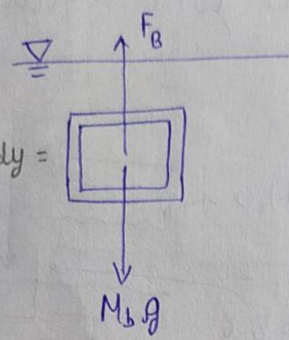
lecture ⑥

Different situations :-

Case - ①

B = Body =



Force analysis

$M_b g = F_B$

$\rho_b \forall_b \cancel{g} = \rho \forall_{displaced} \cancel{g}$

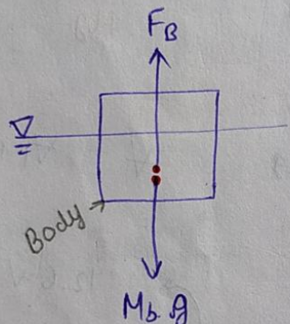
$[\forall_b = \forall_{displaced}]$

$\boxed{\rho_b = \rho}$

ρ = density of liquid

ρ_b = density of body

Case - ②



Force analysis

$M_b g = F_B$

$\rho_b \cdot \forall_b \cdot \cancel{g} = \rho \forall_{displaced} \cdot \cancel{g}$

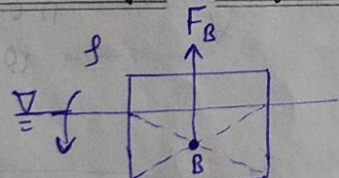
High

low

$\forall_b > \forall_{displaced}$

$\boxed{\rho_b < \rho}$

Center of Buoyancy :-



① It is a point at which buoyant force acts

② this point is exactly same as center of gravity of displaced fluid. ★★★★

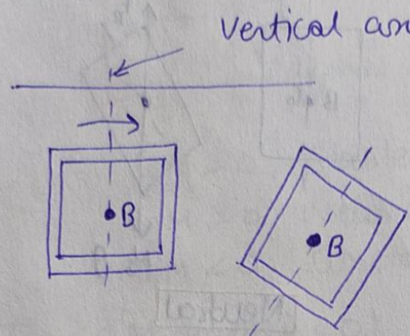
Different geometry

Area same

After same area the CoG will changed

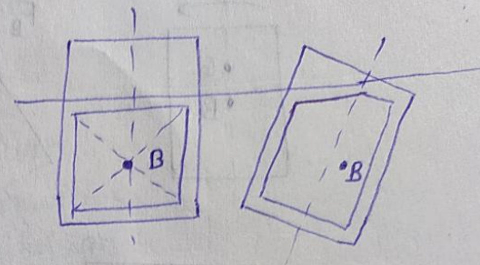
Effect of disturbances :-

Submerge body



- Since the shape of displaced volume remain same during disturbance so center of bouyancy will in the same point on the vertical axis of body.

Floating body

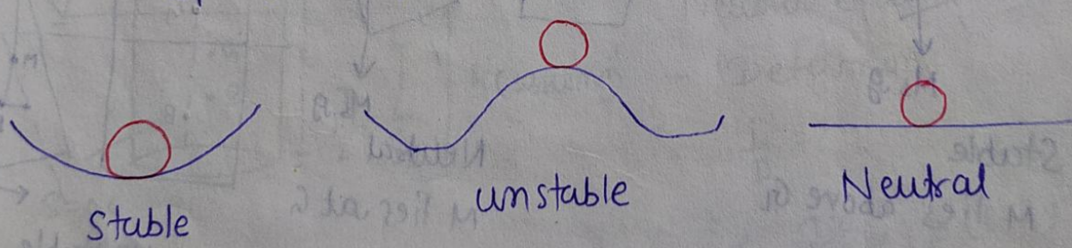


- Since the shape of displaced volm changes during disturbance so center of bouyancy changes it's location and shift from vertical axis of body.

