EXERCISE 9.3

In each of exercise 1 to 5, form a differential Equation representing the a given family of curves by elliminating arbitrary constants a and b.

Q No. 1 $\frac{x}{a}+\frac{y}{b}=1$.

Sol. Given family is $\frac{x}{a}+\frac{y}{b}=1$ ---- (1)

Differentiating (1) we get

$$\frac{1}{a}+\frac{1}{b}y'=0$$

Again diff. w.r.t. x we get

$$0+\frac{1}{b}y''=0 \quad \text{or} \quad y''=0$$

Which is required diff. eqn.

Q No 2 $y^2=a(b^2-x^2)$

Sol Given family is $y^2=a(b^2-x^2)$ ---- (1)

$$\Rightarrow 2y\frac{dy}{dx}=a(-2x) \Rightarrow y\cdot\frac{dy}{dx}=-ax \text{ ---- (2)}$$

$$\Rightarrow y\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{dy}{dx}=-a. \text{ ----- (3)}$$

Elliminating a between (2) and (3)

$$y\cdot\frac{dy}{dx}=x\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)$$

or $yy'=x(yy''+(y')^2)$

Which is required diff. eqn.

Q No. 3 $y=ae^{3x}+be^{-2x}$. ---- (1)

Sol. $\Rightarrow \frac{dy}{dx}=a\cdot e^{3x}\cdot 3+b\cdot e^{-2x}\cdot(-2)$

or $y'=3ae^{3x}-2be^{-2x}$ ---- (2)

(2) $-$ 3 x (1) $\Rightarrow$

$$y'-3y=-5be^{-2x} \text{ ..... (3)}$$

$\Rightarrow$ On diff.

$$y'' - 3y' = -5be^{-2x}(-2)$$

i.e. $y'' - 3y' = 10be^{-2x}$ ---(4)

From (3) and (4)

$$y'' - 3y' = -2(y' - 3y)$$

i.e. $y'' - y' - 6y = 0$ which is required diff. eqn.

QNo 4 $\quad y = e^{2x}(a+bx)$

Sol. : Given family is $y = e^{2x}(a+bx)$ ---(1)

$\Rightarrow \dfrac{dy}{dx} = e^{2x}(b) + (a+bx)e^{2x}.2$

$\Rightarrow y_1 = be^{2x} + 2y$ (Using (1))

$\Rightarrow y_1 - 2y = be^{2x}$ ····(2)

$\Rightarrow y_2 - 2y_1 = b.e^{2x}.2$

$\Rightarrow y'' - 2y' = 2(y' - 2y)$ [Using (2)]

$\Rightarrow y'' - 4y' + 4y = 0$

Which is required diff. eqn.

QNo.5 $\quad y = e^{x}[a\cos x + b\sin x]$

Sol : Given family is $y = e^{x}[a\cos x + b\sin x]$ --(1)

$\Rightarrow \dfrac{dy}{dx} = e^{x}[a(-\sin x) + b(\cos x)] + [a\cos x + b\sin x].e^{x}$

$\Rightarrow \dfrac{dy}{dx} = e^{x}(-a\sin x + b\cos x) + y.$ (Using (1))

$\Rightarrow y' = e^{x}[-a\sin x + b\cos x] + y$ .... (2)

$\Rightarrow y'' = e^{x}[-a\cos x - b\sin x] + [-a\sin x + b\cos x]e^{x} + y'$

$\Rightarrow y'' = -y + (y' - y) + y'$ [Using (1) and (2)]

$\Rightarrow y'' - 2y' + 2y = 0$

Which is required diff. eqn.

Q No. 6. Form the differential Equation of the family of circle touching the y-axis at origin.

Sol. Since the circle touches the y-axis at origin

This means y-axis is tangent to circle at $(0,0)$

$\therefore$ Centre of circle will lie on x-axis (Diameter ⊥r. to tangent line)

$\therefore$ Let centre of circle be $C(h,0)$

$\therefore$ Radius of Circle = Distance between $(0,0)$ and $(h,0)$

$= \sqrt{(h-0)^2+(0-0)^2} = \sqrt{h^2} = |h|$

$\therefore$ Equation of Circle is

$$(x-h)^2+(y-0)^2 = (|h|)^2$$

$\Rightarrow$ $x^2+h^2-2hx+y^2 = h^2$

$\Rightarrow$ $x^2+y^2-2hx = 0$ .... (1)

Differentiating both sides w.r.t x

$$2x+2y\frac{dy}{dx}-2h = 0$$

or $x+yy_1-h = 0$ $\Rightarrow$ $h = (x+yy_1)$

$\therefore$ from (1)

$$x^2+y^2-2(x+yy_1)x = 0$$

$$x^2+y^2-2x^2-2yy_1x = 0$$

or $-x^2+y^2-2xyy_1 = 0$

or $x^2-y^2+2xyy_1 = 0$

Which is required differential eqn.

