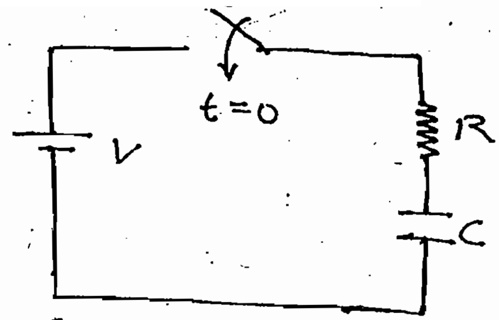
RC circuit with Source



By KVL

$V = iR + \frac{1}{C}\int i\,dt$

Diff. w.r.t t

$0 = R\frac{di}{dt} + \frac{i}{C}$

Divide both sides by R we get

$\frac{di}{dt} + \frac{i}{RC} = 0$

$i(t) = CF + PI$

CF → Transient Response

$\frac{di}{dt} + \frac{i}{R} = 0$

$\boxed{i(t) = A\,e^{-t/RC}}$

PI → steady state response

C → o.c

$i = 0$

$i(t) = CF + PI$

$i(t) = A\,e^{-t/RC} + 0$

$\boxed{A = i(0^+) - i(\infty)}$

$t = 0^-$, $V_c = 0$

$t = 0^+$, $V_c = 0$

By KVL

$t = 0^+$ $\quad V = V_R + V_c$

$= iR + 0$

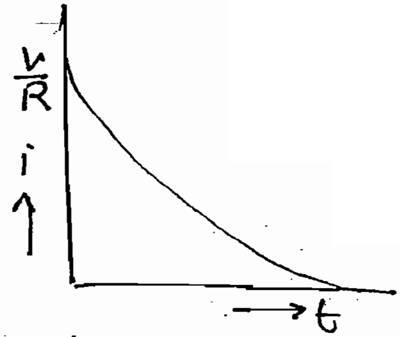
at $t \to t(0^+)$ $\quad i = \frac{V}{R}$

$i(t) = CF + PI$

$A = i(0^+) - i(\infty)$

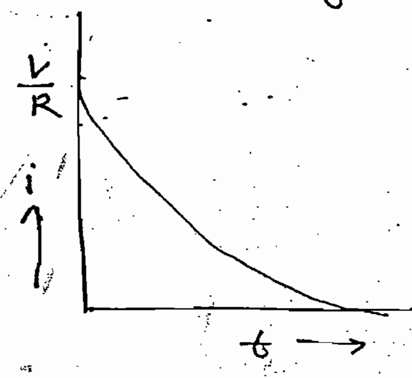
$A = \frac{V}{R} - 0$

$$\boxed{i(t) = \frac{V}{R} e^{-t/RC}}$$



$$V_R = iR$$

$$V_R = V e^{-t/RC}$$



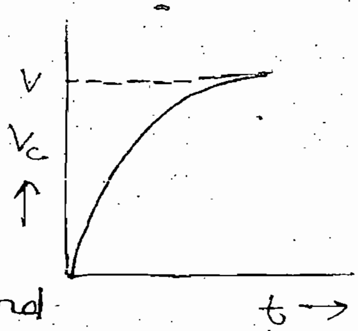
$$V_C = \frac{1}{C}\int_0^t i\,dt$$

$$V_C = \frac{1}{C}\int_0^t \frac{V}{R} e^{-t/RC}\,dt$$

$$V_C = -V e^{-t/RC} + V$$

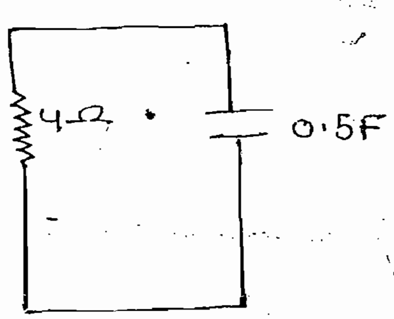
$$\boxed{V_C(t) = [V_C(0^+) - V_C(\infty)]\, e^{-t/RC} + V_C(\infty)}$$

