CBSE Test Paper 01
Chapter 2 Polynomials

1. The zeroes of a polynomial $x^2 + 5x - 24$ are **(1)**
   a. one positive and one negative
   b. both positive
   c. both negative
   d. both equal
2. If 'α' and 'β' are the zeroes of a quadratic polynomial $x^2 - 5x + b$ and $\alpha - \beta = 1$, then the value of 'b' is **(1)**
   a. -6
   b. -5
   c. 5
   d. 6
3. Degree of the polynomial $2x^4+3x^3-5x^2+9x+1$ is **(1)**
   a. 3
   b. 1
   c. 2
   d. 4
4. If α and β are zeros of $x^2 + 5x + 8$, then the value of $(\alpha + \beta)$ is **(1)**
   a. -8
   b. 8
   c. 5
   d. -5
5. Which of the following expressions is not a polynomial? **(1)**
   a. $5x^3 - 3x^2 - \sqrt{x} + 2$
   b. $5x^3 - 3x^2 - x + \sqrt{2}$
   c. $5x^2 - \frac{2}{3}x + 2\sqrt{5}$
   d. $\sqrt{5}x^3 - \frac{3}{5}x + \frac{1}{7}$
6. Find the zeroes of the polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$. **(1)**
7. If the product of the zeros of the polynomial $(ax^2 - 6x - 6)$ is 4. Find the value of a. **(1)**
8. If $x^3 + x^2 - ax + b$ is divisible by $(x^2 - x)$, write the values of a and b. **(1)**

9. Find the zeros of the following quadratic polynomial and verify the relationship between the zeros and the coefficients: $3x^2 - x - 4$. **(1)**
10. Find all the zeroes of $f(x) = x^2 - 2x$. **(1)**
11. If α and β are the zeroes of the polynomial $4x^2 - 2x + (k - 4)$ and $\alpha = \frac{1}{\beta}$, find the value of k. **(2)**
12. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is the reciprocal of the other, find the value of a. **(2)**
13. Find a cubic polynomial whose zeros are 3, $\frac{1}{2}$ and -1. **(2)**
14. If α and β are zeroes of the polynomial $x^2 - p(x+1) + c$ such that $(\alpha+1)(\beta+1) = 0$, then find the value of c. **(3)**
15. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be x + a, find k and a. **(3)**
16. Find the zeroes of the given quadratic polynomials and verify the relationship between the zeroes and the coefficients. $x^2 - 2x - 8$ **(3)**
17. A polynomial g(x) of degree zero is added to polynomial $2x^3 + 5x^2 - 14x + 10$, so that it becomes exactly divisible by $2x - 3$. Find g(x). **(3)**
18. A village of the North-East India is suffering from flood. A group of students decide to help them with food items, clothes etc, So the student collects some amount of rupees, which is represented by $x^4 + x^3 + 8x^2 + ax + b$
    i. If the number of students is represented by $x^2 + 1$, find the values of a and b.
    ii. What values have been depicted by the group of students? **(4)**
19. If α and β are the zeroes of the polynomial $p(x) = 6x^2 + 5x - k$ satisfying the relation, $\alpha - \beta = \frac{1}{6}$, then find the value of k. **(4)**
20. If α and β are the zeroes of polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$. **(4)**

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Solution

1. a. one positive and one negative

   Explanation: $x^2 + 5x - 24$

   = $x^2 + 8x - 3x - 24$

   = $x(x + 8) - 3(x + 8)$ =0

   $(x + 8)(x - 3)$=0

   $\therefore x + 8 = 0$ or $x - 3 = 0$

   $\Rightarrow x = -8$ or $x = 3$

2. d. 6

   Explanation: Here $\alpha + \beta = \frac{-b}{a}$ = $\frac{-(-5)}{1}$ $\alpha + \beta = 5$ ....(i)

   And it is given that $\alpha - \beta = 1$ ....(ii)

