\text{Using } \sin 30 = \frac{1}{2} \text{ and } \sin 120 = \frac{\sqrt{3}}{2}\right).$$

Area of the square with side *AC* = $\sqrt{3}^2$ = 3 sq. cm

68.



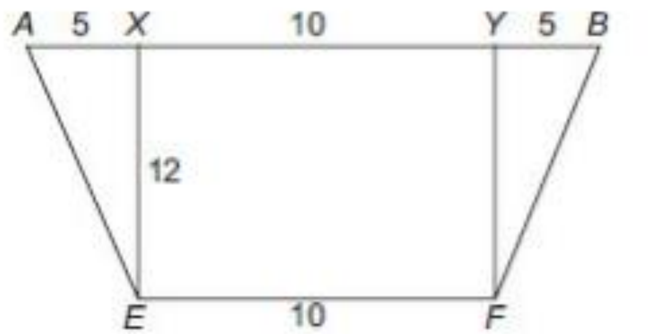
Top and bottom surfaces are trapeziums and the rest are rectangular in shape.

Sum of areas of top and the bottom surfaces = 2(Area of *ABFE*) = 2 $\left[\frac{10+20}{2} \cdot 12\right]$ = 360 square cm.

Note: Here we have used the formula for the area of a trapezium =

$\frac{\text{Sum of parallel Sides}}{2}$ × distance between the parallel sides.

The areas of the lateral sides of the pillar would be equal and both would be rectangles. To find the area of *AEGC*, we need the length of *AE*.



$$AE = \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \text{ cm}$$

Sum of areas of *AEGC* and *BFHD* = 2(Area of *AEGC*) = 2(13 × 20) = 520 sq. cm

Area of *ABCD* = 20 × 20 = 400 sq. cm.

Area of *EFHG* = 10 × 20 = 200 sq. cm.

Sum of all the six surfaces of the pillar = 360 + 520 + 400 + 200 = 1480 sq. cm.

69. Let the length and breadth of the rectangular park are *L* and *B* respectively.

Case I: If we are given the sum of the two lengths and one breadth of the rectangle: as 400:

We have: $2L + B = 400$

$B = 400 - 2L$

Area = $L \times B = L \times (400 - 2L) = 400L - 2L2$

The area is maximum when the differentiation of this expression is 0:

Thus: d(area)/dL = 400 – 4L = 0

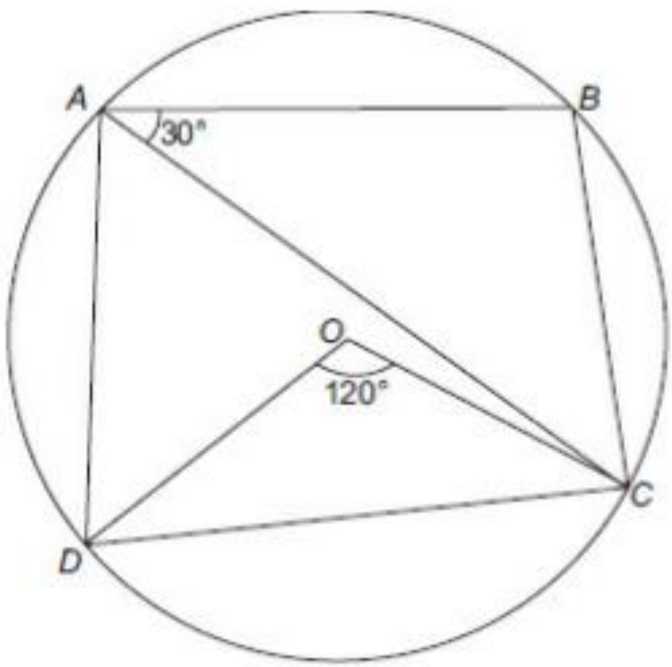
L = 100m → B = 200 m.

Case II: If we are given the sum of the two shorter sides and one longer side as 400:

We have: $2B + L = 400$

$L = 400 - 2B$

70.



$$\angle CAD = \frac{\angle COD}{2} = \frac{120^\circ}{2} = 60^\circ$$

$\angle BAD = \angle CAD + \angle BAC = 60^\circ + 30^\circ = 90^\circ$

Since, the Quadrilateral is a cyclic quadrilateral, the sum of the opposite angles would be 180. Hence, $\angle BCD = 90^\circ$

71. As the distance of P from all the three sides AB, BC and CA are equal so it means 'P' is the in-centre of the triangle ABC.

Let $AC = BC = a$

$$\therefore \quad r = \frac{\Delta}{s} = \left(\frac{\frac{1}{2} \times a \times a}{\frac{a + a + a\sqrt{2}}{2}} \right)$$

$$= \frac{a}{\sqrt{2}}(\sqrt{2} - 1) = 4(\sqrt{2} - 1)$$

$$a = 4\sqrt{2} \text{ cm.}$$

Area of the triangle = $\frac{1}{2} \times 4\sqrt{2} \times 4\sqrt{2}$ sq cm.

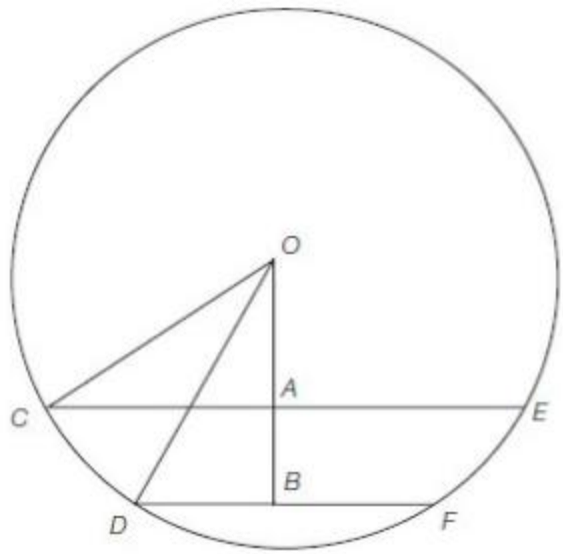
72. Diagonals of the rectangle bisects each other it means point $\left(\frac{2+6}{2}, \frac{3+5}{2}\right) = (4, 4)$ lies on the other diagonal $y = 3x + c$.

Therefore, $4 = 3.4 + c$ or $c = 4 - 12 = -8$.

73. When you cut off the tip of the cone, it creates a frustum of the cone at the bottom. However, the part of the cone that is cut off at the top is similar to the original cone. This thought is crucial to solve this question quickly and gives us the solution thinking in the following manner. Since, the cut is made at a height of 9 ft from the base; it is made at a height of 3 ft from the top. Thus, the cut off part of the cone would be a cone with height 3. The original cone, being a cone of height 12, it means that the new cut-off cone has a height as 1/4th of the original cone. Using the concept of similarity and the concept, that if the lengths are made as 1/4th the original in two similar objects, the volumes would be 1/64th, we can easily solve this question as:

Now, the volume of the original cone = $\frac{1}{3} \times \pi \times r2 \times h = 64\pi$. Volume of the cut off cone = $\frac{1}{64} \times 64\pi = \pi$. Thus, the volume of the frustum of the cone = $64\pi - \pi = 63\pi = 63 \times \frac{22}{7} = 198$ cm3.

74. Let O be the centre of the circle and CE and DF are the parallel chords of lengths 6 cm and 4 cm respectively.



Draw $OB \perp DF$ which cuts CE at A.

As CE and DF are parallel so, $OA \perp CE$.