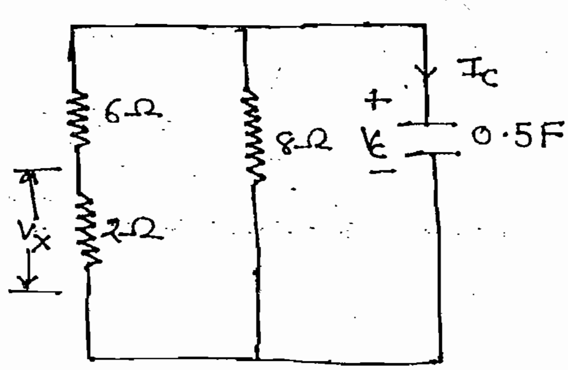
$$V_C(t) = V(1 - e^{-t/RC})$$



Ques :- Find response of V_c, I_c and V_x when initial voltage of the capacitor is 3V

Soln :-





$$V_c = V_0 e^{-t/RC}$$

$$V_c = 3 e^{-t/2}$$

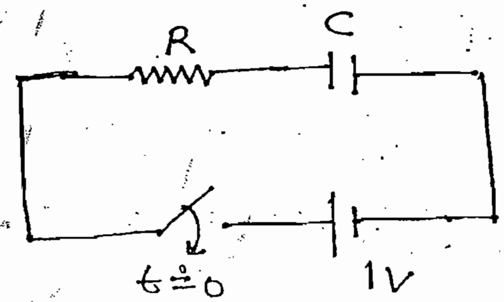
$$V_x = V_c \frac{2}{2+6} \Rightarrow V_x = 3e^{-t/2}\, \tfrac{1}{4}$$

$$\Rightarrow V_x = \frac{3}{4} e^{-t/2}$$

$$I_c = C\frac{dV_c}{dt}$$

$$I_c = \frac{1}{2}\frac{d(3e^{-t/2})}{dt} = 3\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)e^{-t/2}$$

$$\Rightarrow \boxed{I_c = -\frac{3}{4} e^{-t/2}}$$ Ans



Ques:- Find rate of rise of Voltage across the capacitor at $t = 0^+$

(a). RC (b) $\frac{1}{RC}$ (c) 0 (d) $2RC$

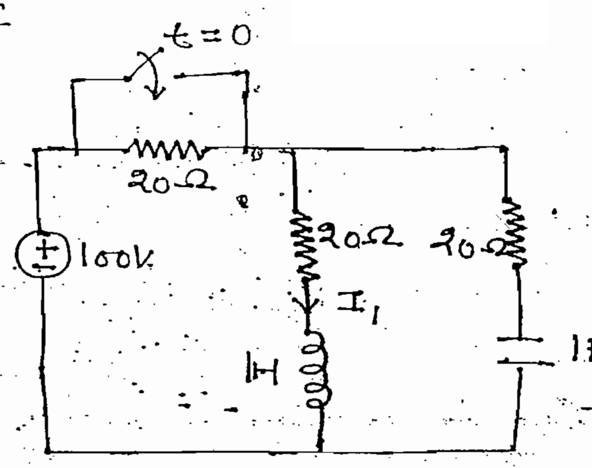
Soln:- $V_c = -Ve^{-t/RC} + V$

$$\frac{dV_c}{dt} = (-V)\left(-\frac{1}{RC}\right)e^{-t/RC} + 0$$

$$\frac{dV_c}{dt} = \frac{V}{RC} e^{-t/RC}$$

$t = 0^+$

$$\frac{dV_c}{dt}(0^+) = \frac{V}{RC} = \frac{1}{RC}$$



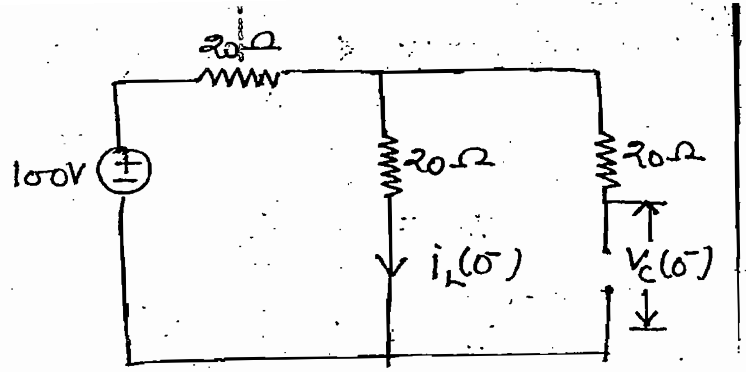
Ques:- Find $\frac{di_1}{dt}$ at $t = 0^+$

