

# BLUE PRINT: CLASS XII MATHS

| CHAPTER'S NAME                     | 1 MARK   | 4 MARKS   | 6 MARKS  | TOTAL |
|------------------------------------|----------|-----------|----------|-------|
| 1. RELATIONS AND FUNCTIONS         | 1        | 1         |          | 5     |
| 2. INVERSE TRIGONOMETRIC FUNCTIONS | 1        | 1         |          | 5     |
| 3. MATRICES                        | 1        |           | 1        | 7     |
| 4. DETERMINANTS                    | 2        | 1         |          | 6     |
| 5. DIFFERENTIATION                 |          | 2         |          | 8     |
| 6 APPLICATION OF DERIVATIVES       |          | 1         | 1        | 10    |
| 7. INTEGRALS                       | 2        | 1         | 1        | 12    |
| 8. APPLICATION OF INTEGRALAS       |          |           | 1        | 6     |
| 9. DIFFERENTIALS EQUATION          |          | 2         |          | 8     |
| 10 . VECTORS                       | 3        | 1         |          | 7     |
| 11 THREE DIMENSAIONAL GEOMETRY     |          | 1         | 1        | 10    |
| 12. LINEAR PROGRAMMING             |          |           | 1        | 6     |
| 13. PROBABILITY                    |          | 1         | 1        | 10    |
| TOTAL                              | 10 ( 10) | 12 ( 48 ) | 7 ( 42 ) | 100   |

**Model question paper**

## Mathematics

Class: 12

Time: 3hrs

Max.marks:100

### General Instructions:

1. All questions are compulsory
2. The question paper consists of 29 questions divided into three sections A,B and C.  
Section A comprises of 10 questions of 1 mark each, Section B comprises of 12 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.
3. Use of calculators is not permitted.

### SECTION A

1. If A is square matrix of order 3 such that  $|\text{adj } A| = 64$ , find  $|A|$
2. If  $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ , find ;  $0 < \theta < \frac{\pi}{2}$  when  $A + A^t = I$ .
3. If  $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ . Find x
4. Let \* be a binary operation on R given by  $a * b = \frac{a^2 - b}{3}$ . Write the value of  $3 * 4$ .
5. Evaluate :  $\sin \{ \pi/3 - \sin^{-1}(-1/2) \}$ .
6. Evaluate  $\int \frac{dx}{\sqrt{x+x}}$ .
7. If  $\int (e^{ax} + bx) dx = \frac{e^{4x}}{4} + \frac{3x^2}{2}$ . find the values of a and b.
8. If  $\vec{a} = i + j$ ,  $\vec{b} = j + k$ , find the unit vector in the direction of  $\vec{a} + \vec{b}$ .
9. If  $|\vec{a}| = \sqrt{3}$ ,  $|\vec{b}| = 2$  and angle between  $\vec{a}$  and  $\vec{b}$  is 60. Find  $(\vec{a} \cdot \vec{b})$
10. What is the cosine of the angle which the vector  $\sqrt{2} i + j + k$  makes with y-axis.

### SECTION B

11. Prove that  $\sin^{-1} \frac{5}{13} + \sin^{-1} \frac{7}{25} = \cos^{-1} \frac{253}{325}$ .

OR

$$\text{Solve for } x : \sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}.$$

12. Consider

$f : R_+ \rightarrow [-5, \infty)$  given by  $f(x) = 9x^2 + 6x - 5$ . Show that  $f$  is invertible. Find inverse of  $f$ ?

13. Show that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

14. If  $x = a \sin pt$  and  $y = b \cos pt$ , find the value of  $\frac{d^2y}{dx^2}$  at  $t = 0$ .

15. If  $y = \sin \left( 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right)$  Prove that  $\frac{dy}{dx} = -\frac{x}{\sqrt{1-x^2}}$

16. Find the intervals in which the function  $f(x) = \sin x - \cos x$ ;  $0 \leq x \leq 2\pi$  is (i) increasing (ii) decreasing

17. Evaluate  $\int \frac{x^2 + 4}{x^4 + x^2 + 16} dx$ .

OR

Evaluate  $\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$ .

18. Form the differential equation representing the family of ellipse having foci on x-axis and centre at the origin.

OR

Form the differential equation of the family of circles having radii 3.

