

36. Permanent Magnets

Short Answer

1. Question

Can we have a single north pole or a single south pole?

Answer

No. A monopole (single pole) does not exist in magnetism. Magnetic poles always occur in pairs.

Magnetic fields of a magnet are formed as loops only and thus, it is necessary to have a beginning and an ending, externally. So, for a real magnet, the magnetic property will not exist without two poles. So, even if we break a magnet into pieces, each piece will form into individual magnets with two poles, North and South poles, each.

2. Question

Do two distinct poles actually exist at two nearby points in a magnetic dipole?

Answer

No. In a magnet, even the concept of poles is relative. Poles are instantaneous and are defined as the points where the magnetic fields are assumed to originate and to end. For a real magnet, which is equivalent to a magnetic dipole, there should be a distance in between the two poles in order to create magnetic field.

The magnetic moments can be defined as the strength of a magnet, that enables the magnet to produce a magnetic field.

For a magnetic pole strength m , and distance in between the two poles as L , the magnetic moment M will be,

$$M = m \times L$$

Since the value of L is Zero in a dipole with nearby points, there won't be any magnetic moments, and hence there would not exist a magnetic dipole with a magnetic field.

3. Question

An iron needle is attracted to the ends of a bar magnet but not to the middle region of the magnet. Is the material making up the ends of a bar magnet different from that of the middle region?

Answer

No. The material of the magnet is homogenous at every point, including at poles and at the center and it is not the material property of magnet that causes this effect.

Iron, from which the needle is made, is ferromagnetic in nature. Hence its magnetic domains get aligned in a magnetic field. When the needle is placed near the poles of the magnet, the opposite magnetic polarity will be induced in the needle. So, whenever it comes to near a pole, it gets attracted. But, when the needle is near the middle portion of the magnet, the two poles will induce two different polarities in the needle. But, the magnetic domains set by one pole gets canceled by the other pole. As a result, the needle will not get attracted to either of the poles, when it is at the center.

4. Question

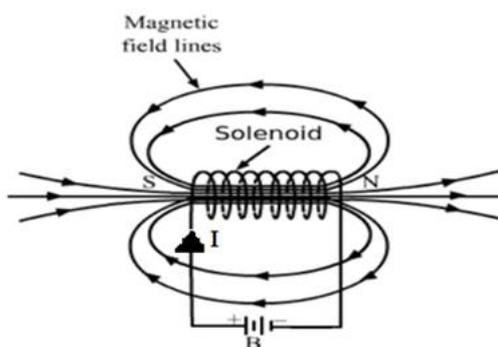
Compare the direction of the magnetic field inside a solenoid with that of the field there if the solenoid is replaced by its equivalent combination of north pole and south pole.

Answer

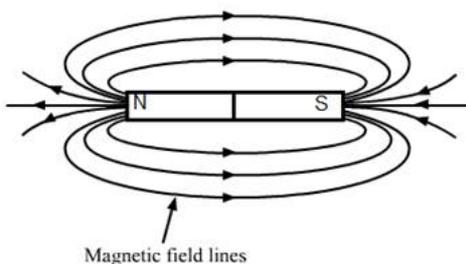
The magnetic field directions in both cases are the same. A current carrying solenoid is equivalent to a bar magnet.

In a magnet (or a combination of north pole and south pole), the magnetic poles would not change, if fixed once, and the direction of magnetic field is from South to North, internally.

In a current carrying solenoid, the magnetic field will be set up according to Ampere's law, and the direction is determined by Right-hand thumb rule, as shown in the figure.



Similarly the direction of magnetic field in a bar magnet is as shown in figure.



In the later case, in the bar magnet, it can be seen that the magnetic field is from South to North at the inside of magnet, while it is North to South at the outside.

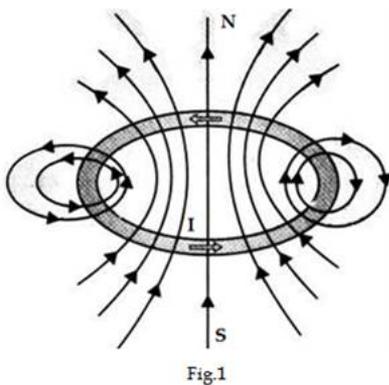
In the case of solenoid, the direction of magnetic field is determined by the direction of current. For the shown setup, with the direction of current as represented, the magnetic field inside the solenoid is from South to North. Even if we reverse the current, the direction of Magnetic field is unaltered but the imaginary South and North of the solenoid will interchange.

In both the cases, for bar magnet and for solenoid, the direction of magnetic field is from South to North in the inside and from North to South in the outside.

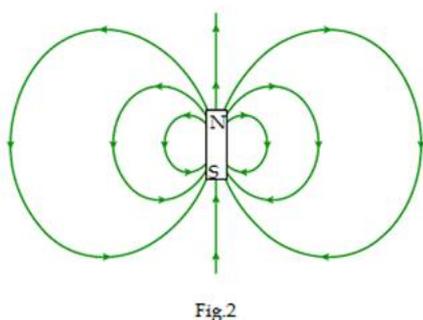
5. Question

Sketch the magnetic field lines for a current-carrying circular loop near its centre. Replace the loop by an equivalent magnetic dipole and sketch the magnetic field lines near the centre of the dipole. Identify the difference.

Answer



The sketch of magnetic field lines associated with a loop and a dipole are shown below in Fig.1 and Fig.2 respectively.



The magnetic field lines at the center of the loop, in Fig.1, is perpendicular to the loop, and the direction is determined by the direction of current.

But in the case of the magnetic dipole, in Fig.2, there is no magnetic field at the center of the dipole. Magnetic field lines start from the North pole and ends at the South pole, externally in the magnetic dipole, and hence they would not pass through center of the dipole.

6. Question

The force on a north pole, $\vec{F} = m\vec{B}$, is parallel to the field \vec{B} . Does it contradict our earlier knowledge that a magnetic field can exert forces only perpendicular to itself.

Answer

Yes, The question statement contradicts our earlier knowledge. But this confusion can be rectified by understanding the above equation.

In the equation, $\vec{F} = m\vec{B}$, m represents the 'Magnetic charge', \vec{F} represents the Magnetic force and \vec{B} represents the magnetic field.

A magnetic charge can be understood by doing the analogy with an electric charge.

A magnetic field, exerts a force on the magnetic charge, which is parallel to the Magnetic field itself and is proportional to the magnitude of magnetic charge and Magnetic field, analogous to electric field exerting force on an electric charge.