   On solving eq. (i) and eq. (ii), we get

   $\alpha + \beta = 5$

   $\alpha - \beta = 1$

   $2a = 6$ (β is cancelled)

   $\alpha = \frac{6}{2}$

   $\alpha = 3$ Put the value of α in eq. (i)

   $\alpha + \beta = 5$

   $\Rightarrow 3 + \beta = 5$

   $\Rightarrow \beta = 5 - 3$

   $\Rightarrow \beta = 2$

   $\therefore \alpha\beta = \frac{c}{a}$

   $\Rightarrow 3 \times 2 = \frac{b}{1} \Rightarrow b = 6$

3. d. 4

   Explanation: The highest power of the variable is 4. So, the degree of the polynomial is 4.

4. d. -5

   Explanation: $x^2 + 5x + 8$

$\alpha+\beta=\frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2}$
$=\frac{-5}{1}$
= -5

5. a. $5x^3-3x^2-\sqrt{x}+2$
   Explanation: $5x^3-3x^2-\sqrt{x}+2$ is not a polynomial because each term of a polynomial should be a product of a constant and one or more variable raised to a positive, zero or integral power. Here $\sqrt{x}$ does not satisfy the condition of being a polynomial.
6. We have to find the zeroes of the polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.
   p(x) = $\sqrt{3}x^2 - 8x + 4\sqrt{3}$
   = $\sqrt{3}x^2 - 6x - 2x + 4\sqrt{3} = 0$
   = $\sqrt{3}(x-2\sqrt{3})-2(x-2\sqrt{3})$
   = $(\sqrt{3}x-2)(x-2\sqrt{3}) = 0$
   $\therefore$ Zeroes = $\frac{2}{\sqrt{3}}, 2\sqrt{3}$
7. According to the question,we have to find the value of a such that the product of the zeros of the polynomial (ax^2 - 6x - 6) is 4.
   Let α and β be the zeros of the polynomial (ax^2 - 6x - 6)
   Then, $\alpha\beta$ = $\frac{\text{constant term}}{\text{coefficient of } x^2}=\frac{-6}{a}$
   But, $\alpha\beta$ = 4 (given).
   $\therefore \quad \frac{-6}{a}=4 \Rightarrow 4a=-6 \Rightarrow a=\frac{-6}{4}=\frac{-3}{2}$
   Hence, a = $\frac{-3}{2}$
8. Since f(x) = $x^3 + x^2 - ax + b$ is divisible by (x^2 - x), we have
   $x^2 - x = 0$
   $\Rightarrow$ x(x - 1) = 0
   $\Rightarrow$ x = 0 or x = 1
   Hence,
   f(0) = 0
   $\Rightarrow x^3 + x^2 - ax + b = 0$
   $\Rightarrow 0^3 + 0^2 - a(0) + b = 0$
   $\Rightarrow$ b = 0
   Also,

f(1) = 0

$\Rightarrow$ $x^3 + x^2 - ax + b = 0$

$\Rightarrow$ $1^3 + 1^2 - a(1) + 0 = 0$

$\Rightarrow$ 1 + 1 - a = 0

$\Rightarrow$ 2 - a = 0

$\Rightarrow$ a = 2

Hence , the value of a and b in given polynomial are a = 2 and b = 0.

9. We have,f(x) = $3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$

= x(3x - 4) + 1(3x - 4)

= (3x - 4))(x + 1)

$\therefore$ f(x) = 0

$\Rightarrow$ (3x - 4)(x +1) = 0

$\Rightarrow$ 3x - 4 = 0 or x + 1 = 0

$\Rightarrow x = \frac{4}{3}$ or x = -1

So, the zeros of f(x) are $\frac{4}{3}$ and -1

Now sum of zeros $= \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(\text{ coefficient of } x)}{(\text{ coefficient of } x^2)}$.