Q No 7. For the differential eqn of the family of parabola's having vertex at origin and axis along +ve y-axis.

Sol. Equation of any parabola of the said type can be written as $x^2 = 4ay$. where a is parameter. ..(1)

$\Rightarrow$ $2x = 4ay_1$ .... (2)

$\Rightarrow$ $x = 2ay_1$ .....(2)

$\Rightarrow$ $4a = 2x/y_1$

$\therefore$ from (1) $\quad x^2 = \frac{2x}{y_1} y.$

or $\quad x = \frac{2y}{y_1}$

or $\quad xy_1 - 2y = 0$

Q No 8. Form the differential eqn of the family of ellipses having foci on y-axis and centre at origin.

Sol.. Equation of ellipse of given type is of the form

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{---(1)}$$

Here a and b are semi-major and semi-minor axes resp.

$$\therefore \quad \frac{2x}{b^2} + \frac{2y}{a^2} \cdot \frac{dy}{dx} = 0$$

or $\quad \frac{1}{b^2} x + \frac{1}{a^2} y y_1 = 0 \quad$ or $\quad \frac{yy_1}{x} = -\frac{a^2}{b^2} \quad \text{--(2)}$

Again differentiating we get

$$\frac{x \cdot \frac{d}{dx}(yy_1) - yy_1 (1)}{x^2} = 0$$

or $\quad x(yy_2 + y_1y_1) - yy_1 = 0$

or. $\quad x(y_1^2 + yy_2) - yy_1 = 0 \quad$ or $\quad x(y'^2 + yy'') - yy' = 0$

which is required diff. eqn.

Q No 9. Form the diff. eqn of family of hyperbolas having foci on x-axis and cetre at origin.

Soln. Equation of the hyperbola of said type is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{----(1)}$$

Differentiatiting both sides

$$\frac{2x}{a^2} - \frac{2yy_1}{b^2} = 0.$$

or. $\quad \frac{yy_1}{x} = \frac{b^2}{a^2}.$

Again differentiating

$$\frac{x\frac{d}{dx}(yy_1) - yy_1 \times 1}{x^2} = 0$$

or $x(yy_2 + y_1 y_1) - yy_1 = 0$

or. $x(y_1^2 + yy_2) - yy_1 = 0.$

Q No 10. Form the diff. eqn of family of circles having centre on Y-axis and radius 3.

Sol. Let centre of circle be $(0,k)$ and $r = 3$

$\therefore$ Its eqn will be

$$(x-0)^2 + (y-k)^2 = 3^2$$

or $x^2 + (y-k)^2 = 9 \quad \cdots (1)$

or $2x + 2(y-k) \cdot y_1 = 0$

or $y - k = -\frac{x}{y_1} \quad \cdots (2)$

$\therefore$ From (1) $x^2 + \left(-\frac{x}{y_1}\right)^2 = 9$

or $x^2(y_1^2 + 1) = 9y_1^2$

Which is required differential eqn.

Q No 11 Which of the following differential eqns. has $y = c_1 e^x + c_2 e^{-x}$ as general solution.

(A) $\frac{d^2y}{dx^2} + y = 0$ (B) $\frac{d^2y}{dx^2} - y = 0$ (C) $\frac{d^2y}{dx^2} + 1 = 0$ (D) $\frac{d^2y}{dx^2} - 1 = 0$.

Sol. The given sol. is $y = c_1 e^x + c_2 e^{-x}$

$$y_1 = c_1 e^x + c_2 e^{-x}(-1)$$

$$y_1 = c_1 e^x - c_2 e^{-x}$$

$$y_2 = c_1 e^x - c_2 e^{-x}(-1)$$

$$y_2 = c_1 e^x + c_2 e^{-x}.$$

or $y_2 = y$

or $y_2 - y = 0 \quad \therefore$ B is correct option

QNo 12. Which of the following differential equation has $y=x$ as one of particular Soln.

(A) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x.$ (B) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x.$

(C) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$ (D) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

Sol. The given sol. is $y=x.$

$\Rightarrow \quad y_1 = 1$

and $y_2 = 0$

$\Rightarrow \quad \frac{dy}{dx} = 1 \quad \frac{d^2y}{dx^2} = 0$

We ~~k~~ note that $y=x$, $\frac{dy}{dx}=1$ and $\frac{d^2y}{dx^2}=0$

Satisfy $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

$\because \quad 0 - x^2(1) + x\cdot x = 0$ is true.

$\therefore$ (C) is the correct option.

Prepared By:

Rupinder Kaur.
Lect. Maths.
GSSS BHARI (FGS)