19. Find the equation of the plane determined by the points A(3,-1,2), B(5,2,4) and C(-1,-1,6). Also find the distance of the point P(6,5,9) from the plane

20. Solve the differential equation  $\frac{dy}{dx} + y \sec^2 x = \tan x \cdot \sec^2 x$ ;  $y(0) = 1$ .

21. If  $a$  and  $b$  are unit vectors and  $\theta$  is the angle between them, then prove that  $\cos \theta/2 = \frac{1}{2} |a + b|$

22. Find the probability distribution of the number of heads in a single throw of three coins.

OR

Three balls are drawn one by one without replacement from a bag containing 5 white and 4 green balls. Find the probability distribution of number of green balls drawn.

## SECTION C

23. Find the matrix  $A$  satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

24. Evaluate  $\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

25. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is  $\sin^{-1} \frac{1}{3}$ .

26. Using method of integration find the area of the region bounded by the lines  $2x + y = 4$ ,  $3x - 2y = 6$  and  $x - 3y + 5 = 0$ .

OR

Using integration, find the area of the region

$$\{(x, y) : |x-1| \leq y \leq \sqrt{5-x^2}\}$$

27. Find the distance of the point (3,4,5) from the plane  $x + y + z = 2$  measured parallel to the line  $2x = y = z$ .

OR

Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1,3,4) from the plane  $2x - y + z + 3 = 0$ .

And also find the image of the point in the plane.

28. A dealer in rural area wishes to purchase a number of sewing machines. He has only rupees 5760.00 to invest and has space for at most 20 items. Electronic sewing machines cost him rupees 360 and manually operated sewing machine rs.240. He can sell an electronic sewing machine at a profit of rupees 22 and a manually operated sewing machine at a profit of Rs.18. Assuming that he can sell all the items he can buy, how should he invest his money in order to maximize his profit. Make it as a Linear Programming Problem and solve it graphically. Justify the values promoted for the selection of the manually operated machines.

29. A student is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

To get the probability as 1, Which value to be promoted among students.

## MARKING SCHEME

### CLASS XII

1.  $|AdjA| = |A|^{n-1} \Rightarrow |A| = \pm 8$  -----(1)

2.  $\pi/3$

3.  $x = +6$  or  $-6$

4.  $5/3$

5.  $1$

6.  $2 \log|1+\sqrt{x}|+c.$

7.  $a = 4$  and  $b = 3$

8.  $i+2j+k/\sqrt{6}$

9.  $\sqrt{3}$

10.  $\frac{1}{2}$

(1 mark each for correct answer for Qs. 1 to 10)

11. Let ,

$$\alpha = \sin^{-1} \frac{5}{13}, \beta = \sin^{-1} \frac{7}{25}, \quad (1)$$

$$\text{then } \cos \alpha = \frac{12}{13}, \cos \beta = \frac{24}{25} \quad (1)$$

$$\therefore \cos(\alpha + \beta) = \frac{253}{325} \quad (1)$$

$$\therefore \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{7}{25} = \cos^{-1} \frac{253}{325} \quad (1)$$

OR

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2} \Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x \quad \frac{1}{2}$$

$$\Rightarrow (1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right) = \cos(2\sin^{-1}x) \quad \frac{1}{2}$$

$$\Rightarrow (1-x) = \cos(2\alpha) \text{ where } \sin^{-1}x = \alpha \text{ or } x = \sin\alpha \quad \frac{1}{2}$$

$$\Rightarrow (1-x) = 1 - 2\sin^2\alpha = 1 - 2x^2, \therefore 2x^2 - x = 0 \quad 1$$

$$\Rightarrow x(2x-1) = 0 \quad \frac{1}{2}$$

$$\therefore x = 0, \frac{1}{2}$$

$$\text{Since } x = \frac{1}{2} \text{ does not satisfy the given equation } \therefore x=0 \quad 1$$