Stability

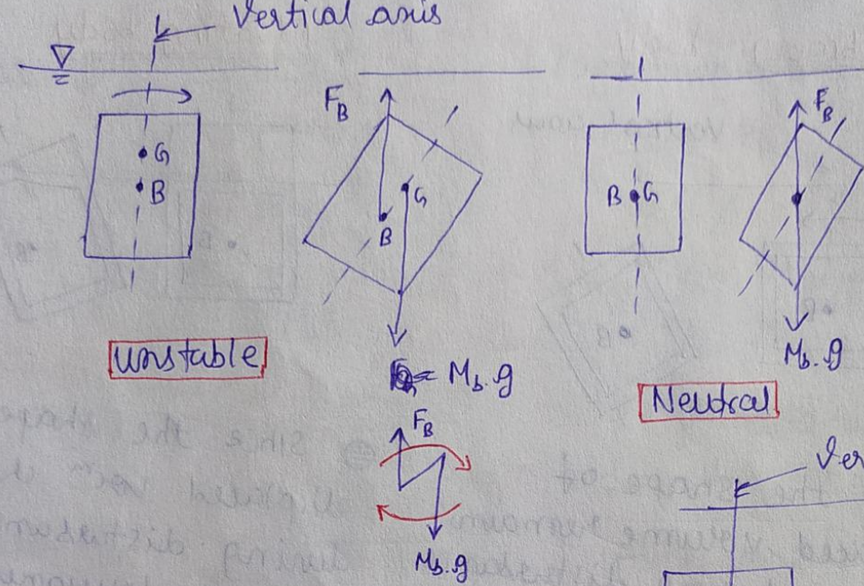
For equilibirium

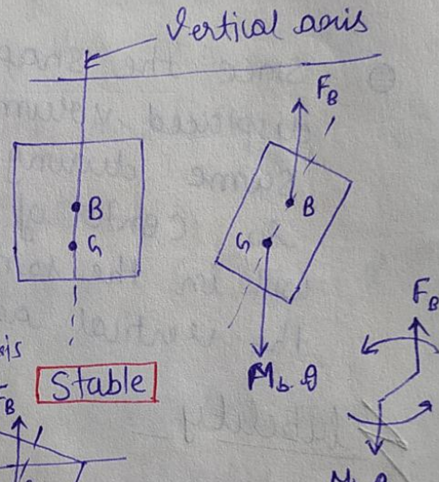
$$\Sigma F = 0 \text{ and } \Sigma M = 0 .$$



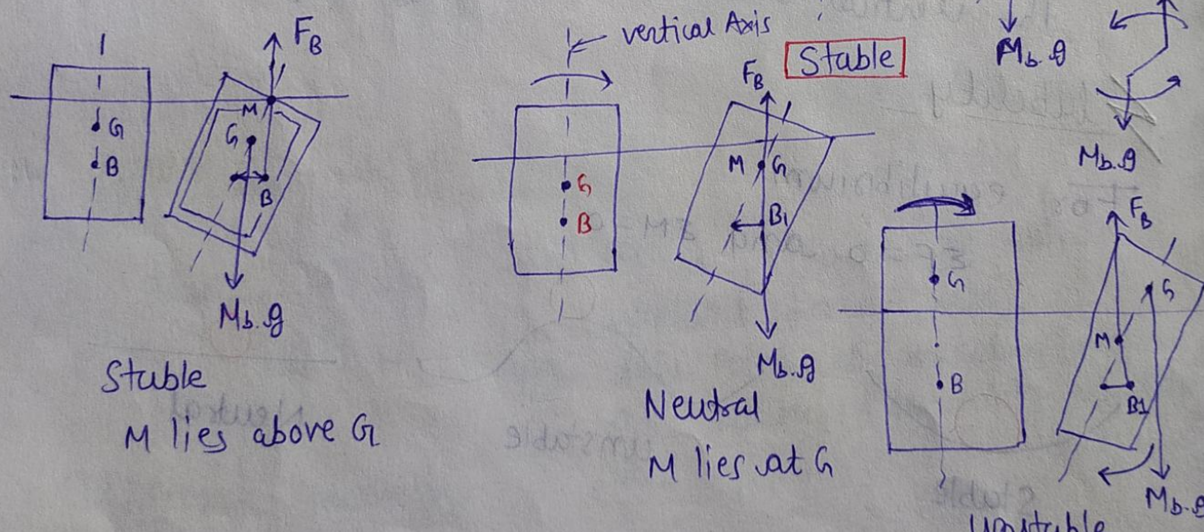
Restoring force is more dominant than disturbing force

