13.6 Volume of a Cuboid

You have already learnt about volumes of certain figures (objects) in earlier classes. Recall that solid objects occupy space. The measure of this occupied space is called the **Volume** of the object.

Note : If an object is solid, then the space occupied by such an object is measured, and is termed the **Volume** of the object. On the other hand, if the object is hollow, then interior is empty, and can be filled with air, or some liquid that will take the shape of its container. In this case, the volume of the substance that can fill the interior is called the **capacity of the container**. In short, the volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. Hence, the unit of measurement of either of the two is cubic unit.

So, if we were to talk of the volume of a cuboid, we would be considering the measure of the space occupied by the cuboid.

Further, the area or the volume is measured as the magnitude of a region. So, correctly speaking, we should be finding the area of a circular region, or volume of a cuboidal region, or volume of a spherical region, etc. But for the sake of simplicity, we say, find the area of a circle, volume of a cuboid or a sphere even though these mean only their boundaries.
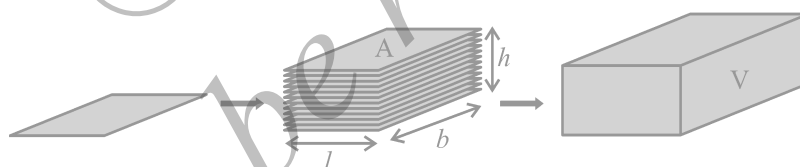

Fig. 13.23

Observe Fig. 13.23. Suppose we say that the area of each rectangle is A, the height up to which the rectangles are stacked is *h* and the volume of the cuboid is V. Can you tell what would be the relationship between V, A and *h*?

The area of the plane region occupied by each rectangle × height

= Measure of the space occupied by the cuboid

So, we get   A × *h* = V

That is,   **Volume of a Cuboid = base area × height = length × breadth × height**

or *l* × *b* × *h*, where *l*, *b* and *h* are respectively the length, breadth and height of the cuboid.

Note : When we measure the magnitude of the region of a space, that is, the space occupied by a solid, we do so by counting the number of cubes of edge of unit length that can fit into it exactly. Therefore, the unit of measurement of volume is cubic unit.

Again | **Volume of a Cube = edge × edge × edge = a^3**

where a is the edge of the cube (see Fig. 13.24).

So, if a cube has edge of 12 cm,

then volume of the cube = 12 × 12 × 12 cm³

= 1728 cm³.

Recall that you have learnt these formulae in earlier classes. Now let us take some examples to illustrate the use of these formulae:
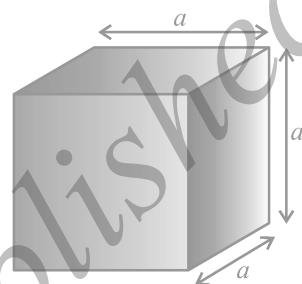
Fig. 13.24

Example 11 : A wall of length 10 m was to be built across an open ground. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm × 12 cm × 8 cm, how many bricks would be required?

Solution : Since the wall with all its bricks makes up the space occupied by it, we need to find the volume of the wall, which is nothing but a cuboid.

Here,   Length = 10 m = 1000 cm

Thickness = 24 cm

Height = 4 m = 400 cm

Therefore,   Volume of the wall = length × thickness × height

= 1000 × 24 × 400 cm³

Now, each brick is a cuboid with length = 24 cm, breadth = 12 cm and height = 8 cm

So, volume of each brick = length × breadth × height

= 24 × 12 × 8 cm³

So, number of bricks required = $\dfrac{\text{volume of the wall}}{\text{volume of each brick}}$

= $\dfrac{1000 \times 24 \times 400}{24 \times 12 \times 8}$

= 4166.6

So, the wall requires 4167 bricks.

Example 12 : A child playing with building blocks, which are of the shape of cubes, has built a structure as shown in Fig. 13.25. If the edge of each cube is 3 cm, find the volume of the structure built by the child.

Fig. 13.25

Solution : Volume of each cube = edge × edge × edge

$$= 3 \times 3 \times 3 \text{ cm}^3 = 27 \text{ cm}^3$$

Number of cubes in the structure = 15

Therefore, volume of the structure = 27 × 15 cm³

$$= 405 \text{ cm}^3$$

EXERCISE 13.5
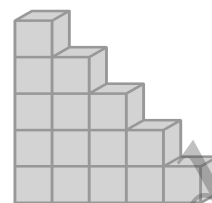
1. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?

2. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? (1 m³ = 1000 *l*)

3. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

4. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of ₹ 30 per m³.

5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

7. A godown measures 40 m × 25 m × 15 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

9. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

13.7 Volume of a Cylinder

Just as a cuboid is built up with rectangles of the same size, we have seen that a right circular cylinder can be built up using circles of the same size. So, using the same argument as for a cuboid, we can see that the volume of a cylinder can be obtained

as : base area × height

= area of circular base × height = $\pi r^2 h$

So, | **Volume of a Cylinder = $\pi r^2 h$** |

where r is the base radius and h is the height of the cylinder.

