

Fill Ups, Subjective Problems of Indefinite Integrals

Fill in the Blanks

Q.1. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \log (9e^{2x} - 4) + C$, then $A = \dots\dots$, $B = \dots\dots$ and $C = \dots\dots$
(1990 - 2 Marks)

Ans. $\frac{-3}{2}, \frac{35}{36}$, any real value

Solutions.

$$\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \ln(9e^{2x} - 4) + C$$

$$\Rightarrow \frac{d}{dx} [Ax + B \ln(9e^{2x} - 4) + C] = \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}$$

$$\Rightarrow A + \frac{18Be^x}{9e^x - 4e^{-x}} = \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}$$

$$\Rightarrow \frac{(9A + 18B)e^x - 4Ae^{-x}}{9e^x - 4e^{-x}} = \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}$$

$$\Rightarrow 9A + 18B = 4; -4A = 6 \Rightarrow A = \frac{-3}{2};$$

$$B = \left(4 + \frac{27}{2}\right) \frac{1}{18} = \frac{35}{36}; C \text{ Can have any real value.}$$

Subjective Problems

Q. 1. Evaluate $\int \frac{\sin x}{\sin x - \cos x} dx$ (1978)

Ans. $\frac{1}{2} \log |\sin x - \cos x| + \frac{x}{2} + C$

Solutions.

$$\begin{aligned}
 I &= \int \frac{\sin x}{\sin x - \cos x} dx = \frac{1}{2} \int \frac{2 \sin x}{\sin x - \cos x} dx \\
 &= \frac{1}{2} \int \frac{\sin x + \cos x + \sin x - \cos x}{\sin x - \cos x} dx \\
 &= \frac{1}{2} \int \frac{\cos x + \sin x}{\sin x - \cos x} dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x - \cos x} dx \\
 &= \frac{1}{2} \log |\sin x - \cos x| + \frac{x}{2} + C
 \end{aligned}$$

Q. 2. Evaluate $\int \frac{x^2 dx}{(a+bx)^2}$ (1979)

Ans. $\frac{1}{b^3} \left[a+bx - 2a \log |a+bx| - \frac{a^2}{a+bx} \right] + C$

Solutions.

Let $I = \int \frac{x^2 dx}{(a+bx)^2}$

Let $a+bx = t \Rightarrow x = \left(\frac{t-a}{b} \right) \Rightarrow dx = \frac{dt}{b}$

$\therefore I = \frac{1}{b^3} \int \frac{(t-a)^2}{t^2} dt = \frac{1}{b^3} \int \frac{t^2 - 2at + a^2}{t^2} dt$

$= \frac{1}{b^3} \int \left(1 - \frac{2a}{t} + \frac{a^2}{t^2} \right) dt = \frac{1}{b^3} \left[t - 2a \log |t| - \frac{a^2}{t} \right] + C$

$= \frac{1}{b^3} \left[a+bx - 2a \log |a+bx| - \frac{a^2}{a+bx} \right] + C$

Q. 3. Evaluate $\int (e^{\log x} + \sin x) \cos x dx$. (1981 - 2 Marks)

Ans. $x \sin x + \cos x - \frac{1}{4} \cos 2x + C$

Solutions.

To evaluate $\int (e^{\log x} + \sin x) \cos x \, dx$

$$= \int (x + \sin x) \cos x \, dx \quad [\text{Using } e^{\log x} = x]$$

$$= \int x \cos x + \frac{1}{2} \int \sin 2x \, dx$$

$$= x \sin x - \int \sin x \, dx + \frac{1}{2} \left(\frac{-\cos 2x}{2} \right)$$

$$= x \sin x + \cos x - \frac{1}{4} \cos 2x + C$$

Q. 4. Evaluate: $\int \frac{(x-1)e^x}{(x+1)^3} \, dx$ (1983 - 2 Marks)

Ans. $\frac{e^x}{(x+1)^2} + C$ 5. $-\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}} + C$

Solutions.

$$I = \int \frac{(x-1)e^x}{(x+1)^3} \, dx = \int \frac{(x+1-2)e^x}{(x+1)^3} \, dx$$

$$= \int \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right] e^x \, dx = \frac{e^x}{(x+1)^2} + C$$

(Using $\int e^x (f(x) + f'(x)) \, dx = e^x f(x)$)