Stability of submerge body →





Stability of Floating body :-



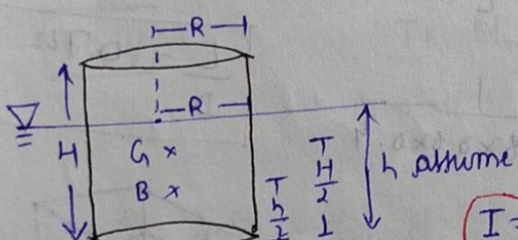
Metacenter (M)

it is point of intersection of vertical line passing through new center of Bouyancy (B_1) with the vertical axis (B_G) of the body under a very small angular tilt given to the body. Ⓔ it is the point about which body is oscillating under a very small angular tilt given to the body.

$$\left[GM = \frac{I}{\forall_{\text{displaced liquid}}} - BG \right]$$

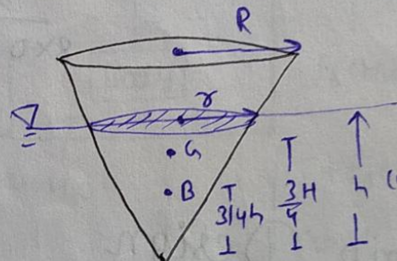
$GM > 0$, Stable equilibrium
$GM = 0$. Neutral equilibrium
$GM < 0$, Unstable equilibrium

I = Moment of inertia of the plane cut by free surface level of liquid.



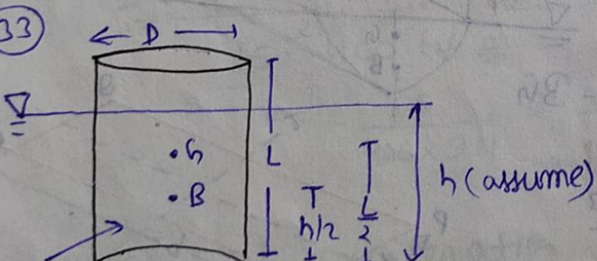
$$I = \frac{\pi R^4}{4}$$

cylinder



$$I = \frac{\pi r^4}{4}$$

Pb - 33



$S = 0.6$

Neutral equilibirium
- Determine $\frac{L}{D} = ?$

$GM = 0$

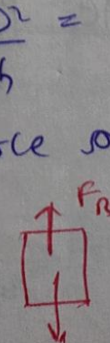
$$\frac{I}{\forall_{\text{displaced}}} - BG = 0$$

$$\frac{\frac{\pi D^4}{64}}{\left(\frac{\pi D^2}{4} \cdot h\right)} - \left(\frac{L}{2} - \frac{h}{2}\right) = 0$$

$$\frac{D^2}{16h} = \frac{1}{2}(L - h)$$

$$\frac{D^2}{8h} = L - h \quad \text{--- (1)}$$

Force analysis →



$$M_b \cdot g = F_B$$

$$\rho_b \, V_b \cdot \cancel{g} = \rho_w \, V_{Dis} \cdot \cancel{g}$$

$$\frac{\rho_b}{\rho_w} \times \frac{\pi D^2}{\cancel{4}} \cdot L = \frac{\pi D^2}{\cancel{4}} \cdot h$$

$\downarrow$

0.6 $\qquad 0.6L = h$ --- (2)

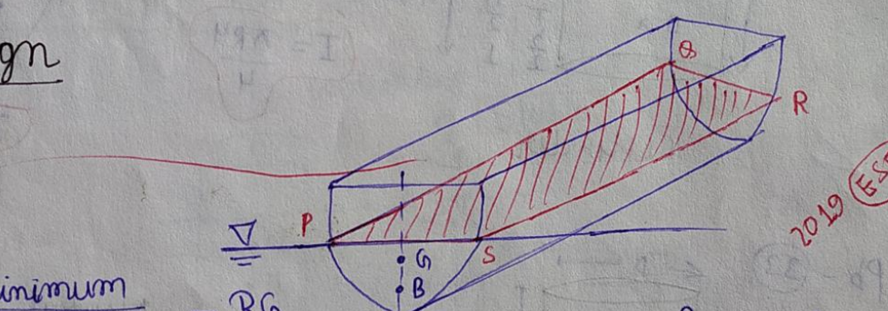
By (1) and (2) eqn

$$\frac{D^2}{8(0.6L)} = L - 0.6L$$

$$\frac{D^2}{8 \times 0.6 \times 0.4} = L^2$$

$$\frac{L^2}{D^2} = \frac{1}{8 \times 0.6 \times 0.4}, \qquad \boxed{\frac{L}{D} = 0.722 \text{ Ans}}$$

