PROOFS IN MATHEMATICS A1

A1.1 Introduction

The ability to reason and think clearly is extremely useful in our daily life. For example, suppose a politician tells you, 'If you are interested in a clean government, then you should vote for me.' What he actually wants you to believe is that if you do not vote for him, then you may not get a clean government. Similarly, if an advertisement tells you, 'The intelligent wear *XYZ* shoes', what the company wants you to conclude is that if you do not wear *XYZ* shoes, then you are not intelligent enough. You can yourself observe that both the above statements may mislead the general public. So, if we understand the process of reasoning correctly, we do not fall into such traps unknowingly.

The correct use of reasoning is at the core of mathematics, especially in constructing proofs. In Class IX, you were introduced to the idea of proofs, and you actually proved many statements, especially in geometry. Recall that a proof is made up of several mathematical statements, each of which is logically deduced from a previous statement in the proof, or from a theorem proved earlier, or an axiom, or the hypotheses. The main tool, we use in constructing a proof, is the process of deductive reasoning.

We start the study of this chapter with a review of what a mathematical statement is. Then, we proceed to sharpen our skills in deductive reasoning using several examples. We shall also deal with the concept of negation and finding the negation of a given statement. Then, we discuss what it means to find the converse of a given statement. Finally, we review the ingredients of a proof learnt in Class IX by analysing the proofs of several theorems. Here, we also discuss the idea of proof by contradiction, which you have come across in Class IX and many other chapters of this book.

A1.2 Mathematical Statements Revisited

Recall, that a 'statement' is a meaningful sentence which is not an order, or an exclamation or a question. For example, 'Which two teams are playing in the

Cricket World Cup Final?' is a question, not a statement. 'Go and finish your homework' is an order, not a statement. 'What a fantastic goal!' is an exclamation, not a statement.

Remember, in general, statements can be one of the following:

- *always true*
- *always false*
- *ambiguous*

In Class IX, you have also studied that in mathematics, **a statement is acceptable only if it is either always true or always false**. So, ambiguous sentences are not considered as mathematical statements.

Let us review our understanding with a few examples.

Example 1 : State whether the following statements are always true, always false or ambiguous. Justify your answers.

(i) The Sun orbits the Earth.

(ii) Vehicles have four wheels.

(iii) The speed of light is approximately 3×10^5 km/s.

(iv) A road to Kolkata will be closed from November to March.

(v) All humans are mortal.

Solution :

(i) This statement is always false, since astronomers have established that the Earth orbits the Sun.

(ii) This statement is ambiguous, because we cannot decide if it is always true or always false. This depends on what the vehicle is — vehicles can have 2, 3, 4, 6, 10, etc., wheels.

(iii) This statement is always true, as verified by physicists.

(iv) This statement is ambiguous, because it is not clear which road is being referred to.

(v) This statement is always true, since every human being has to die some time.

Example 2 : State whether the following statements are true or false, and justify your answers.

(i) All equilateral triangles are isosceles.

(ii) Some isosceles triangles are equilateral.

(iii) All isosceles triangles are equilateral.

(iv) Some rational numbers are integers.

(v) Some rational numbers are not integers.

(vi) Not all integers are rational.

(vii) Between any two rational numbers there is no rational number.

Solution :

(i) This statement is true, because equilateral triangles have equal sides, and therefore are isosceles.

(ii) This statement is true, because those isosceles triangles whose base angles are $60°$ are equilateral.

(iii) This statement is false. Give a counter-example for it.

(iv) This statement is true, since rational numbers of the form $\frac{p}{q}$, where p is an integer and $q = 1$, are integers (for example, $3 = \frac{3}{1}$).

(v) This statement is true, because rational numbers of the form $\frac{p}{q}$, p, q are integers and q does not divide p, are not integers (for example, $\frac{3}{2}$).

(vi) This statement is the same as saying 'there is an integer which is not a rational number'. This is false, because all integers are rational numbers.

(vii) This statement is false. As you know, between any two rational numbers r and s lies $\frac{r+s}{2}$, which is a rational number.

Example 3 : If $x < 4$, which of the following statements are true? Justify your answers.

(i) $2x > 8$ (ii) $2x < 6$ (iii) $2x < 8$

Solution :

(i) This statement is false, because, for example, $x = 3 < 4$ does not satisfy $2x > 8$.

(ii) This statement is false, because, for example, $x = 3.5 < 4$ does not satisfy $2x < 6$.

(iii) This statement is true, because it is the same as $x < 4$.

Example 4 : Restate the following statements with appropriate conditions, so that they become true statements:

(i) If the diagonals of a quadrilateral are equal, then it is a rectangle.

(ii) A line joining two points on two sides of a triangle is parallel to the third side.

(iii) $\sqrt{p}$ is irrational for all positive integers p.

(iv) All quadratic equations have two real roots.

Solution :

(i) If the diagonals of a parallelogram are equal, then it is a rectangle.

(ii) A line joining the mid-points of two sides of a triangle is parallel to the third side.

(iii) $\sqrt{p}$ is irrational for all primes p.

(iv) All quadratic equations have at most two real roots.

Remark : There can be other ways of restating the statements above. For instance, (iii) can also be restated as '$\sqrt{p}$ is irrational for all positive integers p which are not a perfect square'.

EXERCISE A1.1

1. State whether the following statements are always true, always false or ambiguous. Justify your answers.

(i) All mathematics textbooks are interesting.

(ii) The distance from the Earth to the Sun is approximately 1.5×10^8 km.

(iii) All human beings grow old.

(iv) The journey from Uttarkashi to Harsil is tiring.

(v) The woman saw an elephant through a pair of binoculars.

2. State whether the following statements are true or false. Justify your answers.

(i) All hexagons are polygons. (ii) Some polygons are pentagons.

(iii) Not all even numbers are divisible by 2. (iv) Some real numbers are irrational.

(v) Not all real numbers are rational.

3. Let a and b be real numbers such that $ab \neq 0$. Then which of the following statements are true? Justify your answers.

(i) Both a and b must be zero. (ii) Both a and b must be non-zero.

(iii) Either a or b must be non-zero.

4. Restate the following statements with appropriate conditions, so that they become true.

(i) If $a^2 > b^2$, then $a > b$. (ii) If $x^2 = y^2$, then $x = y$.

(iii) If $(x + y)^2 = x^2 + y^2$, then $x = 0$. (iv) The diagonals of a quadrilateral bisect each other.

A1.3 Deductive Reasoning

In Class IX, you were introduced to the idea of deductive reasoning. Here, we will work with many more examples which will illustrate how **deductive reasoning** is used to **deduce** conclusions from given statements that we assume to be true. The given statements are called 'premises' or 'hypotheses'. We begin with some examples.

Example 5 : Given that Bijapur is in the state of Karnataka, and suppose Shabana lives in Bijapur. In which state does Shabana live?