$CA = CE/2 = 3$. $DB = DF/2 = 2$ cm

Let 'r' be the radius of the circle. $OC = OD = r$

In Δ OAC:

$OA = \sqrt{r^2 - 3^2} = \sqrt{r^2 - 9}$

In Δ ODB:

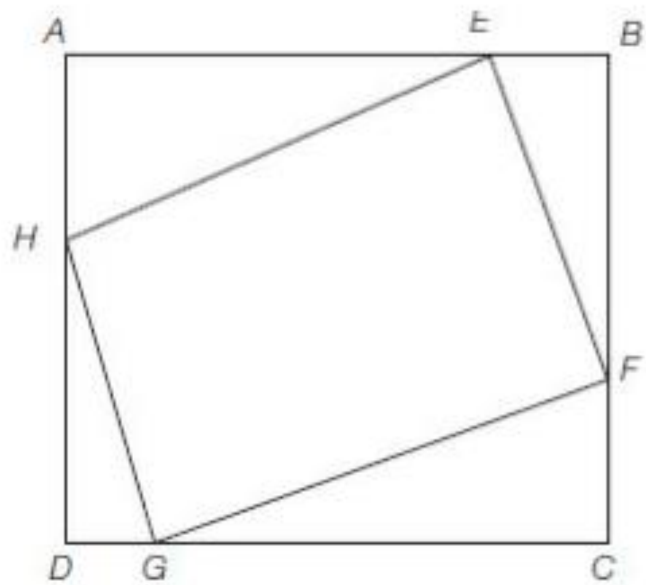
$OB = \sqrt{r^2 - 2^2} = \sqrt{r^2 - 4}$

It is given to us that: $AB = 1 = OB - OA = \sqrt{r^2 - 4} - \sqrt{r^2 - 9}$

Now check the options: The above equation satisfies for $r = \sqrt{13}$ cm.

Hence, Option (b) is correct.

75.



Let $AB = a$

Area of $ABCD = a2$

Area of $EFGH$ = 62.5% of $a2 = 0.625\ a2$

As $EFGH$ is a square, then $EF = FG = GH = HE$. This is possible only when $AE = BF = GC = DH$ and $EB = FC = DG = AH$.

Now check the options:

Option (d): $EB : CG = 1 : 3$

$EB = k, CG = 3k$

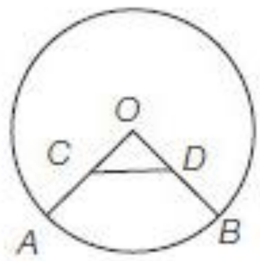
$AB = AE + EB = k + 3k = 4k$. So the area of $ABCD = 16k2$

$EH = \sqrt{k^2 + (3k)^2} = k\sqrt{10}$ cm

Area of $EFGH = 10k2$

In this case, area of $EFGH$ is 62.5% of $ABCD$. Hence, Option (d) is correct.

76.



$OA = OB = 1$ cm

$$R = \pi(1)^2 \times \frac{60^\circ}{360^\circ} = \frac{\pi}{6}$$

$$\text{Area of } \Delta\, OCD = \frac{\sqrt{3}}{4}(OC)^2 = \frac{\pi}{12}$$

$$\sqrt{3}(OC)^2 = \frac{\pi}{3}$$

$$OC = \left[\frac{\pi}{3\sqrt{3}}\right]^{1/2}$$

Hence, Option (d) is correct.

77. Let *ABC* is the equilateral triangle *T*1. After meeting the mid-points of *AB* and *AC*, a second triangle *ADE* is formed which is also an equilateral triangle. Similarly *T*3, *T*4, ... are equilateral triangles.

AB = 24 cm, each side of *T*2 must be 12 cm, each side of *T*3 = 6 cm and so on.

$$\text{Area of } T1 = \frac{\sqrt{3}}{4}(24)^2 \text{ cm}^2$$

$$\text{Area of } T2 = \frac{\sqrt{3}}{4}(12)^2 \text{ cm}^2$$

$$\text{Area of } T3 = \frac{\sqrt{3}}{4}(6)^2 \text{ cm}^2$$

Required sum = $\frac{\sqrt{3}}{4}(24)^2 + \frac{\sqrt{3}}{4}(12)^2 + \frac{\sqrt{3}}{4}(6)^2 + \ldots$ would be the sum of this infinite geometric progression.

$$= = \frac{\sqrt{3}}{4}\left[24^2 + \frac{(24)^2}{4} + \frac{24^2}{16} + \ldots\right] = \frac{\sqrt{3}}{4}\left[\frac{24^2}{1-\frac{1}{4}}\right]$$

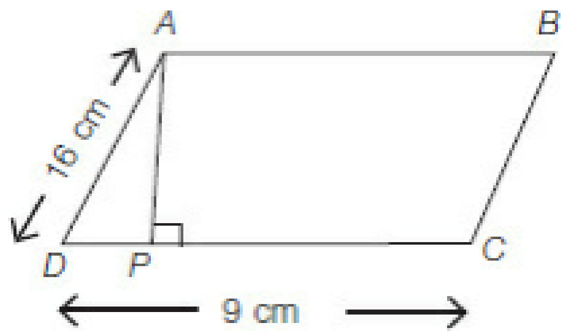
= $192\sqrt{3}$cm2.

Hence, option (a) is correct.

78. The diagonal of the rectangle must be equal to the diameter of the circle.

This is true only for option (d), i.e. $\sqrt{24^2 + 10^2}$ = 26 cm.

79.



16 × 9 × sin $\angle ADC$ = 72

sin $\angle ADC$ = 1/2

$\angle ADC$ = 30º

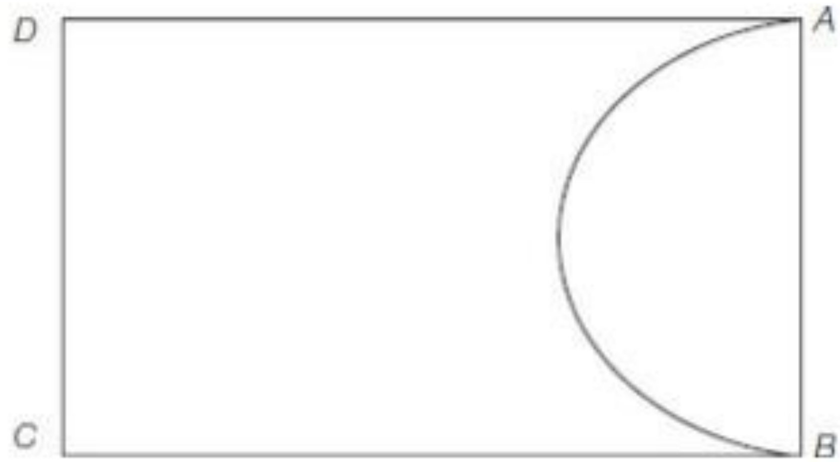
DP = 16 cos 30º = $8\sqrt{3}$ cm.

Area of $\Delta APD = \frac{1}{2} \times 16 \times 8\sqrt{3} \times$ sin $\angle ADP$

$$= \frac{1}{2} \times 16 \times 8\sqrt{3} \times \frac{1}{2} = 32\sqrt{3} \text{ cm}^2$$

Hence, Option (c) is correct.

80.



Area of semicircular portion $AB = 72\pi$

$$\pi\left(\frac{AB}{2}\right)^2 \times \frac{1}{2} = 72\pi$$

$AB = 24\ cm$

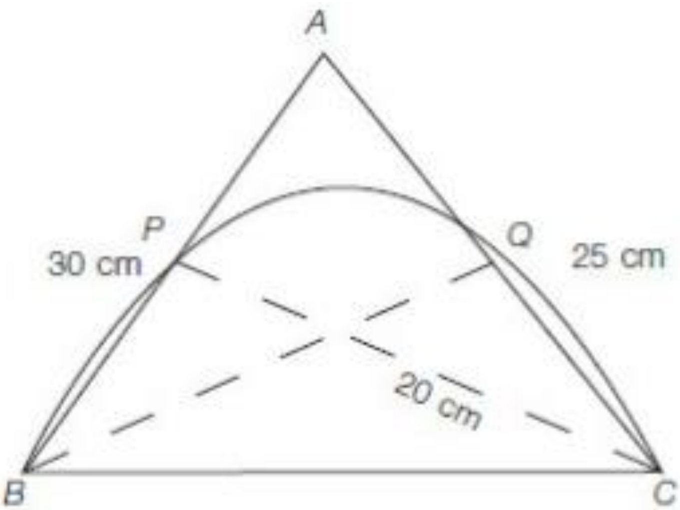
$AB^*AD = 768$

$AD = 768/AB = \frac{768}{24} = 32$ cm

Perimeter of semicircular portion $AB = \pi\left(\frac{AB}{2}\right) = \frac{\pi(24)}{2} = 12\pi$ cm

Perimeter of the leftover portion = $AD + DC + BC$ + Perimeter of semicircular portion $AB = 32 + 24 + 32 + 12\pi = 88 + 12\pi$ cm. Hence, Option (b) is correct.

81.

