ICSE SEMESTER 2 EXAMINATION

SAMPLE PAPER - 5

MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hours

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 10 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

*Attempt **all** questions from **Section A** and **any three** questions from **Section B.***

SECTION A

*(Attempt **all** questions from this Section.)*

Section-A (Attempt all questions)

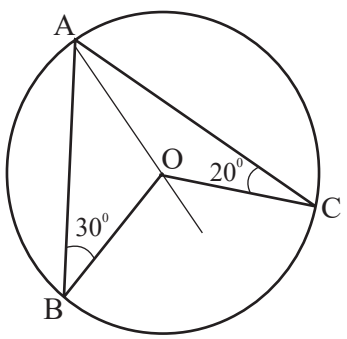
Question 1.

Choose the correct answers to the questions from the given options. (Do not copy the question, write the correct answer only.)

(i) The co-ordinates of the point which divides the line segment joining (-3, 10) and (6, -8) in the ratio 2:7 are:

(a) (1, 6) (b) (1, -6) (c) (-1, -6) (d) (-1, 6)

(ii) In the given Figure O is the centre of the circle and $\angle OBA = 30°$ and $\angle OCA = 20°$, find $\angle BOC$.



(a) 100° (b) 110° (c) 120° (d) 130°

(iii) If the radius and the height of a right circular cylinder are in the ratio 1:1, and its volume is 2744π m^3, then its curved surface area is:

(a) 1232 m^2 (b) 616 m^2 (c) 2464 m^2 (d) 2744 m^2

(iv) A point P (-2 , 3) is reflected in the line $x = 2$, the coordinates of the point of reflection P′ are:

(a) (2, 3) (b) (4, 3) (c) (6, 3) (d) (8, 3)

(v) $\sec A (1 - \sin A)(\sec A + \tan A) =$

(a) $\cos A$ (b) $\sin A$ (c) 1 (d) 0

(vi) A cumulative frequency curve is also known as:

(a) Mean (b) Median (c) Histogram (d) Ogive

(vii) The equation of the line with slope 2 and passing through (1, 2) is:

(a) $y = 2x - 3$ (b) $y = 2x$ (c) $y = 2x + 2$ (d) $y = 2x + 1$

(viii) Volume of a cone is 220 cm^3. The volume of the cylinder having same radius and height as that of the given cone is:

(a) 220 cm^3 (b) 110 cm^3 (c) 660 cm^3 (d) 330 cm^3

(ix) Find the average of the following distribution:

Variate (x)	10	30	50	70	90
Frequency (f)	8	10	10	12	10

(a) 47·5 (b) 51·7 (c) 52·4 (d) 52·8

(x) If a pair of dice is tossed, the probability of getting the sum of the numbers 10 or more, if 6 appears in the first die.

(a) 1 (b) 1/36 (c) 1/2 (d) 0

Section B (Attempt any three questions from this section.)

Question 2.

(i) The mid point M, of the line segment AB is (– 3, –2) as shown in the figure. Find the coordinates of A and B.

(ii) From 25 identical cards, numbered 1, 2, 3, 4, 5, ……, 24, 25: one card is drawn at random. Find the probability that the number on the card drawn is a multiple of:

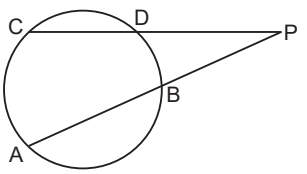
(a) 2 and 3 (b) 3 or 5

(iii) In a cyclic quadrilateral ABCD, $2\angle A = 3\angle C$ and $\angle D = 3\angle B$. Find the measure of each angle of the quadrilateral.

(iv) An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?

Question 3.

(i) In the given figure, AB = 12cm, PB = 8cm and PD =10cm then find the length of CD.



(ii) The total surface area of a right circular cone with slant height 10m is 144π m^2. Calculate

(a) Its radius (b) Its volume

(iii) Prove that $(\text{cosec } A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

(iv) Convert the following in to a less than type distribution and draw an ogive. Also find the median of the data from the graph.

Class	100-120	120-140	140-160	160-180	180-200
Frequency	12	14	8	6	10

Question 4.