Solution : Here we have two premises:

(i) Bijapur is in the state of Karnataka (ii) Shabana lives in Bijapur

From these premises, we deduce that Shabana lives in the state of Karnataka.

Example 6 : Given that all mathematics textbooks are interesting, and suppose you are reading a mathematics textbook. What can we conclude about the textbook you are reading?

Solution : Using the two premises (or hypotheses), we can deduce that you are reading an interesting textbook.

Example 7 : Given that $y = -6x + 5$, and suppose $x = 3$. What is y?

Solution : Given the two hypotheses, we get $y = -6\ (3) + 5 = -13$.

Example 8 : Given that ABCD is a parallelogram, and suppose AD = 5 cm, AB = 7 cm (see Fig. A1.1). What can you conclude about the lengths of DC and BC?

Fig. A1.1

Solution : We are given that ABCD is a parallelogram. So, we deduce that all the properties that hold for a parallelogram hold for ABCD. Therefore, in particular, the property that 'the opposite sides of a parallelogram are equal to each other', holds. Since we know AD = 5 cm, we can deduce that BC = 5 cm. Similarly, we deduce that DC = 7 cm.

Remark : In this example, we have seen how we will often need to find out and use properties hidden in a given premise.

Example 9 : Given that $\sqrt{p}$ is irrational for all primes p, and suppose that 19423 is a prime. What can you conclude about $\sqrt{19423}$?

Solution : We can conclude that $\sqrt{19423}$ is irrational.

In the examples above, you might have noticed that we do not know whether the hypotheses are true or not. We are **assuming** that they are true, and then applying deductive reasoning. For instance, in Example 9, we haven't checked whether 19423 is a prime or not; we assume it to be a prime for the sake of our argument.What we are trying to emphasise in this section is that given a particular statement, how we use deductive reasoning to arrive at a conclusion. What really matters here is that we use the correct process of reasoning, and this process of reasoning does not depend on the trueness or falsity of the hypotheses. However, it must also be noted that if we start with an incorrect premise (or hypothesis), we may arrive at a wrong conclusion.

EXERCISE A1.2

1. Given that all women are mortal, and suppose that A is a woman, what can we conclude about A?
2. Given that the product of two rational numbers is rational, and suppose a and b are rationals, what can you conclude about ab?
3. Given that the decimal expansion of irrational numbers is non-terminating, non-recurring, and $\sqrt{17}$ is irrational, what can we conclude about the decimal expansion of $\sqrt{17}$?
4. Given that $y = x^2 + 6$ and $x = -1$, what can we conclude about the value of y?
5. Given that ABCD is a parallelogram and $\angle B = 80°$. What can you conclude about the other angles of the parallelogram?
6. Given that PQRS is a cyclic quadrilateral and also its diagonals bisect each other. What can you conclude about the quadrilateral?
7. Given that $\sqrt{p}$ is irrational for all primes p and also suppose that 3721 is a prime. Can you conclude that $\sqrt{3721}$ is an irrational number? Is your conclusion correct? Why or why not?

A1.4 Conjectures, Theorems, Proofs and Mathematical Reasoning

Consider the Fig. A1.2. The first circle has one point on it, the second two points, the third three, and so on. All possible lines connecting the points are drawn in each case.

The lines divide the circle into mutually exclusive regions (having no common portion). We can count these and tabulate our results as shown :

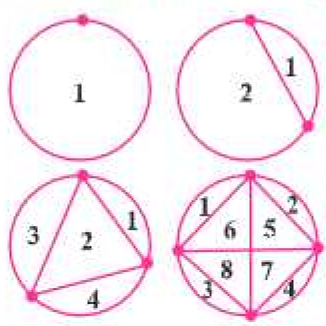


Fig. A1.2

Number of points	Number of regions
1	1
2	2
3	4
4	8
5	
6	
7	

Some of you might have come up with a formula predicting the number of regions given the number of points. From Class IX, you may remember that this intelligent guess is called a '*conjecture*'.

Suppose your conjecture is that given 'n' points on a circle, there are 2^{n-1} mutually exclusive regions, created by joining the points with all possible lines. This seems an extremely sensible guess, and one can check that if $n = 5$, we do get 16 regions. So, having verified this formula for 5 points, are you satisfied that for any n points there are 2^{n-1} regions? If so, how would you respond, if someone asked you, how you can be sure about this for $n = 25$, say? To deal with such questions, you would need a proof which shows beyond doubt that this result is true, or a counter-example to show that this result fails for some 'n'. Actually, if you are patient and try it out for $n = 6$, you will find that there are 31 regions, and for $n = 7$ there are 57 regions. So, $n = 6$, is a counter-example to the conjecture above. This demonstrates the power of a counter-example. You may recall that in the Class IX we discussed that to **disprove a statement, it is enough to come up with a single counter-example**.

You may have noticed that we insisted on a proof regarding the number of regions in spite of verifying the result for $n = 1, 2, 3, 4$ and 5. Let us consider a few more examples. You are familiar with the following result (given in Chapter 5):

$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$. To establish its validity, it is not enough to verify the result for $n = 1, 2, 3$, and so on, because there may be some 'n' for which this result is not true (just as in the example above, the result failed for $n = 6$). What we need is a proof which establishes its truth beyond doubt. You shall learn a proof for the same in higher classes.

Now, consider Fig. A1.3, where PQ and PR are tangents to the circle drawn from P.

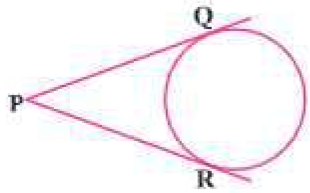


Fig. A1.3

You have proved that PQ = PR (Theorem 10.2). You were not satisfied by only drawing several such figures, measuring the lengths of the respective tangents, and verifying for yourselves that the result was true in each case.

Do you remember what did the proof consist of ? It consisted of a sequence of statements (called **valid arguments**), each following from the earlier statements in the proof, or from previously proved (and known) results independent from the result to be proved, or from axioms, or from definitions, or from the assumptions you had made. And you concluded your proof with the statement PQ = PR, i.e., the statement you wanted to prove. This is the way any proof is constructed.

We shall now look at some examples and theorems and analyse their proofs to help us in getting a better understanding of how they are constructed.

We begin by using the so-called 'direct' or 'deductive' method of proof. In this method, we make several statements. Each is **based on previous statements**. If each statement is logically correct (i.e., a valid argument), it leads to a logically correct conclusion.

Example 10 : The sum of two rational numbers is a rational number.

Solution :

S.No.	Statements	Analysis/Comments
1.	Let x and y be rational numbers.	Since the result is about rationals, we start with x and y which are rational.
2.	Let $x=\frac{m}{n}$, $n \neq 0$ and $y=\frac{p}{q}$, $q \neq 0$ where m, n, p and q are integers.	Apply the definition of rationals.
3.	So, $x+y=\frac{m}{n}+\frac{p}{q}=\frac{mq+np}{nq}$	The result talks about the sum of rationals, so we look at $x + y$.