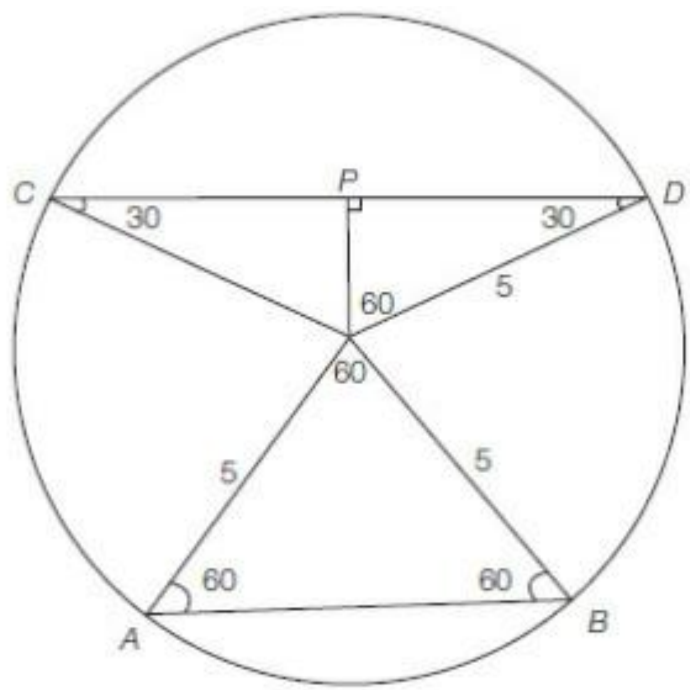


In triangle ABC: $\angle BPC = \angle BQC = 90°$
$BQ \perp AC,\ CP \perp AB$

Area of triangle= $\frac{1}{2} \times AB \times CP = \frac{1}{2} \times AC \times BQ$

$$BQ = \frac{AB \times CP}{AC} = \frac{30 \times 20}{25} = 24 \text{ cm}$$

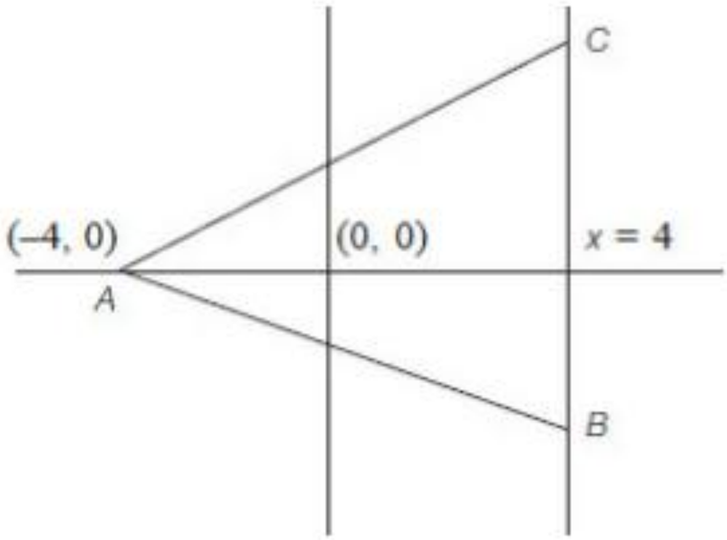
82. Let the centre of the circle be '*O*'. Chord *AB* subtends 60° at the centre of the circle and chord *CD* subtends 120° at the centre of the circle. Draw a perpendicular *OP* from the centre to *CD*, The figure for these would be as shown below.



In the triangle *DOP*, which is a 30 – 60 – 90 triangle, we can see that the side opposite the 90° angle is 5. Hence, the side opposite the 60° angle, i.e. $PD = 2.5\sqrt{3}$.

Also, $CD = 2PD = 5\sqrt{3}$.

83.



The distance between *A* and (0, 0) is minimum possible when *A* lies on the *X*-axis.

In this case, the minimum possible distance = 4. Hence, Option (c) is correct.

84.

Draw $AE \perp CD$

According to the question:

$AE \times CD = 48$ OR $AE \times 8 = 48$ OR $AE = 6$ cm

As, $AD \geq AE$ OR $AD \geq 6$ cm. Hence, Option (c) is correct.

85. Solve this question using options.

For option (d), if we assume the sides of the rectangle as 'n' and '$4n$' respectively, we get:

Area of the rectangle = $4n2$ and its perimeter as $10n$. Hence, the square of the perimeter would be $100n2$. In this situation, the ratio of the area of the rectangle and the square of its perimeter would be 1 : 25. Hence, Option (d) is the correct answer.

86. The number of paths from (1, 1) to (8, 10) via (4, 6) =

[The number of paths from (1, 1) to (4, 6)] × [The number of paths from (4, 6) to (8,10)]

The number of steps from (1, 1) to (4, 6) in x-direction = 4 – 1 = 3.

The number of steps from (1, 1) to (4, 6) in y-direction = 6 – 1 = 5

Hence, the number of paths to reach from (1, 1) to (4, 6) = (3 + 5)c3 = 8c3 = 56.

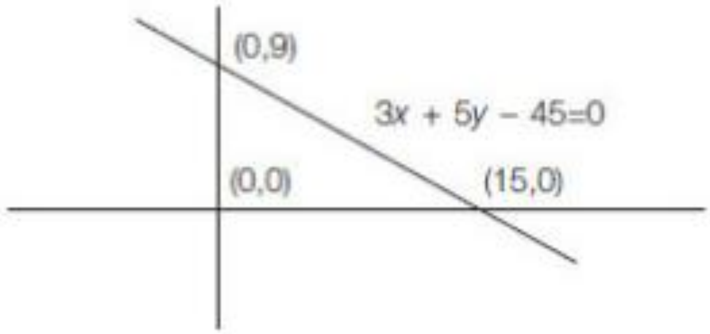
The number of steps from (4, 6) to (8, 10) in x-direction = 8 – 4 = 4.

The number of steps from (4, 6) to (8, 10) in y-direction = 10 – 6 = 4.

Hence, the number of paths to reach from (4, 6) to (8, 10) = (4 + 4)c4 = 8c4 = 70.

The number of paths from (1, 1) to (8, 10) = 56 × 70 = 3920

87.



The above triangle is a right-angle triangle. Circum-radius of the triangle $= \frac{1}{2}$ of the length of the hypotenuse $= \frac{1}{2} \times \sqrt{9^2 + 15^2} = 8.5+$

Hence, the integer closest to $L = 9$.

88. Assuming the dimensions of the brick is a, b and c. Let the lengths of the diagonals be

$3k : 2\sqrt{3}k : \sqrt{15}k$

$a^2 + b^2 = 9k^2$ (1)

$b^2 + c^2 = 12k^2$ (2)

$c^2 + a^2 = 15k^2$ (3)

$2(a^2 + b^2 + c^2) = 36k^2$ or $a^2 + b^2 + c^2 = 18k^2$(4)

Equation (4) – equation (1) : $c^2 = 9k^2$ or $c = 3k$

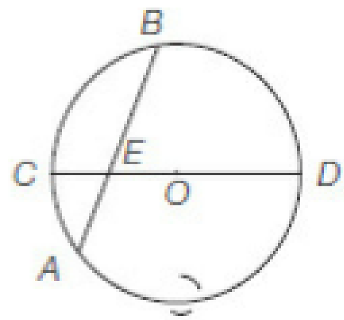
Equation (4) – equation (2) : $a^2 = 6k^2$ or $a = k\sqrt{6}$

Equation (4) – equation (3) : $b^2 = 3k^2$ or $b = k\sqrt{3}$

Hence, the required answer: $\sqrt{3}$

Hence, Option (b) is correct.

89.



$AE \times BE = CE \times DE$

$AE\,(20.5 - AE) = 7 \times 15$

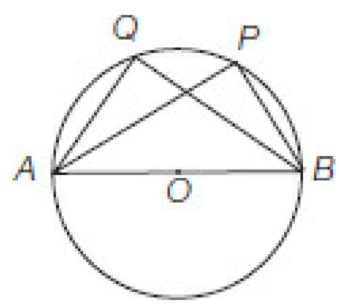
$AE\,(20.5 - AE) = 105$

On solving we get: AE = 10 or 10.5

Required difference = 10.5 – 10 = 0.5 cm

Hence, Option (c) is correct.

90.



ΔAPB & ΔAQB are right angle triangles.