And product of zeros $= \frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{\text{constant term}}{(\text{ coefficient of } x^2)}$

10. f(x) = $x^2 - 2x$

= x (x -2)

f(x) = 0 $\Rightarrow$ x = 0 or x = 2

Hence, zeroes are 0 and 2.

11. Here p(x) = $4x^2 - 2x + k - 4$

Here a=4,b=-2,c=k-4

Given α and β are zeros of the given polynomial

$\alpha = \frac{1}{\beta}$,

$\Rightarrow \beta = \frac{1}{\alpha}$

αβ=1

also αβ= $\frac{c}{a} = \frac{k-4}{4}$

So $\frac{k-4}{4}$ =1

k - 4 = 4

k = 4 + 4 = 8

12. Let α and $\frac{1}{\alpha}$ be the zeros of (a^2 + 9)x^2 + 13x + 6a.

Then, we have

$\alpha \times \frac{1}{\alpha} = \frac{6a}{a^2+9}$

⇒ 1 = $\frac{6a}{a^2+9}$

⇒ a^2 + 9 = 6a

⇒ a^2 - 6a + 9 = 0

⇒ a^2 - 3a - 3a + 9 = 0

⇒ a(a - 3) - 3(a - 3) = 0

⇒ (a - 3) (a - 3) = 0

⇒ $(a - 3)^2$ = 0

⇒ a - 3 = 0

⇒ a = 3

So, the value of a in given polynomial is 3.

13. Let α = 3, β = $\frac{1}{2}$ and γ = -1. Then,

$(\alpha+\beta+\gamma) = \left(3+\frac{1}{2}-1\right) = \frac{5}{2}$,

$(\alpha\beta+\beta\gamma+\gamma\alpha) = \left(\frac{3}{2}-\frac{1}{2}-3\right) = \frac{-4}{2}$ = -2

and $\alpha\beta y = \left\{3\times\frac{1}{2}\times(-1)\right\} = \frac{-3}{2}$

The polynomial with zeros α,β and γ is:

$\mathrm{x}^3-(\alpha+\beta+\gamma)\mathrm{x}^2+(\alpha\beta+\beta\gamma+\gamma\alpha)\mathrm{x}-\alpha\beta\gamma$

$= \mathrm{x}^3-\frac{5}{2}\mathrm{x}^2-2\mathrm{x}+\frac{3}{2}$

Thus, $2x^3$- $5x^2$- 4x + 3 is the desired polynomial.

14. Given, α and β are the zeroes of polynomial $x^2-p(x+1)+c$

which can be written as $x^2-px+c-p$

So, sum of zeroes, $\alpha+\beta=p$ [$\because$ sum of coefficients = $\frac{-(coefficient(x))}{coefficient(x^2)}$]

and product of zeroes $\alpha\beta=c-p$ [$\because$ product of cofficients= $\frac{constant\_term}{coefficient(x^2)}$,]

Also, $(\alpha+1)(\beta+1)=0$

$\alpha\beta+\alpha+\beta+1=0$

$\Rightarrow c-p+p+1=0$

$\Rightarrow c=-1$

15. On dividing x^4 - $6x^3$ - $16x^2$ - 25x + 10 by x^2 - 2x + k

$$
\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 25x + 10 \\
 & -4x^3 + 8x^2 \quad - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 + (4k-25)x + 10 \\
 & (8-k)x^2 - 2(8-k)x + (8-k)k \\
 & - \quad + \quad - \\
\hline
 & (2k-9)x - (8-k)k + 10
\end{array}
$$

∴ Remainder = (2k - 9)x - (8 - k)k + 10

But the remainder is given as x+a.

