Chapter 1. Sets

Question-1

Which of the following are sets?

(i) The collection of all months of a year beginning with letter J.
(ii) The collection of most talented writers of India.
(iii) A team of eleven best cricket batsmen of the world.
(iv) The collection of all boys in your class.
(v) The collection of all natural numbers less than 100.
(vi) The collection of novels written by the river Prem Chand.
(vii) The collection of all even integers.
(viii) The collection of different problems in this chapter.
(ix) A collection of most dangerous animals of the world.

Solution:

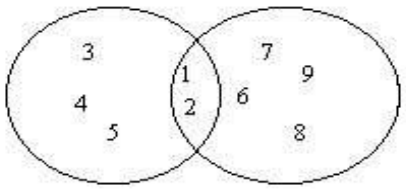
(i), (iv), (v), (vi), (vii) and (viii) are sets.

Question-2

$A \cup B = \{1,2,3,4,5,6,7,8,9\}$
$A \cap B \{1,2\}$ and $A = \{1, 2, 3, 4, 5\}$ find the set B.

Solution:

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A \cap B = \{1, 2\}$ and
$A = \{1, 2, 3, 4, 5\}$



$\therefore B = \{1, 2, 6, 7, 8, 9\}$

Question-3

If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, find the complements of the following sets:

(i) A = {2, 4, 6, 8}
(ii) B = {1, 3, 5, 7, 9}

(iii) C = { 2, 3, 5, 7}

(iv) φ
(v) U

Solution:

(i) A′ = {1, 3, 5, 7, 9}
(ii) B′ = {2, 4, 6, 8}
(iii) C′ = {1, 4, 6, 8, 9}
(iv) φ′ = U
(v) U′ = φ

Question-4

Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol ∈ or ∉ in the blank spaces:

(i) 5 _____ A
(ii) 8 _____ A
(iii) 0 _____ A
(iv) 4 _____A
(v) 2 _____ A
(vi) 10 _____ A

Solution:

(i) 5 __∈ __ A

(ii) 8 __∉ __ A
(iii) 0 __∉__ A
(iv) 4 __∈ __ A
(v) 2 __∈ __ A
(vi) 10 __∉ __ A

Question-5

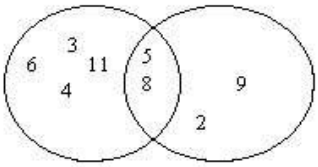
If A ∪ B ={2, 3, 4, 5, 6, 8, 9, 11}, A ∩ B {5,8} and B = {2, 5, 8, 9} find the set A – B

Solution:

A ∪ B ={2, 3, 4, 5, 6, 8, 9, 11},

A ∩ B = {5,8}

B = {2, 5, 8, 9}



∴ A = {3, 4, 5, 6, 8, 11}

∴ A – B = {3, 4, 6, 11}

Question-6

If U is the set of all natural numbers and A' is the set of all composite numbers, what is A?

Solution:

U = {1, 2, 3, 4, 5, 6, ……}

A' = {4, 6,……..}

Then A = {1, 2, 3, 5, ……..}

Question-7

Write the following sets in the roaster form:

(i) A = {x : x is an integer and −3 ≤ x < 7}

(ii) B = {x : x is a natural number less than 6}

(iii) C = {x : x is two digit natural number such that sum of its digits is 8}

(iv) D = {x : x is a prime number which is a divisor of 60}

(v) E = the set of all letters in the word TRIGONOMETRY

(vi) F = the set of all letters in the word SETS.

Solution:

(i) A = {-3, -2, -1, 0, 1, 2, 3, 4, 5, 6}

(ii) B = {1, 2, 3, 4, 5}

(iii) C = {17, 26, 35, 44, 53, 62, 71}

(iv) D = {2, 3, 5}

(v) E = {T, R, I, G, O, N, M, E, R, Y}

(vi) F = {S, E, T}

Question-8

If A = {p, q, r, s} find A ∩ A and A ∪ A.