4.	Using the properties of integers, we see that $mq + np$ and nq are integers.	Using known properties of integers.
5.	Since $n \neq 0$ and $q \neq 0$, it follows that $nq \neq 0$.	Using known properties of integers.
6.	Therefore, $x + y = \frac{mq + np}{nq}$ is a rational number	Using the definition of a rational number.

Remark : Note that, each statement in the proof above is based on a previously established fact, or definition.

Example 11 : Every prime number greater than 3 is of the form $6k + 1$ or $6k + 5$, where k is some integer.

Solution :

S.No.	Statements	Analysis/Comments
1.	Let p be a prime number greater than 3.	Since the result has to do with a prime number greater than 3, we start with such a number.
2.	Dividing p by 6, we find that p can be of the form $6k$, $6k + 1$, $6k + 2$, $6k + 3$, $6k + 4$, or $6k + 5$, where k is an integer.	Using Euclid's division lemma.
3.	But $6k = 2(3k)$, $6k + 2 = 2(3k + 1)$, $6k + 4 = 2(3k + 2)$, and $6k + 3 = 3(2k + 1)$. So, they are not primes.	We now analyse the remainders given that p is prime.
4.	So, p is forced to be of the form $6k + 1$ or $6k + 5$, for some integer k.	We arrive at this conclusion having eliminated the other options.

Remark : In the above example, we have arrived at the conclusion by eliminating different options. This method is sometimes referred to as the **Proof by Exhaustion**.

Theorem A1.1 ***(Converse of the Pythagoras Theorem) :*** *If in a triangle the square of the length of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.*

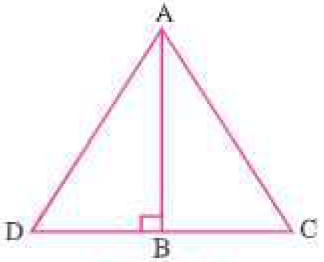


Fig. A1.4

Proof :

S.No.	Statements	Analysis
1.	Let ΔABC satisfy the hypothesis $AC^2 = AB^2 + BC^2$.	Since we are proving a statement about such a triangle, we begin by taking this.
2.	Construct line BD perpendicular to AB, such that BD = BC, and join A to D.	This is the intuitive step we have talked about that we often need to take for proving theorems.
3.	By construction, ΔABD is a right triangle, and from the Pythagoras Theorem, we have $AD^2 = AB^2 + BD^2$.	We use the Pythagoras theorem, which is already proved.
4.	By construction, BD = BC. Therefore, we have $AD^2 = AB^2 + BC^2$.	Logical deduction.
5.	Therefore, $AC^2 = AB^2 + BC^2 = AD^2$.	Using assumption, and previous statement.
6.	Since AC and AD are positive, we have AC = AD.	Using known property of numbers.
7.	We have just shown AC = AD. Also BC = BD by construction, and AB is common. Therefore, by SSS, $\Delta ABC \cong \Delta ABD$.	Using known theorem.
8.	Since $\Delta ABC \cong \Delta ABD$, we get $\angle ABC = \angle ABD$, which is a right angle.	Logical deduction, based on previously established fact. ■

Remark : Each of the results above has been proved by a sequence of steps, all linked together. Their order is important. Each step in the proof follows from previous steps and earlier known results. (Also see Theorem 6.9.)

EXERCISE A1.3

In each of the following questions, we ask you to prove a statement. List all the steps in each proof, and give the reason for each step.

1. Prove that the sum of two consecutive odd numbers is divisible by 4.
2. Take two consecutive odd numbers. Find the sum of their squares, and then add 6 to the result. Prove that the new number is always divisible by 8.
3. If $p \geq 5$ is a prime number, show that $p^2 + 2$ is divisible by 3.

   [**Hint:** Use Example 11].
4. Let x and y be rational numbers. Show that xy is a rational number.
5. If a and b are positive integers, then you know that $a = bq + r$, $0 \leq r < b$, where q is a whole number. Prove that HCF (a, b) = HCF (b, r).

   [**Hint :** Let HCF $(b, r) = h$. So, $b = k_1h$ and $r = k_2h$, where k_1 and k_2 are coprime.]
6. A line parallel to side BC of a triangle ABC, intersects AB and AC at D and E respectively.

   Prove that $\frac{AD}{DB} = \frac{AE}{EC}$.

A1.5 Negation of a Statement

In this section, we discuss what it means to 'negate' a statement. Before we start, we would like to introduce some notation, which will make it easy for us to understand these concepts. To start with, let us look at a statement as a single unit, and give it a name. For example, we can denote the statement 'It rained in Delhi on 1 September 2005' by p. We can also write this by

p: It rained in Delhi on 1 September 2005.

Similarly, let us write

q: All teachers are female.

r: Mike's dog has a black tail.

s: $2 + 2 = 4$.

t: Triangle ABC is equilateral.

This notation now helps us to discuss properties of statements, and also to see how we can combine them. In the beginning we will be working with what we call 'simple' statements, and will then move onto 'compound' statements.

Now consider the following table in which we make a new statement from each of the given statements.

Original statement	New statement
p: It rained in Delhi on 1 September 2005	$\sim p$: It is false that it rained in Delhi on 1 September 2005.
q: All teachers are female.	$\sim q$: It is false that all teachers are female.
r: Mike's dog has a black tail.	$\sim r$: It is false that Mike's dog has a black tail.
s: $2 + 2 = 4$.	$\sim s$: It is false that $2 + 2 = 4$.
t: Triangle ABC is equilateral.	$\sim t$: It is false that triangle ABC is equilateral.

Each new statement in the table is a *negation* of the corresponding old statement. That is, $\sim p$, $\sim q$, $\sim r$, $\sim s$ and $\sim t$ are negations of the statements p, q, r, s and t, respectively. Here, $\sim p$ is read as 'not p'. The statement $\sim p$ negates the assertion that the statement p makes. Notice that in our usual talk we would simply mean $\sim p$ as 'It did not rain in Delhi on 1 September 2005.' However, we need to be careful while doing so. You might think that one can obtain the negation of a statement by simply inserting the word 'not' in the given statement at a suitable place. While this works in the case of p, the difficulty comes when we have a statement that begins with 'all'. Consider, for example, the statement q: All teachers are female. We said the negation of this statement is $\sim q$: It is false that all teachers are female. This is the same as the statement 'There are some teachers who are males.' Now let us see what happens if we simply insert 'not' in q. We obtain the statement: 'All teachers are not female', or we can obtain the statement: 'Not all teachers are female.' The first statement can confuse people. It could imply (if we lay emphasis on the word 'All') that all teachers are male! This is certainly not the negation of q. However, the second statement gives the meaning of $\sim q$, i.e., that there is at least one teacher who is not a female. So, be careful when writing the negation of a statement!