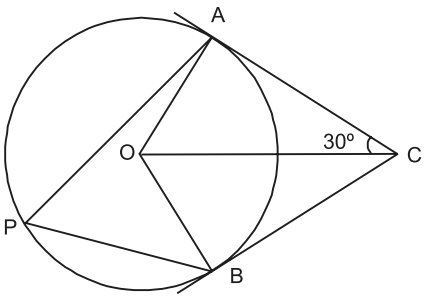
(i) Find the equation of a straight line dropped perpendicularly from a point (1, 5) on the line joining (-2, 3) and (1, -3).

(ii) Draw the histogram and use it to find the mode for the following frequency distribution.

Class	20-40	40-60	60-80	80-100	100-120
Frequency	10	12	7	3	8

(iii) In the given figure, O is the centre of the circle. Tangents at A and B meet at C. If $\angle ACO = 30°$, find :

(a) $\angle BCO$, (b) $\angle AOB$, (c) $\angle APB$.



(iv) The points P (–3, 3) is reflected about Y-axis to get the image Q

(a) Plot the points on a graph paper.

(b) Point Q is reflected about the line y =1 to get image R. Write the coordinates of R.

Question 5.

(i) A solid cone of radius 5 cm and height 8 cm is melted and made into small cylinders of radius 0.5 cm and height 2/3 cm. Find the number of cylinders formed.

(ii) Find the value of : (1 + tan A + sec A) (1 + cot A - cosec A)

(iii) Find the equation of the line passing through the point of intersection of the lines $4x - 7y - 3 = 0$ and $2x - 3y + 1 = 0$ that has equal intercepts on the axes.

(iv) Draw an ogive and for the following distribution and hence find its median.

Class	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	10	8	12	18	6	21	15

Question 6.

(i) 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

(iii) Find the ratio in which the points P(3/4, 5/12) divides the line segments joining the points A(1/2, 3/2) and B(2, – 5).

(iii) From a point on the ground the angle of elevation of the bottom and top of a water tank kept at the top of 20m high tower are 45° and 60° respectively. Find the height of the water tank.

(iv) If the mean of the following data is 14.7. Find the valu of p and q.

Class Interval	0-6	6-12	12-18	18-24	24-30	30-36	36-42	Total
Frequency	10	p	4	7	q	4	1	40

Answers

Section-A

Answer 1.

(i) (d) (-1,6)

Explanation:

$$x = \frac{mx_2 + nx_1}{m+n}, \qquad y = \frac{my_2 + ny_1}{m+n}$$

$$\Rightarrow \quad x = \frac{2\times 6 + 7\times(-3)}{2+7}, \qquad y = \frac{2\times(-8) + 7\times 10}{2+7}$$

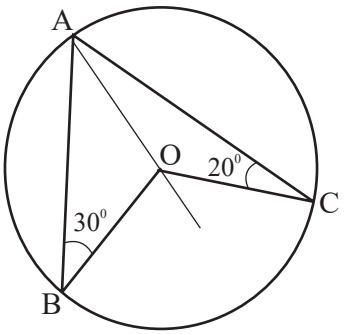
$$\Rightarrow \quad x = \frac{12-21}{9}, \qquad y = \frac{-16+70}{9}$$

$$\Rightarrow \quad x = \frac{-9}{9}, \qquad y = \frac{54}{9}$$

$$\Rightarrow \quad x = -1, \qquad y = 6$$

Hence (-1, 6) are the coordinates of the point

(ii) (a) 100°



Explanation:

OA = OB (Radii of the same circle)

Therefore, ΔOAB is an isosceles triangle.

Hence, $\angle OBA = \angle OAB = 30°$

Similarly, ΔOAC is an isosceles triangle.

Hence, $\angle OCA = \angle OAC = 20°$

Hence, $\angle BAC = \angle BAO + \angle CAO$

$\Rightarrow$ $\angle BAC = 30° + 20° = 50°$

Hence, $\angle BOC = 2\angle BAC = 2 \times 50° = 100°$

(Angle subtended by an are at the centre is twice the angle subtended at any point on the remaining part of the circle)

(iii) (a) 1232 m^2

Explanation:

As the ratio of radius and height is 1:1,

$\therefore$ $r = h$

Given, $V = \pi r^2 h = 2744\pi m^2$

$\Rightarrow$ $r^2 h = 2744$

$\Rightarrow$ $r^3 = 2744$ (as $r = h$)