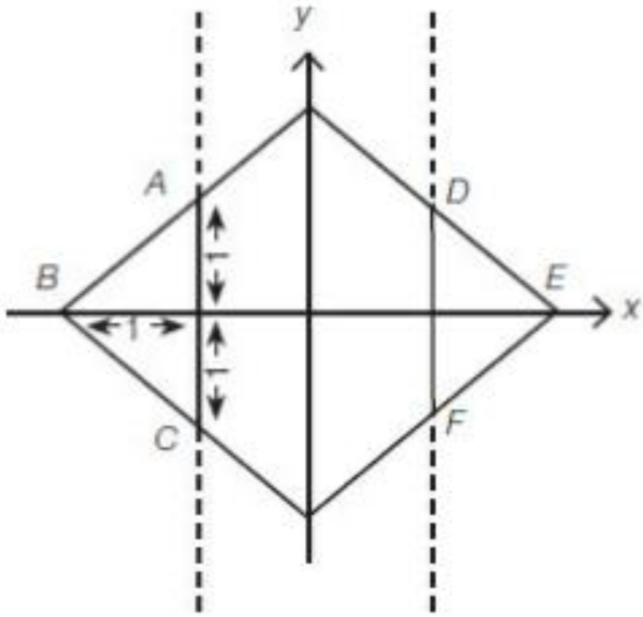
$AP = \sqrt{10^2 - 6^2}$ = 8cm.

AQ = AP/2 = 8/2 = 4 cm.

$BQ = \sqrt{10^2 - 4^2}$ = 9.1 cm.

Hence, Option (c) is correct.

91.



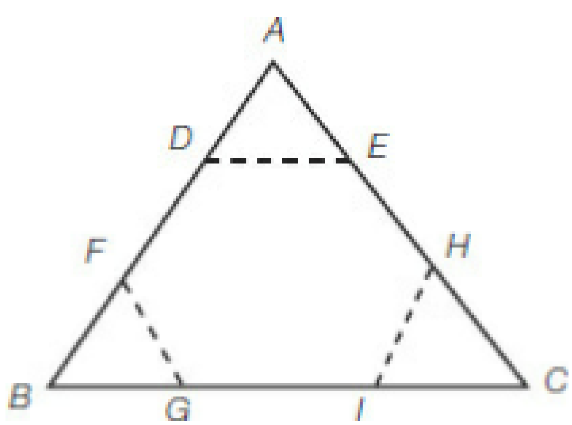
ΔABC and ΔDEF represent the area $|x| + |y| \leq 2$ and $|x| \geq 1$.

Area of $\Delta ABC = \frac{1}{2} \times 1\ (1 + 1) = 1$

Area of $\Delta DEF = \frac{1}{2} \times 1\ (1 + 1) = 1$

Required area = 1 + 1 = 2

92.



Let ABC be an equilateral triangle, with $AB = BC = AC = 6a$. If we connect the points D, E, H, I, G and F on this so as to make the hexagon $DEHIGF$ into a regular hexagon, we would need: $AD = BF = BG = IC = CH = AE = 2a$.

This means that the side of the hexagon would be $2a$.

The area of the triangle =

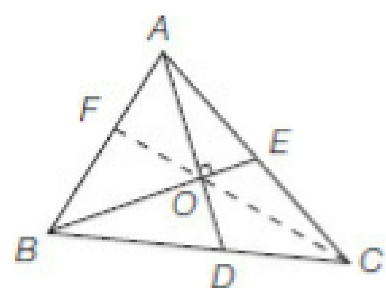
$$\frac{\sqrt{3}}{4}(side)^2 = \frac{\sqrt{3}}{4}(6a)^2 = 9\sqrt{3}a^2$$

The area of the regular hexagon is given by:

$$\frac{6\times\sqrt{3}}{4}(side)^2 = \frac{6\times\sqrt{3}}{4}(2a)^2 = 6\sqrt{3}a^2$$

The required ratio of $H : T = 2 : 3$. Hence, Option (c) is correct.

93.



AREA of ΔABC = 6[Area of ΔAOE]

AD = 12 cm, $AO = \frac{2}{3} \times 12 = 8$ cm.

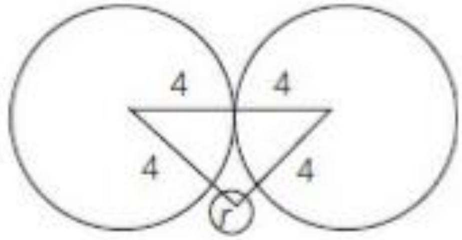
BE = 9 cm, $OE = \frac{1}{3} \times 9 = 3$ cm.

Area of $\Delta AOE = \frac{1}{2} \times 8 \times 3 = 12$ cm2.

Area of $\Delta ABC = 6 \times 12 = 72$ cm2.

Hence, Option (c) is correct.

94.



Let the centres of the three circles be A, B and C (for the smaller circle). Also, assume the radius of the smaller circle to be r. Then, the semi-perimeter of the triangle ABC would be $(8 + 8 + 2r)/2 = 8 + r$.

Using the formula: $\sqrt{s \times (s-a)(s-b)(s-c)}$, for the area of the triangle, we get the area equal to: $\sqrt{(8+r) \times (r)(4)(4)}$.

But the same area can also be derived using base of the triangle as 8 (AB) and height as $(4 - r)$.

Thus, we have: $\sqrt{(8+r) \times (r)(4)(4)} = \frac{1}{2} \times 8 \times (4 - r)$

Solving this, we get $r = 1$ cm.

Hence, Option (b) is correct.

95. Length of the perpendicular is maximum, if the length of rest of the two sides is equal.

In this case, the length of the perpendicular = 20/2 = 10. Hence, option (a) is correct.

96. As the number of cylinders is to be kept minimum, the volume of each cylinder would be the *HCF* of 405, 783 and 351 = 27 cc

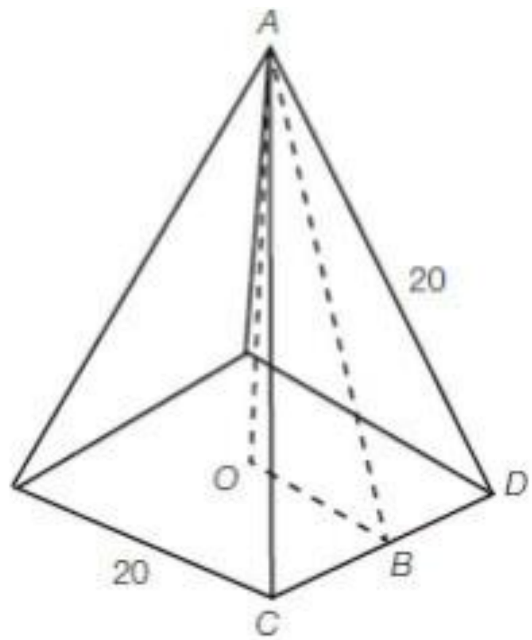
Number of cylinders = $\frac{405}{27} + \frac{783}{27} + \frac{351}{27} = 15 + 29 + 13 = 57$

Since, the volume of each cylinder is 27 cc and the radius is 3 cm, we can get the height of the cylinders as: $\pi r^2 h = \pi \times 3^2 \times h = 27 \rightarrow h = 3/\pi$

Total surface area of the 57 cylinders = 57 × (Curved surface area + area of the flat sides) = $57 \times (2\pi rh + 2\pi r^2) = 114\pi \times \left(\frac{9}{\pi} + 9\right) = 1026(1 + \pi)$ cc

Hence, option (a) is correct.

97.



$AB = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$ (As it is the median of the equilateral triangle ACD.)

$OB = 20/2 = 10$ cm

$AO = \sqrt{(AB^2 - OB^2)} = \sqrt{(300 - 100)} = 10\sqrt{2}$ cm

Hence, option (a) is correct.

98. Interior angle of a 'n' sided polygon = $(n - 2) \times \pi$

According to the question:

$$\frac{(a-2)\pi}{a} \times \frac{3}{2} = \frac{(b-2)\pi}{b}$$

$\frac{(b-2)}{b} = \frac{3}{2} \times \frac{(a-2)}{a}$. Putting $b = 2a$ in this equation, we get:

$$\frac{2a-2}{2a} = \frac{3}{2} \times \frac{a-2}{a}$$

$2a - 2 = 3a - 6$

$a = 4$ and $b = 8$

Interior angle of a polygon with $a + b$ sides =

$$\frac{(a+b-2)\times\pi}{a+b} = \frac{(12-2)\times\pi}{12} = 150°$$

BLOCK V
ALGEBRA

BACK TO SCHOOL...