Solution:

A = {p, q, r, s}

∴ A ∩ A = {p, q, r, s}

∴ A ∪ A = {p, q, r, s}

Question-9

Which of the following statements are true and which are false?

(i) U' = φ

(ii) φ ' = U

(iii) For any two subsets, X and Y of U,
(X ∪ Y)' = X' ∪ Y'

(iv) For any two subsets, X and Y of U,
(X ∩ Y)' = X' ∩ Y'

(v) For any two subsets, S and T of U,
(S ∪ T)' = S' ∩ T'

(vi) For any two subsets S and T of U,
(S ∩ T)' = S' ∪ T'

Solution:

(i) True

(ii) True

(iii) False

(iv) False

(v) True

(vi) True

Question-10

If A = {x : x is a letter in the word, 'follow'} and
B = {x : x is a letter in the word, 'wolf'}, show that A = B.

Solution:

Clearly, A = {f, o , l, w} and B = {w, o, l, f}.

Since every element of A is in B and every element of B is in A, so A = B.

Question-11

Express the following sets by using the set builder method:

(i) A = { 1, 3, 5, 7, 9}

(ii) B = {2, 4, 6, 8}

(iii) C = {-1, 1}

(iv) D = {1, 5, 10, 15, ...}

(v) E = {14, 21, 28, 35, 42, ...,98}

Solution:

(i) A = { x : x is an odd natural number, x £ 9}

(ii) B = { x : x is an even natural number, x ≤ 8}

(iii) C= {x : x is an odd natural number and |x| < 2}

(iv) D = {x : x is a natural number multiple of 5 and x = 1}

(v) E = {x : x is a multiple of 7 and 7<x<100}

Question-12

Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60^0, what is A?

Solution:

U = The set of all triangles in a plane.

A = The set of all triangles with at least one angle different from 60^0.

A' = The set of all equilateral triangles.

Question-13

List all the elements of the following sets:

(i) A = {x : x is an odd natural number}

(ii) B = {x : x is an integer, -1/2<x<9/2}

(iii) C = {x : x is an integer, $x^2 \leq 4$}

(iv) D = {x : x is a letter in the word "LOYAL"}

(v) E = {x : x is a month of a year not having 31 days}

(vi) F = {x : x is a consonant in the English alphabet which precedes k}

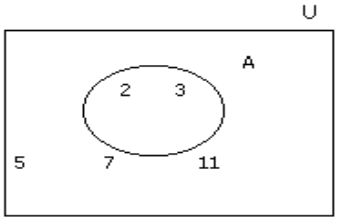
Solution:

(i) A = {1, 3, 5, 7, 9, 11, 13….}
(ii) B = {0, 1, 2, 3, 4}
(iii) C = {-2, -1, 1, 2}
(iv) D = {L, O, Y, A}
(v) E = {February, April, June, September, November}
(vi) F = {b, c, d, f, g, h, j}

Question-14

Represent the following sets in a Venn diagram: U = {2, 3, 5, 7, 11}, A = {2, 3}

Solution:



Question-15

Match each of the sets on the left described in the roster form with the same set on the right described in set builder form:

(i) {1, 2, 3, 6} **(a) { x : x is a prime number and a divisor of 6 }**

(ii) {2, 3} **(b) {x : x is an odd natural number less than 10}**

(iii) {H, A, Y, R, N} **(c) {x : x is a natural number and divisor of 6.}**

(iv) {1, 3, 5, 7, 9} **(d) {x : x is a letter of the word 'HARYANA'.}**

Solution:

(i) {1, 2, 3, 6} (c) {x : x is a natural number and divisor of 6.}

(ii){2, 3} (a) { x : x is a prime number and a divisor of 6 }

(iii) {H, A, Y, R, N} (d) {x : x is a letter of the word 'HARYANA'.}

(iv) {1, 3, 5, 7, 9} (b) {x : x is an odd natural number less than 10}

Question-16

Which of the following sets is finite or infinite?