$\Rightarrow$ $r = 14$ m

$\therefore$ Curved surface area $= 2\pi rh$

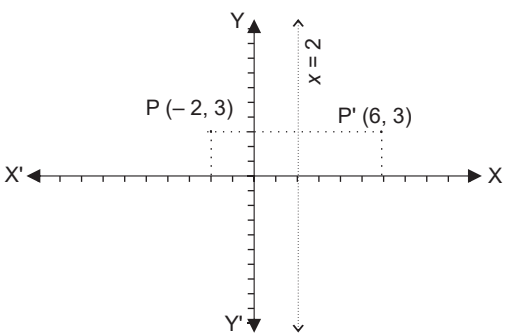
$= 2 \times \pi \times 14 \times 14$

$= 2 \times (22/7) \times 14 \times 14$

$= 1232\ m^2$

(iv) (c) (6, 3)

Explanation:



(v) (c) 1

Explanation:

L.H.S. = sec A (1– sin A) (sec A + tan A)

$$= \frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$$

$$= \frac{(1-\sin A)(1+\sin A)}{\cos^2 A}$$

$$= \frac{(1-\sin^2 A)}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A} = 1$$

(vi) (d) Ogive

Explanation:

(vii) (b) $y = 2x$

Explanation:

Let the equation be $y = mx + c$

As the slope is 2, $y = 2x + c$

as it is passing through (1,2), substituting this in the given equation

$$2 = 2 \times 1 + c$$

$\Rightarrow$ $c = 0$ Hence, the equation is $y = 2x$

(viii) (c) 660 cm^2

Explanation:

Volume of cone = $(1/3)\pi r^2 h = 220\ cm^3$

Volume of cylinder = $\pi r^2 h = 3 \times 220 = 660\ cm^3$ ($\because$ radius and height of cylinder are as same as that of cone)

(ix) (c) 52·4

Explanation :

$$\Sigma f = 8 + 10 + 10 + 12 + 10 = 50$$

$$\Sigma fx = 80 + 300 + 500 + 840 + 900$$

$$= 2620$$

$\therefore$ $$\text{Average} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{2620}{50}$$

$$= 52{\cdot}4.$$

(x) (c) 1/2

Explanation:

Total number of outcomes = 6 {(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Possible outcomes are {(6, 4), (6, 5), (6, 6)} so as the sum is 10 or more

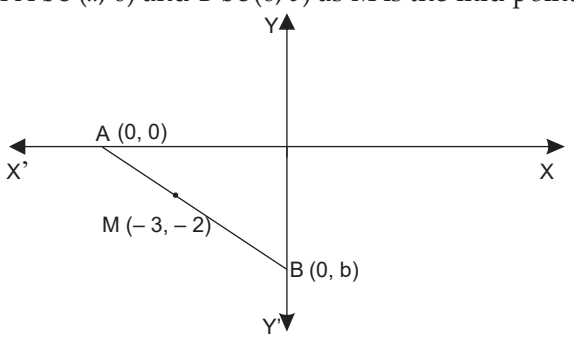
Hence, the probability of getting the sum 10 or more

$$= 3/6 = 1/2$$

Section B

Answer 2.

(i) Let the co-ordinates of A be $(a, 0)$ and B be $(0, b)$ as M is the mid point of AB, are



$$-3 = \frac{a+0}{2} \text{ and } -2 = \frac{b+0}{2}$$

$$a = -6 \text{ and } b = -4$$

Hence, coordinates of A are (-6, 0)

and B are $(0, -4)$

(ii) (a) P(multiple of 2 and 3) = P (multiple of 6)

Total number of outcomes = 25

possible outcomes are { 6, 12, 18, 24}

$\therefore$ No of possible outcomes = 4

$\therefore$ P(multiple of 2 and 3) = 4/25

Total number of outcomes = 25

possible outcomes are { 3, 6, 9, 12, 15, 18, 21, 24, 10, 20,25}

$\therefore$ No. of possible outcomes = 11

P (multiple of 3 or 5) = 11/25

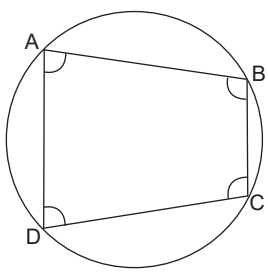
(iii) $2\angle A = 3\angle C$ (Given)

Since, $\angle A + \angle C = 180°$ (Opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow$ $2\angle A + 2\angle C = 360°$ (multiplying the equation by 2)

