Areas

Exercise 7A

Question 1:

Here, b = 24 cm and h = 14.5 cm

Area of triangle =
$$\left(\frac{1}{2} \times \text{base} \times \text{height}\right)$$
 sq units
$$= \left(\frac{1}{2} \times 24 \times 14.5\right) \text{cm}^2$$

$$= 174 \text{ cm}^2$$

Question 2:

Let height = x and base = 3x
Area of triangle =
$$\left(\frac{1}{2} \times \text{base} \times \text{height}\right)$$
 sq units
∴ Area of triangle = $\frac{1}{2} \times x \times 3x$
= $\frac{3}{2}x^2$

Hence, height = 300 m and base = 900 m.

We know that, 1 hectare = 10000 sq metre
Rate of sowing the field per hectare = Rs.58
Total cost of sowing the triangular field = Rs.783 $\Rightarrow \qquad \text{Total cost} = \text{Area of the triangular field} \times \text{Rs. 58}$ $\Rightarrow \qquad \frac{3}{2}x^2 \times \frac{58}{10000} = 783$ $\Rightarrow \qquad x^2 = \frac{783}{58} \times \frac{2}{3} \times 10000 \text{ sq metre}$ $\Rightarrow \qquad x^2 = 90000 \text{ sq metre}$ $\Rightarrow \qquad x = 300 \text{ m}$

Question 3:

Here, a = 42 cm, b = 34 cm and c = 20 cm
Therefore,
$$s = \frac{42 + 34 + 20}{2} = 48$$

Area = $\sqrt{S(S-a)(S-b)(S-c)}$
= $\sqrt{48(48 - 42)(48 - 34)(48 - 20)}$
= $\sqrt{48 \times 6 \times 14 \times 28}$
= $\sqrt{4 \times 4 \times 3 \times 3 \times 2 \times 14 \times 14 \times 2}$
= $4 \times 3 \times 2 \times 14$
= 336 cm²
Longest side = 42 cm
 $\Rightarrow b = 42$ cm

Let h be the height corresponding to the longest side.

Area of the triangle
$$=\frac{1}{2} \times b \times h$$

$$\Rightarrow \frac{1}{2} \times b \times h = 336$$

$$\Rightarrow 42 \times h = 336 \times 2$$

$$\Rightarrow h = \frac{336 \times 2}{42} = 16 \text{ cm}$$

Question 4:

Here, a = 18 cm, b = 24 cm and c = 30 cm
Therefore,
$$s = \frac{18 + 24 + 30}{2} = 36$$

Area = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{36(36-18)(36-24)(36-30)}$
= $\sqrt{36 \times 18 \times 12 \times 6}$
= $\sqrt{6 \times 6 \times 6 \times 3 \times 3 \times 4 \times 6}$

$$= 6 \times 6 \times 3 \times 2$$
$$= 216 \text{ cm}^2$$

Smallest side = 18 cm

Let h be the height corresponding to the smallest side.

Area of the triangle
$$=\frac{1}{2} \times b \times h$$

$$\Rightarrow \frac{1}{2} \times b \times h = 216$$

$$\Rightarrow 18 \times h = 216 \times 2$$

$$\Rightarrow h = \frac{216 \times 2}{18} = 24 \text{ cm}$$

Question 5:

Here, a = 91 m, b = 98 m and c = 105 m
Therefore,
$$s = \frac{91 + 98 + 105}{2} = \frac{294}{2} = 147$$

Area = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{147(147 - 91)(147 - 98)(147 - 105)}$
= $\sqrt{147 \times 56 \times 49 \times 42}$
= $\sqrt{49 \times 3 \times 7 \times 2 \times 2 \times 2 \times 49 \times 7 \times 3 \times 2}$
= 49 × 3 × 2 × 2 × 7
= 4116 m²

Longest side = $105m \Rightarrow b=105$

Let h be the height corresponding to the longest side.