But, from Lorentz law, we know that Magnetic force is perpendicular to magnetic field.

$$\vec{F} = q\vec{v} \times \vec{B}$$

Where q represents an electric charge, v represents the velocity of movement. By this relation, the magnetic force is perpendicular to the magnetic field.

In this law, a moving electric charge is used to define the magnetic force, which is not used in the former equation. So the above cases cannot be compared directly.

The non-existence of a monopole in magnetism makes the former equation obsolete and hence Lorentz equation is widely used for practical cases.

Hence, even though the two concepts seem to contradict while comparing the orientations of magnetic field and magnetic force, they are not effectively the same equations.

7. Question

Two bar magnets are placed close to each other with their opposite poles facing each other. In absence of other forces, the magnets are pulled towards each other and their kinetic energy increases. Does it contradict our earlier knowledge that magnetic forces cannot do any work and hence cannot increase kinetic energy of a system?

Answer

Yes. It does contradict our earlier knowledge that magnetic forces cannot do any work and hence cannot increase kinetic energy of a system.

But, our earlier knowledge about the above situation comes from the Lorentz equation, which is $\vec{F} = q\vec{v} \times \vec{B}$, we understood that the a "Uniform magnetic field" can not do any work on the charge, q .

But this equation would not apply when the magnetic field is non-uniform. In the given case, when two opposite poles are placed proximal, each of the magnets experiences a non-uniform magnetic field and there will be instantaneous magnetic moment and that will produce the movement.

To summarize, both the poles are in the influence of an external force, which is the cause for the kinetic energy dissipated, and it is not the internal magnetic force.

8. Question

Magnetic scalar potential is defined as

$$U(\vec{r}_2) - U(\vec{r}_1) = - \int_{\vec{r}_1}^{\vec{r}_2} \vec{B} \cdot d\vec{l}.$$

Apply this equation to a closed curve enclosing a long straight wire. The RHS of the above equation is then $-\mu_0 i$ by Ampere's law. We see that $U(\vec{r}_2) \neq U(\vec{r}_1)$ even when $\vec{r}_2 = \vec{r}_1$. Can we have a magnetic scalar potential in this case?

Answer

No, a magnetic scalar potential is not possible in this case.

The magnetic scalar potential, analogous to electric scalar potential, is calculated between two different point. But Ampere's law is used for a closed, plane curve only.

For a straight long wire, the magnetic field is distributed surrounding the wire. In that case, if we calculate the magnetic potential by the given expression, over a closed curve, the beginning and the end points will coincide and the potential becomes zero.

Hence, In the given case, even though the RHS gives a non-zero value, it is not possible to have magnetic scalar potential.

9. Question

Can the earth's magnetic field be vertical at a place? What will happen to a freely suspended magnet at such a place? What is the value of dip here?

Answer

Yes, The magnetic field of the earth is vertical at poles. So, a freely suspended magnet will try to align with earth's magnetic field. Hence the magnet will stay in a vertical position, with its south pole pointing towards Earth's south pole, which is magnetic north.

As angle of dip is the inclination between the horizontal and the magnetic field lines of earth, poles will have an angle of dip of value 90°

10. Question

Can the dip at a place be (a) zero (b) 90° ?

Answer

Dip is the angle made by the magnetic field lines of earth with the horizontal

a) Yes, As the magnetic field lines at the equator of earth is horizontal, the dip is 0°

b) Yes, As the magnetic field lines at poles of earth is vertical, the dip is 90°

The angle of Dip can vary from 0° to 90°

11. Question

The reduction factor K of a tangent galvanometer is written on the instrument. The manual says that the current is obtained by multiplying this factor to $\tan \theta$. The procedure works well at Bhuwaneshwar. Will the procedure work if the instrument is taken to Nepal? If there is some error, can it be corrected by correcting the manual or the instrument will have to be taken back to the factory?

Answer

Yes. The procedure will work if it is taken to Nepal as well.

The tangent Galvanometer is used to find out current or earth's magnetic field. It works on the equation,

$$B = B_H \tan \theta \text{ (eqn. 1)}$$

Where B is a known magnetic field produced, B_H is the horizontal component of geomagnetic field, and θ is the angle shown in the galvanometer dial.

As the magnetic field, B is produced by the current passing through a loop according to Ampere's law, is,

$$B = \frac{\mu_0 n i}{2r}$$

Where n is the number of loops, i is the current and r is the radius of the loop.

Substituting this in eqn.1, we can rearrange it to find $\tan \theta$ as,

$$\tan \theta = \frac{\mu_0 n i}{2r \times B_H}$$

Also,

$$i = k \times \tan \theta$$

Where k is the reduction factor and is a constant. So, by correcting the data for k and the value of $\tan \theta$, the measurements can be accurately conducted.

So, despite the place, we can calculate the current depending on the $\tan \theta$ value. So, any error caused can be rectified by the angle corrections and there is no need to bring the instrument back to the factory.

Objective I

1. Question

A circular loop carrying a current is replaced by an equivalent magnetic dipole. A point on the axis of the loop is in

- A. end-on position
- B. broadside-on position
- C. both
- D. none

Answer

Any point on the axis of a bar magnet (or a magnetic dipole) is called as a point at "End-on position". In the given case, when the loop is replaced by a dipole, and hence the point on the axis of the loop will come at an axial position of dipole as well. So, the point can be considered at 'end-on position'.

2. Question

A circular loop carrying a current is replaced by an equivalent magnetic dipole. A point on the loop is in

- A. end-on position
- B. broadside-on position
- C. both
- D. none

Answer

Any point on the equatorial line of a bar magnet (or a magnetic dipole) is called as a point at "Broadside-on position". So, when the current carrying circular loop is replaced with a magnetic dipole, the point on the loop comes at the equatorial position, Thus the point is said to be located at broadside-on position.

3. Question

When a current in a circular loop is equivalently replaced by a magnetic dipole,

- A. The pole strength m of each pole is fixed
- B. the distance d between the poles is fixed
- C. the product md is fixed
- D. none of the above

Answer

When we replace a current carrying circular loop with an equivalent magnetic dipole, the quantity that is not altered is 'Magnetic moment'.

We know that, for a circular loop of area A and current I , the magnetic moment is,

$$M = IA$$

So, when we replace it with a dipole of pole strength m and in-between distance d , the magnetic dipole should be same as the first case, hence

$$M = IA = md$$

So, the product of pole strength, m and distance d should be fixed, even though they are not fixed individually.