$\Rightarrow$ $3\angle C + 2\angle C = 360°$

$\Rightarrow$ $5\angle C = 360°$



$\therefore$ $\angle C = 72°$ and $\angle A = 180° - 72° = 108°$

$\angle D = 3\angle B$ (Given)

$\Rightarrow$ Since, $\angle B + \angle D = 180°$ (Opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow$ $\angle B + 3\angle B = 180°$

$\Rightarrow$ $4\angle B = 180°$

$\Rightarrow$ $\angle B = 45°$

$\therefore$ $\angle D = 3\angle B = 3 \times 45° = 135°$

Hence, $\angle A = 108°$, $\angle B = 45°$,

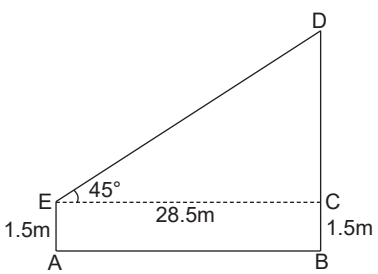
$\angle C = 72°$ and $\angle D = 135°$

(iv) $\tan E = DC/EC$

$\Rightarrow$ $\tan 45° = DC/28.5$

$\Rightarrow$ $1 = DC/28.5$

$\Rightarrow$ $DC = 28.5m$



$\therefore$ Height of Chimney = DB

$\Rightarrow$ $DB = DC + CB$

$\Rightarrow$ $DB = 28.5 + 1.5$

$\Rightarrow$ $DB = 30m$

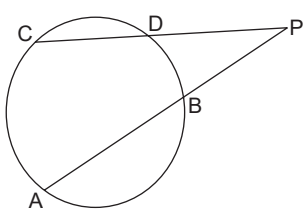
Answer 3.

(i) Given, $AB = 12$ cm, $PB = 8$ cm

Hence, $PA = AB + PB = 12 + 8$

$= 20$ cm

$PD = 10$ cm



From the figure, $PB \times PA = PD \times PC$

$\Rightarrow$ $8 \times 20 = 10 \times PC$

$\Rightarrow$ $PC = 16$

$\Rightarrow$ $CD = PC - PD$

$\Rightarrow$ $CD = 16 - 10$

$\Rightarrow$ $CD = 6$ cm

(ii) Total surface area of a cone = $\pi r^2 + \pi r l = 144\ \pi$

$\Rightarrow \quad \pi r(r + l) = 144\ \pi$

$\Rightarrow \quad r(r + 10) = 144$

$\Rightarrow \quad r^2 + 10r - 144 = 0$

$\Rightarrow \quad r^2 + 18r - 8r - 144 = 0$

$r(r + 18) - 8(r + 18) = 0$

$(r - 8)(r + 18) = 0$

$\therefore \quad r = 8$ or $r = -18$ ($r = -18$ is not possible as it is negative)

Hence, $r = 8$ m

We know that, $h^2 = l^2 - r^2$

$\therefore \quad h^2 = 10^2 - 8^2$

$= 100 - 64$

$= 36$

$\Rightarrow \quad h = 6$ m

$\therefore$ Volume $= (1/3)\ \pi r^2 h = (1/3) \times \pi \times 8 \times 8 \times 6 = 128\ \pi\ m^3$

(iii) L.H.S. = (cosec A - sin A) (sec A - cos A) (tan A+cot A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \frac{1 - \sin^2 A}{\sin A} \times \frac{1 - \cos^2 A}{\cos A} \times \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

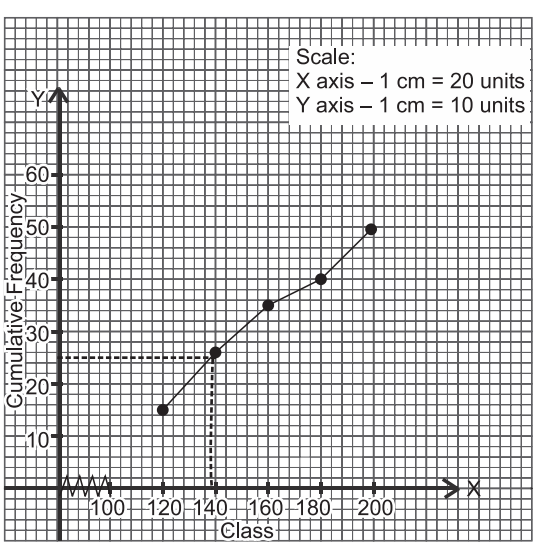
$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} \times \frac{1}{\sin A \cos A}$$

$$= \frac{\sin^2 A \cos^2 A}{\sin^2 A \cos^2 A} = 1$$

= R.H.S.