(i) The set of the months of a year.

(ii) {1, 2, 3, ……}

(iii) {1, 2, 3, ……., 99, 100}

(iv) The set of positive integers greater than 100.

(v) The set of prime numbers less that 99.

Solution:

(i) Finite set

(ii) Infinite set

(iii) Finite set

(iv) Infinite set

(v) Finite set

Question-17

Represent the following sets in a Venn diagram:

U = { x : x is a natural number and 2 ≤ x ≤ 8.}

A = {x : x ∈ U and x divides 18}

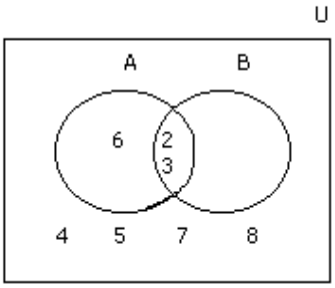
B = {x: x ∈ U and x is a prime divisor of 18}

Solution:

U = {2, 3, 4, 5, 6, 7, 8}

A = {2, 3, 6}

and B = {2, 3}



Question-18

Which of the following sets is finite or infinte?

(i) The set of lines which are parallel to the x-axis.

(ii) The set of letters in the English alphabet.

(iii) The set of numbers which are multiples of 5.

(iv) The set of animals living on earth.

(v) The set of circles in plane passing through the origin.

Solution:

(i) Infinite set

(ii) Finite set

(iii) Infinite set

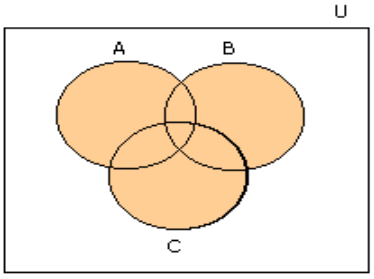
(iv) Finite set

(v) Infinite set

Question-19

If A, B and C are three subsets of the universal set U, draw a venn diagram showing A ∪ (B ∪ C)

Solution:



Question-20

Which of the following are examples of the null set?

(i) Set of odd natural numbers divisible by 2.

(ii) Set of even prime numbers.

(iii) {x : x is a natural number, x<5 and simultaneously x>7}

(iv) {y : y is a point common to any parallel lines}

Solution:

(i) Null set.

(ii) It is not a null set because 2 is a even prime number.

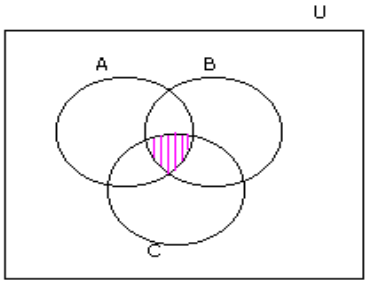
(iii) Null set.

(iv) Null set.

Question-21

If A, B and C are three subsets of the universal set U, draw a venn diagram showing (A ∩ B) ∩ C.

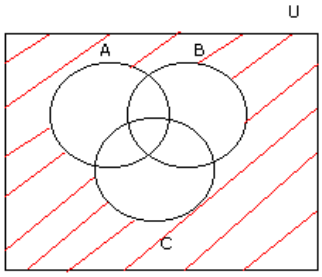
Solution:



Question-22

If A, B and C are three subsets of the universal set U, draw a venn diagram showing [(A ∪ B) ∪ C]'

Solution:



Question-23

In the following, state whether A = B or not:

(i) A = {a, b, c, d} B = {d, c, b, a}

(ii) A = {4, 8, 12, 16} B = {8, 4, 16, 18}

(iii) A = {2, 4, 6, 8, 10} B = {x :x is positive even integer less than 10}

(iv) A = {x : x is a multiple of 10} B = {10, 15, 20, 25, 30,.....}

Solution:

(i) A = B

(ii) A ≠ B, because element 12 of set A is not present in set B and element 18 of set B is not present in set A.