Q. 5. Evaluate the following $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ (1984 - 2 Marks)

Ans. $-\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}} + C$

Solutions.

$$\text{Let } I = \int \frac{dx}{x^3 x^2 \left(1 + \frac{1}{x^4}\right)^{3/4}}$$

$$\text{Put } 1 + \frac{1}{x^4} = t \Rightarrow \frac{-4}{x^5} dx = dt \Rightarrow \frac{dx}{x^5} = -\frac{dt}{4}$$

$$\begin{aligned} \therefore I &= \int \frac{-dt}{4t^{3/4}} = \frac{-1}{4} \left(\frac{t^{-3/4+1}}{-\frac{3}{4}+1} \right) + C \\ &= -t^{1/4} + C = -\left(1 + \frac{1}{x^4}\right)^{1/4} + C \end{aligned}$$

Q. 6. Evaluate the following $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$ **(1985 - 2½ Marks)**

Ans. $-2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x}\sqrt{1-x} + C$

Solutions.

$$I = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$$

$$\text{Put } x = \cos^2 \theta \Rightarrow dx = -2 \cos \theta \sin \theta d\theta$$

$$\begin{aligned} \therefore I &= -\int \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} \cdot 2 \sin \theta \cos \theta d\theta \\ &= -\int \frac{\sin \theta / 2}{\cos \theta / 2} \cdot 2 \cdot 2 \sin(\theta / 2) \cos(\theta / 2) \cos \theta d\theta \\ &= -2 \int (1 - \cos \theta) \cos \theta d\theta \end{aligned}$$

$$= -2 \int (\cos \theta - \cos^2 \theta) d\theta$$

$$= -2 \int \left(\cos \theta - \frac{1 + \cos 2\theta}{2} \right) d\theta$$

$$= -2 \left[\sin \theta - \frac{1}{2} \left(\theta + \frac{\sin 2\theta}{2} \right) + C \right]$$

$$= -2\sqrt{1-x} + \left[\cos^{-1} \sqrt{x} + \sqrt{x}\sqrt{1-x} \right] + C$$

$$\text{[Using } \sin \theta = \sqrt{1-x} \text{]}$$

$$= -2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x}\sqrt{1-x} + C$$

Q. 7. Evaluate: $\int \left[\frac{(\cos 2x)^{1/2}}{\sin x} \right] dx$ (1987 - 6 Marks)

Ans. $\frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2} + \sqrt{1 - \tan^2 x}}{\sqrt{2} - \sqrt{1 - \tan^2 x}} \right| - \log(\cot x + \sqrt{\cot^2 x - 1}) + C$

Solutions.

$$I = \int \frac{\sqrt{\cos 2x}}{\sin x} dx = \int \frac{\sqrt{\cos^2 x - \sin^2 x}}{\sin^2 x} dx = \int \frac{\sqrt{\cot^2 x - 1}}{\cot^2 x} dx$$

$$\text{Let } \cot x = \sec \theta \Rightarrow -\operatorname{cosec}^2 x dx = \sec \theta \tan \theta d\theta$$

$$\text{We get, } I = \int \sqrt{\sec^2 \theta - 1} \cdot \frac{\sec \theta \tan \theta}{-(1 + \sec^2 \theta)} d\theta$$

$$= -\int \frac{\sec \theta \cdot \tan^2 \theta}{1 + \sec^2 \theta} d\theta = -\int \frac{\sin^2 \theta}{\cos \theta + \cos^3 \theta} d\theta$$

$$= -\int \frac{1 - \cos^2 \theta}{\cos \theta (1 + \cos^2 \theta)} d\theta = -\int \frac{(1 + \cos^2 \theta) - 2\cos^2 \theta}{\cos \theta (1 + \cos^2 \theta)} d\theta$$

$$= -\int \sec \theta d\theta + 2 \int \frac{\cos \theta}{1 + \cos^2 \theta} d\theta$$

$$= -\log |\sec \theta + \tan \theta| + 2 \int \frac{\cos \theta}{2 - \sin^2 \theta} d\theta$$

$$= -\log |\sec \theta + \tan \theta| + 2 \cdot \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + \sin \theta}{\sqrt{2} - \sin \theta} \right| + C$$

$$= -\log |\cot x + \sqrt{\cot^2 x - 1}| + \frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2} + \sqrt{1 - \tan^2 x}}{\sqrt{2} - \sqrt{1 - \tan^2 x}} \right| + C$$