4. Question

Let r be the distance of a point on the axis of a bar magnet from its centre. The magnetic field at such a point is proportional to

A. $\frac{1}{r}$

B. $\frac{1}{r^2}$

C. $\frac{1}{r^3}$

D. none of these

Answer

For a bar magnet with length $2l$, and magnetic moment M , the magnetic field on any point in the axis at a distance r from the centre can be expressed as,

$$B = \frac{\mu_0 Mr}{2\pi(r^2 - l^2)^2}$$

Hence, we can understand that, magnetic field is inversely proportional to $(r^2 - l^2)^2$, that is,

$$B \propto \frac{1}{(r^2 - l^2)^2}$$

Hence option D is the correct option.

5. Question

Let r be the distance of a point on the axis of a magnetic dipole from its centre. The magnetic field at such a point is proportional to

A. $\frac{1}{r}$

B. $\frac{1}{r^2}$

C. $\frac{1}{r^3}$

D. none of these

Answer

For a bar magnet with length $2l$, and magnetic moment M , the magnetic field on any point in the axis at a distance r from the center can be expressed as,

$$B = \frac{\mu_0 Mr}{2\pi(r^2 - l^2)^2}$$

But, in the case of dipole, the length in-between the poles are very small. So, $r \gg l$. Hence the above expression will become,

$$B = \frac{\mu_0 Mr}{2\pi(r^2)^2}$$

Or,

$$B = \frac{\mu_0 M}{2\pi r^3}$$

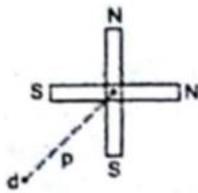
Hence, we can see that, in the case of dipole, the magnetic field is inversely proportional to the third power of r , that is

$$B \propto \frac{1}{r^3}$$

Thus, option C is the correct option.

6. Question

Two short magnets of equal dipole moments M are fastened perpendicularly at their centres figure. The magnitude of the magnetic field at a distance d from the centre on the bisector of the right angle is



A. $\frac{\mu_0}{4\pi} \frac{M}{d^3}$

B. $\frac{\mu_0}{4\pi} \frac{\sqrt{2}M}{d^3}$

C. $\frac{\mu_0}{4\pi} \frac{2\sqrt{2}M}{d^3}$

$$D. \frac{\mu_0}{4\pi} \frac{2M}{d^3}$$

Answer

We know that the resultant magnetic field on a point P due to a dipole can be expressed as,

$$B = \frac{\mu_0 M}{4\pi r^3} \sqrt{1 + 3\cos^2\theta}$$

Where, M is the magnetic moment, r is the perpendicular distance of the point P from the centre of the dipole. θ is the angle made by the line connecting the centre and the point P with that of axis of the dipole.

In the given question, as the point is on the angular bisector of the right angle, θ is 45° . Hence the above expression becomes,

$$B_{1,2} = \frac{\mu_0 M}{4\pi r^3} \sqrt{1 + 3\cos^2 45}$$

Or,

$$B_{1,2} = \frac{\mu_0 2M}{4\pi r^3}$$

Here the point P will be affected by both the crossed dipoles, hence the total magnetic field will be,

$$B_{Resultant} = \sqrt{B_1^2 + B_2^2}$$

Or,

$$B_{Resultant} = \sqrt{\left(\frac{\mu_0 2M}{4\pi r^3}\right)^2 + \left(\frac{\mu_0 2M}{4\pi r^3}\right)^2}$$

Or,

$$B_{Resultant} = \frac{\mu_0 2\sqrt{2}M}{4\pi r^3}$$

Thus, option C is the correct option.

7. Question

Magnetic meridian is

- A. a point
- B. a line along north-south
- C. a horizontal plane

D. a vertical plane

Answer

Magnetic meridian at a place can be defined as a vertical plane passing through the axis of a freely suspended magnet. It is neither a plane nor a point.

8. Question

A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It

- A. will stay in north-south direction only
- B. will stay in east-west direction only
- C. will become rigid showing no movement
- D. will stay in any position

Answer

The magnetic field of the earth is vertical at poles. So, a freely suspended magnet, or a compass needle will align vertically at poles. But, the compass needle, in the given situation is restricted from any vertical movements. But, the horizontal motion is unprecedented and can be oriented to any direction. Hence, the needle will take any position depending on any external horizontal forces or other conditions.

9. Question

A dip circle is taken to geomagnetic equator. The needle is allowed to move in vertical plane perpendicular to the magnetic meridian. The needle will stay

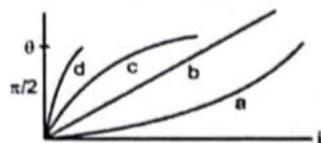
- A. in horizontal direction only
- B. in vertical direction only
- C. in any direction except vertical and horizontal
- D. in any direction it is released.

Answer

The earth's magnetic field at the geomagnetic equator is horizontal. Hence, the needle of a dip circle will align according to the earth's magnetic field, if suspended freely. But, in this case, movement in the horizontal direction is restricted but is free to move in vertical direction. Since there is no magnetic field in the vertical direction at the geomagnetic equator, the needle is restricted to move only in the vertical plane perpendicular to the magnetic meridian. Hence, the needle will assume any direction once it is released.

10. Question

Which of the following four graphs may best represent the current-deflection relation in a tangent galvanometer?



Answer

We know that for a tangent galvanometer,

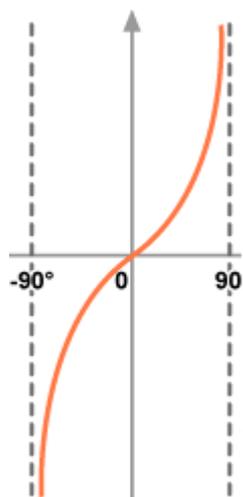
$$i = k \tan \theta$$

Or,

$$i \propto \tan \theta$$

Where θ represents the deflection, i represents the current and k is the reduction constant.

From the above relation, we can see that the current is proportional to the tangent of the deflection angle. Hence the graph showing their relation should be equivalent to $\tan \theta$ graph, which is,



So, from the given curves, one that is similar to the above graph is curve(C).

11. Question

A tangent galvanometer is connected directly to an ideal battery. If the number of turns in the coil is double, the deflection will

- A. increase
- B. decrease
- C. remain unchanged
- D. either increase of decrease

Answer

We know that for a tangent galvanometer, the relation connecting current, i and the deflection, θ is,

$$i = k \tan \theta$$

Or,

$$\theta = \tan^{-1}(i/k)$$

Where k is the reduction constant and is fixed for a galvanometer.

So, we can see that deflection depends only on i and k . Hence, the deflection won't change when the number of turns is changed.