So, how do we decide that we have the correct negation? We use the following criterion.

Let p be a statement and $\sim p$ its negation. Then $\sim p$ is false whenever p is true, and $\sim p$ is true whenever p is false.

For example, if it is true that Mike's dog has a black tail, then it is false that Mike's dog does not have a black tail. If it is false that 'Mike's dog has a black tail', then it is true that 'Mike's dog does not have a black tail'.

Similarly, the negations for the statements s and t are:

s: $2 + 2 = 4$; negation, $\sim s$: $2 + 2 \neq 4$.

t: Triangle ABC is equilateral; negation, $\sim t$: Triangle ABC is not equilateral.

Now, what is $\sim(\sim s)$? It would be $2 + 2 = 4$, which is s. And what is $\sim(\sim t)$? This would be 'the triangle ABC is equilateral', i.e., t. In fact, **for any statement p, $\sim(\sim p)$ is p.**

Example 12 : State the negations for the following statements:

(i) Mike's dog does not have a black tail.

(ii) All irrational numbers are real numbers.

(iii) $\sqrt{2}$ is irrational.

(iv) Some rational numbers are integers.

(v) Not all teachers are males.

(vi) Some horses are not brown.

(vii) There is no real number x, such that $x^2 = -1$.

Solution :

(i) It is false that Mike's dog does not have a black tail, i.e., Mike's dog has a black tail.

(ii) It is false that all irrational numbers are real numbers, i.e., some (at least one) irrational numbers are not real numbers. One can also write this as, 'Not all irrational numbers are real numbers.'

(iii) It is false that $\sqrt{2}$ is irrational, i.e., $\sqrt{2}$ is not irrational.

(iv) It is false that some rational numbers are integers, i.e., no rational number is an integer.

(v) It is false that not all teachers are males, i.e., all teachers are males.

(vi) It is false that some horses are not brown, i.e., all horses are brown.

(vii) It is false that there is no real number x, such that $x^2 = -1$, i.e., there is at least one real number x, such that $x^2 = -1$.

Remark : From the above discussion, you may arrive at the following **Working Rule** for obtaining the negation of a statement :

(i) First write the statement with a 'not'.

(ii) If there is any confusion, make suitable modification , specially in the statements involving 'All' or 'Some'.

EXERCISE A1.4

1. State the negations for the following statements :
   - (i) Man is mortal.
   - (ii) Line l is parallel to line m.
   - (iii) This chapter has many exercises.
   - (iv) All integers are rational numbers.
   - (v) Some prime numbers are odd.
   - (vi) No student is lazy.
   - (vii) Some cats are not black.
   - (viii) There is no real number x, such that $\sqrt{x} = -1$.
   - (ix) 2 divides the positive integer a.
   - (x) Integers a and b are coprime.
2. In each of the following questions, there are two statements. State if the second is the negation of the first or not.
   - (i) Mumtaz is hungry.
     Mumtaz is not hungry.
   - (ii) Some cats are black.
     Some cats are brown.
   - (iii) All elephants are huge.
     One elephant is not huge.
   - (iv) All fire engines are red.
     All fire engines are not red.
   - (v) No man is a cow.
     Some men are cows.

A1.6 Converse of a Statement

We now investigate the notion of the converse of a statement. For this, we need the notion of a 'compound' statement, that is, a statement which is a combination of one or more 'simple' statements. There are many ways of creating compound statements, but we will focus on those that are created by connecting two simple statements with the use of the words 'if' and 'then'. For example, the statement 'If it is raining, then it is difficult to go on a bicycle', is made up of two statements:

p: It is raining

q: It is difficult to go on a bicycle.

Using our previous notation we can say: If p, then q. We can also say 'p implies q', and denote it by $p \Rightarrow q$.

Now, supose you have the statement 'If the water tank is black, then it contains potable water.' This is of the form $p \Rightarrow q$, where the hypothesis is p (the water tank is black) and the conclusion is q (the tank contains potable water). Suppose we interchange the hypothesis and the conclusion, what do we get? We get $q \Rightarrow p$, i.e., if the water in the tank is potable, then the tank must be black. This statement is called the **converse** of the statement $p \Rightarrow q$.

In general, the **converse** of the statement $p \Rightarrow q$ is $q \Rightarrow p$, where p and q are statements. Note that $p \Rightarrow q$ and $q \Rightarrow p$ are the converses of each other.

Example 13 : Write the converses of the following statements :

(i) If Jamila is riding a bicycle, then 17 August falls on a Sunday.

(ii) If 17 August is a Sunday, then Jamila is riding a bicycle.

(iii) If Pauline is angry, then her face turns red.

(iv) If a person has a degree in education, then she is allowed to teach.

(v) If a person has a viral infection, then he runs a high temperature.

(vi) If Ahmad is in Mumbai, then he is in India.

(vii) If triangle ABC is equilateral, then all its interior angles are equal.

(viii) If x is an irrational number, then the decimal expansion of x is non-terminating non-recurring.

(ix) If $x - a$ is a factor of the polynomial $p(x)$, then $p(a) = 0$.

Solution : Each statement above is of the form $p \Rightarrow q$. So, to find the converse, we first identify p and q, and then write $q \Rightarrow p$.

(i) p: Jamila is riding a bicycle, and q: 17 August falls on a Sunday. Therefore, the converse is: If 17 August falls on a Sunday, then Jamila is riding a bicycle.

(ii) This is the converse of (i). Therefore, its converse is the statement given in (i) above.

(iii) If Pauline's face turns red, then she is angry.

(iv) If a person is allowed to teach, then she has a degree in education.

(v) If a person runs a high temperature, then he has a viral infection.

(vi) If Ahmad is in India, then he is in Mumbai.

(vii) If all the interior angles of triangle ABC are equal, then it is equilateral.

(viii) If the decimal expansion of x is non-terminating non-recurring, then x is an irrational number.

(ix) If $p(a) = 0$, then $x - a$ is a factor of the polynomial $p(x)$.

Notice that we have simply written the converse of each of the statements above without worrying if they are true or false. For example, consider the following statement: If Ahmad is in Mumbai, then he is in India. This statement is true. Now consider the converse: If Ahmad is in India, then he is in Mumbai. This need not be true always – he could be in any other part of India.

In mathematics, especially in geometry, you will come across many situations where $p \Rightarrow q$ is true, and you will have to decide if the converse, i.e., $q \Rightarrow p$, is also true.

Example 14 : State the converses of the following statements. In each case, also decide whether the converse is true or false.

(i) If n is an even integer, then $2n + 1$ is an odd integer.

(ii) If the decimal expansion of a real number is terminating, then the number is rational.