This block of chapters (consisting of Functions, Inequalities, Quadratic and other Equations and Logarithms) has always been a source of discomfort for students who have had negative experiences with Mathematics. At the same time, students who have had a positive experience with Mathematics throughout their school, junior college and college years find these chapters extremely easy.

Throughout my experience in training students preparing for the CAT exam and other aptitude exams like CMAT, SNAP, IIFT, MAT, XAT, etc., I have always critically tried to understand the key mental representations that differentiate the two categories of students I have mentioned above—how is it that two students who are basically similarly talented with similar IQs be so different when it comes to Higher Maths?

The reason I am talking about this issue is that Block V consists of some of the most critical areas of the entire QA Section for the CAT exam. Besides the XLRI entrance test, XAT also has a very high focus on this block of chapters. Hence, this is one area where you simply cannot allow yourselves to remain in the '*I cannot solve Block V questions*' category if you want to seriously take a crack at the CAT Exam. However, to enable students who are weak at Maths to move from the 'I cannot' to the 'I can' category in this block of chapters, you will first have to believe that this can happen.

And the good news is that in my experience of having trained and mentored lacs of students for aptitude examinations, I have seen a lot of mediocre students in Maths transit from being initially poor at the chapters of this block to becoming strong in the same.

If you belong to such a category of students, what you first need to understand is that the reason for your inability in Higher Maths lies in the way you have approached it up to now. There is simply no reason that a better approach will not help you.

WHAT APPROACH CHANGE DO YOU NEED?

To put it simply—rather than trying to study these Higher Maths chapters in so called Mathematical language, you need to study the same in plain logical language (with as little Maths as possible). The following write-up is an attempt at the same. My attempt in the following part of this section is to give you in as simple language as possible some of the missing technology that makes you consider yourself weak at Maths (especially this block of chapters).

The following is a list of start-up issues you need to resolve in your mind in order to make yourself comfortable with this block of chapters.

ISSUE 1: The $X - Y$ Plot

ISSUE 2: Locus of Points

ISSUE 3: What is an Equation?

ISSUE 4: Equations and Functions

ISSUE 5: How Inequalities Relate to Graphs of Functions?

Let us now start to look at these issues one by one

ISSUE 1: The X – Y Plot: The need for an $X - Y$ plot essentially arises out of the need to measure anything which cannot be measured in a single dimension.

For instance, any object that can be represented on a straight line (like a rope, a tape, etc.) could be easily represented on the number line itself. However, when you are required to represent any two-dimensional figures (like table top, a book, etc.), simply one-dimensional representations (which can be done on the number line only) are not sufficient.

You are required to be able to represent things in two dimensions.

For this purpose, a vertical line is drawn from point '0' of the number line. This line is perpendicular to the number line and is called the Y-axis (which in plain language terms can be understood as the axis which provides the vital second dimension of measurement which helps us describe plain figures better.)

In this situation, the horizontal line (which we know as the number line) is called the X-axis and the diagram which emerges represents what is known as the $X - Y$ plane and is also called as the $X - Y$ plot.

The number line (X-axis) has a positive measurement direction on the right side of zero (and keeps going to a mathematically defined, in reality non-existent point called infinity). On the left side of zero, the X-axis shows negative measurements and keeps going to negative infinity.

In a similar manner, the Y-axis starts from 0 and moves upwards to positive infinity and moves downwards to negative infinity.

The following figures illustrate this point

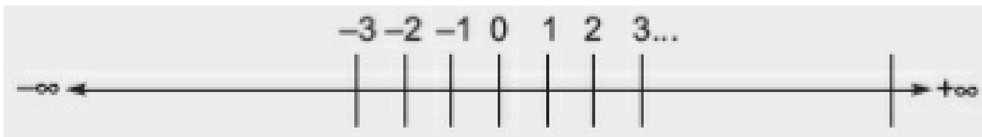


Fig. 1 Number line (also the *X*-axis)

As can be seen the $X-Y$ plot divides the plane into four4 parts (quadrants). The origin is the point where both X and Y are zero.

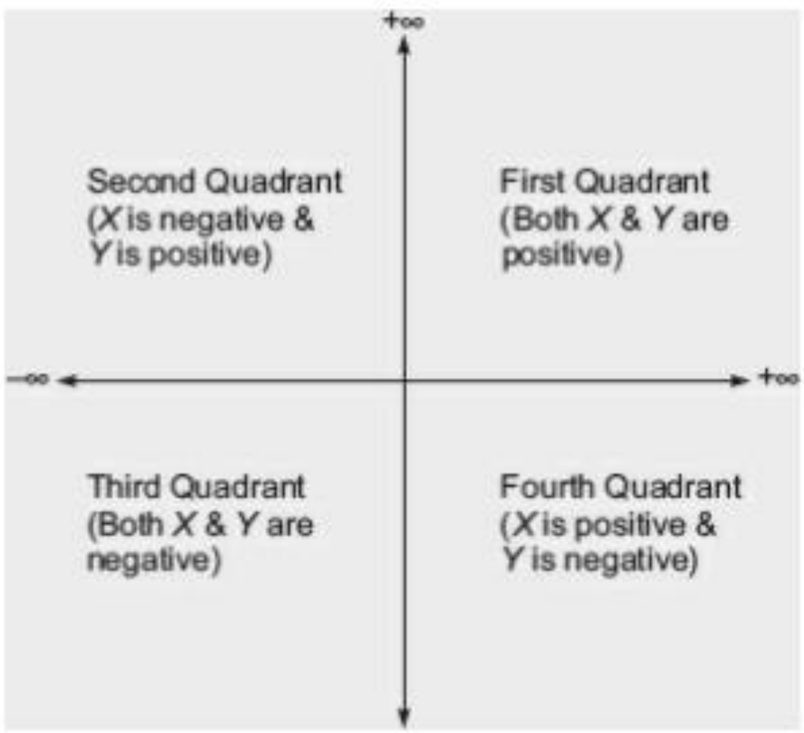


Fig. 2 *X* – *Y* plot

Plotting a Point on the $X - Y$ Plot

Suppose, we have to plot the point (2, 3) on the $X - Y$ plot. In order to do so, we first need to understand what $x = 2$ means on the $X - Y$ chart. On the $X - Y$ plot, if you start from the $x = 2$ point on the X-axis and draw a line parallel to the Y-axis, you will get a set of points which represent a constant value of x (=2). Similarly for $y = 3$, draw a line parallel to the X-axis from the $y = 3$ point on the Y-axis.

In this situation, the point of intersection between these two lines will be the point (2, 3).

The readers are encouraged to try to draw the figure as described above.

ISSUE 2: Locus of Points In the above situation we have come across a set of points which represents $x = 2$. In Mathematics lingo, a set of points is also referred to as a locus of points.

ISSUE 3: What is an Equation? Consider the following:

(a) $2x + 3 = 0$

(b) $x^2 - 5x + 6 = 0$

(c) $4x^3 - 3x^2 - 11x - 18 = 0$

In each of the above cases, we have a situation where there is equality between what is written on the left side of the equality sign and the right side of the equality sign, i.e. LHS = RHS (Left Hand Side = Right Hand Side).

This is the basic definition of an equation.

All Equations have Solutions

Consider the first equation above:

Here you have: $2x + 3 = 0 \rightarrow 2x = -3 \rightarrow x = -3/2$

i.e. $x = -1.5$

This means that at the value $x = -1.5$, the condition of the equation (i.e. LHS = RHS) is satisfied. This value of x is called the 'solution' or the 'root' of the equation.

(**Note:** The equation we have just seen had a highest power of 'x' as 1. Such equations are called as linear equations. [We shall come to this issue of why they are called as linear equations shortly]. What you need to remember at this stage is that linear equations have only one solution. (or root as mathematicians prefer to call it)

ISSUE 4: Equations and Functions: Before we move ahead into the second and third equations given above, let us first try to connect equations and functions.