12. Question

If the current is doubled, the deflection is also doubled in

- A. a tangent galvanometer
- B. a moving-coil galvanometer
- C. both
- D. none

Answer

We know that for a moving-coil galvanometer, the relation between current, i and deflection, θ can be expressed as,

$$i = \frac{k}{nAB} \theta$$

Where k is the torsional constant, A is the area of the coil, n is the number of turns and B is the strength of Magnetic field.

From the above expression, we can see that the current is directly proportional to the deflection. And hence, if the current is doubled, the deflection will also get doubled.

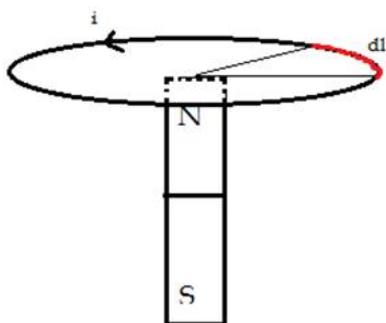
13. Question

A very long bar magnet is placed with its north pole coinciding with the centre of a circular loop carrying an electric current i . The magnetic field due to the magnet at a point on the periphery of the wire is B . The radius of the loop is a . The force on the wire is

- A. very nearly $2\pi aiB$ perpendicular to the plane of the wire
- B. $2\pi aiB$ in the plane of the wire
- C. πaiB along the magnet
- D. zero

Answer

When a bar magnet's north pole is placed at the centre of a current carrying loop, the magnetic field will get distributed in such a way that the magnetic field will lie in the plane of the loop. So, the magnetic force will be perpendicular to the plane of the loop.



Now, let's take a current element idl , and the magnetic field associated with it is B . Then the magnetic force on the element is,

$$dF = B \times idl$$

On integration throughout the loop of radius a ,

$$F = \int_0^{2\pi a} B idl$$

Or,

$$F = 2\pi aiB$$

Hence the force acting on the wire is $2\pi aiB$, which is directed in the perpendicular direction of the plane of the loop.

Objective II

1. Question

Pick the correct options.

- A. Magnetic field is produced by electric charges only.
- B. Magnetic poles are only mathematical assumptions having no real existence
- C. A north pole is equivalent to a clockwise current and a south pole is equivalent to an anticlockwise current.
- D. A bar magnet is equivalent to a long, straight current.

Answer

Magnetic Fields are produced by electric charges. Hence A is true. And magnetic poles are a theoretical concept thus B is true. A clockwise current gives a south pole and so C is incorrect. A bar magnet is equivalent to a solenoid so D is wrong.

2. Question

A horizontal circular loop carries a current that looks clockwise when viewed from above. It is replaced by an equivalent magnetic dipole consisting of a south pole S and a north pole N.

- A. The line SN should be along a diameter of the loop.
- B. The line SN should be perpendicular to the plane of the loop.
- C. The south pole should be below the loop.
- D. The north pole should be below the loop.

Answer

Clockwise current is equivalent to a south pole. hence, the SN line should be perpendicular to the plane of the loop with South above the loop and north below it. Thus options B and D are correct.

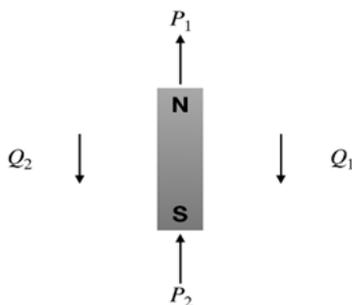
3. Question

Consider a magnetic dipole kept in the north to south direction. Let P_1, P_2, Q_1, Q_2 be four points at the same distance from the dipole towards north, south, east and west of the dipole respectively. The directions of the magnetic field due to the dipole are the same at

- A. P_1 and P_2
- B. Q_1 and Q_2
- C. P_1 and Q_1
- D. P_2 and Q_2

Answer

The following diagram explains the configuration.



4. Question

Consider the situation of the previous problem. The directions of the magnetic field due to the dipole are opposite at

- A. P_1 and P_2
- B. Q_1 and Q_2
- C. P_1 and Q_1
- D. P_2 and Q_2

Answer

Again from the above diagram, it is evident that the field are opposite at P_1 and Q_1 and P_2 and Q_2

5. Question

To measure the magnetic moment of a bar magnet, one may use

- A. a tangent galvanometer
- B. a deflection galvanometer if the earth's horizontal field is known
- C. an oscillation magnetometer if the earth's horizontal field is known
- D. both deflection and oscillation magnetometer if the earth's horizontal field is not known.

Answer

A deflection or oscillation galvanometer can be used to know the magnetic moment of a bar magnet. For a deflection magnetometer, the formula to calculate moment is

$$\frac{M}{B_H} = \frac{4\pi(d^2 - l^2)}{\mu_0 2d}$$

And for a vibrational magnetometer is

$$MB_H = 4\pi^2 \frac{I}{T^2}$$

Where, M is the magnetic moment, B_H is the horizontal magnetic field, T is the time period.

Combining both equations, we can get rid of B_H . Hence, B, C and D are true.

Exercises

1. Question

A long bar magnet has a pole strength of 10 Am. Find the magnetic field at a point on the axis of the magnet at a distance of 5 cm from the north pole of the magnet.

Answer

Given: - $M = 10\text{Am}$

$$r = 5\text{cm} = 0.05\text{m} = 5 \times 10^{-2}\text{m}$$

we now that, $\mu_0 = 1.257 \times 10^{-6}$ henry per meter

$$\pi = 3.142$$

We need to find the magnetic field due to magnetic charge B is given by

$$B = \frac{\mu_0}{4\pi} \times \frac{M}{r}$$

$$B = \frac{1.257 \times 10^{-6}}{4 \times 3.142} \times \frac{10}{(5 \times 10^{-2})^2}$$

$$B = 1 \times 10^{-7} \times \frac{10}{2.5 \times 10^{-3}}$$

$$B = 1 \times 10^{-7} \times 4000$$

$$B = 4 \times 10^{-4} \text{ T}$$

2. Question

Two long bar magnets are placed with their axes coinciding in such a way that the north pole of the first magnet is 2.0 cm from the south pole of the second. If both the magnets have a pole strength of 10 Am, find the force exerted by one magnet on the other.