Soln:- Step-(1):-

Develop eq. circuit at $t = 0^-$ and find initial current of conductor and initial voltage of Capacitor

→ In the above ckt at $t = 0^-$ for DC source inductor behaves as S.C and C behaves as as O.C

$i_L(0^-) = \dfrac{100}{20+20} = 2.5A$



$V_c(0^-) = 2.5 \times 20$
$= 50$

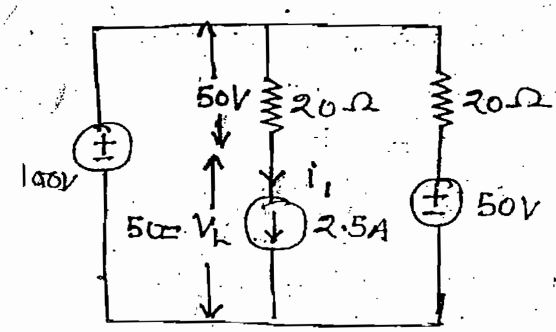
Step-2:—

Develop eq. ckt at $t=0^+$ and indicate initial current of the inductor and initial voltage of the capacitor

$t = 0^+$

$i_L(0^+) = i_L(0^-) = 2.5A$

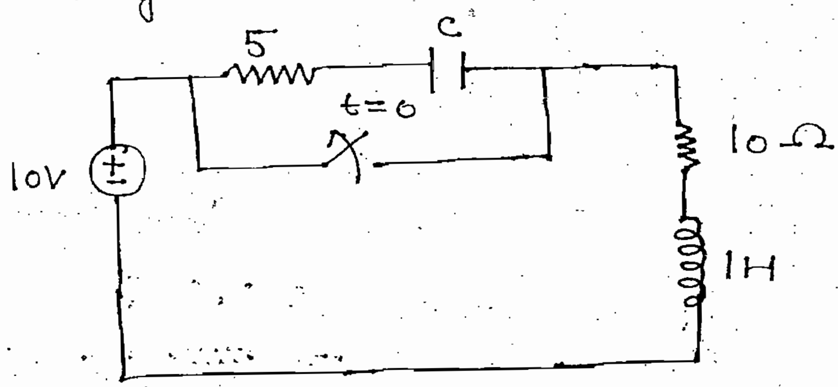
$V_c(0^+) = V_c(0^-) = 50V$



$V_L = L\dfrac{di}{dt}$

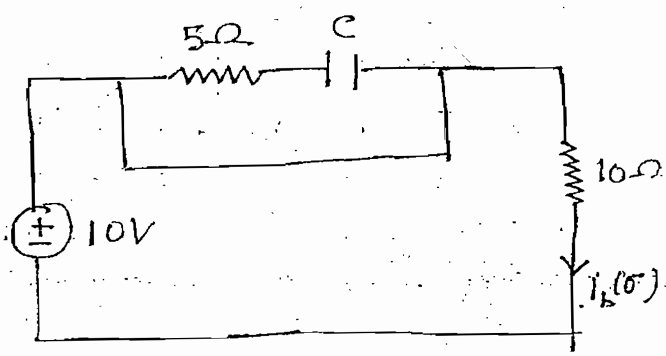
$\Rightarrow 50 = 1\dfrac{di}{dt} \Rightarrow \dfrac{di}{dt} = 50\ A/s$ Ans

Ques:- Find voltage across the switch at $t=0^+$



Soln:- At $t=0^-$

→ Develop equivalent circuit at $t=0^-$



→ In the above ckt at $t=0^-$ capacitor is uncharged & for DC source inductor behaves as a S.C