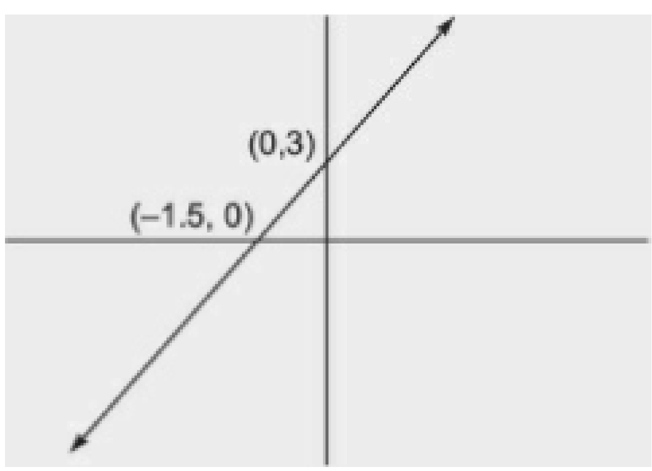
If you remember, we studied in school a chapter called as the "Equation of a Straight Line". For instance, parallel to the equation $2x + 3 = 0$, we have $y = 2x + 3$ (which was defined as the equation of a straight line – hence $2x + 3 = 0$ is called a linear equation).

In $y = 2x + 3$, we have a consistent relationship between the values of x and the corresponding values of y that we could get.

Thus, for $y = 2x + 3$, we can create the following table giving us the specific points which satisfy the equation as below:

x	–3	–2	–1	0	1	2
y	–3	–1	+1	3	5	7

If you plot these points on the $X – Y$ plane, you will get the following figure.



All the points in the table above will lie on this line. The locus of points which are commonly represented by $y = 2x + 3$ is represented by the line above.

In higher Mathematics instead of calling expressions like $y = 2x + 3$ as linear equations, they are called as linear functions. (Perhaps the reason for this is that in higher Maths, we are interested in identifying relationships and behaviours of interdependent variables especially how a change in one variable changes the other variables. Hence, while equations which are of the form $f(x) = 0$ deal with only one variable x, functions represent relationships between two or more variables. They are denoted in Maths as $y = f(x)$ [Reads as: y is a function of x] or $y = f(a, b)$ [Read as: y is function of a and b].

In general, whenever we have any expression in x (denoted by $f(x)$) we can use it either as

$y = f(x)$ [which is what a function is !!]

$f(x) = 0$ [an equation]

Other Principles related to functions and equations

(a) Every function of the form $y = f(x)$ can be plotted on the $X - Y$ plane.

For instance, consider what we did with the function $y = 2x + 3$. [This form of writing a function is called as its analytical representation, while the graph plotted is called as the graphical representation of the function.]

We first created a table where we started putting values of x and getting the respective value for y. Thus, we got a locus of points (as shown in the table above) which represented the function $y = 2x + 3$. This representation of the function is called as the analytical representation of the function.

Further, the line representing $y = 2x + 3$ was plotted on the $X - Y$ plane. This is called as the graphical representation of the function.

(b) The point/points where the graph of the function $y = f(x)$ cuts the x-axis, represents the solution of the equation $f(x) = 0$.

Thus, if you noticed $y = 2x + 3$ cuts the x axis at $x = -1.5$. At the same time the root or solution of the equation $2x + 3 = 0$ is $x = -1.5$.

This is not accidental but will happen in each and every case. Thus, in general you can state that the point where the graph of a function $y = f(x)$ cuts the x-axis is the solution of the equation $f(x) = 0$. (**Note:** This will be true for every expression in x-irrespective of the highest power of 'x' in the expression.)

It is a consequence of this logic, that linear functions will cut the x-axis only once (since linear equations have only one solution.)

(c) **Quadratic Equations and Quadratic Functions:** Let us now pick up the equation: $x^2 - 5x + 6 = 0$.

If you solve this equation you will get the solution of this equation at two values of x. viz. at $x = 2$ as well as at $x = 3$.

[Remember, the root/solution of an equation is that value of the variable which makes the LHS = RHS in the equation. You can easily see that at both $x = 2$ and $x = 3$ the expression $x^2 - 5x + 6$ yields a value of 0.

At $x = 2$, $x^2 - 5x + 6 \rightarrow 2^2 - 5 \rightarrow 2 + 6 = 0$

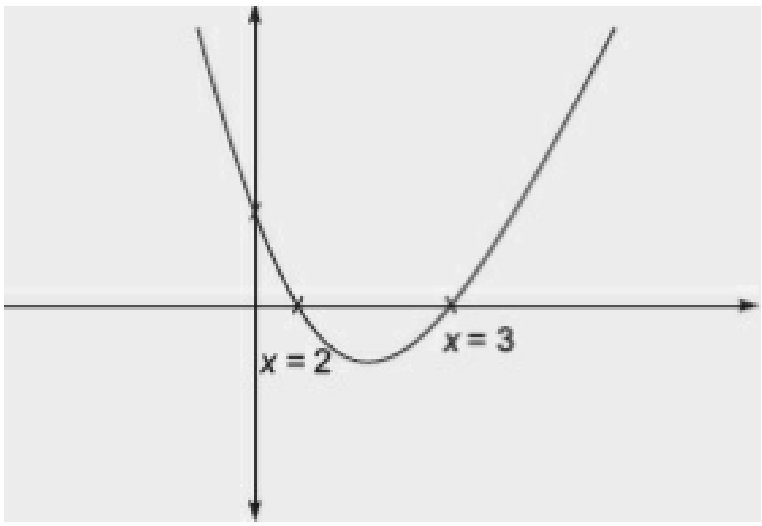
At $x = 3$, $x^2 - 5x + 6 \rightarrow 3^2 - 5 \rightarrow 3 + 6 = 0$

At this point you can predict that the graph of the expression $y = x^2 - 5x + 6$ will cut the x axis at both these points viz. $x = 2$ and $x = 3$.

If you now try to plot $y = x^2 - 5x + 6$, your tabular representation will be:

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
y	56	42	30	20	12	6	2	0	0	2	6

This locus of points will look like the figure below:



Naturally, the graph cuts the x axis at $x = 2$ and $x = 3$.

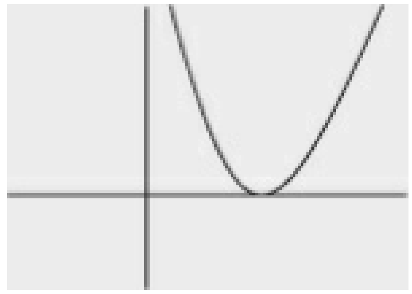
Special note on Quadratic equations

Since all quadratic equations have two solutions, graph of the function for the same expression should cut the x-axis at two points.

However, this might not always be true.

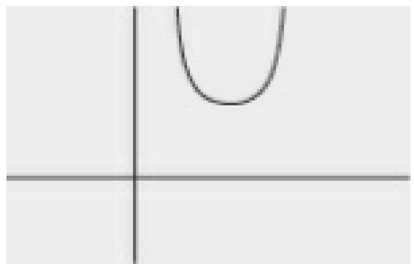
For instance, look at the following graphs.

Case 1:



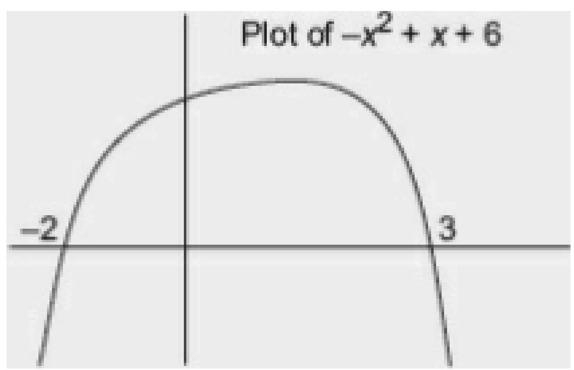
In this case the roots are said to be real and equal.