Answer

Given: - pole strength, $M_1 = M_2 = 10\text{Am}$

$$r_2 = 2 \text{ cm} = 0.02\text{m} = 2 \times 10^{-2}\text{m}$$

We need to find the force exerted by one magnet of the other F is given by

$$F = \frac{\mu_0}{4\pi} \times \frac{M_1 M_2}{r_2}$$

$$F = \frac{1.257 \times 10^{-6}}{4 \times 3.142} \times \frac{10^2}{(2 \times 10^{-2})^2}$$

$$F = 1 \times 10^{-7} \times \frac{100}{4 \times 10^{-4}}$$

$$F = 1 \times 10^{-7} \times 250000$$

$$F = 2.5 \times 10^{-2} \text{ N}$$

3. Question

A uniform magnetic field of $0.20 \times 10^{-3} \text{ T}$ exists in the space. Find the change in the magnetic scalar potential as one moves through 50 cm along the field.

Answer

$$\text{Given, } B = 0.20 \times 10^{-3} \text{ T}$$

$$r = 50\text{cm} = 50 \times 10^{-2}$$

we need to find the change in the magnetic scalar ΔV is given by

$$B = -\frac{dv}{dl}$$

$$dv = -Bdl$$

on integration on both side we get

$$\int dv = -\int_{r_1}^{r_2} Bdl$$

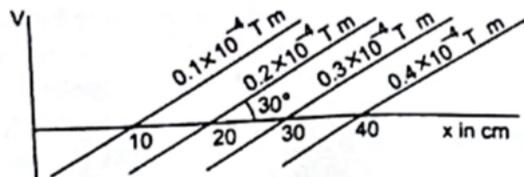
$$\Delta V = B \times \Delta r$$

$$\Delta V = -0.20 \times 10^{-3} \times 50 \times 10^{-2}$$

$$\Delta V = -0.1 \times 10^{-3}$$

4. Question

Figure shows some of the equipotential surfaces of the magnetic scalar potential. Find the magnetic field B at a point in the region.



Answer

$$\text{Given, change in potential, } dv = 0.1 \times 10^{-4} \text{ T} - m$$

$$\text{perpendicular distance, } dx = 10 \sin 30^\circ = 10 \times \frac{1}{2} = 5\text{cm} = 5 \times 10^{-2} \text{m}$$

we now that relation between the potential and the field is given by

$$B = -\frac{dv}{dx}$$

$$B = -\frac{0.1 \times 10^{-4}}{5 \times 10^{-2}} = -2 \times 10^{-4} \text{ T}$$

$$B = -2 \times 10^{-4} \text{ T}$$

Since B is perpendicular to equipotential surface where it is angle is 120° with over x-axis

5. Question

The magnetic field at a point, 10 cm away from a magnetic dipole, is found to be 2.0×10^{-4} T. Find the magnetic moment of the dipole if the point is

- (a) in end-on position of the dipole and
- (b) in broadside-on position of the dipole.

Answer

Given, $B = 2 \times 10^{-4} T$

$$d = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$$

(a) End-on-position is: -

$$B = \frac{\mu_0}{4\pi} \times \frac{2M}{d^3}$$

$$2 \times 10^{-4} = \frac{10^{-7} \times 2M}{(10 \times 10^{-2})^3}$$

$$M = 1 \text{ A-m}$$

(b) Broadside-on-position: -

$$B = \frac{\mu_0}{4\pi} \times \frac{M}{d^3}$$

$$2 \times 10^{-4} = \frac{10^{-7} \times M}{(10 \times 10^{-2})^3}$$

$$M = 2 \text{ A-m}$$

6. Question

Show that the magnetic field at a point due to a magnetic dipole is perpendicular to the magnetic axis if the line joining the point with the centre of the dipole makes an angle of $\tan^{-1}(\sqrt{2})$ with the magnetic axis.

Answer

$$\theta = \tan^{-1} \sqrt{2} \tan \theta = \sqrt{2}$$

$$\tan^2 \theta = 2$$

we can written as, $\tan \theta \times \tan \theta = 2$

$$\tan\theta = \frac{2}{\tan\theta} = 2\cot\theta$$

$$\cot\theta = \frac{\tan\theta}{2} \text{-----(1)}$$

we know that, $\tan\alpha = \frac{\tan\theta}{2} \text{-----(2)}$

Now from equation (1) and (2) we get

$$\tan\alpha = \cot\theta$$

$$\Rightarrow \tan\alpha = \tan(90-\theta)$$

$$\alpha = 90-\theta$$

$\theta + \alpha = 90^\circ$ hence we proved that the magnetic field at a point due to a magnetic dipole is perpendicular to the magnetic axis

7. Question

A bar magnet has a length of 8 cm. The magnetic field at a point at a distance 3 cm from the centre in the broadside-on position is found to be 4×10^{-6} T. Find the pole strength of the magnet.

Answer

Given, $B = 4 \times 10^{-6}$ T

$$2l = 8 \text{ cm}$$

$$l = 4 \text{ cm}$$

$$d = 3 \text{ cm} = 3 \times 10^{-2}$$

we need to find the value of the pole strength of the magnetthe magnetic field due

to the dipole on the equatorial point B is given by $B = \frac{\mu_0}{4\pi} \times \frac{M \times 2l}{(d^2+l^2)^{\frac{3}{2}}}$

$$4 \times 10^{-6} = 10^{-7} \times \frac{M \times (2 \times 4 \times 10^{-2})}{[(9 \times 10^{-4}) + (16 \times 10^{-4})]^{\frac{3}{2}}}$$

$$M = \frac{(4 \times 10^{-6} \times 2.5 \times 10^{-3})}{(10^{-7} \times 8 \times 10^{-2})}$$

$$M = 6.25 \times 10^{-2} \text{ Am}$$

8. Question

A magnetic dipole of magnetic moment 1.44 A m^2 is placed horizontally with the north pole pointing towards north. Find the position of the neutral point if the

horizontal component of the earth's magnetic field is $18 \mu\text{T}$.

Answer

Given, $B = 18\mu\text{T}$

$$M=1.44\text{Am}^2$$

We know for a magnetic dipole with its pole pointing to the north, neutral point always lies in the broadside-on position.

let d be the perpendicular distance of the neutral point from midpoint of the magnet

the magnetic field due to the dipole at the broadside-on position(B) is given by

$$B = \frac{\mu_0 M}{4\pi d^3}$$

$$18 \times 10^{-6} = \frac{10^{-7} \times 1.44}{d^3}$$

$$d^3 = \frac{10^{-7} \times 1.44}{18 \times 10^{-6}}$$

$$d = \sqrt[3]{8 \times 10^{-3}}$$

$$d = 0.2\text{m} = 20\text{cm}$$

9. Question

A magnetic dipole of magnetic moment 0.72 A m^2 is placed horizontally with the north pole pointing towards south. Find the position of the neutral point if the horizontal component of the earth's magnetic field is $18 \mu\text{T}$.