$i_L(0^-) = \dfrac{10}{10} = 1A$, $V_c(0^-) = 0$

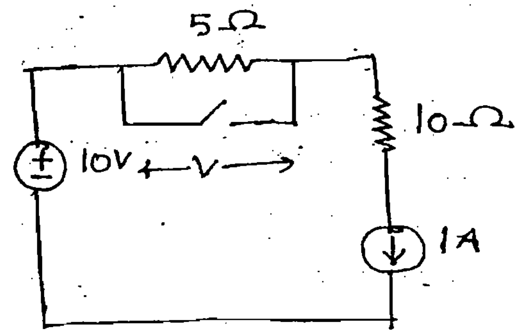
Step-2:-

At $t = 0^+$

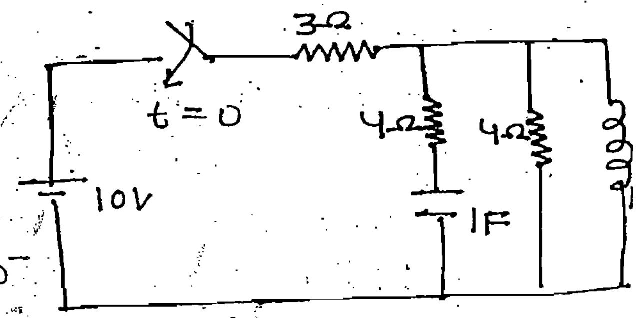
$i_L(0^+) = i_L(0^-) = 1A$

$V_c(0^+) = V_c(0^-) = 0$

$V(0^+) = 5 \times 1 = 5V$



Ques:- Find I in capacitor and voltage across inductor at $t = 0^+$



Soln:- Step (I):-

Develop eq. ckt. at $t = 0^-$

→ In the above ckt at $t = 0^-$ capacitor and inductor are uncharged elements.