Example 13 : The pillars of a temple are cylindrically shaped (see Fig. 13.26). If each pillar has a circular base of radius 20 cm and height 10 m, how much concrete mixture would be required to build 14 such pillars?

Solution : Since the concrete mixture that is to be used to build up the pillars is going to occupy the entire space of the pillar, what we need to find here is the volume of the cylinders.

Fig. 13.26

Radius of base of a cylinder = 20 cm

Height of the cylindrical pillar = 10 m = 1000 cm

So,   volume of each cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 20 \times 20 \times 1000 \text{ cm}^3$$

$$= \frac{8800000}{7} \text{ cm}^3$$

$$= \frac{8.8}{7} \text{ m}^3 \text{ (Since } 1000000 \text{ cm}^3 = 1\text{m}^3\text{)}$$

Therefore, volume of 14 pillars = volume of each cylinder × 14

$$= \frac{8.8}{7} \times 14 \text{ m}^3$$
$$= 17.6 \text{ m}^3$$

So, 14 pillars would need 17.6 m³ of concrete mixture.

Example 14 : At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses (see Fig. 13.27) of radius 3 cm up to a height of 8 cm, and sold for ₹ 3 each. How much money does the stall keeper receive by selling the juice completely?
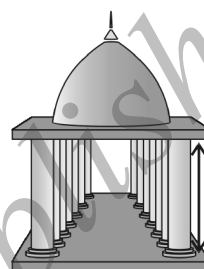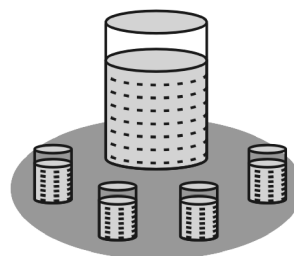
Fig. 13.27

Solution : The volume of juice in the vessel

= volume of the cylindrical vessel

= $\pi R^2 H$

(where R and H are taken as the radius and height respectively of the vessel)

= $\pi \times 15 \times 15 \times 32$ cm³

Similarly, the volume of juice each glass can hold = $\pi r^2 h$

(where r and h are taken as the radius and height respectively of each glass)

= $\pi \times 3 \times 3 \times 8$ cm³

So, number of glasses of juice that are sold

$$= \frac{\text{volume of the vessel}}{\text{volume of each glass}}$$

$$= \frac{\pi \times 15 \times 15 \times 32}{\pi \times 3 \times 3 \times 8}$$

= 100

Therefore, amount received by the stall keeper = ₹ 3 × 100

= ₹ 300

EXERCISE 13.6

Assume $\pi = \dfrac{22}{7}$, unless stated otherwise.

1. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? (1000 cm³ = 1l)

2. The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm³ of wood has a mass of 0.6 g.

3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

4. If the lateral surface of a cylinder is 94.2 cm² and its height is 5 cm, then find (i) radius of its base   (ii) its volume. (Use $\pi = 3.14$)

5. It costs ₹ 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of ₹ 20 per m², find

   (i) inner curved surface area of the vessel,

   (ii) radius of the base,

   (iii) capacity of the vessel.

6. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

13.8 Volume of a Right Circular Cone

In Fig 13.28, can you see that there is a right circular cylinder and a right circular cone of the same base radius and the same height?
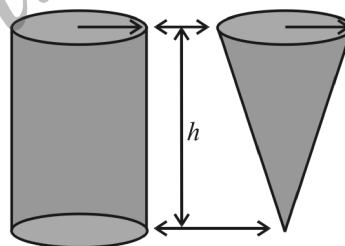

Fig. 13.28

Activity : Try to make a hollow cylinder and a hollow cone like this with the same base radius and the same height (see Fig. 13.28). Then, we can try out an experiment that will help us, to see practically what the volume of a right circular cone would be!
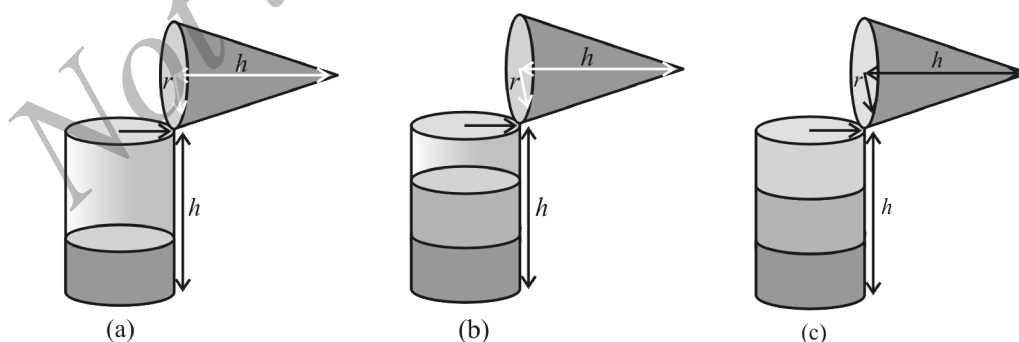

(a)   (b)   (c)

Fig. 13.29

So, let us start like this.