(iv)

Class	Frequency	Cumulative Frequency
100-120	12	12
120-140	14	26
140-160	8	34
160-180	6	40
180-200	10	50
Total = 50		



Median $= \frac{N}{2}$ th term

$= \frac{50}{2}$ th term

= 25th term

= 138.5 (From the graph)

Answer 4.

(i) (a) Slope of the line joining the points (– 2, 3) and (1, – 3) is

$$\frac{y_1 - y_2}{x_2 - x_1} = \frac{(-3)-3}{1-(-2)} = \frac{-6}{3} = -2$$

Hence, the slope of the line perpendicular to this line is 1/2. $(\because m_1 m_2 = -1)$

Let the equation of the line be $y = (1/2)\,x + c$

as it is passing through (1, 5),

$\therefore \quad 5 = (1/2)1 + c$

$\Rightarrow \quad 5 - (1/2) = c$

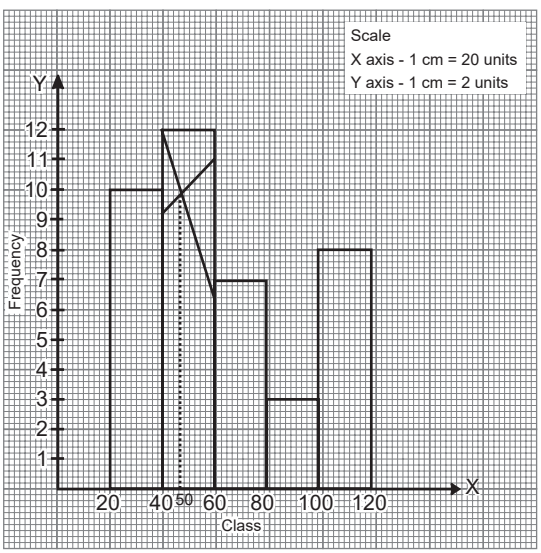
$\Rightarrow \quad c = 9/2$

Hence, equation is $y = (1/2)x + (9/2)$

i.e $\quad 2y = x + 9$

$\Rightarrow \quad x - 2y + 9 = 0$

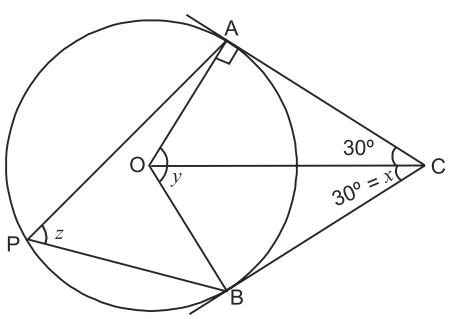
(ii)



From the histogram, Mode = 45

(iii) Given, ∠ACO = 30°

(a) $\angle BCO = \angle ACO = 30°$ (Since, AC and BC are tangents)



(b) $\angle OAC = 90° = \angle OBC$ [$\because$ Tangent $\perp$ Radius]

In ΔAOC,

$\angle AOC + \angle OAC + \angle ACO = 180°$ [Angles of ΔAOC]

$\Rightarrow \quad \angle AOC + 90° + 30° = 180°$

$\Rightarrow \quad \angle AOC = 180° - 120°$

$= 60°$

$\therefore$ $\angle BOC = 60°$ [$\because \angle BOC = \angle AOC$]