$t = 0^-$, $V_c(0^-) = 0$

$i_L(0^-) = 0$

Step-(II):-

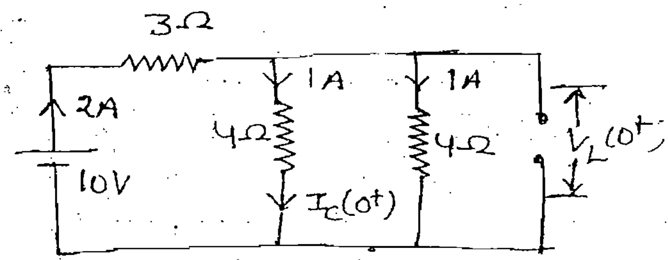
At $t = 0^+$

$i_L(0^+) = i_L(0^-) = 0$

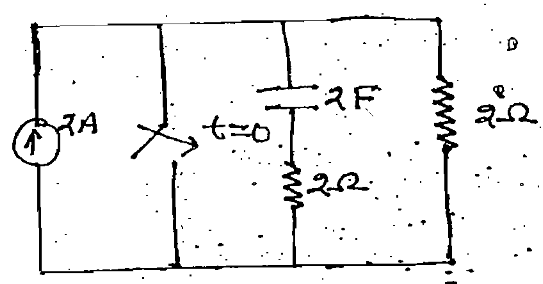
$V_c(0^+) = V_c(0^-) = 0$

$I_c(0^+) = 1A$

$V_L(0^+) = 4 \times 1 = 4V$

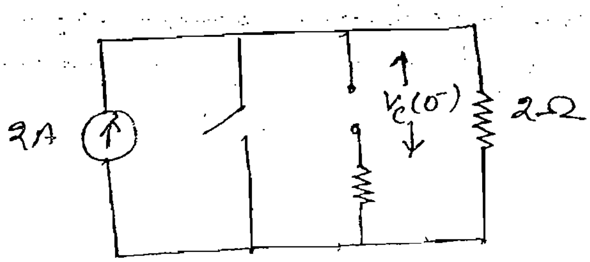


Ques:- Find initial voltage and final voltage of the capacitor



Soln:- Step (I):-

$t = 0^-$



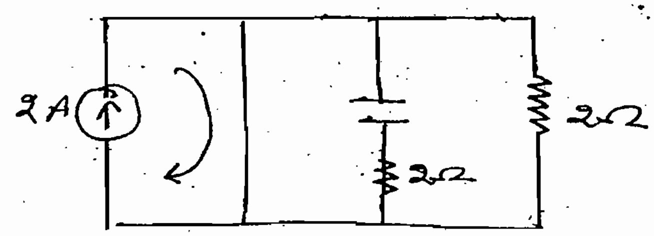
$V_c(0^-) = 2 \times 2 = 4V$

Step-(II):-

At $t = 0^+$

$V_c(0^+) = V_c(0^-) = 4V$

$V_c(\infty) = 0$

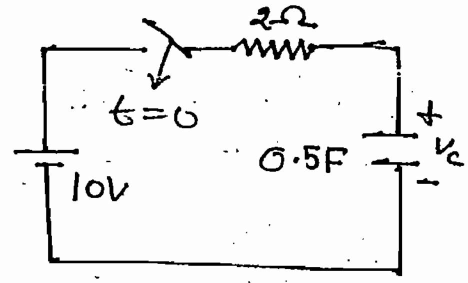


Ques:- Find

(I) Energy of Capacitor at $t = \infty$

(II) current in the capacitor at t = 1sec

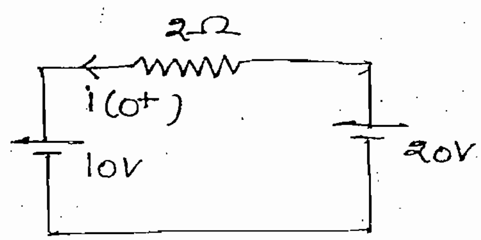
(III) Energy of the resistor from 0 to ∞ interval



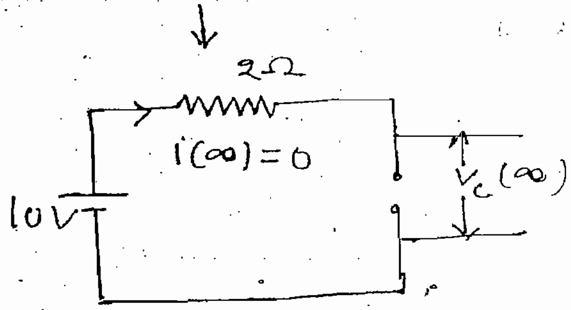
When initial charge of the capacitor is 10C

Soln:- At $t = 0^+$ $(\because Q_{ini} = 10C)$

$$V_0 = \frac{Q_0}{C} = \frac{10}{0.5} \Rightarrow V_0 = 20$$



At $t = \infty$



$V_c(\infty) = 10V$

$$W_c(\infty) = \frac{1}{2} C V_c^2(\infty) = \frac{1}{2} \cdot \frac{1}{2} \cdot (10)^2 = 25J$$

(II) $i(t) = [i(0^+) - i(\infty)] e^{-t/RC} + i(\infty)$

$\Rightarrow i(t) = [-5-0] e^{-t/1} + 0$

$i(t) = -5e^{-t}$

At t = 1sec

$i(1s) = -5e^{-1} = -\frac{5}{e}$

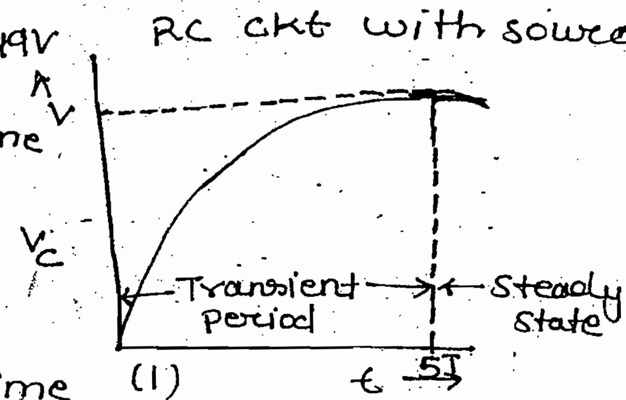
$= -1.84$

(iii) $W_R = \int_0^\infty P\,dt = \int_0^\infty i^2 R\,dt = \int_0^\infty (-5e^{-t})^2\, 2\,dt$

$\Rightarrow W_R = 25\text{J}$, Ans.