Fill the cone up to the brim with sand once, and empty it into the cylinder. We find that it fills up only a part of the cylinder [see Fig. 13.29(a)].

When we fill up the cone again to the brim, and empty it into the cylinder, we see that the cylinder is still not full [see Fig. 13.29(b)].

When the cone is filled up for the third time, and emptied into the cylinder, it can be seen that the cylinder is also full to the brim [see Fig. 13.29(c)].

With this, we can safely come to the conclusion that three times the volume of a cone, makes up the volume of a cylinder, which has the same base radius and the same height as the cone, which means that the volume of the cone is one-third the volume of the cylinder.

So,

$$\boxed{\textbf{Volume of a Cone} = \frac{1}{3}\pi r^2 h}$$

where r is the base radius and h is the height of the cone.

Example 15 : The height and the slant height of a cone are 21 cm and 28 cm respectively. Find the volume of the cone.

Solution : From $l^2 = r^2 + h^2$, we have

$$r = \sqrt{l^2 - h^2} = \sqrt{28^2 - 21^2} \text{ cm} = 7\sqrt{7} \text{ cm}$$

So, volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7\sqrt{7} \times 7\sqrt{7} \times 21$ cm³

$$= 7546 \text{ cm}^3$$

Example 16 : Monica has a piece of canvas whose area is 551 m². She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 m², find the volume of the tent that can be made with it.

Solution : Since the area of the canvas = 551 m² and area of the canvas lost in wastage is 1 m², therefore the area of canvas available for making the tent is (551 – 1) m² = 550 m².

Now, the surface area of the tent = 550 m² and the required base radius of the conical tent = 7 m

Note that a tent has only a curved surface (the floor of a tent is not covered by canvas!!).

Therefore, curved surface area of tent = 550 m^2.

That is, $\pi r l = 550$

or, $\dfrac{22}{7} \times 7 \times l = 550$

or, $l = 3\dfrac{550}{22}$ m = 25 m

Now, $l^2 = r^2 + h^2$

Therefore, $h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2}$ m $= \sqrt{625 - 49}$ m $= \sqrt{576}$ m

$= 24$ m

So, the volume of the conical tent $= \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 24$ m$^3 = 1232$ m^3.

EXERCISE 13.7

Assume $\pi = \dfrac{22}{7}$, unless stated otherwise.

1. Find the volume of the right circular cone with
   (i) radius 6 cm, height 7 cm   (ii) radius 3.5 cm, height 12 cm

2. Find the capacity in litres of a conical vessel with
   (i) radius 7 cm, slant height 25 cm   (ii) height 12 cm, slant height 13 cm

3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base.
   (Use $\pi = 3.14$)

4. If the volume of a right circular cone of height 9 cm is 48 π cm^3, find the diameter of its base.

5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find
   (i) height of the cone   (ii) slant height of the cone
   (iii) curved surface area of the cone

7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

13.9 Volume of a Sphere

Now, let us see how to go about measuring the volume of a sphere. First, take two or three spheres of different radii, and a container big enough to be able to put each of the spheres into it, one at a time. Also, take a large trough in which you can place the container. Then, fill the container up to the brim with water [see Fig. 13.30(a)].

Now, carefully place one of the spheres in the container. Some of the water from the container will over flow into the trough in which it is kept [see Fig. 13.30(b)]. Carefully pour out the water from the trough into a measuring cylinder (i.e., a graduated cylindrical jar) and measure the water over flowed [see Fig. 13.30(c)]. Suppose the radius of the immersed sphere is r (you can find the radius by measuring the diameter of the sphere). Then evaluate $\frac{4}{3}\pi r^3$. Do you find this value almost equal to the measure of the volume over flowed?

(a)   (b)   (c)
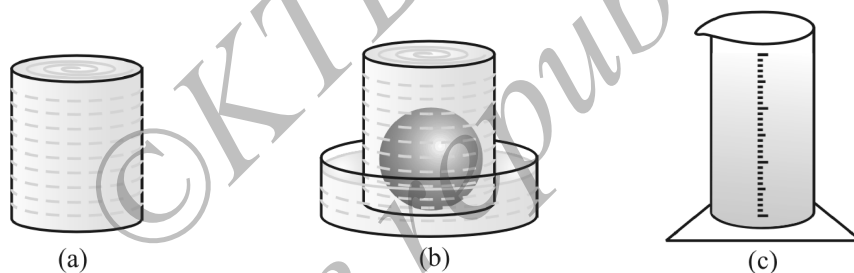
Fig. 13.30

Once again repeat the procedure done just now, with a different size of sphere. Find the radius R of this sphere and then calculate the value of $\frac{4}{3}\pi R^3$. Once again this value is nearly equal to the measure of the volume of the water displaced (over flowed) by the sphere. What does this tell us? We know that the volume of the sphere is the same as the measure of the volume of the water displaced by it. By doing this experiment repeatedly with spheres of varying radii, we are getting the same result, namely, the volume of a sphere is equal to $\frac{4}{3}\pi$ times the cube of its radius. This gives us the idea that

$$\boxed{\textbf{Volume of a Sphere} = \frac{4}{3}\pi r^3}$$

where r is the radius of the sphere.