(iii) If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.

(iv) If each pair of opposite sides of a quadrilateral is equal, then the quadrilateral is a parallelogram.

(v) If two triangles are congruent, then their corresponding angles are equal.

Solution :

(i) The converse is 'If $2n + 1$ is an odd integer, then n is an even integer.' This is a false statement (for example, $15 = 2(7) + 1$, and 7 is odd).

(ii) 'If a real number is rational, then its decimal expansion is terminating', is the converse. This is a false statement, because a rational number can also have a non-terminating recurring decimal expansion.

(iii) The converse is 'If a transversal intersects two lines in such a way that each pair of corresponding angles are equal, then the two lines are parallel.' We have assumed, by Axiom 6.4 of your Class IX textbook, that this statement is true.

(iv) 'If a quadrilateral is a parallelogram, then each pair of its opposite sides is equal', is the converse. This is true (Theorem 8.1, Class IX).

(v) 'If the corresponding angles in two triangles are equal, then they are congruent', is the converse. This statement is false. We leave it to you to find suitable counter-examples.

EXERCISE A1.5

1. Write the converses of the following statements.

   (i) If it is hot in Tokyo, then Sharan sweats a lot.

   (ii) If Shalini is hungry, then her stomach grumbles.

   (iii) If Jaswant has a scholarship, then she can get a degree.

   (iv) If a plant has flowers, then it is alive.

   (v) If an animal is a cat, then it has a tail.

2. Write the converses of the following statements. Also, decide in each case whether the converse is true or false.
   (i) If triangle ABC is isosceles, then its base angles are equal.
   (ii) If an integer is odd, then its square is an odd integer.
   (iii) If $x^2 = 1$, then $x = 1$.
   (iv) If ABCD is a parallelogram, then AC and BD bisect each other.
   (v) If a, b and c, are whole numbers, then $a + (b + c) = (a + b) + c$.
   (vi) If x and y are two odd numbers, then $x + y$ is an even number.
   (vii) If vertices of a parallelogram lie on a circle, then it is a rectangle.

A1.7 Proof by Contradiction

So far, in all our examples, we used direct arguments to establish the truth of the results. We now explore 'indirect' arguments, in particular, a very powerful tool in mathematics known as 'proof by contradiction'. We have already used this method in Chapter 1 to establish the irrationality of several numbers and also in other chapters to prove some theorems. Here, we do several more examples to illustrate the idea.

Before we proceed, let us explain what a *contradiction* is. In mathematics, a contradiction occurs when we get a statement p such that p is true and $\sim p$, its negation, is also true. For example,

p: $x = \frac{a}{b}$, where a and b are coprime.

q: 2 divides both 'a' and 'b'.

If we assume that p is true and also manage to show that q is true, then we have arrived at a contradiction, because q implies that the negation of p is true. If you remember, this is exactly what happened when we tried to prove that $\sqrt{2}$ is irrational (see Chapter 1).

How does proof by contradiction work? Let us see this through a specific example.

Suppose we are given the following :

All women are mortal. A is a woman. Prove that A is mortal.

Even though this is a rather easy example, let us see how we can prove this by contradiction.

- Let us assume that we want to establish the truth of a statement p (here we want to show that p : 'A is mortal' is true).

- So, we begin by assuming that the statement is not true, that is, we assume that the negation of p is true (i.e., A is not mortal).
- We then proceed to carry out a series of logical deductions based on the truth of the negation of p. (Since A is not mortal, we have a counter-example to the statement 'All women are mortal.' Hence, it is false that all women are mortal.)
- If this leads to a contradiction, then the contradiction arises because of our faulty assumption that p is not true. (We have a contradiction, since we have shown that the statement 'All women are mortal' and its negation, 'Not all women are mortal' is true at the same time. This contradiction arose, because we assumed that A is not mortal.)
- Therefore, our assumption is wrong, i.e., p has to be true. (So, A is mortal.)

Let us now look at examples from mathematics.

Example 15 : The product of a non-zero rational number and an irrational number is irrational.

Solution :

Statements	Analysis/Comment
We will use proof by contradiction. Let r be a non-zero rational number and x be an irrational number. Let $r=\frac{m}{n}$, where m, n are integers and $m \neq 0$, $n \neq 0$. We need to prove that rx is irrational.	
Assume rx is rational.	Here, we are assuming the negation of the statement that we need to prove.
Then $rx=\frac{p}{q}$, $q \neq 0$, where p and q are integers.	This follow from the previous statement and the definition of a rational number.
Rearranging the equation $rx=\frac{p}{q}$, $q \neq 0$, and using the fact that $r=\frac{m}{n}$, we get $x=\frac{p}{rq}=\frac{np}{mq}$.	

Since np and mq are integers and $mq \neq 0$, x is a rational number.	Using properties of integers, and definition of a rational number.
This is a contradiction, because we have shown x to be rational, but by our hypothesis, we have x is irrational.	This is what we were looking for — a contradiction.
The contradiction has arisen because of the faulty assumption that rx is rational. Therefore, rx is irrational.	Logical deduction.

We now prove Example 11, but this time using proof by contradiction. The proof is given below:

Statements	**Analysis/Comment**
Let us assume that the statement is note true.	As we saw earlier, this is the starting point for an argument using 'proof by contradiction'.
So we suppose that there exists a prime number $p > 3$, which is not of the form $6n + 1$ or $6n + 5$, where n is a whole number.	This is the negation of the statement in the result.
Using Euclid's division lemma on division by 6, and using the fact that p is not of the form $6n + 1$ or $6n + 5$, we get $p = 6n$ or $6n + 2$ or $6n + 3$ or $6n + 4$.	Using earlier proved results.
Therefore, p is divisible by either 2 or 3.	Logical deduction.
So, p is not a prime.	Logical deduction.
This is a contradiction, because by our hypothesis p is prime.	Precisely what we want!
The contradiction has arisen, because we assumed that there exists a prime number $p > 3$ which is not of the form $6n + 1$ or $6n + 5$.	
Hence, every prime number greater than 3 is of the form $6n + 1$ or $6n + 5$.	We reach the conclusion. ■

Remark : The example of the proof above shows you, yet again, that there can be several ways of proving a result.