Answer

Given :- $M = 0.72\text{Am}^2$

$$B = 18\mu\text{T}$$

When the magnet is such that the its north pole faces the geographic south of earth, the natural point lies along the axial line of the magnet

$$B = \frac{\mu_0 \times 2M}{4\pi d^3}$$

$$18 \times 10^{-6} = \frac{10^{-7} \times 0.72}{d^3}$$

$$d^3 = \frac{10^{-7} \times 2 \times 0.72}{18 \times 10^{-6}}$$

$$d = \sqrt[3]{8 \times 10^{-3}}$$

$$d = 0.2m = 20cm$$

10. Question

A magnetic dipole of magnetic moment $0.72\sqrt{2} \text{ A m}^2$ is placed horizontally with the north pole pointing towards east. Find the position of the neutral point if the horizontal component of the earth's magnetic field is $18 \mu\text{T}$.

Answer

$$\text{Given, } M = 0.72\sqrt{2} \text{ Am}^2$$

$$B_H = 18\mu\text{T} = 18 \times 10^{-6} \text{ T}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \times \frac{M}{d^3}$$

$$\therefore B_H = \vec{B} \text{ at point } d$$

$$\Rightarrow 18 \times 10^{-6} = \frac{10^{-7} \times 0.72 \times \sqrt{2}}{d^3}$$

$$d^3 = \frac{10^{-7} \times 0.72 \times 1.414}{18 \times 10^{-6}}$$

$$d = 0.2m = 20cm$$

11. Question

The magnetic moment of the assumed dipole at the earth's centre is $8.0 \times 10^{22} \text{ Am}^2$. Calculate the magnetic field B at the geomagnetic poles of the earth. Radius of the earth is 6400 km .

Answer

$$\text{Given, } M = 8 \times 10^{22} \text{ Am}^2$$

$$R = 6400 \text{ km}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \times \frac{2M}{d^3}$$

$$\vec{B} = \frac{10^{-7} \times 2 \times 8 \times 10^{22}}{(64)^3 \times 10^{15}}$$

$$\vec{B} = 6 \times 10^{-5} \text{ T}$$

12. Question

If the earth's magnetic field has a magnitude $3.4 \times 10^{-5} \text{ T}$ at the magnetic equator of the earth, what would be its value at the earth's geomagnetic poles?

Answer

Given, magnetic field at the magnetic equator, $\vec{B}_1 = 3.4 \times 10^{-5} \text{ T}$

let M be the magnetic moment of earth's magnetic dipole and R be the distance of the observation point from the centre of earth magnetic dipole.

As the point on the magnetic equator is on the equatorial position of earth magnet, the magnetic field at the equatorial point(B)is given by,

$$\vec{B}_1 = \frac{\mu_0}{4\pi} \times \frac{2M}{R^3}$$

$$3.4 \times 10^{-5} = \frac{10^{-7} \times M}{R^3}$$

$$M = 3.4 \times 10^2 R^3 \text{ --- (1)}$$

$$\vec{B}_2 \text{ at the pole} \Rightarrow \vec{B}_2 = \frac{\mu_0}{4\pi} \times \frac{2M}{d^3}$$

$$\vec{B}_2 = 10^{-7} \times \frac{2 \times 3.4 \times 10^2 R^3}{R^3}$$

$$\vec{B}_2 = 6.8 \times 10^{-5} \text{ T}$$

13. Question

The magnetic field due to the earth has a horizontal component of $26 \mu\text{T}$ at a place where the dip is 60° . Find the vertical component and the magnitude of the field.

Answer

Given, $B_H = 26 \mu\text{T} = 26 \times 10^{-6} \text{ T}$

$\delta = 60^\circ$

(a) for horizontal component

we know that

$$\vec{B}_H = B \times \cos \delta$$

$$\Rightarrow 26 \times 10^{-6} = B \times \text{Cos}60$$

$$B = \frac{26 \times 10^{-6}}{0.5}$$

$$\vec{B} = 52 \times 10^{-6} \text{ T}$$

$$\vec{B} = 52\mu T$$

(b) for vertical component

$$\vec{B}_v = B \times \sin \delta$$

$$\vec{B}_v = 52 \times 10^{-6} \times \sin 60$$

$$\vec{B}_v = 44.98 \times 10^{-6}$$

$$\vec{B}_v = 45\mu T$$

14. Question

A magnetic needle is free to rotate in a vertical plane which makes an angle of 60° with the magnetic meridian. If the needle stays in a direction making an angle of $\tan^{-1}(2\sqrt{3})$ with the horizontal, what would be the dip at that place?

Answer

Given, the angle made by the magnetic meridian with the plane of rotation of the needle $\theta = 60^\circ$

angle made by the needle with the horizontal

$$\delta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$\tan \delta = \tan^{-1}(\cos \theta)$$

$$\tan \delta = \frac{2}{\sqrt{3}} \cos 60$$

$$\tan \delta = \frac{2}{\sqrt{3}} \times \frac{1}{2}$$

$$\tan \delta = \frac{1}{\sqrt{3}}$$

$$\delta = \tan^{-1} 0.5773$$

$$\delta = 30^\circ$$

15. Question

The needle of a dip circle shows an apparent dip of 45° in a particular position and 53° when the circle is rotated through 90° . Find the true dip.

Answer

If δ_1 and δ_2 be the apparent dips shown by the dip circle in the 2 perpendicular positions

$$\delta_1 = 45^\circ \text{ and } \delta_2 = 53^\circ$$

To find δ

the true dip (δ) is given by

$$\cot^2 \delta = \cot^2 \delta_1 + \cot^2 \delta_2$$

$$\cot^2 \delta = \cot^2 45 + \cot^2 53$$

$$\cot^2 \delta = 1.56$$

$$\delta = \cot^{-1}(1.56)$$

$$\delta = 38.6^\circ \cong 39^\circ$$

16. Question

A tangent galvanometer shows a deflection of 45° when 10 mA of current is passed through it. If the horizontal component of the earth's magnetic field is $B_H = 3.6 \times 10^{-5}$ T and radius of the coil is 10 cm, find the number of turns in the coil.