Later, in higher classes it can be proved also. But at this stage, we will just take it as true.

Since a hemisphere is half of a sphere, can you guess what the volume of a hemisphere will be? Yes, it is $\dfrac{1}{2}$ of $\dfrac{4}{3}\pi r^3 = \dfrac{2}{3}\pi r^3$.

So,
$$\boxed{\textbf{Volume of a Hemisphere} = \dfrac{\mathbf{2}}{\mathbf{3}}\boldsymbol{\pi} \, \boldsymbol{r}^{\mathbf{3}}}$$

where r is the radius of the hemisphere.

Let us take some examples to illustrate the use of these formulae.

Example 17 : Find the volume of a sphere of radius 11.2 cm.

Solution : Required volume = $\dfrac{4}{3}\pi r^3$

$$= \dfrac{4}{3}\times\dfrac{22}{7}\times 11.2 \times 11.2 \times 11.2 \ \text{cm}^3 = 5887.32 \ \text{cm}^3$$

Example 18 : A shot-putt is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm³, find the mass of the shot-putt.

Solution : Since the shot-putt is a solid sphere made of metal and its mass is equal to the product of its volume and density, we need to find the volume of the sphere.

Now, volume of the sphere = $\dfrac{4}{3}\pi r^3$

$$= \dfrac{4}{3}\times\dfrac{22}{7}\times 4.9 \times 4.9 \times 4.9 \ \text{cm}^3$$

$$= 493 \ \text{cm}^3 \ (\text{nearly})$$

Further, mass of 1 cm³ of metal is 7.8 g.

Therefore, mass of the shot-putt = 7.8 × 493 g

$$= 3845.44 \ \text{g} = 3.85 \ \text{kg} \ (\text{nearly})$$

Example 19 : A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

Solution : The volume of water the bowl can contain

$$= \dfrac{2}{3}\pi r^3$$

$$= \dfrac{2}{3}\times\dfrac{22}{7}\times 3.5 \times 3.5 \times 3.5 \ \text{cm}^3 = 89.8 \ \text{cm}^3$$

EXERCISE 13.8

Assume $\pi = \dfrac{22}{7}$, unless stated otherwise.

1. Find the volume of a sphere whose radius is
    (i) 7 cm   (ii) 0.63 m

2. Find the amount of water displaced by a solid spherical ball of diameter
    (i) 28 cm   (ii) 0.21 m

3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3?

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

7. Find the volume of a sphere whose surface area is 154 cm^2.

8. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹ 498.96. If the cost of white-washing is ₹ 2.00 per square metre, find the
    (i)   inside surface area of the dome,   (ii) volume of the air inside the dome.

9. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the
    (i)   radius r' of the new sphere,   (ii) ratio of S and S′.

10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

EXERCISE 13.9 (Optional)*

1. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see Fig. 13.31). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.
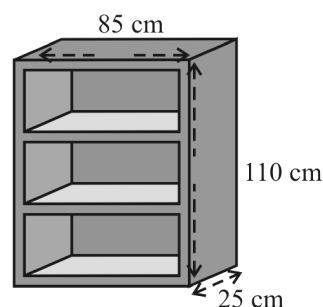
Fig. 13.31

*These exercises are not from examination point of view.

2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in Fig 13.32. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.
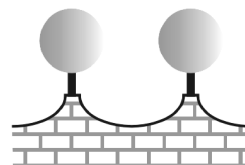
Fig. 13.32

3. The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

13.10 Summary

In this chapter, you have studied the following points:

1. **Surface area of a cuboid = 2 (lb + bh + hl)**

2. **Surface area of a cube = 6a^2**

3. **Curved surface area of a cylinder = 2πrh**

4. **Total surface area of a cylinder = 2πr(r + h)**

5. **Curved surface area of a cone = πrl**

6. **Total surface area of a right circular cone = πrl + πr^2, i.e., πr(l + r)**

7. **Surface area of a sphere of radius r = 4πr^2**

8. **Curved surface area of a hemisphere = 2πr^2**

9. **Total surface area of a hemisphere = 3πr^2**

10. **Volume of a cuboid = $l \times b \times h$**

11. **Volume of a cube = a^3**

12. **Volume of a cylinder = $\pi r^2 h$**

13. **Volume of a cone = $\dfrac{1}{3}\pi r^2 h$**

14. **Volume of a sphere of radius r = $\dfrac{4}{3}\pi r^3$**

15. **Volume of a hemisphere = $\dfrac{2}{3}\pi r^3$**

[Here, letters l, b, h, a, r, etc. have been used in their usual meaning, depending on the context.]