Time Constant :-

RC ckt with source

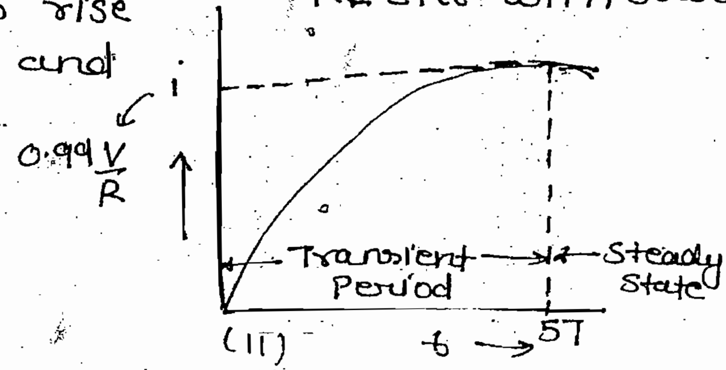


→ Time constant can be define either w.r.t charging or discharging action of energy storage element.

↔ Time constant is the time taken for response to rise 63.2% of max value and it is given by

$$\boxed{T = \frac{L}{R} \text{ sec} \qquad T = RC}$$

RC ckt with sourc



$$V_c(t) = V(1 - e^{-t/T})$$

(I) $V_c(t) = V(1 - e^{-t/RC})$

(II) $i(t) = \frac{V}{R}(1 - e^{-Rt/L})$

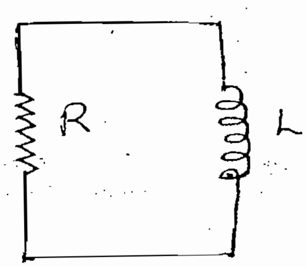
(1) $t = T$ $\quad V_c = V(1 - e^{-1}) = 0.632V$

$t = 5T$ $\quad V_c = V(1 - e^{-5}) = 0.99V$

$R_1 = 10\Omega$ $\quad P_1 = I^2 R_1$ $\quad t = 10\text{sec}$

$R_2 = 20\Omega$ $\quad P_2 = i^2 R_2$ $\quad t = 7\text{sec}$

$R_3 = 100\Omega$ $\quad P_3 = i^2 R_3$ $\quad t = 2\text{sec}$.

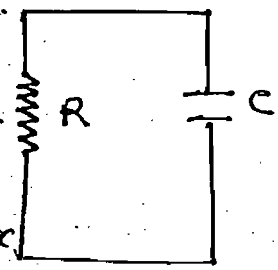


Hence $T = \frac{L}{R}$

$R_1 = 10\Omega \qquad P_1 = \frac{V^2}{R_1} \qquad t = 10sec.$

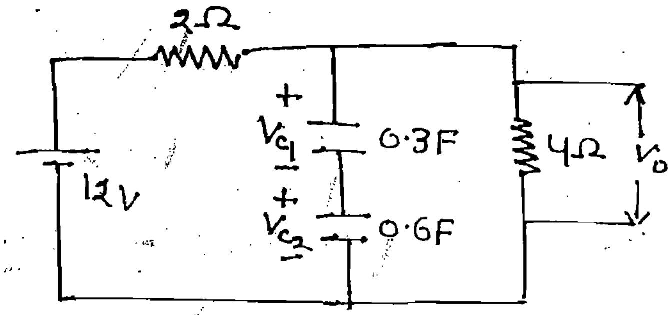
$R_2 = 20\Omega \qquad P_2 = \frac{V^2}{R_2} \qquad t = 15sec.$