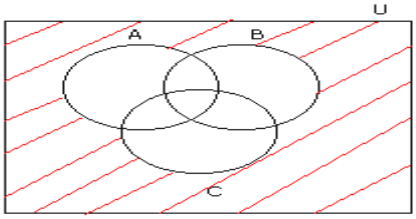
(iii) A = B

(iv) A ≠B, because set B consists of elements that are multiple of 5.

Question-24

If A, B and C are three subsets of the universal set U, draw a venn diagram showing (A' ∩ B') ∩ C'

Solution:



Question-25

Are the following pair of sets equal? Give reasons.

(i) A = {2, 3}
B = {x: x is a solution of $x^2 + 5x + 6 = 0$}

(ii) A = {x:x is a letter in the word FOLLOW}
B = {y:y is a letter in the word WOLF}

Solution:

(i) A = {2, 3}

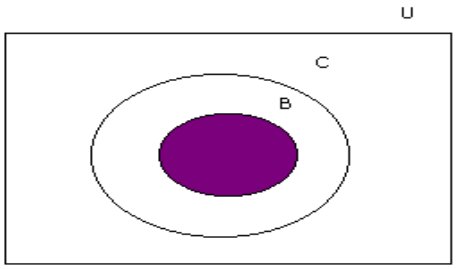
B = {x: x is a solution of $x^2 + 5x + 6 = 0$} = {2, 3}
Therefore the above pair are equal sets.

(ii) A = {x:x is a letter in the word FOLLOW} = {F, O, L, W}
B = {y:y is a letter in the word WOLF} = {W, O, L, F}
Therefore the above pair are equal sets.

Question-26

If A, B, and C are three subsets of the universal set U, draw Venn diagrams for the following:B ∩ C, when B ⊂ C.

Solution:



Question-27

From the sets given below, select equal sets and equivalent sets

A = {0, a}	**B = {1, 2, 3, 4}**
C = {4, 8, 12}	**D = {3, 1, 2, 4}**
E = {1, 0}	**F = {8, 4, 12}**
G = {1, 5, 7, 11}	**H = {a, b}**

Solution:

Equal sets:
(i) B = D
(ii) C = F;
Equivalent sets:

(i) A, E, H;
(ii) D, G;

Question-28

Given that A = {6, 7, 8, 9, 10} and B = {2, 3, 4, 5}. Write down all ordered pairs (a, b) such that a is divisible by b and hence write down the set ordered pairs given the relation 'is a multiple of' from A and B.

Solution:

A = {6, 7, 8, 9, 10} and B = {2, 3, 4, 5}

List of all ordered pairs : (6, 2), (6, 3), (6, 4), (6, 5), (7, 2), (7, 3), (7, 4), (7, 5), (8, 2), (8, 3), (8, 4), (8, 5), (9, 2), (9, 3), (9, 4), (9, 5), (10, 2), (10, 3), (10, 4), (10, 5)

The ordered pairs (a, b) such that a is divisible by b: (6, 2), (6, 3), (8, 2), (8, 4), (9, 3), (10, 2), (10,5)

Question-29

Which of the following statements are true?

(i) The set of all cats is contained in the set of all animals.
(ii) The set of all isosceles triangles is contained in the set of all equilateral triangles.
(iii) The set of all rectangles is contained in the set of all squares.
(iv) The sets A = {1} and B = {{1}} are equal.
(v) The sets A = {x : x is a letter in the word "TITLE"} and B = {x : x is a letter in the word "LITTLE"} are equal.