Answer

Given,

horizontal components of earth's magnetic field,

$$B_H = 3.6 \times 10^{-5} \text{ T}$$

deflection shown by the tangent galvanometer, $\theta = 45^\circ$

radius of coil, $r = 10 \text{ cm} = 0.1 \text{ m}$

current through the galvanometer, $I = 10 \text{ mA} = 10 \times 10^{-3} \text{ A} = 10^{-2} \text{ A}$

number of turn's in the coil, $n = ?$

$$B_H \tan \theta = \frac{\mu_0 n I}{2r}$$

$$n = \frac{B_H \tan \theta 2r}{\mu_0 I}$$

$$n = \frac{3.6 \times 10^{-5} \times \tan 45 \times 2 \times 0.1}{4\pi \times 10^{-7} \times 10^{-2}}$$

$$n = 0.5723 \times 10^3$$

$$n = 573$$

17. Question

A moving-coil galvanometer has a 50-turn coil of size 2 cm × 2 cm. It is suspended between the magnetic poles producing a magnetic field of 0.5 T. Find the torque on the coil due to the magnetic field when a current of 20 mA passes through it.

Answer

Given,

number of turns in the coil $n=50$

area of the cross section of the coil, $A = 4 \text{ cm}^2$

magnetic field strength due to the presence of the pole $B = 0.5 \text{ T}$

current flow through the coil $I = 20 \text{ mA} = 20 \times 10^{-3}$

$\zeta = ?$

$$\zeta = nI(\vec{A} \times \vec{B})$$

$$\zeta = nI(AB \sin 90^\circ)$$

$$\zeta = 50 \times 20 \times 10^{-3} \times 4 \times 10^{-4} \times 0.5$$

$$\zeta = 2 \times 10^{-4} \text{ N - m}$$

18. Question

A short magnet produces a deflection of 37° in a deflection magnetometer in Tan-A position when placed at a separation of 10 cm from the needle. Find the ratio of the magnetic moment of the magnet to the earth's horizontal magnetic field.

Answer

Given,

separation between the magnet and the needle, $d = 10 \text{ cm} = 0.1 \text{ m}$

deflection in the magnetometer in the given position when placed in the magnetic field of a short magnet, $\theta = 37^\circ$

$$\frac{M}{B_H} = ?$$

let M be the magnetic moment of the magnet and B_H be earth's horizontal magnetic field.

according to magnetometer theory

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \times \frac{(d^2 - l^2)^2}{2d} \times \tan \theta$$

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \times \frac{d^4}{2d} \times \tan \theta$$

$$\frac{M}{B_H} = \frac{4 \times 3.142}{4 \times 3.142 \times 10^{-7}} \times \frac{(0.1)^4}{(2 \times 0.1)} \times \tan 37^\circ$$

$$\frac{M}{B_H} = 0.5 \times 0.75 \times 1 \times 10^4$$

$$\frac{M}{B_H} = 3.75 \times 10^3 \text{ A m}^2 / \text{T}$$

19. Question

The magnetometer of the previous problem is used with the same magnet in Tan-B position. Where should the magnet be placed to produce a 37° deflection of the needle?

Answer

Given,

separation between the magnet and the needle $d = 10\text{cm} = 0.1 \text{ m}$

deflection in the magnetometer in the given position when placed in the magnetic field of a short magnet, $\theta = 37^\circ$

let M be the magnetic moment of the magnet and B_H be earth's horizontal magnetic field.

according to magnetometer theory

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \times \frac{(d^2 - l^2)^2}{2d} \times \tan \theta$$

for short magnet $\rightarrow d \gg \gg \gg l$

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \times \frac{d^4}{2d} \times \tan \theta$$

$$\frac{M}{B_H} = \frac{4 \times 3.142}{4 \times 3.142 \times 10^{-7}} \times \frac{(0.1)^4}{(2 \times 0.1)} \times \tan 37^\circ$$

$$\frac{M}{B_H} = 0.5 \times 0.75 \times 1 \times 10^4$$

deflection in the magnetometer $\theta = 37^\circ$

From the magnetometer theory in Tan-B position, we have

$$\frac{M}{B_H} = 3.75 \times 10^3 \text{ A m}^2 / \text{T} \text{-----(1)}$$

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \times (d^2 + l^2)^{\frac{3}{2}} \times \tan \theta \quad l \ll \ll \ll d$$

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \times d^3 \times (\tan \theta)$$

$$3.75 \times 10^3 = \frac{4 \times 3.142}{4 \times 3.142 \times 10^{-7}} \times d^3 \times \tan 37$$

$$d^3 = \frac{3.75 \times 10^3 \times 10^{-7}}{0.75}$$

$$d = \sqrt[3]{5 \times 10^{-4}}$$

$$d = 0.079370 \text{ m} = 7.9370 \text{ cm}$$

20. Question

A deflection magnetometer is placed with its arms in north-south direction. How and where should a short magnet having $M/B_H = 40 \text{ A m}^2\text{T}^{-1}$ be placed so that the needle can stay in any position?

Answer

$$\text{given, } \frac{M}{B_H} = 40 \text{ A m}^2/\text{T}$$

$d = ?$

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \times \frac{(d^2 - l^2)^2}{2d} \times \tan \theta$$

$d \gg \gg \gg l$

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \times \frac{d^3}{2}$$

$$40 = \frac{4 \times 3.142}{4 \times 3.142 \times 10^{-7}} \times \frac{d^3}{2}$$

$$d^3 = 40 \times 10^{-7} \times 2$$

$$d = \sqrt[3]{8 \times 10^{-6}}$$

$$d = 0.02 \text{ m} = 2 \text{ cm}$$

21. Question

A bar magnet takes $\pi/10$ second to complete one oscillation in an oscillation magnetometer. The moment of inertia of the magnet about the axis of rotation is $1.2 \times 10^{-4} \text{ kg m}^2$ and the earth's horizontal magnetic field is $30 \mu\text{T}$. Find the magnetic moment of the magnet.

Answer

-

$$\text{Given, } T = \frac{\pi}{10} \text{ sec}$$

the earth's horizontal magnetic field, $B_H = 30\mu T = 30 \times 10^{-6}$

The moment of inertia of the magnet about the axis of rotation

$$I = 1.2 \times 10^{-4} \text{ kg/m}^2$$

the time period of a magnetometer is given by,

$$T = 2\pi \sqrt{\frac{I}{MB_H}}$$

$$\frac{\pi}{10} = 2\pi \sqrt{\frac{1.2 \times 10^{-4}}{M \times 30 \times 10^{-6}}}$$

$$\left(\frac{1}{20}\right)^2 = \frac{1.2 \times 10^{-4}}{M \times 30 \times 10^{-6}}$$

$$M = \frac{1.2 \times 10^{-4} \times 400}{30 \times 10^{-6}}$$

$$M = 1600 \text{ Am}^2$$

22. Question

The combination of two bar magnets makes 10 oscillations per second in an oscillation magnetometer when like poles are tied together and 2 oscillations per second when unlike poles are tied together. Find the ratio of the magnetic moments of the magnets. Neglect any induced magnetism.