Area of the triangle
$$=\frac{1}{2} \times b \times h$$

$$\Rightarrow \frac{1}{2} \times b \times h = 4116$$

$$\Rightarrow$$
 105 × h = 2 × 4116

$$\Rightarrow$$
 h = $\frac{2 \times 4116}{105}$ = 78.4 m

Question 6:

Let the sides of the triangle be 5x, 12x and 13x.

Its perimeter = (5x + 12x + 13x) = 30x

$$\Rightarrow x = \frac{150}{30} = 5 \text{ m}$$

Thus, sides of the triangle are;

$$5x = 5 \times 5 = 25 \text{ m}$$

$$12x = 12 \times 5 = 60 \text{ m}$$

$$13x = 13 \times 5 = 65 \text{ m}$$

Let a = 25 m, b = 60 m and c = 65 m.

Now

$$S = \frac{1}{2}(a+b+c)$$
$$= \left(\frac{25+60+65}{2}\right) m = \frac{150}{2} = 75 m.$$

:. area of the triangle =
$$\sqrt{s(s-a)(s-b)(s-c)}$$

= $\sqrt{75(75-25)(75-60)(75-65)}$

$$= \sqrt{75 \times 50 \times 15 \times 10}$$

$$= \sqrt{25 \times 3 \times 25 \times 2 \times 5 \times 3 \times 5 \times 2}$$

$$= \sqrt{25 \times 25 \times 5 \times 5 \times 3 \times 3 \times 2 \times 2}$$
$$= 25 \times 5 \times 3 \times 2 = 750 \text{ sq m}.$$

: area of the triangle = 750 sq m.

Question 7:

Let the sides of the triangle be 25x, 17x and 12x.

Then, its perimeter = (25x + 17x + 12z) = 54x

$$\Rightarrow \qquad \qquad x = \frac{540}{54} = 10\text{m}.$$

Thus, sides of the triangle are:

 $25x = 25 \times 10 = 250 \text{ m}$

$$17x = 17 \times 10 = 170 \text{ m}$$

$$12x = 12 \times 10 = 120 \text{ m}$$

Let, a = 250 m, b = 170 m and c = 120 m

Now,

$$s = \frac{1}{2}(a+b+c)$$
$$= \left(\frac{250 + 170 + 120}{2}\right)m$$
$$= \left(\frac{540}{2}\right)m = 270 m$$

$$\therefore \text{ area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{270(270-250)(270-170)(270-120)}$$

$$= \sqrt{3 \times 3 \times 3 \times 10 \times 10 \times 2 \times 10 \times 10 \times 5 \times 3}$$

$$= 3 \times 3 \times 10 \times 10 \times 10 = 9000 \text{ m}^2$$

.. Cost of ploughing the field at the rate of Rs. 18.80 per 10 m²

$$= \frac{18.80}{10} \times 9000 = \text{Rs. } 16920$$

:. Cost of ploughing the field = Rs. 16920.

Question 8:

One side of a triangular field = 85 m

Second side of a triangular field = 154 m

Let the third side of a triangular field be x m

$$\Rightarrow$$
 $X = 324 - 239$

:. the third side = 85 m

Let a = 85 m, b = 154 m and c = 85 m

Now

$$S = \frac{1}{2}(a+b+c)$$
$$= \left(\frac{85+154+85}{2}\right) = \frac{324}{2} = 162$$

: area of the triangle = $\sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{162 (162 - 85) (162 - 154) (162 - 85)}$$

$$= \sqrt{162 \times 77 \times 8 \times 77}$$

$$= \sqrt{2 \times 9 \times 9 \times 7 \times 11 \times 2 \times 2 \times 2 \times 7 \times 11}$$

$$= \sqrt{11 \times 11 \times 9 \times 9 \times 7 \times 7 \times 2 \times 2 \times 2 \times 2}$$

$$= 11 \times 9 \times 7 \times 2 \times 2 = 2772 \text{ m}^2$$

: area of triangle = 2772 m²

Also, area of triangle = $\frac{1}{2} \times base \times height$

$$2772 = \frac{1}{2} \times 154 \times h = 77h$$

h =
$$\frac{2772}{77}$$
 = 36 m

:. the length of the perpendicular from the opposite vertex on the side measuring 154 m = 36 m.