12. let  $x_1, x_2 \in \mathbb{R}^+$  s.t  $f(x_1) = f(x_2) \rightarrow$  prove that  $x_1 = x_2$  (1)  
f is 1-1

$$9x^2 + 6x - 5 = y$$

$$\Rightarrow (3x+1)^2 - 6 = y$$

$$3x+1 = \sqrt{y+6}$$

$$x = \frac{\sqrt{(y+6)}-1}{3} = f^{-1}(y)$$

Therefore for every  $y \in \mathbb{I}-5, \infty)$  ,  $\exists (\sqrt{y+6}-1)/3 \in \mathbb{R}_+$  s.t

$$f(\sqrt{y+6}-1)/3 = y. \text{ f is onto.} \quad (1\frac{1}{2})$$

$$\text{since f is 1-1 and onto, f is invertible} \quad (\frac{1}{2})$$

Therefore  $f^{-1}(x)$ : from  $\mathbb{I}-5, \infty)$  to  $\mathbb{R}_+$  defined as  $f^{-1}(x) = (\sqrt{x+6}-1)/3$  (1)

13.

$$\Delta = abc \begin{vmatrix} \frac{1}{a} + 1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} + 1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} \quad (1)$$

$$C_1 + C_2 + C_3 \Rightarrow$$

$$\Delta = abc \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b} + 1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c} + 1 \end{vmatrix} \quad (1)$$

$$\Rightarrow \Delta = abc \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad (1)$$

$$= abc \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \quad (1)$$

14.

$$\frac{dx}{dt} = ap \cos pt \quad \frac{dy}{dt} = -bp \sin pt \quad (1)$$

$$\frac{dy}{dx} = -\frac{b}{a} \tan pt \quad (1)$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a^2} \sec^3 pt \quad (1)$$

$$\frac{d^2y}{dx^2} \text{ at } t=0 = -\frac{b}{a^2} \quad (1)$$

15.

.Let  $x = \cos t$

$$Y = \sin \left[ 2 \tan^{-1} \sqrt{\frac{1-\cos t}{1+\cos t}} \right] = \sin \left[ 2 \tan^{-1} \left( \tan \frac{t}{2} \right) \right] \quad (1 \frac{1}{2})$$

$$Y = \sin t = \sqrt{1-x^2} \quad (1)$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}} \quad (1 \frac{1}{2})$$

16.  $f'(x) = \cos(x) + \sin(x)$

$$f'(x) = 0 \rightarrow \tan(x) = -1 \quad (1)$$

$x = 3\pi/4$  and  $7\pi/4$

Intervals are  $[0, 3\pi/4)$ ,  $(3\pi/4, 7\pi/4)$ ,  $(7\pi/4, 2\pi]$  (1)

$[0, 3\pi/4)$ -  $f'$  is positive, so  $f(x)$  is increasing

$(3\pi/4, 7\pi/4)$ -  $f'$  is negative, so  $f(x)$  is decreasing

$(7\pi/4, 2\pi]$ -  $f'$  is positive, so  $f(x)$  is increasing (2)

17. Consider

$$\int \frac{x^2+4}{x^4+x^2+16} dx = \int \frac{1+\frac{4}{x^2}}{x^2+1+\frac{16}{x^2}} dx \quad \text{-----(1)}$$

$$= \int \frac{dt}{t^2+3^2}, \text{ where } t = x - \frac{4}{x} \quad \text{-----(1)}$$

$$= \frac{1}{3} \tan^{-1} \frac{t}{3} + c \quad \text{-----(1)}$$

$$= \frac{1}{3} \tan^{-1} \frac{x^2-4}{3x} + c \quad \text{-----(1)}$$

OR

consider

$$\int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx = \int_0^{\frac{\pi}{2}} \frac{x + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} dx \quad \text{-----(1)}$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} x \sec^2 \frac{x}{2} dx + \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} dx \quad \text{-----(1)}$$

By solving further we get given =

$$\frac{\pi}{2} \tan \frac{\pi}{4} = \frac{\pi}{2} \quad \text{-----(2)}$$

18. The equation of the ellipse is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (1)

$$2x/a^2 + 2y/b^2 dy/dx = 0$$

$$(y/x) dy/dx = -b^2/a^2 \quad (1)$$

Differentiating again and getting the differential equation as

$$(xy) d^2y/dx^2 + x(dy/dx)^2 - y dy/dx = 0 \quad (2)$$

OR

The equation of the family of circles is  $(x-a)^2 + (y-b)^2 = 9$  -----(i) 1/2

$$\Rightarrow 2(x-a) + 2(y-b) \frac{dy}{dx} = 0 \text{ or } (x-a) = -(y-b) \frac{dy}{dx} \quad \text{-----(ii)} \quad 1$$