STATISTICS

14.1 Introduction

Everyday we come across a lot of information in the form of facts, numerical figures, tables, graphs, etc. These are provided by newspapers, televisions, magazines and other means of communication. These may relate to cricket batting or bowling averages, profits of a company, temperatures of cities, expenditures in various sectors of a five year plan, polling results, and so on. These facts or figures, which are numerical or otherwise, collected with a definite purpose are called *data*. Data is the plural form of the Latin word *datum*. Of course, the word 'data' is not new for you. You have studied about data and data handling in earlier classes.

Our world is becoming more and more information oriented. Every part of our lives utilises data in one form or the other. So, it becomes essential for us to know how to extract meaningful information from such data. This extraction of meaningful information is studied in a branch of mathematics called *Statistics*.

The word 'statistics' appears to have been derived from the Latin word 'status' meaning 'a (political) state'. In its origin, statistics was simply the collection of data on different aspects of the life of people, useful to the State. Over the period of time, however, its scope broadened and statistics began to concern itself not only with the collection and presentation of data but also with the interpretation and drawing of inferences from the data. Statistics deals with collection, organisation, analysis and interpretation of data. The word 'statistics' has different meanings in different contexts. Let us observe the following sentences:

1. May I have the latest copy of 'Educational Statistics of India'.

2. I like to study 'Statistics' because it is used in day-to-day life.

In the first sentence, statistics is used in a plural sense, meaning numerical data. These may include a number of educational institutions of India, literacy rates of various

states, etc. In the second sentence, the word 'statistics' is used as a singular noun, meaning the subject which deals with the collection, presentation, analysis of data as well as drawing of meaningful conclusions from the data.

In this chapter, we shall briefly discuss all these aspects regarding data.

14.2 Collection of Data

Let us begin with an exercise on gathering data by performing the following activity.

Activity 1 : Divide the students of your class into four groups. Allot each group the work of collecting one of the following kinds of data:

(i)   Heights of 20 students of your class.

(ii) Number of absentees in each day in your class for a month.

(iii) Number of members in the families of your classmates.

(iv) Heights of 15 plants in or around your school.

Let us move to the results students have gathered. How did they collect their data in each group?

(i)   Did they collect the information from each and every student, house or person concerned for obtaining the information?

(ii) Did they get the information from some source like available school records?

In the first case, when the information was collected by the investigator herself or himself with a definite objective in her or his mind, the data obtained is called *primary data*.

In the second case, when the information was gathered from a source which already had the information stored, the data obtained is called *secondary data*. Such data, which has been collected by someone else in another context, needs to be used with great care ensuring that the source is reliable.

By now, you must have understood how to collect data and distinguish between primary and secondary data.

EXERCISE 14.1

1. Give five examples of data that you can collect from your day-to-day life.

2. Classify the data in Q.1 above as primary or secondary data.

14.3 Presentation of Data

As soon as the work related to collection of data is over, the investigator has to find out ways to present them in a form which is meaningful, easily understood and gives its main features at a glance. Let us now recall the various ways of presenting the data through some examples.

Example 1 : Consider the marks obtained by 10 students in a mathematics test as given below:

| 55 | 36 | 95 | 73 | 60 | 42 | 25 | 78 | 75 | 62 |

The data in this form is called *raw data*.

By looking at it in this form, can you find the highest and the lowest marks?

Did it take you some time to search for the maximum and minimum scores? Wouldn't it be less time consuming if these scores were arranged in ascending or descending order? So let us arrange the marks in ascending order as

| 25 | 36 | 42 | 55 | 60 | 62 | 73 | 75 | 78 | 95 |

Now, we can clearly see that the lowest marks are 25 and the highest marks are 95.

The difference of the highest and the lowest values in the data is called the *range* of the data. So, the range in this case is 95 – 25 = 70.

Presentation of data in ascending or descending order can be quite time consuming, particularly when the number of observations in an experiment is large, as in the case of the next example.

Example 2 : Consider the marks obtained (out of 100 marks) by 30 students of Class IX of a school:

10	20	36	92	95	40	50	56	60	70
92	88	80	70	72	70	36	40	36	40
92	40	50	50	56	60	70	60	60	88

Recall that the number of students who have obtained a certain number of marks is called the *frequency* of those marks. For instance, 4 students got 70 marks. So the frequency of 70 marks is 4. To make the data more easily understandable, we write it

in a table, as given below:

Table 14.1

Marks	Number of students (i.e., the frequency)
10	1
20	1
36	3
40	4
50	3
56	2
60	4
70	4
72	1
80	1
88	2
92	3
95	1
Total	30

Table 14.1 is called *an ungrouped frequency distribution table,* or simply a *frequency distribution table.* Note that you can use also *tally marks* in preparing these tables, as in the next example.