Case 2:

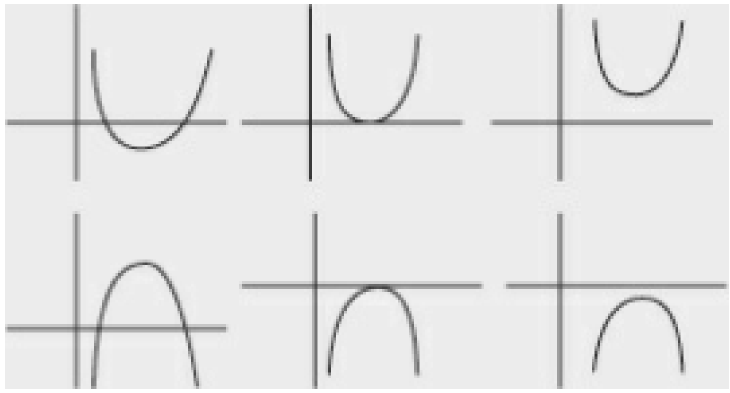


In this case the roots of the quadratic equation are imaginary with respect to the *x*-axis.

(Note: You also need to know that quadratic functions can also look inverted as shown below. This occurs when the coefficient of x^2 is negative.)



In general, quadratic equations then can take any of six shapes (with respect to the *x*-axis).

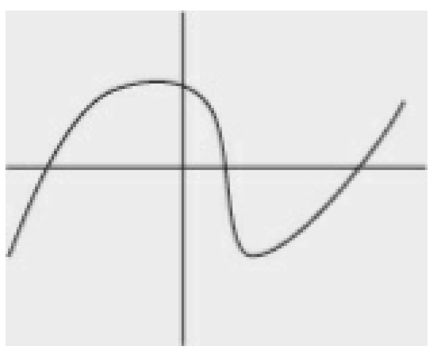


(d) How cubic functions look: Cubic functions have x^3 as their highest powers of x. Cubic equations have three solutions and hence cubic functions will cut the x-axis thrice.

They can look like:

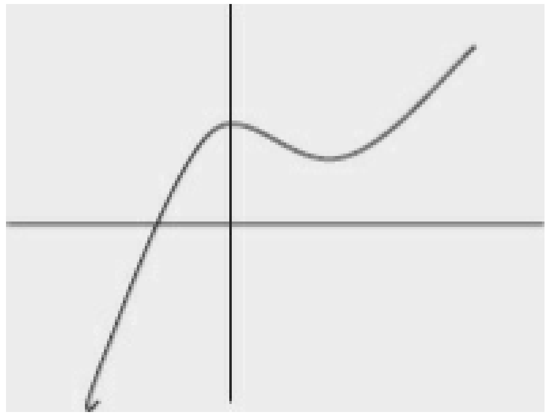
CASE 1: When the Coefficient of x^3 is Positive

(a) Cubic functions with three distinct real roots (e.g. $y = x^3 - 6x^2 + 11x - 6$)



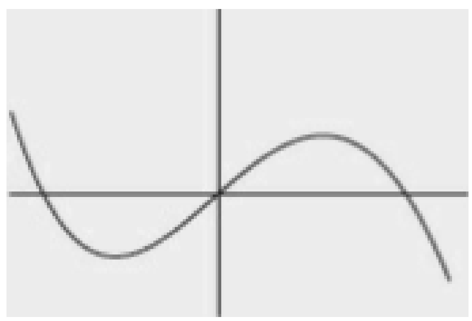
(b) Cubic function with three real and equal roots. (e.g.: $y = (x - 1)^3$)

(c) Cubic function with one real and two imaginary roots

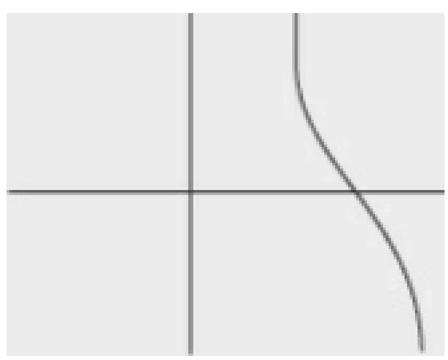


CASE 2: When the Coefficient of x^3 is Negative

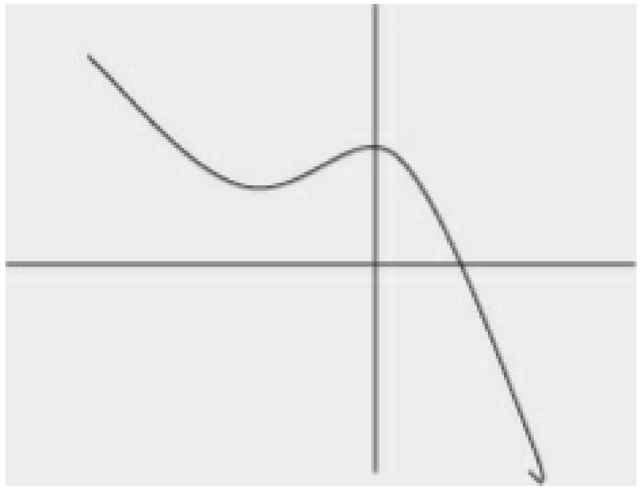
(a) Cubic function with three distinct real roots



(b) Cubic function with three real and equal roots

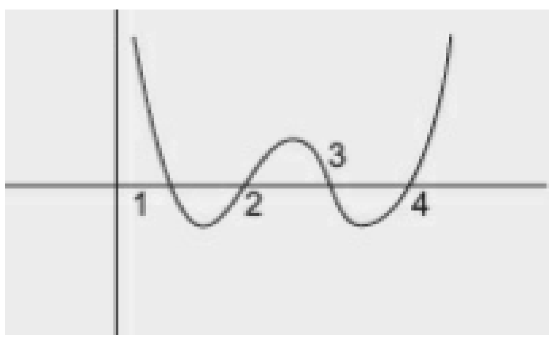


(c) Cubic function with one real and two imaginary roots

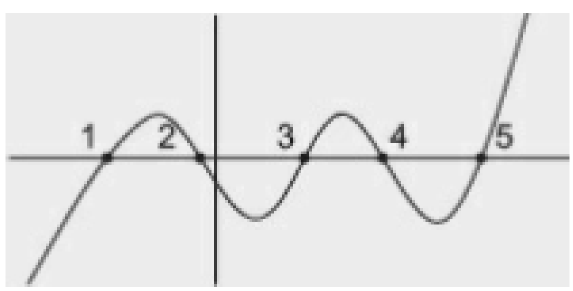


You can similarly visualise functions with highest power 4, 5 and so on to have shapes such that they can cut the x – axis four times, five times and so on. The following are standard figures for expressions with

(a) Degree 4 (i.e. x^4) [4 roots]



(b) Degree 5 [5 roots]



Key Note

Graphical representations and ability to understand how an expression in x will look on the $X - Y$ plot is a very key factor in your moving towards proficiency and self-dependence in this crucial block of chapters. Hence, it is encouraged to try to plot the graphs of as many $y = f(x)$ functions as one can imagine. To give you a target, it is recommended a minimum of 100 graphs to be plotted by you. Only then would you be able to move to a level of comfort in visualising graphs.

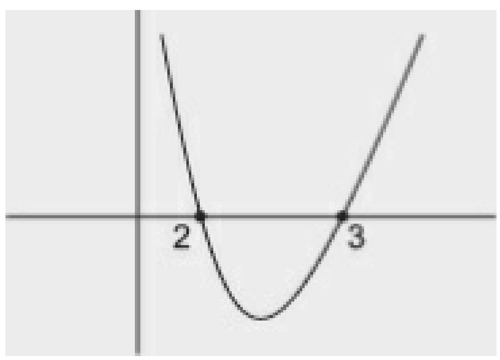
ISSUE 5: How Inequalities Relate to Graphs of Functions?

We have just seen the relationship between a function and an equation. While a function is written in the $y = f(x)$ form, an equation is written in the $f(x) = 0$ form.

An inequality (or an inequation) can be written in the form $f(x) > 0$ or $f(x) \geq 0$.

Thus, if we say $x^2 - 5x + 6 > 0$, we are looking for those values of x where the above expression yields values above 0.

Going back to the graph for $x^2 - 5x + 6$, it is obvious that the function is greater than 0 when $x > 3$ or $x < 2$.



In such a case, the solution of the inequality $x^2 - 5x + 6 > 0$ becomes $x > 3$ or $x < 2$.

Understand here that when we are solving an inequality all we are bothered about is to see at what values of x does the inequality hold true.

CAT Scan 1

Directions for Questions 1 to 3: Answer the questions on the basis of the tables given below.