$$\Rightarrow 1 + (y-b) \frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^2 = 0 \Rightarrow (y-b) = - \frac{[1 + \left( \frac{dy}{dx} \right)^2]}{\frac{d^2y}{dx^2}} \quad \text{-----(iii)} \quad 1$$

from (ii),  $(x-a) = + \frac{[1 + \left( \frac{dy}{dx} \right)^2]}{\frac{d^2y}{dx^2}} \cdot \frac{dy}{dx}$  1/2

putting in (i) to get  $\left[ \frac{1 + \left( \frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}} \right]^2 \left( \frac{dy}{dx} \right)^2 + \left[ \frac{1 + \left( \frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}} \right]^2 = 9$  1/2

or  $\left[ \frac{1 + \left( \frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}} \right]^2 \left[ \left( \frac{dy}{dx} \right)^2 + 1 \right] = 9 \Rightarrow \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^3 = 9 \left( \frac{d^2y}{dx^2} \right)^2$  1/2



19. Equation of the plane

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$

$$\text{i.e., } 3x - 4y + 3z - 19 = 0$$

(2)

Now the perp. Distance from (6,5,9) to this plane is

$$\begin{aligned} &= \frac{|3 \cdot 6 - 4 \cdot 5 + 3 \cdot 9 - 19|}{\sqrt{9 + 16 + 9}} \\ &= \frac{6}{\sqrt{34}} \text{ units} \end{aligned} \quad \text{----- (2)}$$

20. Which is in linear differential equation

$$\text{For finding I.F.} = e^{\int \sec^2 x \, dx} = e^{\tan x} \quad \text{----- (1)}$$

$$\text{Solution y.I.F.} = \int e^{\tan x} \cdot \tan x \cdot (\sec(x))^2 \, dx + c \quad (1)$$

$$= \tan x \cdot e^{\tan x} - e^{\tan x} + c \quad \text{----- (1)}$$

$$\text{When } x=0, y=1 \Rightarrow c=2 \text{ and writing the completed solution } \text{----- (1)}$$

$$21. \text{ Consider } |\hat{a} + \hat{b}|^2 = 1 + 1 + 2 \cos \theta \quad \text{----- (1)}$$

$$= 2(1 + \cos \theta) \quad \text{----- (1)}$$

$$= 4 \cos^2 \frac{\theta}{2}$$

$$\Rightarrow \cos \frac{\theta}{2} = \frac{1}{2} |\hat{a} + \hat{b}| \quad \text{----- (2)}$$

22. Let X be the number of heads, X=0,1,2,3

$$P(\text{having head}) = p = 1/2 \quad q = 1/2, \quad n = 3 \quad \text{.-} \quad \text{----- (1)}$$

$$\text{Now } P(X=0) = {}^3C_0 p^0 q^3 = q^3 = \frac{1}{8} \quad (1)$$

Probability distribution

|      |     |     |     |     |
|------|-----|-----|-----|-----|
| X    | 0   | 1   | 2   | 3   |
| P(X) | 1/8 | 3/8 | 3/8 | 1/8 |

----- (2)

OR

|        |                     |                                |                                |                     |   |
|--------|---------------------|--------------------------------|--------------------------------|---------------------|---|
| X :    | 0                   | 1                              | 2                              | 3                   | 1 |
| P(X) : | $\frac{5c_3}{9c_3}$ | $\frac{5c_2 \cdot 4c_1}{9c_3}$ | $\frac{5c_1 \cdot 4c_2}{9c_3}$ | $\frac{4c_3}{9c_3}$ | 1 |
|        | $= \frac{5}{42}$    | $\frac{10}{21}$                | $\frac{5}{14}$                 | $\frac{1}{21}$      | 2 |

23.

$$\begin{aligned} \text{Let } B &= \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} \\ |B| &= 1 \neq 0 \quad \text{and} \quad |C| = -1 \neq 0 \\ \text{adj}B &= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \quad \text{adj}C = \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} \quad \text{-----} (3) \\ B^{-1} &= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \quad \text{and} \quad C^{-1} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \\ \therefore A = B^{-1}C^{-1} &= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}. \quad \text{-----} (3) \end{aligned}$$

24.

$$\begin{aligned} I &= \int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x} \\ I &= \int_0^{\pi} \frac{(\pi - x) \, dx}{a^2 \cos^2 x + b^2 \sin^2 x} \end{aligned} \quad (1)$$