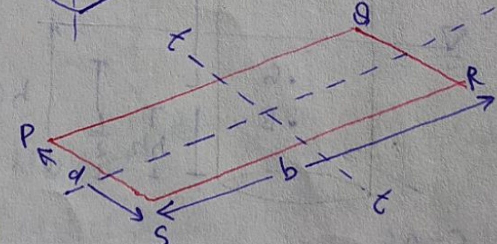
Ship Design



2019 ESE

(Metacenter height)

$$GM = \frac{I_{minimum}}{V_{\text{displaced liquid}}} - BG$$



$$I_{ll} = \frac{bd^3}{12}, \quad I_{tt} = \frac{db^3}{12}$$

longitudinal, transverse

$I_{Rolling} <<<< I_{pitching}$

time period of oscillations →

$$T = 2\pi\sqrt{\frac{K^2}{g(G.M)}} \quad = \quad K = \text{Radius of Gyration}$$

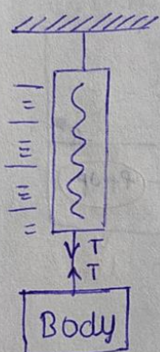
$$T \propto \frac{1}{\sqrt{GM}}$$

	GM
• Passenger ship	0.4 - 1 m
• War ship	

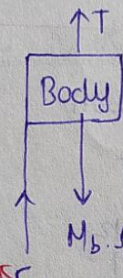
Weight of body

- Real wt
- Apparent wt.

Real wt.



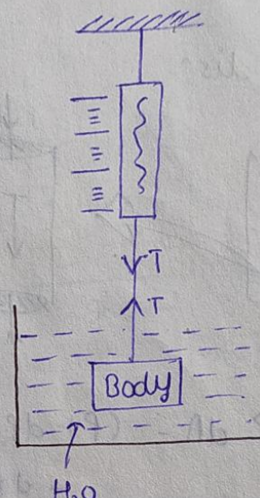
Free analysis



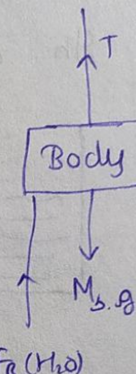
Neglect ~~$F_{B(Air)}$~~

$\underbrace{T}_{\text{Reading}} = \underbrace{M_b.g}_{\text{Real wt}}$

Apparent wt



Force analysis



$T + F_{B(H_2O)} = M_b.g$

$T = M_b.g - F_{B(H_2O)}$

T → Reading or apparent wt.

$M_b.g$ → Real wt

(Pb) - 37

$$T + F_B = M_b.g$$

$$30 + (800)\forall.g = M_b.g$$

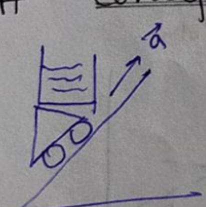
$$-15 \pm (1200)\forall.g = M_b.g$$

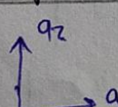
$$15 = 400(\forall.g)$$

$$\forall = \frac{15}{400 \times 9.81} = 3.82 \times 10^{-3}\ m^3$$

$$= \boxed{3.82 \text{ litre}}$$

concept of acceleration vessel containing liquid.