Question 9:

Let a = 13 cm, B = 13 cm and c = 20 cm Now, $s = \frac{1}{2} (a + b + c)$ $= \left(\frac{13 + 13 + 20}{2}\right) \text{Cm} = \frac{46}{2} = 23 \text{ cm}$ $\therefore \text{ area of the triangle} = \sqrt{s(s - a)(s - b)(s - c)}$ $= \sqrt{23(23 - 13)(23 - 13)(23 - 20)}$ $= \sqrt{23 \times 10 \times 10 \times 3}$ $= 10\sqrt{69}$ $= 10 \times 8.306 = 83.06 \text{ cm}^2$

 \therefore area of an isosceles triangle = 83.06 cm²

Question 10:

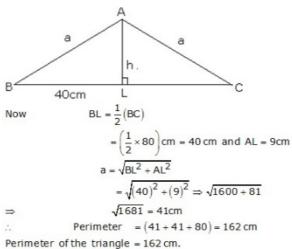
Let ΔABC be an isosceles triangle and Let $AL \perp BC$. Given that BC = 80 cm and Area of $\Delta ABC = 360$ cm²

$$\frac{1}{2} \times BC \times AL = 360 \text{ cm}^2$$

$$\Rightarrow \qquad \frac{1}{2} \times 80 \times h = 360 \text{ cm}^2$$

$$\Rightarrow \qquad 40 \times h = 360 \text{ cm}^2$$

$$\Rightarrow \qquad h = \frac{360}{40} = 9 \text{ cm}$$



Question 11:

In an isosceles triangle, the lateral sides are of equal length. Let the length of lateral side be \mathbf{x} cm.

Then, base =
$$\frac{3}{2} \times x \text{ cm}$$
 [given]

(i) Length of each side of the triangle:

Perimeter of an isosceles triangle = 42 cm

$$\Rightarrow$$
 $x + x + \frac{3}{2}x = 42 \text{ cm}$

$$\Rightarrow$$
 2x + 2x + 3x = 84 cm

$$7x = 84$$

$$\Rightarrow x = \frac{84}{7} = 12 \, \text{cm}$$

:. length of lateral side = 12 cm

And base =
$$\frac{3}{2}x = \frac{3}{2} \times 12 = 18cm$$

: the length of each side of the triangle = 12 cm, 12 cm and 18 cm.

(ii) Area of the triangle:

Let a = 12 cm, b = 12 cm and c = 18 cm.

Now,

$$S = \frac{1}{2}(a+b+c)$$

= $\left(\frac{12+12+18}{2}\right)$ cm = $\left(\frac{42}{2}\right)$ cm
= 21 cm

: area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{21(21-12)(21-12)(21-18)}$$

$$= \sqrt{21 \times 9 \times 9 \times 3}$$

$$= \sqrt{3 \times 7 \times 9 \times 9 \times 3}$$

$$= 27\sqrt{7} = 71.42 \text{ cm}^2 \qquad (\sqrt{7} = 2.64)$$

: area of the triangle = 71.42 cm².

(iii) Height of the triangle :

Area of a triangle = $\frac{1}{2}$ xbase x height

$$71.42 \, \text{cm}^2 = \frac{1}{2} \times 18 \, \text{xh}$$

$$\Rightarrow 71.42 \, \text{cm}^2 = 9 \, \text{xh}$$

⇒
$$h = \frac{71.42}{9} = 7.94 \text{ cm}$$

: the height of the triangle = 7.94cm.

Question 12:

Let a be the length of a side of an equilateral triangle.