Example 3 : 100 plants each were planted in 100 schools during Van Mahotsava. After one month, the number of plants that survived were recorded as :

95	67	28	32	65	65	69	33	98	96
76	42	32	38	42	40	40	69	95	92
75	83	76	83	85	62	37	65	63	42
89	65	73	81	49	52	64	76	83	92
93	68	52	79	81	83	59	82	75	82
86	90	44	62	31	36	38	42	39	83
87	56	58	23	35	76	83	85	30	68
69	83	86	43	45	39	83	75	66	83
92	75	89	66	91	27	88	89	93	42
53	69	90	55	66	49	52	83	34	36

To present such a large amount of data so that a reader can make sense of it easily, we condense it into groups like 20-29, 30-39, . . ., 90-99 (since our data is from 23 to 98). These groupings are called 'classes' or 'class-intervals', and their size is called the *class-size* or *class width*, which is 10 in this case. In each of these classes, the least number is called the *lower class limit* and the greatest number is called the *upper class limit,* e.g., in 20-29, 20 is the 'lower class limit' and 29 is the 'upper class limit'.

Also, recall that using tally marks, the data above can be condensed in tabular form as follows:

Table 14.2

Number of plants survived	Tally Marks	Number of schools (frequency)
20 - 29	|||	3
30 - 39	ℕ ℕ |||	14
40 - 49	ℕ ℕ ||	12
50 - 59	ℕ |||	8
60 - 69	ℕ ℕ ℕ |||	18
70 - 79	ℕ ℕ	10
80 - 89	ℕ ℕ ℕ ℕ |||	23
90 - 99	ℕ ℕ ||	12
Total		100

Presenting data in this form simplifies and condenses data and enables us to observe certain important features at a glance. This is called *a grouped frequency distribution table*. Here we can easily observe that 50% or more plants survived in 8 + 18 + 10 + 23 + 12 = 71 schools.

We observe that the classes in the table above are non-overlapping. Note that we could have made more classes of shorter size, or fewer classes of larger size also. For instance, the intervals could have been 22-26, 27-31, and so on. So, there is no hard and fast rule about this except that the classes should not overlap.

Example 4 : Let us now consider the following frequency distribution table which gives the weights of 38 students of a class:

Table 14.3

Weights (in kg)	Number of students
31 - 35	9
36 - 40	5
41 - 45	14
46 - 50	3
51 - 55	1
56 - 60	2
61 - 65	2
66 - 70	1
71 - 75	1
Total	38

Now, if two new students of weights 35.5 kg and 40.5 kg are admitted in this class, then in which interval will we include them? We cannot add them in the ones ending with 35 or 40, nor to the following ones. This is because there are gaps in between the upper and lower limits of two consecutive classes. So, we need to divide the intervals so that the upper and lower limits of consecutive intervals are the same. For this, we find the difference between the upper limit of a class and the lower limit of its succeeding class. We then add half of this difference to each of the upper limits and subtract the same from each of the lower limits.

For example,   consider the classes 31 - 35 and 36 - 40.

The lower limit of 36 - 40 = 36

The upper limit of 31 - 35 = 35

The difference = 36 – 35 = 1

So,   half the difference = $\frac{1}{2}$ = 0.5

So the new class interval formed from 31 - 35 is (31 – 0.5) - (35 + 0.5), i.e., 30.5 - 35.5. Similarly, the new class formed from the class 36 - 40 is (36 – 0.5) - (40 + 0.5), i.e., 35.5 - 40.5.

Continuing in the same manner, the continuous classes formed are:

30.5-35.5, 35.5-40.5, 40.5-45.5, 45.5-50.5, 50.5-55.5, 55.5-60.5, 60.5 - 65.5, 65.5 - 70.5, 70.5 - 75.5.

Now it is possible for us to include the weights of the new students in these classes. But, another problem crops up because 35.5 appears in both the classes 30.5 - 35.5 and 35.5 - 40.5. In which class do you think this weight should be considered?

If it is considered in both classes, it will be counted twice.

By convention, we consider 35.5 in the class 35.5 - 40.5 and not in 30.5 - 35.5. Similarly, 40.5 is considered in 40.5 - 45.5 and not in 35.5 - 40.5.

So, the new weights 35.5 kg and 40.5 kg would be included in 35.5 - 40.5 and 40.5 - 45.5, respectively. Now, with these assumptions, the new frequency distribution table will be as shown below:

Table 14.4

Weights (in kg)	Number of students
30.5-35.5	9
35.5-40.5	6
40.5-45.5	15
45.5-50.5	3
50.5-55.5	1
55.5-60.5	2
60.5-65.5	2
65.5-70.5	1
70.5-75.5	1
Total	40

Now, let us move to the data collected by you in Activity 1. This time we ask you to present these as frequency distribution tables.

Activity 2 : Continuing with the same four groups, change your data to frequency distribution tables.Choose convenient classes with suitable class-sizes, keeping in mind the range of the data and the type of data.

EXERCISE 14.2

1. The blood groups of 30 students of Class VIII are recorded as follows:

    A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,

    A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.

    Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students?

2. The distance (in km) of 40 engineers from their residence to their place of work were found as follows:

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

    Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0-5 (5 not included). What main features do you observe from this tabular representation?