Q. 8. Evaluate $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$ (1989 - 3 Marks)

Ans. $\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + c$

Solutions.

$$I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx = \int \frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} dx$$

$$= \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx = \sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{\sin 2x}}$$

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$ also $(\sin x - \cos x)^2 = t^2 \Rightarrow 1 - \sin 2x = t^2$

$$\Rightarrow \sin 2x = 1 - t^2 \quad \therefore I = \sqrt{2} \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \sqrt{2} \sin^{-1} t + C = \sqrt{2} \sin^{-1} (\sin x - \cos x) + C$$

Q. 9. Find the indefinite integral $\int \left(\frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} + \frac{\ln(1 + \sqrt[6]{x})}{\sqrt[3]{x} + \sqrt{x}} \right) dx$ **(1992 - 4 Marks)**

Ans.

$$\frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + 2x^{1/2} - \frac{12}{5}x^{5/12} + 3x^{1/3} - 4x^{1/4} + 6x^{1/6} - 12x^{1/12}$$

$$+ 12 \log |x^{1/2} + 1| + 6 \left\{ \left[\frac{(1+x^{1/6})^3}{3} - \frac{3}{2}(1+x^{1/6})^2 + 3(1+x^{1/6}) \right] \right.$$

$$\left. \ln(1+x^{1/6}) - \left[\frac{(1+x^{1/6})^3}{9} - \frac{3}{4}(1+x^{1/6})^2 + 3(1+x^{1/6}) \right] \right\}$$

Solutions.

$$\text{Let } I = \int \left(\frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} + \frac{\ln(1 + \sqrt[6]{x})}{\sqrt[3]{x} + \sqrt{x}} \right) dx$$

$$= \int \frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} dx + \int \frac{\ln(1 + \sqrt[6]{x})}{\sqrt[3]{x} + \sqrt{x}} dx$$

$$I = I_1 + I_2 \quad \dots(1)$$

$$\text{where } I_1 = \int \frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} dx$$

$$\text{Let } x = y^{12} \text{ so that } dx = 12 y^{11} dy$$

$$\begin{aligned} \therefore I_1 &= \int \frac{12y^{11}}{y^4 + y^3} dy = 12 \int \frac{y^8}{1+y} dy \\ &= 12 \int \left(y^7 - y^6 + y^5 - y^4 + y^3 - y^2 + y - 1 + \frac{1}{y+1} \right) dy \\ &= 12 \left[\frac{y^8}{8} - \frac{y^7}{7} + \frac{y^6}{6} - \frac{y^5}{5} + \frac{y^4}{4} - \frac{y^3}{3} + \frac{y^2}{2} - y + \log |y+1| \right] + C_1 \\ &= \frac{3}{2} x^{2/3} - \frac{12}{7} x^{7/12} + 2x^{1/2} - \frac{12}{5} x^{5/12} + 3x^{1/3} \\ &\quad - 4x^{1/4} + 6x^{1/6} - 12x^{1/12} + 12 \log |x^{1/12} + 1| + C_1 \\ &\quad \dots(2) \end{aligned}$$

$$I_2 = \int \frac{\ln(1+(x)^{1/6})}{(x)^{1/3} + (x)^{1/2}} dx$$

$$\text{Let } x = z^6 \text{ so that } dx = 6z^5 dz$$

$$= \int \frac{\ln(1+z)}{z^2 + z^3} \cdot 6z^5 dz = \int \frac{6z^3 \ln(z+1)}{z+1} dz$$

$$\text{Put } z+1=t \quad \Rightarrow \quad dz = dt$$

$$\begin{aligned} \therefore I_2 &= \int \frac{6(t-1)^3 \ln t}{t} dt = 6 \int \left(t^2 - 3t + 3 - \frac{1}{t} \right) \ln t dt \\ &= 6 \left[\int (t^2 - 3t + 3) \ln t dt - \int \frac{1}{t} \ln t dt \right] \\ &= 6 \left[\left(\frac{t^3}{3} - \frac{3t^2}{2} + 3t \right) \ln t - \int \left(\frac{t^3}{3} - \frac{3t^2}{2} + 3t \right) \cdot \frac{1}{t} dt - \frac{(\ln t)^2}{2} \right] \\ &= 6 \left[\left(\frac{t^3}{3} - \frac{3t^2}{2} + 3t \right) \ln t - \int \left(\frac{t^2}{3} - \frac{3}{2}t + 3 \right) dt - \frac{(\ln t)^2}{2} \right] \\ &= 6 \left[\left(\frac{t^3}{3} - \frac{3t^2}{2} + 3t \right) \ln t - \left(\frac{t^3}{9} - \frac{3t^2}{4} + 3t \right) - \frac{(\ln t)^2}{2} \right] + C_2 \end{aligned}$$