∴ Area of an equilateral triangle =
$$\frac{\sqrt{3} \times a^2}{4}$$
 sq units

Area of the equilateral triangle = $36\sqrt{3}$ cm² [given]

$$\Rightarrow \frac{\sqrt{3} \times a^2}{4} = 36 \times \sqrt{3}$$

$$\Rightarrow a^2 = \frac{36 \times \sqrt{3} \times 4}{\sqrt{3}}$$

$$\Rightarrow a^2 = 36 \times 4 = 144$$

$$\therefore a = \sqrt{144} = 12 \text{ cm}$$

Perimeter of an equilateral triangle = $3 \times a$

Since, a = 12 cm,

Perimeter = (3×12) cm = 36 cm

Question 13:

Let a be the length of the side of an equilateral triangle

:. Area of an equilateral triangle = $\frac{\sqrt{3}}{4}$ a² sq units

Area of the equilateral triangle = $81\sqrt{3}$ cm² [given]

$$\Rightarrow 81\sqrt{3} \text{ cm}^2 = \frac{\sqrt{3}}{4} \text{ a}^2$$

$$\Rightarrow \qquad \qquad a^2 = \frac{81\sqrt{3} \times 4}{\sqrt{3}} = 324$$

$$\Rightarrow \qquad \qquad a = \sqrt{324} = 18 \text{ cm}$$

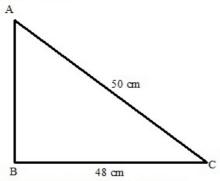
Height of an equilateral triangle = $\frac{\sqrt{3}}{2}$ a

Since a = 18 cm,

Height of the equilateral triangle = $\frac{\sqrt{3}}{2} \times 18 = 9\sqrt{3}$ cm.

Question 14:

Base of the right triangle is BC = 48 cm Hypotenuse of the right triangle is AC = 50 cmLet AB = x cm



By Pythagoras Theorem, we have,

$$AC^2 = AB^2 + BC^2$$

That is we have

$$50^2 = x^2 + 48^2$$

$$\Rightarrow$$
 $x^2 = 50^2 - 48^2$

$$\Rightarrow$$
 $x^2 = 2500 - 2304 = 196$

$$\Rightarrow \qquad x = \sqrt{196} = 14 \text{cm}$$

 \therefore Area of the right angle triangle = $\frac{1}{2} \times base \times height$

$$= \frac{1}{2} \times 48 \times 14$$

$$= (24 \times 14) \text{ cm}^2 = 336 \text{ cm}^2$$

 \therefore Area of the triangle = 336 cm²

Question 15:

(i) Area of an equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

Where a is the side of the equilateral triangle

area =
$$\frac{\sqrt{3}}{4} \times 8^2$$

= $\frac{\sqrt{3}}{4} \times 64 \Rightarrow \sqrt{3} \times 16$
= 1.732×16
= $27.712 = 27.71$ cm². [correct upto 2] decimal places]

Question 16:

Let a be the side of an equilateral triangle.

:. Height of an equilateral triangle = $\frac{\sqrt{3}}{2}$ a units

Height of an equilateral triangle = 9cm [given]

$$\Rightarrow \frac{\sqrt{3}}{2} a = 9$$

$$\Rightarrow a = \frac{9 \times 2}{\sqrt{3}}$$

$$\Rightarrow = \frac{9 \times 2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \quad \text{[Rationalizing the denominator]}$$

$$\Rightarrow = \frac{9 \times 2\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$\Rightarrow a = 6\sqrt{3}$$

$$\Rightarrow base = 6\sqrt{3}$$

Area of the equilateral triangle = $\frac{1}{2} \times base \times height$

$$= \frac{1}{2} \times 6\sqrt{3} \times 9 \quad [\because \text{ base} = 6\sqrt{3} \text{ and height} = 9\text{cm}]$$
$$= 27\sqrt{3}$$