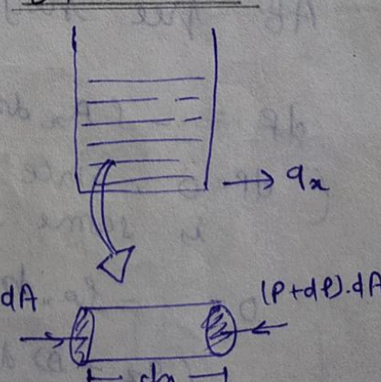




$\vec{a}$ = vector quantity

$\vec{a} = a_x\hat{i} + a_z\hat{k}$

In x-direcn



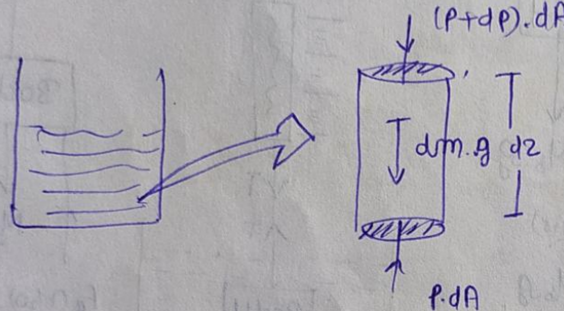
$$P.dA - (P + dP).dA = dm.a_x$$

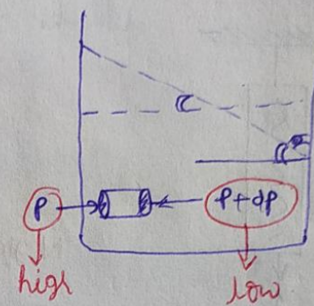
$$-dP.dA = \rho.d\forall.a_x$$

$$dP.dA = -\rho[dA.dx].a_x$$

$$\boxed{\frac{dP}{dx} = -\rho a_x}$$

In Z- dir^n





$$P\,dA - (P + dP)\,dA - dm.g = dm.a_z$$

$$-dP.dA = dm(a_z + g)$$

$$dP.dA = \rho(dA.dz)(a_z + g)$$

$$\boxed{\frac{dP}{dz} = -\rho(a_z + g)}$$

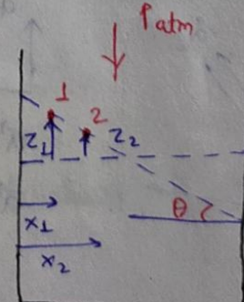
$P = f^n(x)$ and also $P = f^n(z)$

combine $\quad P = f^n(x, z)$ $\quad -\rho a_x \quad -\rho(a_z + g)$

Math $\quad dP = \frac{\partial P}{\partial x}.dx + \frac{\partial P}{\partial z}.dz$

$$dP = -\rho.a_x.dx - \rho(a_z + g).dz$$

At free surface ⟶



$$dP = -\rho a_x dx - \rho(a_z + g).dz$$

($dP = 0$, since the pressure is same at all the points)

$$0 = -\rho a_x dx - \rho(a_z + g)\,dz$$

$$(a_z + g)\,dz = -a_x.dx$$

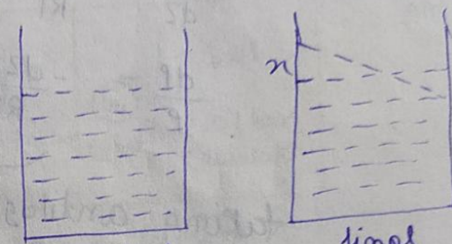
$$-\frac{dz}{dx} = \frac{a_x}{a_z + g}$$

Int. it

$$-\frac{(z_2-z_1)}{(x_2-x_1)} = \frac{a_x}{a_z+g}$$

$$\boxed{\tan\theta = \frac{a_x}{a_z+g}}$$

Conservation of Volume :-

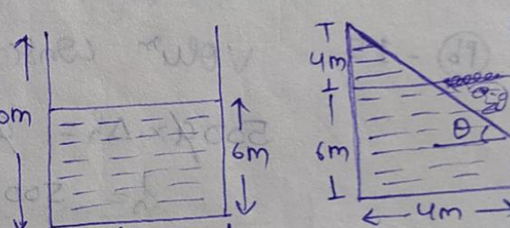


initial   final

$\boxed{x=y}$

Problem ①

Determine $a_x = ??$



width = 3m

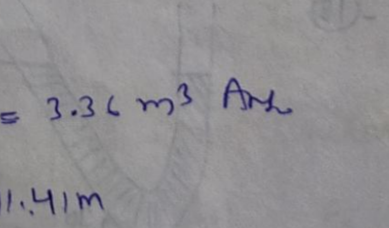
$a_x = ?$ 3 (max)

$$\tan\theta = \frac{a_x}{\cancel{a_z}^{0}+g}$$

$$\frac{8}{4} = \frac{a_x}{9.81}$$

$a_x = 19.62\ m/s^2$

Determine V spillout = ?



$$V_{final} = \left[10\times4 - \left(\frac{1}{2}\times8.56\times4\right)\right]\times3$$

$= 68.64\ m^3$

$V_{initial} = 6\times4\times3 = 72\ m^3$

$V_{spilled} = 72 - 68.64 = 3.36\ m^3$ Ans

② $a_x = 28\ m/s^2$, $y = 11.41m$

$$V_{final} = \left(\frac{1}{2}\times3.5\times10\right)\times3$$

$= 52.3\ m^3$

$V_{initial} = 6\times4\times3 = 72\ m^3$