3. The relative humidity (in %) of a certain city for a month of 30 days was as follows:

98.1	98.6	99.2	90.3	86.5	95.3	92.9	96.3	94.2	95.1
89.2	92.3	97.1	93.5	92.7	95.1	97.2	93.3	95.2	97.3
96.2	92.1	84.9	90.2	95.7	98.3	97.3	96.1	92.1	89

    (i)   Construct a grouped frequency distribution table with classes 84 - 86, 86 - 88, etc.

    (ii) Which month or season do you think this data is about?

    (iii) What is the range of this data?

4. The heights of 50 students, measured to the nearest centimetres, have been found to be as follows:

161	150	154	165	168	161	154	162	150	151
162	164	171	165	158	154	156	172	160	170
153	159	161	170	162	165	166	168	165	164
154	152	153	156	158	162	160	161	173	166
161	159	162	167	168	159	158	153	154	159

    (i)   Represent the data given above by a grouped frequency distribution table, taking the class intervals as 160 - 165, 165 - 170, etc.

    (ii) What can you conclude about their heights from the table?

5. A study was conducted to find out the concentration of sulphur dioxide in the air in

parts per million (ppm) of a certain city. The data obtained for 30 days is as follows:

0.03	0.08	0.08	0.09	0.04	0.17
0.16	0.05	0.02	0.06	0.18	0.20
0.11	0.08	0.12	0.13	0.22	0.07
0.08	0.01	0.10	0.06	0.09	0.18
0.11	0.07	0.05	0.07	0.01	0.04

(i) Make a grouped frequency distribution table for this data with class intervals as 0.00 - 0.04, 0.04 - 0.08, and so on.

(ii) For how many days, was the concentration of sulphur dioxide more than 0.11 parts per million?

6. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:

0	1	2	2	1	2	3	1	3	0
1	3	1	1	2	2	0	1	2	1
3	0	0	1	1	2	3	2	2	0

Prepare a frequency distribution table for the data given above.

7. The value of π upto 50 decimal places is given below:

3.14159265358979323846264338327950288419716939937510

(i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.

(ii) What are the most and the least frequently occurring digits?

8. Thirty children were asked about the number of hours they watched TV programmes in the previous week. The results were found as follows:

1	6	2	3	5	12	5	8	4	8
10	3	4	12	2	8	15	1	17	6
3	2	8	5	9	6	8	7	14	12

(i) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals as 5 - 10.

(ii) How many children watched television for 15 or more hours a week?

9. A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows:

2.6	3.0	3.7	3.2	2.2	4.1	3.5	4.5
3.5	2.3	3.2	3.4	3.8	3.2	4.6	3.7
2.5	4.4	3.4	3.3	2.9	3.0	4.3	2.8
3.5	3.2	3.9	3.2	3.2	3.1	3.7	3.4
4.6	3.8	3.2	2.6	3.5	4.2	2.9	3.6

Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from the interval 2 - 2.5.

14.4 Graphical Representation of Data

The representation of data by tables has already been discussed. Now let us turn our attention to another representation of data, i.e., the graphical representation. It is well said that one picture is better than a thousand words. Usually comparisons among the individual items are best shown by means of graphs. The representation then becomes easier to understand than the actual data. We shall study the following graphical representations in this section.

(A) Bar graphs

(B) Histograms of uniform width, and of varying widths

(C) Frequency polygons

(A) Bar Graphs

In earlier classes, you have already studied and constructed bar graphs. Here we shall discuss them through a more formal approach. Recall that a bar graph is a pictorial representation of data in which usually bars of uniform width are drawn with equal spacing between them on one axis (say, the *x*-axis), depicting the variable. The values of the variable are shown on the other axis (say, the *y*-axis) and the heights of the bars depend on the values of the variable.

Example 5 : In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained:
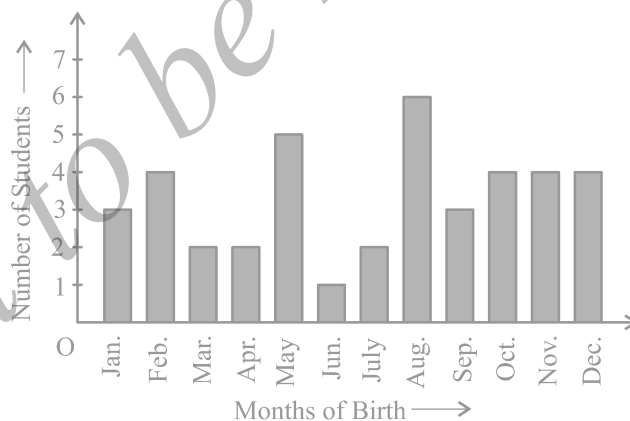

Fig. 14.1

Observe the bar graph given above and answer the following questions:

(i) How many students were born in the month of November?

(ii) In which month were the maximum number of students born?

Solution : Note that the variable here is the 'month of birth', and the value of the variable is the 'Number of students born'.

(i) 4 students were born in the month of November.

(ii) The Maximum number of students were born in the month of August.

Let us now recall how a bar graph is constructed by considering the following example.