$$= 6 \left[\left\{ \frac{(1+x^{1/6})^3}{3} - \frac{3}{2}(1+x^{1/6})^2 + 3(1+x^{1/6}) \right\} \right.$$

$$\left. \ln(1+x^{1/6}) - \left\{ \frac{(1+x^{1/6})^3}{9} - \frac{3}{4}(1+x^{1/6})^2 + 3(1+x^{1/6}) \right\} + \frac{[\ln(1+x^{1/6})^2]}{2} \right] + C_2 \quad \dots(3)$$

Thus we get the value of I on substituting the values of I₁ and I₂ from (2) and (3) in equation (1).

Q. 10. Find the indefinite integral $\int \cos 2\theta \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$ **(1994 - 5 Marks)**

Ans. $\frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \frac{1}{2} \ln \sec 2\theta + C$

Solutions.

$$\text{Let } I = \int \cos 2\theta \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$$

Now we observe that

$$\frac{d}{d\theta} \left\{ \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) \right\}$$

$$= \frac{d}{d\theta} [\ln(\cos \theta + \sin \theta) - \ln(\cos \theta - \sin \theta)]$$

$$= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} - \frac{-\cos \theta - \sin \theta}{\cos \theta - \sin \theta}$$

$$= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} + \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{2}{\cos 2\theta}$$

$\therefore$ Integrating I with respect to θ , by parts we get

$$\begin{aligned}
 I &= \frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \int \frac{\sin 2\theta}{2} \cdot \frac{2}{\cos 2\theta} d\theta \\
 &= \frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \int \tan 2\theta d\theta \\
 &= \frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \frac{1}{2} \ln \sec 2\theta + C
 \end{aligned}$$

Q. 11. Evaluate $\int \frac{(x+1)}{x(1+xe^x)^2} dx$. (1996 - 2 Marks)

Ans. $\log \left(\frac{1+xe^x}{xe^x} \right) - \frac{1}{1+xe^x} + C$

Solutions.

$$I = \int \frac{(x+1)}{x(1+xe^x)^2} dx = \int \frac{e^x(x+1)}{xe^x(1+xe^x)^2} dx$$

$$\text{Put } 1+xe^x = t \Rightarrow (xe^x + e^x) dx = dt$$

$$\therefore I = \int \frac{dt}{(t-1)t^2} = \int \left(\frac{1}{1-t} + \frac{1}{t} + \frac{1}{t^2} \right) dt$$

$$= -\log |1-t| + \log |t| - \frac{1}{t} + C$$

$$= \log \left| \frac{t}{1-t} \right| - \frac{1}{t} + C = \log \left| \frac{1+xe^x}{-xe^x} \right| - \frac{1}{1+xe^x} + C$$

$$= \log \left(\frac{1+xe^x}{xe^x} \right) - \frac{1}{1+xe^x} + C$$

Q. 12. Integrate $\int \frac{x^3+3x+2}{(x^2+1)^2(x+1)} dx$. (1999 - 5 Marks)

Ans. $-\frac{1}{2} \log |x+1| + \frac{1}{4} \log(x^2+1) + \frac{3}{2} \tan^{-1} x + \frac{x}{1+x^2} + C + \frac{(\ln(1+x^{1/6}))^2}{2} \Big] + C$

Solutions.

$$I = \int \frac{x^3 + 3x + 2}{(x^2 + 1)^2(x + 1)} dx.$$

$$\frac{x^3 + 3x + 2}{(x^2 + 1)^2(x + 1)} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2}$$