Solution:

(i) True
(ii) False
(iii) False
(iv) False
(v) True

Question-30

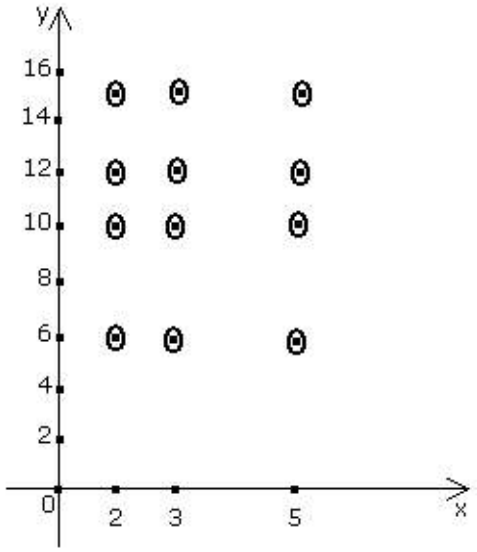
Let R be the relation defined by 'divides" from A = {2, 3, 5} to B = {6, 10, 12, 15}. Represent R (i) as a set of ordered pairs (ii) as a graph and (iii) by an arrow diagram.

Solution:

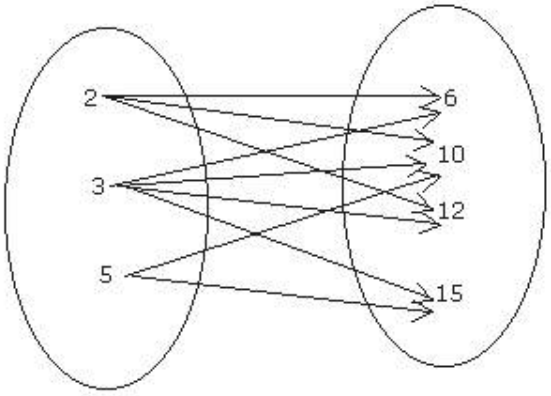
A = {2, 3, 5} to B = {6, 10, 12, 15}

(i) R = {(2, 6), (2, 10), (2, 12), (2, 15), (3, 6), (3, 10), (3, 12), (3, 15), (5, 6), (5, 10), (5, 12), (5, 15)}

(ii)



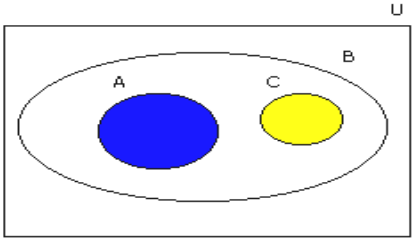
(iii)



Question-31

A and C are disjoint sets and both A and C are subsets of B, draw Venn diagrams for the following.

Solution:



Question-32

Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:

(i) {2, 3, 4} ..... {1, 2, 3, 4, 5,}

(ii) {a, b, c}..... {b, c, d}

(iii) {x : x is a student of Class 9 of your school.} .....{x: x is a student of your school}

(iv) {x : x is a circle in the plane} .....{x : x is a circle with radius 1}

(v) {x : x is a triangle in the plane} ..... { x : x is a rectangle in the plane}

(vi) {x : x is an equilateral triangle in the plane}.....{ x : x is a triangle in the plane}

(vii) {x : x is an even natural number} ..... {x : x is an integer}

Solution:

(i) {2, 3, 4} ...$\subset$.... {1, 2, 3, 4, 5,}

(ii) {a, b, c}...$\not\subset$..... {b, c, d}

(iii) {x : x is a student of Class 9 of your school.} ...$\subset$....{x; x is a student of your school.}

(iv) {x : x is a circle in the plane.} ...$\not\subset$... {x : x is a circle with radius 1.}

(v) {x : x is a triangle in the plane.} ...$\not\subset$... { x : x is a rectangle in the plane.}

(vi) {x : x is an equilateral triangle in the plane .}... $\subset$..... { x : x is a triangle in the plane.}

(vii) {x : x is an even natural number.} ...$\subset$... {x : x is an integer.}

Question-33

If A and B are two sets such that A has 21 elements, B has 17 elements, and A ∪ B has 21 elements, how many elements does A ∩ B have?