Answer

given,

Number of oscillations per second made by the combination of bar magnets with like poles, $f_1 = 10 \text{ s}^{-1}$

Number of oscillations per second made by the combination of bar magnets with unlike poles, $f_2 = 2 \text{ s}^{-1}$

The frequency of oscillations in the magnetometer (ν) is given by

$$T = 2\pi \sqrt{\frac{I}{MB_H}}$$

$$\Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{MB_H}{I}} \dots \dots \dots (1)$$

(1) When like poles are tied together, the effective magnetic moment is $M = M_1 - M_2$

(2) When unlike poles are tied together, the effective magnetic moment is $M = M_1 + M_2$

As the frequency of oscillations is directly proportional to the magnetic moment,

$$\Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}}$$

$$\left(\frac{10}{2}\right)^2 = \frac{M_1 - M_2}{M_1 + M_2}$$

$$\frac{M_1 - M_2}{M_1 + M_2} = \frac{25}{1}$$

$$\frac{M_1 - M_2 + M_1 + M_2}{M_1 + M_2 - M_1 - M_2} = \frac{25 + 1}{25 - 1}$$

$$\frac{M_1}{M_2} = -\frac{26}{24} = -\frac{13}{12}$$

$$\frac{M_1}{M_2} = -\frac{13}{12}$$

23. Question

A short magnet oscillates in an oscillation magnetometer with a time period of 0.10 s where the earth's horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire placed 20 cm east of the magnet. Find the new time period.

Answer

Given, Time period of oscillation, $T_1 = 0.10$ sec

Horizontal component of Earth's magnetic field,

$$B_H = 24\mu\text{T} = 24 \times 10^{-6}\text{T}$$

Downward current in the vertical wire, $I = 18\text{A}$

Distance of wire from the magnet, $d = 20\text{cm} = 0.2\text{m}$

When a current-carrying wire is placed near the magnet, the effective magnetic field gets changed. Now the net magnetic field can be obtained by subtracting the magnetic field due to the wire from Earth's magnetic field.

$$\vec{B} = \vec{B}_H - \vec{B}_{Wire}$$

$$\Rightarrow \vec{B} = \vec{B}_H - \frac{\mu_0 I}{2\pi r}$$

$$\vec{B} = 24 \times 10^{-6} - \frac{4\pi \times 10^{-7} \times 18}{2\pi \times 0.2}$$

$$\vec{B} = 24 \times 10^{-6} - \frac{2 \times 10^{-7} \times 18}{0.2}$$

$$\vec{B} = 14 \times 10^{-6} T$$

Time period of the coil (T) is given by

$$T = 2\pi \sqrt{\frac{I}{MB_H}}$$

Let T_1 and T_2 be the time periods of the coil in the absence of the wire and in the presence the wire respectively. As time period is inversely proportional to magnetic field,

$$\frac{T_1}{T_2} = \sqrt{\frac{B}{B_H}}$$

$$\Rightarrow \frac{0.1}{T_2} = \sqrt{\frac{14 \times 10^{-6}}{24 \times 10^{-6}}}$$

$$T_2 = \sqrt{\frac{0.01 \times 14}{24}}$$

$$T_2 = 0.076 \text{ sec}$$

24. Question

A bar magnet makes 40 oscillations per minute in an oscillation magnetometer. An identical magnet is demagnetized completely and is placed over the magnet in the magnetometer. Find the time taken for 40 oscillations by this combination. Neglect any induced magnetism.

Answer

Given,

$$T_1 = \frac{1}{40} \text{ min Here } I' = 2I$$

$$T_2 = ?$$

$$T = 2\pi \sqrt{\frac{I}{MB_H}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{I}{I'}}$$

$$\Rightarrow \frac{1}{40T_2} = \sqrt{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{1600T_2} = \frac{1}{2}$$

$$T_2 = \frac{1}{800}$$

$$T_2 = 0.03536 \text{ min}$$

for 1 oscillation time taken = 0.03536 min

for 40 oscillation time taken = $40 \times 0.03536 = 1.414 = 2 \text{ min}$

25. Question

A short magnet makes 40 oscillations per minute when used in an oscillation magnetometer at a place where the earth's horizontal magnetic field is $25 \mu\text{T}$. Another short magnet of magnetic moment 1.6 A m^2 is placed 20 cm east of the oscillating magnet. Find the new frequency of oscillation if the magnet has its north pole

(a) towards north and

(b) towards south.

Answer

Given, $v_1 = 40 \text{ oscillation/min}$

$$B_H = 25 \mu\text{T}$$

M of second magnet = 1.6 Am^2

(a) for north pole facing north

$$\gamma_1 = \frac{1}{2\pi} \sqrt{\frac{MB_H}{I}}, \gamma_2 = \frac{1}{2\pi} \sqrt{\frac{M(B_H - B)}{I}}$$

$$B = \frac{\mu_0}{4\pi} \times \frac{M}{d^3}$$

$$B = \frac{10^{-7} \times 1.6}{8 \times 10^{-3}} = 20 \mu\text{T}$$

$$\frac{\gamma_1}{\gamma_2} = \sqrt{\frac{B_H}{(B_H - B)}}$$

$$\Rightarrow \frac{40}{\gamma_2} = \sqrt{\frac{25}{(25-20)}}$$

$$\frac{40}{\gamma_2} = \sqrt{\frac{25}{5}}$$

$$\gamma_2 = \frac{40}{\frac{5}{\sqrt{5}}} = 17.88$$

$$\gamma_2 = 17.88 = 18 \text{ oscillation/min}$$

(b) for north pole facing south

$$\gamma_1 = \frac{1}{2\pi} \sqrt{\frac{MB_H}{I}}, \gamma_2 = \frac{1}{2\pi} \sqrt{\frac{M(B_H - B)}{I}}$$

$$B = \frac{\mu_0}{4\pi} \times \frac{M}{d^3}$$

$$B = \frac{10^{-7} \times 1.6}{8 \times 10^{-3}} = 20 \mu T$$

$$\frac{\gamma_1}{\gamma_2} = \sqrt{\frac{B_H}{(B_H + B)}}$$

$$\Rightarrow \frac{40}{\gamma_2} = \sqrt{\frac{25}{(25+20)}}$$

$$\frac{40}{\gamma_2} = \sqrt{\frac{25}{45}}$$

$$\gamma_2 = \frac{40}{\frac{5}{\sqrt{45}}} = 53.6656$$

$$\gamma_2 = 53.6656 = 54 \text{ oscillation/min}$$