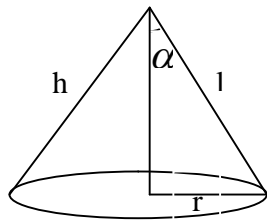
$$2I = \pi \int_0^\pi \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \quad (1)$$

$$\begin{aligned} I &= \pi \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \\ I &= \pi \int_0^{\pi/2} \frac{\sec^2 x \, dx}{a^2 + b^2 \tan^2 x} \end{aligned} \quad (1)$$

For getting the answer as

$$\frac{\pi^2}{2ab} \quad (3)$$

25. Let  $r, h, l, S$  and  $V$  be the radius, height, slant height, surface area and the volume of the cone.



$S =$

$$\pi r l + \pi r^2$$

$$l = \frac{S - \pi r^2}{\pi r} \quad \text{----- (1)}$$

$$\text{and } V = \frac{1}{3} \pi r^2 h \Rightarrow V^2 = \frac{1}{9} \pi^2 r^4 h^2 \quad \text{----- (1)}$$

$$\frac{dV^2}{dr} = 0$$

$$\frac{1}{9} (2rS^2 - 8S\pi r^3) = 0$$

$$\text{For max or min } \Rightarrow \frac{r}{l} = \frac{1}{3} \quad \text{----- (2)}$$

$$\text{now } \frac{d^2V^2}{dr^2} (\text{at } S = 4\pi r^2) < 0$$

$\therefore V^2$  is maximum.

$\therefore V$  is maximum.

$$\text{----- (1)}$$

$$\sin \alpha = \frac{r}{l} = \frac{1}{3}$$

Now

$$\alpha = \sin^{-1}\left(\frac{1}{3}\right).$$

$$\text{----- (1)}$$

26. Let  $AB \rightarrow 2x + y = 4$

$$BC \rightarrow 3x - 2y = 6$$

$$\text{and } AC \rightarrow x - 3y + 5 = 0$$

Solving 1 and 2

$$B(2,0), C(4,3) \text{ and } A(1,2) \quad \left(1\frac{1}{2}\right)$$

For the correct figure (1)

$$\text{Area of triangle} = \int_1^4 \frac{x+5}{3} dx - \int_1^2 4 - 2x dx - \int_2^4 \frac{3x-6}{2} dx \quad \left(1\frac{1}{2}\right)$$

For integrating and getting area=7/2 sq.units

(2)

OR

Equations of curves are

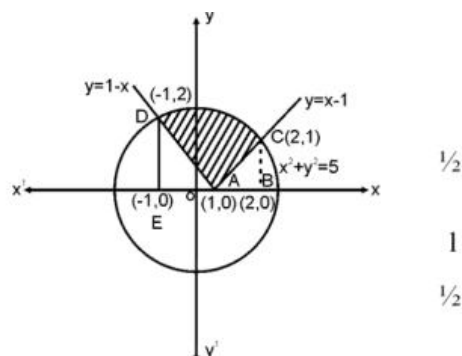
$$x^2 + y^2 = 5 \text{ and } y = \begin{cases} 1-x, & x < 1 \\ x-1, & x > 1 \end{cases}$$

correct figure

Points of intersection are C(2, 1)

D(-1, 2)

Required Area = Area of region (EABCDE) - Area of (ADEA) - Area of (ABCA)



$$= \int_{-1}^2 \sqrt{5-x^2} dx - \int_{-1}^1 (1-x) dx - \int_1^2 (x-1) dx -$$

1

$$= \left[ \frac{x}{2} \sqrt{5-x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_{-1}^2 - \left[ x - \frac{x^2}{2} \right]_{-1}^1 - \left[ \frac{x^2}{2} - x \right]_1^2$$

1

$$= \left[ \left\{ 1 + \frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}} \right\} - \left\{ -\frac{1}{2} \times 2 + \frac{5}{2} \sin^{-1} \left( \frac{-1}{\sqrt{5}} \right) \right\} \right] - \left[ \left( 1 - \frac{1}{2} \right) - \left( -1 - \frac{1}{2} \right) \right] - \left[ (2-2) - \left( \frac{1}{2} - 1 \right) \right]$$