$\therefore$ $\angle AOB = \angle AOC + \angle BOC$

$= 60° + 60°$

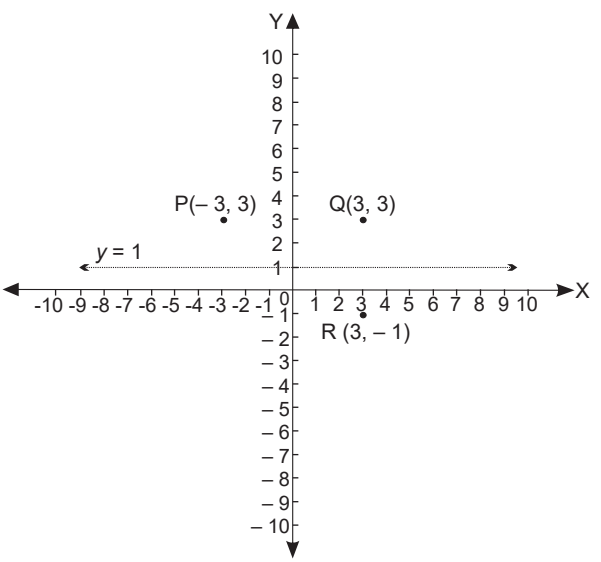
$= 120°$

(c) $\angle APB = \frac{1}{2}\angle AOB$ [Angle subtended at remaining part of the circle is half the angle subtended at the centre]

$= \frac{1}{2} \times 120°$

$= 60°$

(iv)



(a) Points P(– 3, 3) and Q(3, 3) are plotted on the graph

(b) Coordinates of R are (3, -1)

Answer 5.

(i) For cone,

Radius (r_1) = 5 cm, height (h_1) = 8 cm

$\therefore$ Volume of cone (V_1) = $\frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\pi(5)^2 \times 8$

For Cylinder, Radius (r_2) = 0.5 cm

Height (h_2) = $\frac{2}{3}$ cm.

$\therefore$ Volume of cylinder (V_2) = $\pi r_2^2 h_2$

$= \pi.(0.5)^2.\ 2/3$

$= \frac{\pi(0.5)^2 2}{3}$

$\because$ Number of cylinders = $\frac{\text{Volume of cone}}{\text{Volume of cylinder}}$

$\therefore$ Number of cylinders $= \dfrac{V_1}{V_2}$

$$= \frac{\frac{1}{3}\pi(5)^2 \times 8}{\pi(0.5)^2 \frac{2}{3}}$$

$$= \frac{25 \times 8 \times 3}{3 \times 0.5 \times 0.5 \times 2}$$

$$= \frac{25 \times 8 \times 100}{2 \times 5 \times 5}$$

$$= 400$$

(ii) $(1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A)$

$$= \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)\left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)$$

$$= \left(\frac{\cos A + \sin A + 1}{\cos A}\right)\left(\frac{\cos A + \sin A - 1}{\sin A}\right)$$

$$= \left[\frac{(\cos A + \sin A)^2 - 1}{\cos A \sin A}\right]$$

$$= \frac{\cos^2 A + \sin^2 A + 2\sin A \cos A - 1}{\cos A \sin A}$$

$$= \frac{1 + 2\sin A \cos A - 1}{\cos A \sin A}$$

$$= \frac{2\sin A \cos A}{\cos A \sin A}$$

$$= 2$$

(iii) Equations of the given lines are

$$4x - 7y - 3 = 0 \quad \ldots 1$$

$$2x - 3y + 1 = 0 \quad \ldots 2$$

Multiplying equation by 2

$$4x - 6y = -2 \quad \ldots 3$$

$$4x - 7y = 3 \quad \ldots 1$$

$y = -5$ (by subtracting equation 1 from equation 3)

substituting $y = -5$ in equation (1)

$$4x - 7(-5) = 3$$

$\Rightarrow \quad 4x = -32$

$\Rightarrow \quad x = -8$

Hence, the line is passing through $(-8, -5)$

Let the line meets X axis at $(a, 0)$ and Y axis at $(0, a)$

Let the equation of the line be $(x/a) + (y/a) = 1$

i.e., $x + y = a$

$\Rightarrow \quad -8 - 5 = a$

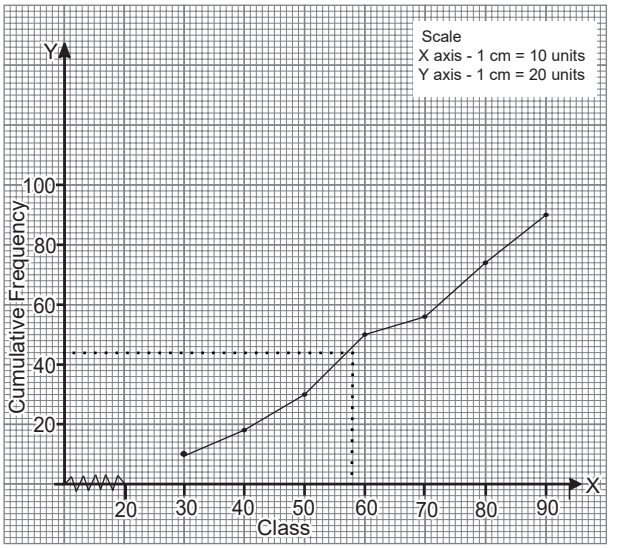
$\Rightarrow \quad a = -13$

Hence, the equation of the line is

$$x + y + 13 = 0$$

(iv)