Solution:

We have n(A) = 12

n(B) = 17

n(A ∪ B) = 21

By using the formula,

n(A ∪ B) = n(A) + n(B) - n(A ∩ B)

We have 21 = 12 + 17 - n(A ∩ B)

∴ n(A ∩ B) = 29 -21

∴ n(A ∩ B) = 8

Question-34

If P = {2, 3, 4, 8, 9} write down the sets of ordered pairs representing the relations (i) is a factor of (ii) is divisible by (iii) is a multiple of 2 on P.

Solution:

P = {2, 3, 4, 8, 9}

(i) The sets of ordered pairs representing "is a factor of" : {(2, 2), (2, 4), (2, 8), (3, 3), (3, 9), (4, 4), (4, 8), (8, 8), (9, 9)}

(ii) The sets of ordered pairs representing "is divisible by" : {(2, 2), (4, 2), (8, 2), (4, 4), (8, 4), (8, 8), (9, 3), (9, 9), (3, 3)}

(iii) The sets of ordered pairs representing "is a multiple of 2 on P" : {((2, 2), (4, 2)), (8, 2)}

Question-35

Examine whether the following statements are true or false:

(i) {a, b} ⊄ {b, c, a}

(ii) {a, e} ⊂ { x : x is a vowel in the English alphabet.}

(iii) {1, 2, 3} ⊂ {1, 2, 3}

(iv) {a} ⊂ {a, b, c}

(v) {a} ∈ {a, b, c}

(vi) {x : x is an even natural number less than 6.} ⊂ { x : x is a natural number which divides 36}

Solution:

(i) False, because elements a and b are present in that set.

(ii) True

(iii) True

(iv) True

(v) False

(vi) True

Question-36

Describe the relation R defined from A to B where A = {- 1, 2, 3, 4} to B = {-2, 4, 6} by the set R = {(- 1, - 2), (2, 4), (3, 6)}.

Solution:

A = {- 1, 2, 3, 4} to B = {-2, 4, 6}
set R = {(- 1, - 2), (2, 4), (3, 6)} is the relation "is half of"

Question-37

If A and B are disjoint sets, show that n(A ∪ B) = n(A) + n(B)

Solution:

We know that, if A and B are disjoint sets then $n(A \cap B) = \varphi$.
Hence, by using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
We have $n(A \cup B) = n(A) + n(B) - \varphi$
$\therefore n(A \cup B) = n(A) + n(B)$

Example: Let A = {1, 2} and B = {3, 4},
then $A \cup B = \{1, 2, 3, 4\}$ and $A \cap B = \varphi$
Now $n(A) = 2, n(B) = 2, n(A \cup B) = 4$ and $n(A \cap B) = \varphi$
Hence, $n(A \cup B) = n(A) + n(B)$

Question-38

Let A = {1, 2, {3, 4}, 5}. Which of the following statements are false and why?

(i) $\{3, 4\} \subset A$

(ii) $\{3, 4\} \in A$

(iii) $\{\{3, 4\}\} \subset A$

(iv) $1 \in A$

(v) $1 \subset A$

(vi) $\{1, 2, 5\} \subset A$

(vii) $\{1, 2, 5\} \in A$

(viii) $\{1, 2, 3\} \subset A$

(ix) $\varphi \in A$

(x) $\{\varphi\} \subset A$

Solution:

(i) False, {3,4} is an element not a set.
(ii) True
(iii) True
(iv) True
(v) False, 1 is an element not a set.
(vi) True
(vii) False, {1, 2, 5} is a set not an element.
(viii) False, 3 is an element of set contained in A.
(ix) False, φ is not an element of A.
(x) False, φ is not an element of A.

Question-39

Write the power set of A = {3,6,9}.

Solution:

P(A)= {φ , {3}, {9},{6},{3,6},{3,9},{6,9},{3,6,9}}.