Area of the equilateral triangle = $27 \times 1.732 = 46.764$

$$=46.76$$
cm²

[Correct to 2 places of decimal]

Question 17:

Let a=50cm, b=20cm and c=50cm. Let us find s:

$$s = \frac{1}{2}(a+b+c)$$

$$= \left(\frac{50+20+50}{2}\right)cm = \left(\frac{120}{2}\right)cm$$

$$= 60 cm$$

Now, area of one triangular piece of cloth

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{60(60-50)(60-20)(60-50)}$$

$$= \sqrt{60 \times 10 \times 40 \times 10}$$

$$= \sqrt{6 \times 10 \times 10 \times 4 \times 10 \times 10}$$

$$= \sqrt{10 \times 10 \times 10 \times 10 \times 2 \times 2 \times 2 \times 3}$$

$$= 10 \times 10 \times 2\sqrt{6}$$

$$= 200\sqrt{6} = 200 \times 2.45 = 490 \text{ cm}^2$$

: area of one piece of cloth = 490 cm²

Now area of 12 pieces = $(12 \times 490) \text{ cm}^2 = 5880 \text{ cm}^2$

Question 18:

Let, a = 16 cm, b = 12 and c = 20 cm

Let us now find s:

$$s = \frac{1}{2}(a+b+c)$$

$$= \left(\frac{16+12+20}{2}\right) cm = \left(\frac{48}{2}\right) cm$$

$$= 24 cm$$

 \therefore Area of one tile = 96 cm²

$$\Rightarrow$$
 Area of 16 tiles = 96 × 16 = 1536 cm²

Cost of polishing the tiles per sq.cm = Re.1

Thus, the total cost of polishing all the tiles = Rs. (1×1536)

= Rs. 1536.

Question 19:

Consider the right triangle ABC.

By Pythagoras Theorem, we have,

$$BC = \sqrt{AB^2 - AC^2}$$

$$= \sqrt{17^2 - 15^2}$$

$$= \sqrt{289 - 225}$$

$$= \sqrt{64}$$

$$= 8 \text{ cm}$$

Perimeter of quad. ABCD = 17 + 9 + 12 + 8 = 46 cm

Area of triangle $\triangle ABC = \frac{1}{2} \times base \times height$

$$= \frac{1}{2} \times BC \times AC$$
$$= \frac{1}{2} \times 8 \times 15$$
$$= 60 \text{ cm}^2$$

For area of triangle ACD,

Let a = 15 cm, b = 12 cm and c = 9 cm

Therefore,
$$s = \frac{a+b+c}{2} = \frac{15+12+9}{2} = 18 \text{ cm}$$

Area of
$$\triangle ACD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18(18-15)(18-12)(18-9)}$$

$$= \sqrt{18 \times 3 \times 6 \times 9}$$

$$= \sqrt{18 \times 18 \times 3 \times 3}$$

$$= 18 \times 3 = 54 \text{ cm}^2$$

Thus the area of quad. ABCD = Area of \triangle ABC + Area of \triangle ACD = 60 + 54 = 114 cm².

Question 20:

Perimeter of quad. ABCD =
$$34 + 29 + 21 + 42 = 126$$
 cm Area of triangle BCD = $\frac{1}{2} \times 20 \times 21 = 210$ cm² For area of triangle ABD, Let a = 42 cm, b = 20 cm and c = 34 cm Therefore, s = $\frac{42 + 20 + 34}{2} = \frac{96}{2} = 48$ cm Area of ABD = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{48(48-42)(48-20)(48-34)}$ = $\sqrt{48 \times 6 \times 28 \times 14}$ = $\sqrt{16 \times 3 \times 3 \times 2 \times 2 \times 14 \times 14}$ = $4 \times 3 \times 2 \times 14 = 336$ cm² Area of quad. ABCD = Area Δ ABD + Area Δ BCD

Thus the area of quad. ABCD = $336 + 210 = 546 \text{ cm}^2$.