$R_3 = 100\Omega \qquad P_3 = \frac{V^2}{R_3} \qquad t = 50sec$



Hence $\quad T = RC$

Ques:- Find V_0 response for $t>0$



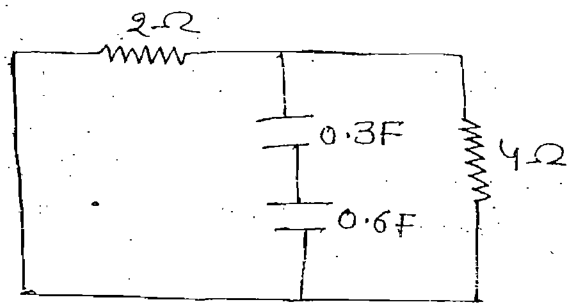
$V_{c_1}(0) = 24V$

$V_{c_2}(0) = 6V$

(a) $8 + 22e^{-3.75t}$ (b) $8 + 22e^{-t/3.75}$

(c) $8 + 22e^{-t/1.2}$ (d) $8 + 22e^{-1.2t}$

Soln:- For time constant, deactivate all the Independent sources



$R_{eq} = \frac{2\times4}{2+4}$

$C_{eq} = \frac{0.3\times0.6}{0.3+0.6}$

$T = R_{eq}C_{eq} = RC$

$V(t) = [V(0^+) - V(\infty)]e^{-t/RC} + V(\infty)$

$V_0(t) = [V_0(0^+) - V_0(\infty)]e^{-t/RC} + V_0(\infty)$

$V_0 = V_{c_1} + V_{c_2}$

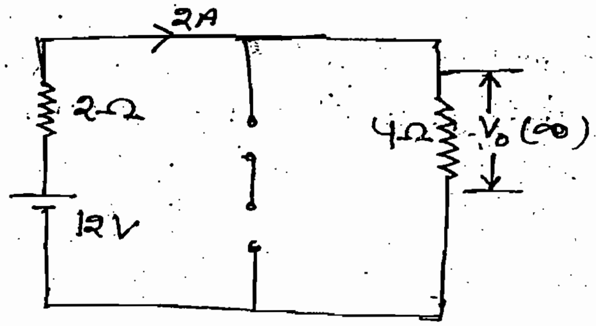
$t = 0^+$

$V_0(0^+) = V_{c_1}(0^+) + V_{c_2}(0^+) = 24 + 6 = 30$

At $t = \infty$

At $t=\infty$

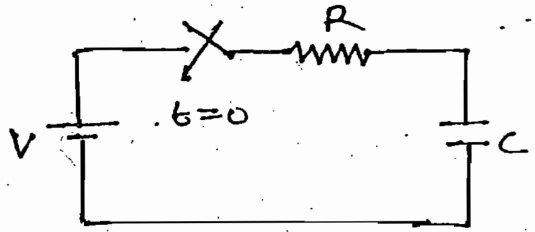
$V_0(\infty) = 2\times 4 = 8$



$V_0(t) = [30-8]e^{-3.75t} + 8$

$= 8 + 22e^{-3.75t}$

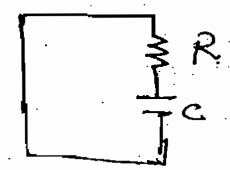
(I)



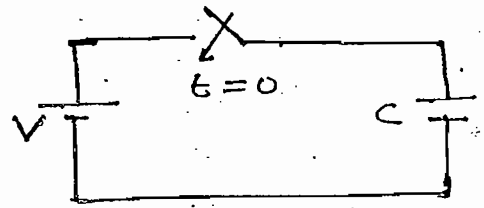
At $t=0^-$, $V_c=0$

$t=0^+$, $V_c=0$

$T=RC$



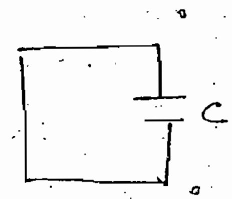
(II)



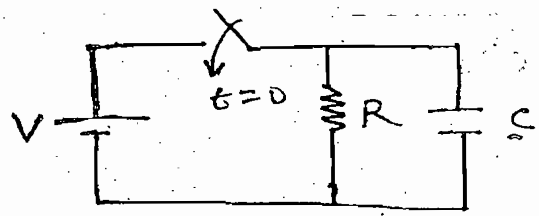
$t=0^-$, $V_c=0$

$t=0^+$, $V_c=V$

$T = RC = 0$



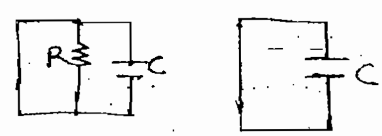
(III)