Theorem A1.2 : *Out of all the line segments, drawn from a point to points of a line not passing through the point, the smallest is the perpendicular to the line.*

Proof :

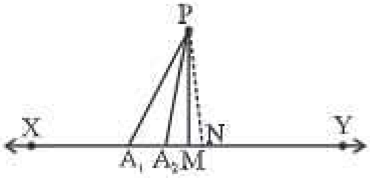


Fig. A1.5

Statements	Analysis/Comment
Let XY be the given line, P a point not lying on XY and PM, PA_1, PA_2, . . . etc., be the line segments drawn from P to the points of the line XY, out of which PM is the smallest (see Fig. A1.5).	Since we have to prove that out of all PM, PA_1, PA_2, . . . etc., the smallest is perpendicular to XY, we start by taking these line segments.
Let PM be not perpendicular to XY	This is the negation of the statement to be proved by contradiction.
Draw a perpendicular PN on the line XY, shown by dotted lines in Fig. A1.5.	We often need constructions to prove our results.
PN is the smallest of all the line segments PM, PA_1, PA_2, . . . etc., which means PN < PM.	Side of right triangle is less than the hypotenuse and known property of numbers.
This contradicts our hypothesis that PM is the smallest of all such line segments.	Precisely what we want!
Therefore, the line segment PM is perpendicular to XY.	We reach the conclusion.

EXERCISE A1.6

1. Suppose $a + b = c + d$, and $a < c$. Use proof by contradiction to show $b > d$.
2. Let r be a rational number and x be an irrational number. Use proof by contradiction to show that $r + x$ is an irrational number.
3. Use proof by contradiction to prove that if for an integer a, a^2 is even, then so is a. [**Hint :** Assume a is not even, that is, it is of the form $2n + 1$, for some integer n, and then proceed.]
4. Use proof by contradiction to prove that if for an integer a, a^2 is divisible by 3, then a is divisible by 3.
5. Use proof by contradiction to show that there is no value of n for which 6^n ends with the digit zero.
6. Prove by contradiction that two distinct lines in a plane cannot intersect in more than one point.

A1.8 Summary

In this Appendix, you have studied the following points :

1. Different ingredients of a proof and other related concepts learnt in Class IX.
2. The negation of a statement.
3. The converse of a statement.
4. Proof by contradiction.

MATHEMATICAL MODELLING A2

A2.1 Introduction

- An adult human body contains approximately 1,50,000 km of arteries and veins that carry blood.
- The human heart pumps 5 to 6 litres of blood in the body every 60 seconds.
- The temperature at the surface of the Sun is about 6,000° C.

Have you ever wondered how our scientists and mathematicians could possibly have estimated these results? Did they pull out the veins and arteries from some adult dead bodies and measure them? Did they drain out the blood to arrive at these results? Did they travel to the Sun with a thermometer to get the temperature of the Sun? Surely not. Then how did they get these figures?

Well, the answer lies in **mathematical modelling,** which we introduced to you in Class IX. Recall that a mathematical model is a mathematical description of some real-life situation. Also, recall that mathematical modelling is the process of creating a mathematical model of a problem, and using it to analyse and solve the problem.

So, in mathematical modelling, we take a real-world problem and convert it to an equivalent mathematical problem. We then solve the mathematical problem, and interpret its solution in the situation of the real-world problem. And then, it is important to see that the solution, we have obtained, 'makes sense', which is the stage of validating the model. Some examples, where mathematical modelling is of great importance, are:

(i) Finding the width and depth of a river at an unreachable place.

(ii) Estimating the mass of the Earth and other planets.

(iii) Estimating the distance between Earth and any other planet.

(iv) Predicting the arrival of the monsoon in a country.

(v) Predicting the trend of the stock market.

(vi) Estimating the volume of blood inside the body of a person.

(vii) Predicting the population of a city after 10 years.

(viii) Estimating the number of leaves in a tree.

(ix) Estimating the ppm of different pollutants in the atmosphere of a city.

(x) Estimating the effect of pollutants on the environment.

(xi) Estimating the temperature on the Sun's surface.

In this chapter we shall revisit the process of mathematical modelling, and take examples from the world around us to illustrate this. In Section A2.2 we take you through all the stages of building a model. In Section A2.3, we discuss a variety of examples. In Section A2.4, we look at reasons for the importance of mathematical modelling.

A point to remember is that here we aim to make you aware of an important way in which mathematics helps to solve real-world problems. However, you need to know some more mathematics to really appreciate the power of mathematical modelling. In higher classes some examples giving this flavour will be found.

A2.2 Stages in Mathematical Modelling

In Class IX, we considered some examples of the use of modelling. Did they give you an insight into the process and the steps involved in it? Let us quickly revisit the main steps in mathematical modelling.

Step 1 (Understanding the problem) : Define the real problem, and if working in a team, discuss the issues that you wish to understand. Simplify by making assumptions and ignoring certain factors so that the problem is manageable.

For example, suppose our problem is to estimate the number of fishes in a lake. It is not possible to capture each of these fishes and count them. We could possibly capture a sample and from it try and estimate the total number of fishes in the lake.

Step 2 (Mathematical description and formulation) : Describe, in mathematical terms, the different aspects of the problem. Some ways to describe the features mathematically, include:

- define variables
- write equations or inequalities
- gather data and organise into tables
- make graphs
- calculate probabilities

For example, having taken a sample, as stated in Step 1, how do we estimate the entire population? We would have to then mark the sampled fishes, allow them to mix with the remaining ones in the lake, again draw a sample from the lake, and see how many of the previously marked ones are present in the new sample. Then, using ratio and proportion, we can come up with an estimate of the total population. For instance, let us take a sample of 20 fishes from the lake and mark them, and then release them in the same lake, so as to mix with the remaining fishes. We then take another sample (say 50), from the mixed population and see how many are marked. So, we gather our data and analyse it.

One major assumption we are making is that the marked fishes mix uniformly with the remaining fishes, and the sample we take is a good representative of the entire population.

Step 3 (Solving the mathematical problem) : The simplified mathematical problem developed in Step 2 is then solved using various mathematical techniques.

For instance, suppose in the second sample in the example in Step 2, 5 fishes are marked. So, $\frac{5}{50}$, i.e., $\frac{1}{10}$, of the population is marked. If this is typical of the whole population, then $\frac{1}{10}$th of the population = 20.

So, the whole population = $20 \times 10 = 200$.

Step 4 (Interpreting the solution) : The solution obtained in the previous step is now looked at, in the context of the real-life situation that we had started with in Step 1.

For instance, our solution in the problem in Step 3 gives us the population of fishes as 200.

Step 5 (Validating the model) : We go back to the original situation and see if the results of the mathematical work make sense. If so, we use the model until new information becomes available or assumptions change.

Sometimes, because of the simplification assumptions we make, we may lose essential aspects of the real problem while giving its mathematical description. In such cases, the solution could very often be off the mark, and not make sense in the real situation. If this happens, we reconsider the assumptions made in Step 1 and revise them to be more realistic, possibly by including some factors which were not considered earlier.

For instance, in Step 3 we had obtained an estimate of the entire population of fishes. It may not be the actual number of fishes in the pond. We next see whether this is a good estimate of the population by repeating Steps 2 and 3 a few times, and taking the mean of the results obtained. This would give a closer estimate of the population.

Another way of visualising **the process of mathematical modelling** is shown in Fig. A2.1.