1

$$= 1 + \frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}} + 1 - \frac{5}{2} \sin^{-1} \left( \frac{-1}{\sqrt{5}} \right) - 2 - \frac{1}{2}$$

$$= -\frac{1}{2} + \frac{5}{2} \left[ \sin^{-1} \frac{2}{\sqrt{5}} - \sin^{-1} \left( \frac{-1}{\sqrt{5}} \right) \right]$$

1/2

27. Line is  $x/1=y/2=z/2$

----- (1)

Line PQ through P(3,4,5) and II to the given line is

----- (1)

$$(x-3)/1 = (y-4)/2 = (z-5)/2 = \lambda$$

General point on the line is  $Q(\lambda + 3, 2\lambda + 4, 2\lambda + 5)$

If this point lies on the plane  $x+y+z=2$ ,

$$\therefore \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 2$$

----- (2)

$$\Rightarrow \lambda = -2$$

$$\therefore Q(1, 0, 1)$$

----- (2)

$$\therefore PQ = \sqrt{(3-1)^2 + (4-0)^2 + (5-1)^2} = 6.$$

OR

Let Q be the foot of perpendicular from P to the plane and P' (x, y, z) be the image of P in the plane.

∴ The equations of line through P and Q is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1}$$

The coordinates of Q (for some value of  $\lambda$ ) are

$$(2\lambda+1, -\lambda+3, \lambda+4)$$

Since Q lies on the plane, ∴  $2(2\lambda+1) - 1(-\lambda+3) + (\lambda+4) + 3 = 0$

Solving to get  $\lambda = -1$

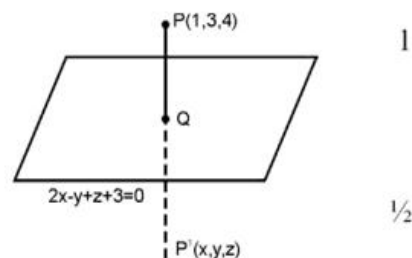
∴ coordinates of foot of perpendicular (Q) are (-1, 4, 3)

Perpendicular distance (PQ) =  $\sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6}$  units

Since Q is mid point of PP'

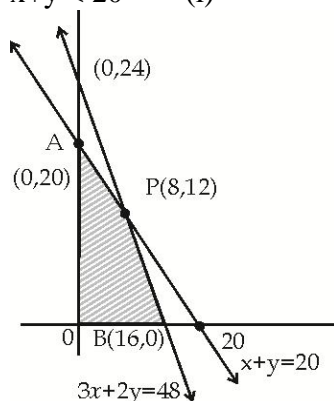
$$\therefore \frac{x+1}{2} = -1, \frac{y+3}{2} = 4, \frac{z+4}{2} = 3 \Rightarrow x = -3, y = 5, z = 2$$

∴ Image of P is (-3, 5, 2)



28. Suppose number of electronic operated machine = x and number of manually operated sewing machines = y

$$x+y < 20 \text{ --- (i)}$$



$$\text{and, } 360x + 240y < 5760 \text{ or } 3x+2y < 48$$

$$x > 0, y > 0$$

$$\text{To maximise } Z = 22x + 18y$$

Corners of feasible region are A (0,20), P(8,12),

B(16,0)

$$ZA = 18 \times 20 = 360, ZP = 22 \times 8 + 18 \times 12 = 392, ZB = 352$$

$$\text{--- (ii)}$$

2

Z is maximum at  $x=8$  and  $y=12$

The dealer should invest in 8 electric and 12 manually operated machines (½)

Keeping the 'save environment' factor in mind the manually operated machine should be promoted so that energy could be saved. (1)

29.

Let E be the event that the student reports that 6 occurs in the throwing of die and let  $S_1$  be the event that 6 occurs and  $S_2$  be the event that 6 does not occur.

$$P(S_1)=1/6, P(S_2)=5/6, \quad (1)$$

$$P(E/S_1)=3/4, P(E/S_2)=1/4 \quad (2)$$

$$P(S_1/E) = \frac{P(S_1) \cdot P\left(\frac{E}{S_1}\right)}{P(S_1) \cdot P\left(\frac{E}{S_1}\right) + P(S_2) \cdot P\left(\frac{E}{S_2}\right)} = 3/8 \quad (2)$$

To get the probability as one, the student should always speak truth.

The value to be promoted among students is truth value. (1)