Two binary operations (+) and * are defined over the set {*i*, *j*, *k*, *l*, *m*} as per the following tables:

(+)	*i*	*j*	*k*	*l*	*m*
i	*i*	*j*	*k*	*l*	*m*
j	*j*	*k*	*l*	*m*	*i*
k	*k*	*l*	*m*	*i*	*j*
l	*l*	*m*	*i*	*j*	*k*
m	*m*	*i*	*j*	*k*	*l*

*	*i*	*j*	*k*	*l*	*m*
i	*i*	*i*	*i*	*i*	*i*
j	*i*	*j*	*k*	*l*	*m*
k	*i*	*k*	*m*	*j*	*l*
l	*i*	*l*	*j*	*l*	*k*
m	*i*	*m*	*l*	*k*	*j*

Thus, according to the first table $k\ (+)\ l = i$; while according to the second table $l * m = k$, and so on. Also, let $k_2 = k * k$, $13 = l * l * l$, and so on.

1. What is the smallest positive integer n such that $k_n = l$?

   (a) 4

   (b) 5

   (c) 2

   (d) 3

2. Upon simplification, m (+) [k * {k (+) (k * k)}] equals

   (a) j

   (b) k

   (c) l

   (d) m

3. Upon simplification, {i_{10} * (k_{10} (+) l_9)} (+) j_8 equals

   (a) j

   (b) k

   (c) l

   (d) m

4. What is the sum of n terms in the series: log m + log (m_2/n) + log (m_4/n_3) + ...?

   (a) log $[n_{(n-1)} / m_{(n+1)}]_{n/2}$

   (b) log $[m_m/n_n]_{n/2}$

   (c) log $[m_{(1-n)}/n_{(1-m)}]_{n/2}$

   (d) log $[m_{(n+1)}/n_{(n-1)}]_{n/2}$

5. If n is such that $36 \le n \le 72$, then $x = (n^2 + 2\sqrt{n}(n + 4) + 16)/(n + 4\sqrt{n} + 4)$ satisfies:

   (a) $20 < x < 54$

   (b) $23 < x < 58$

(c) $25 < x < 58$

(d) $28 < x < 60$

6. A real number x is such that it satisfies the condition: $2–1/m < x \leq 4 + 1/m$, for every positive integer m. Which of the following best describes the range of values that x can take?

(a) $1 < x < 5$

(b) $1 < x \leq 4$

(c) $1 < x \leq 5$

(d) $1 \leq x \leq 4$

7. If $1/3 \log_3 M + 3 \log_3 N = 1 + \log_{0.008} 5$, then

(a) $M^9 = 9/N$

(b) $N^9 = 9/M$

(c) $M^3 = 3/N$

(d) $N^9 = 3/M$

CAT Scan 2

1. If $f(x) = \log \{(1 + x)/(1 – x)\}$, then $f(x) + f(y)$ is

(a) $f(x + y)$

(b) $f\{(x + y)/(1 + xy)\}$

(c) $(x + y) f\{1/(1 + xy)\}$

(d) $f(x) + f(y)/(1 + xy)$

2. The n^{th} element of a series is represented as

$X_n = (–2)^n X_{n–1}$

If $X_0 = x$ and $x > 0$, then the following is always true.

(a) X_n is positive if n is even

(b) X_n is positive if n is odd

(c) X_n is negative if n is even

(d) None of these

3. The number of roots common between the two equations $x^3 + 3x^2 + 4x + 7 = 0$ and $x^3 + 2x^2 + 7x + 5 = 0$ is

(a) 0

(b) 1

(c) 2

(d) 3

4. If $|y| \geq 1$ and $x = -|a|\, y$, then which one of the following is necessarily true?

(a) $a - xy < 0$

(b) $a - xy \geq 0$

(c) $a - xy > 0$

(d) $a - xy \leq 0$

5. Consider the sets $T_n = \{n, n + 1, n + 2, n + 3, n + 4\}$, where $n = 1, 2, 3, \ldots, 96$. How many of these sets do not contain 6 or any integral multiple thereof (i.e. any one of the numbers 6, 12, 18, ...)?

(a) 16

(b) 15

(c) 14

(d) 13

6. If $13m + 1 < 2n$, and $n + 3 = 5y^2$, then

(a) m is necessarily less than n

(b) m is necessarily greater than n

(c) m is necessarily equal to n

(d) None of the above is necessarily true

7. If both a and b belong to the set {1, 2, 3, 4}, then the number of equations of the form

$ax^2 + bx + 1 = 0$ having real roots is

(a) 10

(b) 7

(c) 6

(d) 12

CAT Scan 3

1. If three positive real numbers x, y and z satisfy $y - x = z - y$ and $xyz = 4$, then what is the minimum possible value of y?

(a) $2^{1/3}$

(b) $2^{2/3}$

(c) $2^{1/4}$

(d) $2^{3/4}$

2. There are 12 towns grouped into four zones with three towns per zone. It is intended to connect the towns with telephone lines such that every

two towns are connected with three direct lines if they belong to the same zone, and with only one direct line otherwise. How many direct telephone lines are required?

(a) 72

(b) 90

(c) 96

(d) 144

3. If $\log_{10} x + \log_{10} x = 2 \log_x 10$, then a possible value of x is given by

(a) 10

(b) 1/10

(c) 1/100

(d) None of these

4. Find the area enclosed by the region in sq. units described by $0 \le x \le 5$ and $0 \le |y| \le 8$.

(a) 40

(b) 80

(c) 160

(d) 120

5. The value of the expression [(1/2) + (1/100)] + [(1/2) + (2/100)] + [(1/2) + (3/100)] + ...[(1/2) + (103/100)] is equal to {Where [x] is the Greatest Integer Function of x}

(a) 53

(b) 54

(c) 52

(d) None of these

6. The number of non-negative real roots of $2^x - x - 1 = 0$ equals

(a) 0

(b) 1

(c) 2

(d) 3

7. Let a, b, c and d be four integers such that $a + b + c + d = 4m + 1$ where m is a positive integer. Given m, which one of the following is necessarily true?

(a) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$

(b) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$

(c) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$

(d) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$

CAT Scan 4

1. The 288th term of the series $a, b, b, c, c, c, d, d, d, d, e, e, e, e, e, f, f, f, f, f, f$ ...... is

(a) u

(b) v

(c) w

(d) x

2. Find the area enclosed by the curve $|x| + |y| = 4$.

(a) 24

(b) 28

(c) 32

(d) 36

3. Let $y = \min \{(x + 8), (6 - x)\}$. If x e R, what is the maximum value of y?

(a) 8

(b) 7

(c) 6

(d) 10

4. Let S be the set of integers {3, 11, 19, 27, ... 451, 459, 467} and M be a subset of S such that no two elements of M add up to 470. The maximum possible number of elements in M is:

(a) 32

(b) 28

(c) 29

(d) 30

Directions for Questions 5 and 6: We are given that $f(x) = f(y)$ and $f(x, y) = x + y$, if $x, y > 0$

$$f(x, y) = xy, \text{ if } x, y = 0$$

$$f(x, y) = x - y, \text{ if } x, y < 0$$

$f(x, y) = 0$, otherwise

5. **Find the value of the following function:** $f[f(2, 0), f(-3, 2)] + f[f(-6, -3), f(2, 3)]$.

   (a) 0

   (b) 2

   (c) –8

   (d) None of these

6. **Find the value of the following function.** $\{f[f(1, 2), f(2, 3)]\} \times \{f[f(1.6, 2.9), f(-1, -3)]\}$

   (a) 12

   (b) 36

   (c) 48

   (d) 52

7. **Given that** $f(a) = a(a + 1)(a + 2)$ *where* $a = 1, 2, 3, \ldots$. *Then find* $S = f(1) + f(2) + f(3) + \ldots\ldots + f(10)$.

   (a) 4200

   (b) 4290

   (c) 4400

   (d) None of these

ANSWER KEY

CAT Scan 1

1. (d)

2. (a)
3. (a)
4. (d)
5. (d)
6. (c)
7. (b)

CAT Scan 2

1. (b)
2. (d)
3. (a)
4. (b)
5. (a)
6. (d)
7. (b)

CAT Scan 3

1. (b)
2. (b)
3. (a)
4. (b)
5. (b)
6. (c)
7. (b)

CAT Scan 4

1. (d)
2. (c)
3. (b)
4. (d)

5. (a)
6. (d)
7. (b)