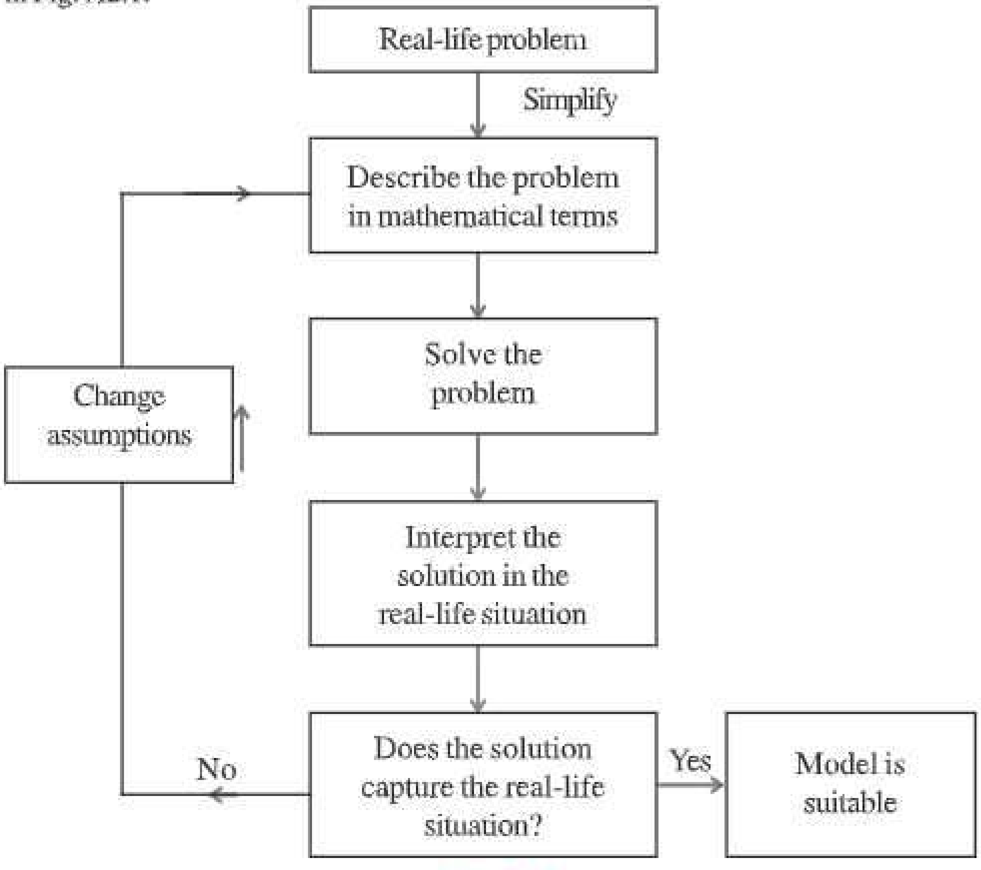


Fig. A2.1

Modellers look for a balance between simplification (for ease of solution) and accuracy. They hope to approximate reality closely enough to make some progress. The best outcome is to be able to predict what will happen, or estimate an outcome, with reasonable accuracy. Remember that different assumptions we use for simplifying the problem can lead to different models. So, there are no perfect models. There are good ones and yet better ones.

EXERCISE A2.1

1. Consider the following situation.

A problem dating back to the early 13th century, posed by Leonardo Fibonacci asks how many rabbits you would have if you started with just two and let them reproduce. Assume that a pair of rabbits produces a pair of offspring each month and that each pair of rabbits produces their first offspring at the age of 2 months. Month by month the number of pairs of rabbits is given by the sum of the rabbits in the two preceding months, except for the 0th and the 1st months.

Month	Pairs of Rabbits
0	1
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	34
9	55
10	89
11	144
12	233
13	377
14	610
15	987
16	1597

After just 16 months, you have nearly 1600 pairs of rabbits!

Clearly state the problem and the different stages of mathematical modelling in this situation.

A2.3 Some Illustrations

Let us now consider some examples of mathematical modelling.

Example 1 (**Rolling of a pair of dice**) **:** Suppose your teacher challenges you to the following guessing game: She would throw a pair of dice. Before that you need to guess the sum of the numbers that show up on the dice. For every correct answer, you get two points and for every wrong guess you lose two points. What numbers would be the best guess?

Solution :

Step 1 (**Understanding the problem**) **:** You need to know a few numbers which have higher chances of showing up.

Step 2 (**Mathematical description**) **:** In mathematical terms, the problem translates to finding out the probabilities of the various possible sums of numbers that the dice could show.

We can model the situation very simply by representing a roll of the dice as a random choice of one of the following thirty six pairs of numbers.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The first number in each pair represents the number showing on the first die, and the second number is the number showing on the second die.

Step 3 (**Solving the mathematical problem**) **:** Summing the numbers in each pair above, we find that possible sums are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. We have to find the probability for each of them, assuming all 36 pairs are equally likely.

We do this in the following table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Observe that the chance of getting a sum of a seven is 1/6, which is larger than the chances of getting other numbers as sums.

Step 4 (Interpreting the solution) : Since the probability of getting the sum 7 is the highest, you should repeatedly guess the number seven.

Step 5 (Validating the model) : Toss a pair of dice a large number of times and prepare a relative frequency table. Compare the relative frequencies with the corresponding probabilities. If these are not close, then possibly the dice are biased. Then, we could obtain data to evaluate the number towards which the bias is.

Before going to the next example, you may need some background.

Not having the money you want when you need it, is a common experience for many people. Whether it is having enough money for buying essentials for daily living, or for buying comforts, we always require money. To enable the customers with limited funds to purchase goods like scooters, refrigerators, televisions, cars, etc., a scheme known as an *instalment scheme* (*or plan*) is introduced by traders.

Sometimes a trader introduces an *instalment* scheme as a marketing strategy to allure customers to purchase these articles. Under the instalment scheme, the customer is not required to make full payment of the article at the time of buying it. She/he is allowed to pay a part of it at the time of purchase, and the rest can be paid in instalments, which could be monthly, quarterly, half-yearly, or even yearly. Of course, the buyer will have to pay more in the instalment plan, because the seller is going to charge some interest on account of the payment made at a later date (called *deferred payment*).

Before we take a few examples to understand the instalment scheme, let us understand the most frequently used terms related to this concept.

The *cash price* of an article is the amount which a customer has to pay as full payment of the article at the time it is purchased. *Cash down payment* is the amount which a customer has to pay as part payment of the price of an article at the time of purchase.

Remark : If the instalment scheme is such that the remaining payment is completely made within one year of the purchase of the article, then simple interest is charged on the deferred payment.

> In the past, charging interest on borrowed money was often considered evil, and, in particular, was long prohibited. One way people got around the law against paying interest was to borrow in one currency and repay in another, the interest being disguised in the exchange rate.

Let us now come to a related mathematical modelling problem.