Classes	Frequency	Cumulative Frequency:
20-30	10	10
30-40	8	18
40-50	12	30
50-60	18	48
60-70	6	54
70-80	21	75
80-90	15	90
	90	



$$\text{Median} = \frac{N}{2}\text{th term} = \frac{90}{2}\text{th term}$$

$$= 45\text{th term}$$

$$= 58 \text{ (from graph)}$$

Answer 6.

(i) $$\text{Total number of pens} = 12 + 132 = 144$$

$$\text{No. of good pens} = 132$$

$$\text{P(getting a good pen)} = \frac{\text{No. of good pens}}{\text{Total number of pens}}$$

$$= \frac{132}{144}$$

$$= \frac{11}{12}$$

(ii) Let the ratio be k:1.

By section formula, $$x = \frac{mx_2 + nx_1}{m+n},\ y = \frac{my_2 + ny_1}{mn}$$

$$\therefore \quad \frac{3}{4} = \frac{k(2)+1(1/2)}{k+1}$$

$$\Rightarrow \quad 3(k+1) = 8k + 2$$

$$\Rightarrow \quad 3k + 3 = 8k + 2$$

$$\Rightarrow \quad 5k = 1$$

$$\Rightarrow \quad k = 1/5$$

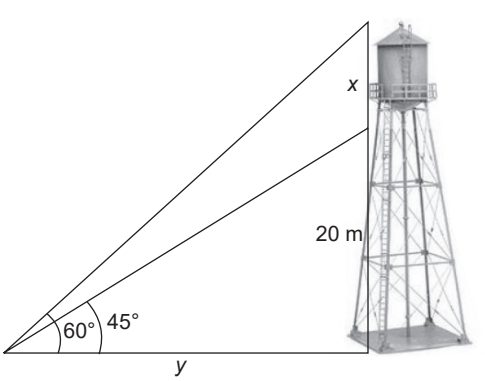
Hence the ratio is 1:5.

(iii) Let the height of the water tank be x.

$$\tan 45° = \frac{20}{y}$$

$$\Rightarrow \quad 1 = \frac{20}{y}$$

$$\Rightarrow \quad y = 20 \text{ m}$$



and $\tan 60° = \frac{20+x}{y}$

$\Rightarrow \sqrt{3} = \frac{20+x}{y}$

$\Rightarrow \sqrt{3}y = 20 + x$

$\Rightarrow 20\sqrt{3} = 20 + x$

$\Rightarrow x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$

$= 20 \times (1.732\text{-}1)$

$= 0.732 \text{ X } 20$

$\Rightarrow x = 14.64$ m

(iv)

Class Interval	Mid Value (x_i)	Frequency (f_i)	($f_i x_i$)
0 - 6	3	10	30
6 -12	9	p	9p
12 - 18	15	4	60
18 - 24	21	7	147
24 - 30	27	q	27q
30 - 36	33	4	132
36 - 42	39	1	39
		Total: $40 = 26 + p + q$	$408 + 9p + 27q$

$\because \Sigma f_i = 40 = 26 + p + q$

$\Rightarrow p + q = 14$ ...1

Given Mean = 14.7

$\Rightarrow \frac{\Sigma f_i x_i}{\Sigma f_i} = 14.7$

$\Rightarrow \frac{408 + 9p + 27q}{40} = 14.7$

$\Rightarrow 408 + 9p + 27q = 40 \times 14.7$

$\Rightarrow 408 + 9p + 27q = 588$

$\Rightarrow 9p + 27q = 588 - 408$

$\Rightarrow 9p + 27q = 180$

$\Rightarrow p + 3p = 20$ (on dividing the equation by 9)

$\Rightarrow 140 - q + 3q = 20$ (from equation 1)

$\Rightarrow 14 + 2q = 20$

$\Rightarrow 2q = 6$

$\Rightarrow q = 3$

$\because p + q = 14$

Hence, $p = 14 - 3 = 11$

$\therefore \quad p = 11$ and $q = 3$

❑❑