On comparing their coefficients,

2k - 9 = 1

⇒ k = 10

⇒ k = 5 and,

-(8 - k)k + 10 = a

⇒ a = -(8 - 5)5 + 10 = -15 + 10 = -5

Hence, k = 5 and a = -5

16. Let p(x) = x^2 - 2x - 8

By the method of splitting the middle term,

$x^2 - 2x - 8 = x^2 - 4x + 2x - 8$

$= x(x-4) + 2(x-4) = (x-4)(x+2)$

For zeroes of p(x),

p(x) = 0

$\Rightarrow (x-4)(x+2) = 0$

$\Rightarrow x - 4 = 0$ or $x + 2 = 0$

$\Rightarrow x = 4$ or $x = -2$

$\Rightarrow x = 4, -2$

So, the zeroes of p(x) are 4 and -2.

We observe that, Sum of its zeroes

= 4 + (-2) = 2

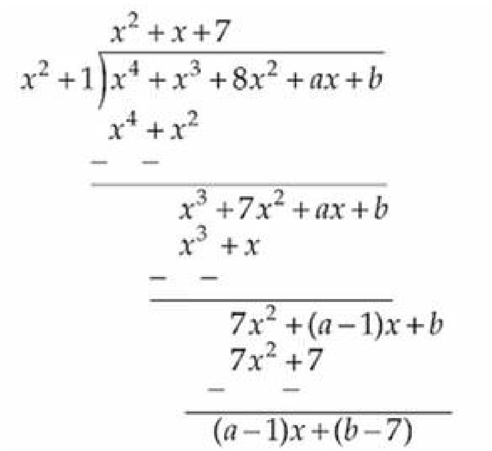
$= \frac{-(-2)}{1} = \frac{-\text{(Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of its zeroes

$= 4x(-2) = -8 = \frac{-8}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, relation between zeroes and coefficients is verified.

17. According to the question, g(x) of degree degree zero is added to the polynomial $(2x^3 + 2x^2 - 14x + 10)$
such that it becomes completely divisible by $2x - 3$.
Let g(x)=k, then $2x^3 + 5x^2 - 14x + 10$+ k will be exactly divisible by $2x - 3$.
P(x)= $2x^3 + 5x^2 - 14x + 10 + k$
We know dividend = quotient x divisor + remainder
On dividing $2x^3 + 5x^2 - 14x + 10 + k$ by $2x - 3$, we get quotient $x^2 + 4x - 1$ and remainder = k+7
The degree of g(x) is zero then g(x) = 0
$\Rightarrow k + 7 = 0 \Rightarrow k = -7$
$\therefore$ g(x) = -7
18. i. First we divide $x^4 + x^3 + 8x^2 + ax + b$ by $x^2 + 1$ as follows:



Since $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$, therefore remainder = 0
i.e. $(a - 1)x + (b - 7) = 0$
or $(a - 1)x + (b - 7) = 0x + 0$
Equating the corresponding terms, We have
$a - 1 = 0$ and $b - 7 = 0$
i.e. $a = 1$ and $b = 7$
ii. Common good, Social responsibility
19. According to the question, α and β are zeroes of p(x) = 6x^2 - 5x + k
So, Sum of zeroes = $\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ....(i)

$\alpha - \beta = \frac{1}{6}$ (Given) ....(ii)

Adding equations (i) and (ii) , we get

$2\alpha = 1$

or, $\alpha = \frac{1}{2}$

On putting the value of α in equation (ii), we get

$\frac{1}{2} - \beta = \frac{1}{6}$

$\beta = \frac{1}{2} - \frac{1}{6}$

$\beta = \frac{2}{6} = \frac{1}{3}$

$\therefore \alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Hence, k = 1

20. Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$.

Hence, $\alpha + \beta = -\frac{2}{3}$

and $\alpha\beta = \frac{1}{3}$

Now, for the new polynomial,

Sum of zeroes = $\frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$

$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$

$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$

$\therefore$ Sum of zeroes = $\frac{4/3}{2/3} = 2$

Product of zeroes = $\left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$

$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$

$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$

$= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{3}{3}} = 3$

Hence, Required polynomial = x^2 - (Sum of zeroes)x + Product of zeroes

$= x^2 - 2x + 3$