Example 2 : Juhi wants to buy a bicycle. She goes to the market and finds that the bicycle she likes is available for ₹ 1800. Juhi has ₹ 600 with her. So, she tells the shopkeeper that she would not be able to buy it. The shopkeeper, after a bit of calculation, makes the following offer. He tells Juhi that she could take the bicycle by making a payment of ₹ 600 cash down and the remaining money could be made in two monthly instalments of ₹ 610 each. Juhi has two options one is to go for instalment scheme or to make cash payment by taking loan from a bank which is available at the rate of 10% per annum simple interest. Which option is more economical to her?

Solution :

Step 1 (Understanding the problem) : What Juhi needs to determine is whether she should take the offer made by the shopkeeper or not. For this, she should know the two rates of interest — one charged in the instalment scheme and the other charged by the bank (i.e., 10%).

Step 2 (Mathematical description) : In order to accept or reject the scheme, she needs to determine the interest that the shopkeeper is charging in comparison to the bank. Observe that since the entire money shall be paid in less than a year, simple interest shall be charged.

We know that the cash price of the bicycle = ₹1800.

Also, the cashdown payment under the instalment scheme = ₹ 600.

So, the balance price that needs to be paid in the instalment scheme = ₹ (1800 – 600) = ₹1200.

Let r % per annum be the rate of interest charged by the shopkeeper.

Amount of each instalment = ₹ 610

Amount paid in instalments = ₹ 610 + ₹610 = ₹1220

Interest paid in instalment scheme = ₹1220 – ₹1200 = ₹20 (1)

Since, Juhi kept a sum of ₹1200 for one month, therefore,

Principal for the first month = ₹ 1200

Principal for the second month = ₹ (1200 – 610) = ₹590

Balance of the second principal ₹590 + interest charged (₹20) = monthly instalment (₹610) = 2nd instalment

So, the total principal for one month = ₹ 1200 + ₹590 = ₹1790

Now,
$$\text{interest} = ₹\ \frac{1790 \times r \times 1}{100 \times 12} \qquad (2)$$

Step 3 (Solving the problem) : From (1) and (2)

$$\frac{1790 \times r \times 1}{100 \times 12} = 20$$

or $$r = \frac{20 \times 1200}{1790} = 13.14 \text{ (approx.)}$$

Step 4 (Interpreting the solution) : The rate of interest charged in the instalment scheme = 13.14 %.

The rate of interest charged by the bank = 10%

So, she should prefer to borrow the money from the bank to buy the bicycle which is more economical.

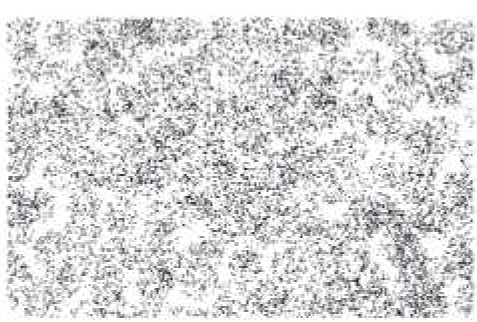
Step 5 (Validating the model) : This stage in this case is not of much importance here as the numbers are fixed. However, if the formalities for taking loan from the bank such as cost of stamp paper, etc., which make the effective interest rate more than what it is the instalment scheme, then she may change her opinion.

Remark : Interest rate modelling is still at its early stages and validation is still a problem of financial markets. In case, different interest rates are incorporated in fixing instalments, validation becomes an important problem.

EXERCISE A2.2

In each of the problems below, show the different stages of mathematical modelling for solving the problems.

1. An ornithologist wants to estimate the number of parrots in a large field. She uses a net to catch some, and catches 32 parrots, which she rings and sets free. The following week she manages to net 40 parrots, of which 8 are ringed.
   (i) What fraction of her second catch is ringed?
   (ii) Find an estimate of the total number of parrots in the field.



2. Suppose the adjoining figure represents an aerial photograph of a forest with each dot representing a tree. Your purpose is to find the number of trees there are on this tract of land as part of an environmental census.

3. A T.V. can be purchased for ₹ 24000 cash or for ₹ 8000 cashdown payment and six monthly instalments of ₹ 2800 each. Ali goes to market to buy a T.V., and he has ₹ 8000 with him. He has now two options. One is to buy TV under instalment scheme or to make cash payment by taking loan from some financial society. The society charges simple interest at the rate of 18% per annum simple interest. Which option is better for Ali?

A2.4 Why is Mathematical Modelling Important?

As we have seen in the examples, mathematical modelling is an interdisciplinary subject. Mathematicians and specialists in other fields share their knowledge and expertise to improve existing products, develop better ones, or predict the behaviour of certain products.

There are, of course, many specific reasons for the importance of modelling, but most are related in some ways to the following :

- *To gain understanding*. If we have a mathematical model which reflects the essential behaviour of a real-world system of interest, we can understand that system better through an analysis of the model. Furthermore, in the process of building the model we find out which factors are most important in the system, and how the different aspects of the system are related.
- *To predict, or forecast, or simulate.* Very often, we wish to know what a real-world system will do in the future, but it is expensive, impractical or impossible to experiment directly with the system. For example, in weather prediction, to study drug efficacy in humans, finding an optimum design of a nuclear reactor, and so on.

  Forecasting is very important in many types of organisations, since predictions of future events have to be incorporated into the decision-making process. For example:

  In marketing departments, reliable forecasts of demand help in planning of the sale strategies.

  A school board needs to able to forecast the increase in the number of school going children in various districts so as to decide where and when to start new schools.

  Most often, forecasters use the past data to predict the future. They first analyse the data in order to identify a pattern that can describe it. Then this data and pattern is extended into the future in order to prepare a forecast. This basic strategy is employed in most forecasting techniques, and is based on the assumption that the pattern that has been identified will continue in the future also.

- *To estimate.* Often, we need to estimate large values. You've seen examples of the trees in a forest, fish in a lake, etc. For another example, before elections, the contesting parties want to predict the probability of their party winning the elections. In particular, they want to estimate how many people in their constituency would vote for their party. Based on their predictions, they may want to decide on the campaign strategy. Exit polls have been used widely to predict the number of seats, a party is expected to get in elections.

EXERCISE A2.3

1. Based upon the data of the past five years, try and forecast the average percentage of marks in Mathematics that your school would obtain in the Class X board examination at the end of the year.

A2.5 Summary

In this Appendix, you have studied the following points :

1. A mathematical model is a mathematical description of a real-life situation. Mathematical modelling is the process of creating a mathematical model, solving it and using it to understand the real-life problem.
2. The various steps involved in modelling are : understanding the problem, formulating the mathematical model, solving it, interpreting it in the real-life situation, and, most importantly, validating the model.
3. Developed some mathematical models.
4. The importance of mathematical modelling.