Question-40

Describe the relation, domain and range if (i) R = {(1, 1), (8, 2), (27, 3), (64, 4)} (ii) R = {(Delhi, India), (Paris, France), (Karachi, Pakistan)} (iii) R = {(4, - 2), (9, - 3), (1, 1), (4, 2), (1, - 1), (9, 3)}

Solution:

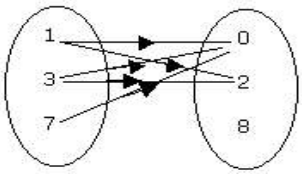
(i) R = {(1, 1), (8, 2), (27, 3), (64, 4)}
R is the relation "is the cube of "
Domain = {1, 8, 27, 64}
Range = {1, 2, 3, 4}
(ii) R = {(Delhi, India), (Paris, France), (Karachi, Pakistan)}
R is the relation "is the capital of"
Domain = {Delhi, Paris, Karachi}
Range = {India , France, Pakistan}
(iii) R = {(4, - 2), (9, - 3), (1, 1), (4, 2), (1, - 1), (9, 3)}
R is the relation "is the square of"
Domain = {1, 4, 9}
Range = (- 3, - 2, - 1, 2 , 3}

Question-41

If $x \in \{1, 3, 7\}$, $y \in \{0, 2, 8\}$ and R is the relation such that $x + y < 8$, represent R (i) as a set of ordered pairs and (ii) by an arrow diagram.

Solution:

(i) $x \in \{1, 3, 7\}$, $y \in \{0, 2, 8\}$, R is such that $x + y < 8$.
The set of ordered pairs = {(1, 0), (1, 2), (3, 0), (3, 2), (7, 0)}
(ii)



Question-42

Which of the following sets are equal ?

A = {x : x ∈ N, x < 3}, B = {1, 2}, C = {3, 1}

D = {x : x ∈ N, x is odd, x < 5}, E = {1, 2, 1}, F = {1, 1, 3}

Solution:

A = {1, 2}, B = {1, 2}, C = {3, 1},
D = {1, 3}, E = {1, 2, 1}, F = {1, 1, 3}
A, B, E and C, D, F are equal sets.

Question-43

If A and B are two sets such that A∨ B has 25 elements, A has 10 elements, and B has 37 elements, how many elements does A ∧ B have?

Solution:

$n(A \cup B) = 25; n(A) = 10 ; n(B) = 37$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore 25 = 10 + 37 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 12$

$\therefore A \wedge B$ has 12 elements.

Question-44

In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea. Find :

(i) how many drink tea and coffee both:

(ii) how many drink coffee but not tea.

Solution:

Let A be the set of those persons who drink tea and let B be the set of those persons who drink coffee. Then,

$A \cap B$ = set of persons who drink both tea and coffee.

$A - B$ = set of persons who drink tea but not coffee.

$B - A$ = set of persons who drink coffee but not tea

$\therefore n(A \cup B) = 52, n(A-B) = 16$ and $n(A) = 33$

Now, $n(A-B) + n(A \cap B) = n(A)$

$\therefore n(A \cap B) = n(A) - n(A - B) = (33 - 16) = 17$

Thus, 17 persons drink tea and coffee both.

Now, $n(A) = 33$, $n(A \cup B) = 52$ and $n(A \cap B) = 17$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore n(A) = 33$, $n(A \cup B) = 52$ and $n(A \cap B) = 17$.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(B) = n(A \cup B) + n(A \cap B) - n(A)$

$\Rightarrow n(B) = (52 + 17 - 33) = 36$

Also, $n(B-A) + n(A \cap B) = n(B)$

$\Rightarrow n(B-A) = n(B) - n(A \cap B) = (36 - 17) = 19$

$\therefore$ 19 persons drink coffee but not tea.

Question-45

Represent the relation R from A = {2, 4, 5, 7} to B = {3, 5, 6, 8, 10} by an arrow diagram given a R b if b = a + 1 where a ∈ A and b ∈ B.

Solution:

A = {2, 4, 5, 7} to B = {3, 5, 6, 8, 10}