$t=0^-$, $V_c=0$

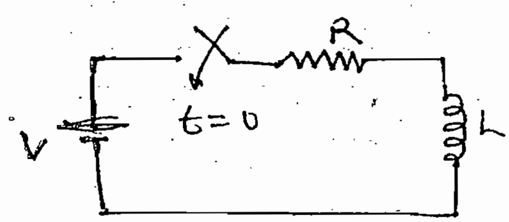
$t=0^+$, $V_c=V$

$T=RC=0$



$T=RC=0$

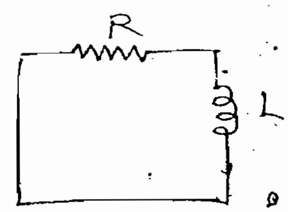
(IV)



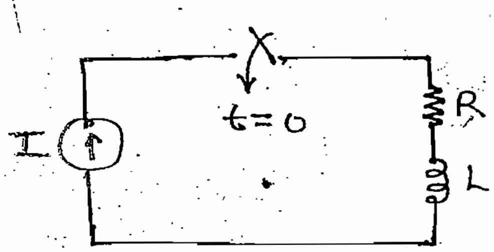
$t=0^-$, $i=0$

$t=0^+$, $i=0$

$T = L/R$



(V)

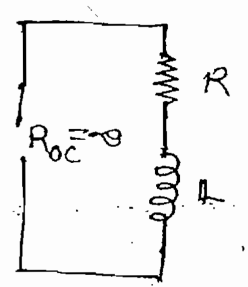


$t=0^-$, $i=0$

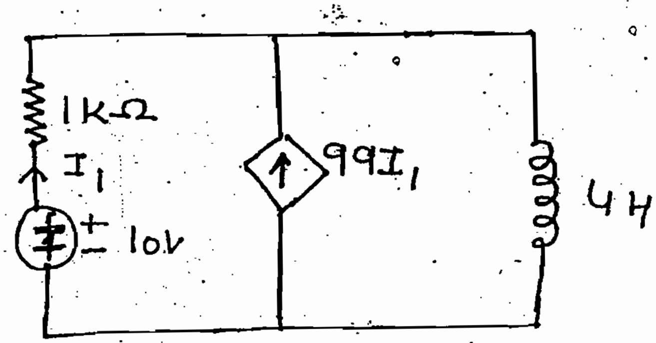
$t=0^+$, $i=I$

$R_{eq} = R_{oc}+R = \infty$

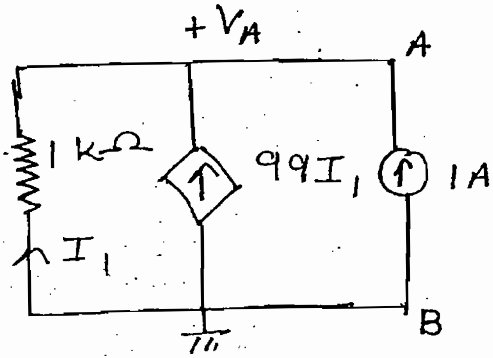
$T = \dfrac{L}{R_{eq}} = 0$

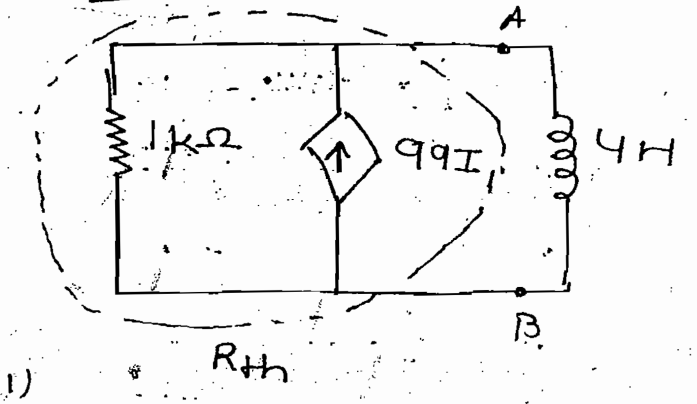


Ques:- Find time constant of the circuit shown



Soln:-





$$\frac{V_A}{1\times10^3} = 99I_1 + 1 \quad -(I)$$

$$I_1 = -\frac{V_A}{1\times10^3} \quad -(II)$$

From (I) & (II)

$V_A = 10V$

$$R_{th} = \frac{V_A}{I_S} = \frac{10}{1} = 10\Omega$$

$$T = \frac{L}{R_{th}} = \frac{4}{10}$$