Question 21:

Consider the right triangle ABD. By Pythagoras Theorem, we have

$$AB = \sqrt{BD^2 - AD^2}$$

$$AB = \sqrt{26^2 - 24^2}$$

$$= \sqrt{676 - 576}$$

$$= \sqrt{100}$$

$$AB = 10 \text{ cm}$$

$$\Rightarrow \text{base} = 10 \text{ cm}$$

Area of the triangle ABD = $\frac{1}{2}$ × base × height

$$\Rightarrow$$
 Area of \triangle ABD= $\frac{1}{2} \times 10 \times 24 \ [\because base = 10 cm, height = 24 cm]$

$$\Rightarrow$$
 Area of \triangle ABD=120cm²

Area of equilateral triangle BCD =
$$\frac{\sqrt{3}}{4}$$
 a²
 \Rightarrow = $\frac{1.73}{4} (26)^2$ [a = 26cm, $\sqrt{3}$ = 1.73]

$$\Rightarrow = \frac{1.7}{4} (20) [d = 200TI, \sqrt{3} = 1.7]$$

$$\Rightarrow = 292.37 \text{ cm}^2$$

Area of quad. ABCD = Area of
$$\triangle$$
ABD + Area of \triangle BCD = 120 + 292.37 = 412.37 cm².

Question 22:

Consider the triangle ABC, Let a = 26 cm, b = 30 cm and c = 28 cm
$$s = \frac{26 + 30 + 28}{2} = \frac{84}{2} = 42 \text{ cm}$$
 Area of ABC = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{42(42-26)(42-30)(42-28)}$ = $\sqrt{42 \times 16 \times 12 \times 14}$ = $\sqrt{14 \times 3 \times 16 \times 4 \times 3 \times 16 \times 4}$ = $14 \times 3 \times 4 \times 2$ = 336 cm²

In a parallelogram , diagonal divides the parallelogram in two equal area therefore

$$\therefore$$
 Area of quad. ABCD = Area of ΔABC + Area of ΔACD = Area of ΔABC \times 2 = 336 \times 2 = 672 cm².

Question 23:

Consider the triangle ABC,

Let a = 10 cm, b = 16 cm and c = 14 cm

$$s = \frac{10 + 16 + 14}{2} = \frac{40}{2} = 20$$
Area of ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{20(20-10)(20-16)(20-14)}$$

$$= \sqrt{20 \times 10 \times 4 \times 6}$$

$$= \sqrt{10 \times 2 \times 10 \times 4 \times 3 \times 2}$$

$$= \sqrt{10 \times 10 \times 4 \times 2 \times 2 \times 3}$$

$$= 10 \times 2 \times 2 \times \sqrt{3}$$

$$= 40\sqrt{3} \text{ cm}^2$$

In a parallelogram , diagonal divides the parallelogram in two equal area therefore

∴ Area of quad. ABCD = Area of
$$\triangle$$
ABC + Area of \triangle ACD = Area of \triangle ABC × 2 = $40\sqrt{3}$ × 2 = $80\sqrt{3}$ cm² = 138.4 cm² [∴ $\sqrt{3}$ = 1.73]

Question 24:

Area of triangle ABD =
$$\frac{1}{2} \times \text{base} \times \text{height}$$

= $\frac{1}{2} \times \text{BD} \times \text{AL}$
= $\frac{1}{2} \times 64 \times 16.8$
= 537.6 cm^2
Area of triangle BCD = $\frac{1}{2} \times \text{base} \times \text{height}$
= $\frac{1}{2} \times \text{BD} \times \text{CM}$
= $\frac{1}{2} \times 64 \times 13.2$
= 422.4 cm^2
Area of quad. ABCD = Area of $\triangle \text{ABD} + \text{Area of } \triangle \text{BCD}$
= $537.6 + 422.4 = 960 \text{ cm}^2$.