Comparing and solving, we get,

$$A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}, D = 0, E = 2.$$

$$\begin{aligned} \therefore I &= -\frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{x+1}{x^2+1} dx + 2 \int \frac{dx}{(x^2+1)^2} \\ &= -\frac{1}{2} \log|x+1| + \frac{1}{4} \log(x^2+1) + \frac{1}{2} \tan^{-1} x + 2I_1 + C \end{aligned}$$

$$\text{where } I_1 = \int \frac{dx}{(x^2+1)^2}, \text{ putting } x = \tan \theta,$$

$$\begin{aligned} I_1 &= \int \frac{\sec^2 \theta}{\sec^4 \theta} d\theta = \int (\cos^2 \theta) d\theta = \frac{1}{2} \int (1 + \cos 2\theta) d\theta \\ &= \frac{1}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right) = \frac{1}{2} \tan^{-1} x + \frac{1}{4} \cdot \frac{2x}{1+x^2} \end{aligned}$$

$$\therefore I = -\frac{1}{2} \log|x+1| + \frac{1}{4} \log(x^2+1) + \frac{3}{2} \tan^{-1} x + \frac{x}{1+x^2} + C$$

Where C is constant of integration.

Q. 13. Evaluate $\int \sin^{-1} \left(\frac{2x+2}{\sqrt{4x^2+8x+13}} \right) dx.$ **(2001 - 5 Marks)**

Ans. $(x+1) \tan^{-1} \left(\frac{2x+2}{3} \right) - \frac{3}{4} \log(4x^2+8x+13) + C$

Solutions.

$$\begin{aligned}
 I &= \int \sin^{-1} \left(\frac{2x+2}{\sqrt{4x^2+8x+13}} \right) dx \\
 &= \int \sin^{-1} \left[\frac{x+1}{\sqrt{x^2+2x+\frac{13}{4}}} \right] dx \\
 &= \int \sin^{-1} \left[\frac{x+1}{\sqrt{(x+1)^2+(3/2)^2}} \right] dx
 \end{aligned}$$

Put $x+1 = \frac{3}{2} \tan \theta$, $dx = \frac{3}{2} \sec^2 \theta d\theta$

$$\begin{aligned}
 \therefore I &= \int \sin^{-1} \left[\frac{\left(\frac{3}{2} \tan \theta\right)}{\sqrt{\frac{9}{4} \tan^2 \theta + \frac{9}{4}}} \right] \frac{3}{2} \sec^2 \theta d\theta \\
 &= \frac{3}{2} \int \sin^{-1} \left[\frac{\sin \theta \cos \theta}{\cos \theta \cdot 1} \right] \sec^2 \theta d\theta \\
 &= \frac{3}{2} \int \theta \sec^2 \theta d\theta = \frac{3}{2} \left[\theta \tan \theta - \int \tan \theta d\theta \right] \\
 &= \frac{3}{2} [\theta \tan \theta - \log |\sec \theta|] + C \\
 I &= \frac{3}{2} \left[\frac{2}{3} (x+1) \tan^{-1} \left[\frac{2}{3} (x+1) \right] - \log \sqrt{1 + \frac{4}{9} (x+1)^2} \right] + C \\
 &= (x+1) \tan^{-1} \left(\frac{2x+2}{3} \right) - \frac{3}{4} \log(9 + 4x^2 + 8x + 4) + \frac{3}{4} \log 9 + C \\
 &= (x+1) \tan^{-1} \left(\frac{2x+2}{3} \right) - \frac{3}{4} \log(4x^2 + 8x + 13) + C
 \end{aligned}$$

Q. 14. For any natural number m,

evaluate $\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{1/m} dx, x > 0.$

(2002 - 5 Marks)

Ans.
$$\frac{1}{6} \frac{(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{m+1}{m}}}{m+1} + C$$

Solutions.

$$\begin{aligned} &= \int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{1/m} dx \\ &= \int (x^{3m} + x^{2m} + x^m) \left[\frac{2x^{3m} + 3x^{2m} + 6x^m}{x^m} \right]^{1/m} dx \\ &= \int \left(\frac{x^{3m} + x^{2m} + x^m}{x} \right) (2x^{3m} + 3x^{2m} + 6x^m)^{1/m} dx \end{aligned}$$

Put $2x^{3m} + 3x^{2m} + 6x^m = y$

$$\therefore I = \frac{1}{6m} \int y^{1/m} dy = \frac{1}{6m} \left(\frac{y^{1/m+1}}{1/m+1} \right) + C$$

$$= \frac{1}{6} \left(\frac{y^{\frac{m+1}{m}}}{m+1} \right) + C$$

$$= \frac{1}{6} \frac{(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{m+1}{m}}}{m+1} + C$$