Example 6 : A family with a monthly income of Rs 20,000 had planned the following expenditures per month under various heads:

<div align="center">Table 14.5</div>

Heads	Expenditure (in thousand rupees)
Grocery	4
Rent	5
Education of children	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Draw a bar graph for the data above.

Solution : We draw the bar graph of this data in the following steps. Note that the unit in the second column is thousand rupees. So, '4' against 'grocery' means Rs 4000.

1. We represent the Heads (variable) on the horizontal axis choosing any scale, since the width of the bar is not important. But for clarity, we take equal widths for all bars and maintain equal gaps in between. Let one Head be represented by one unit.

2. We represent the expenditure (value) on the vertical axis. Since the maximum expenditure is Rs 5000, we can choose the scale as 1 unit = Rs 1000.

3. To represent our first Head, i.e., grocery, we draw a rectangular bar with width 1 unit and height 4 units.

4. Similarly, other Heads are represented leaving a gap of 1 unit in between two consecutive bars.

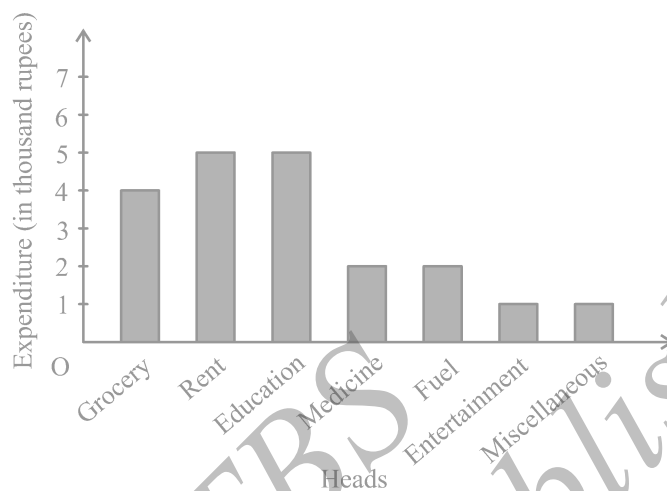   The bar graph is drawn in Fig. 14.2.

Fig. 14.2

Here, you can easily visualise the relative characteristics of the data at a glance, e.g., the expenditure on education is more than double that of medical expenses. Therefore, in some ways it serves as a better representation of data than the tabular form.

Activity 3 : Continuing with the same four groups of Activity 1, represent the data by suitable bar graphs.

Let us now see how a frequency distribution table for *continuous* class intervals can be represented graphically.

(B) Histogram

This is a form of representation like the bar graph, but it is used for continuous class intervals. For instance, consider the frequency distribution Table 14.6, representing the weights of 36 students of a class:

Table 14.6

Weights (in kg)	Number of students
30.5 - 35.5	9
35.5 - 40.5	6
40.5 - 45.5	15
45.5 - 50.5	3
50.5 - 55.5	1
55.5 - 60.5	2
Total	36

Let us represent the data given above graphically as follows:

(i)   We represent the weights on the horizontal axis on a suitable scale. We can choose the scale as 1 cm = 5 kg. Also, since the first class interval is starting from 30.5 and not zero, we show it on the graph by marking a *kink* or a break on the axis.

(ii)   We represent the number of students (frequency) on the vertical axis on a suitable scale. Since the maximum frequency is 15, we need to choose the scale to accomodate this maximum frequency.

(iii) We now draw rectangles (or rectangular bars) of width equal to the class-size and lengths according to the frequencies of the corresponding class intervals. For example, the rectangle for the class interval 30.5 - 35.5 will be of width 1 cm and length 4.5 cm.

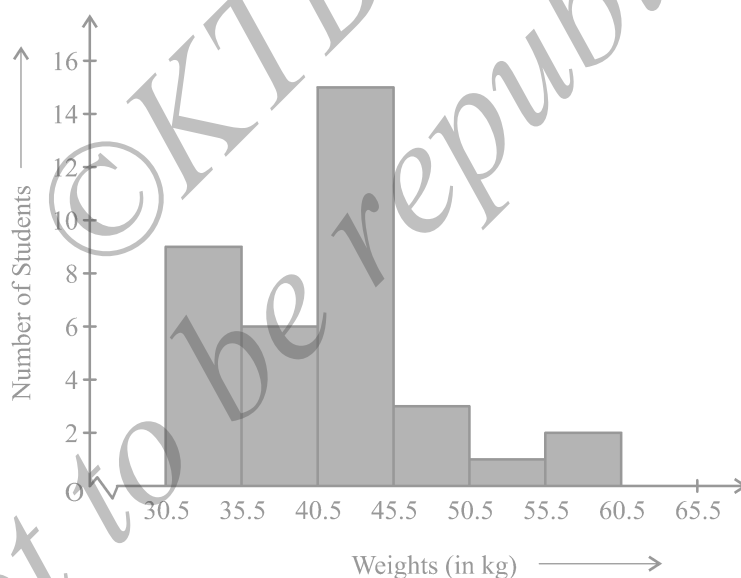(iv)   In this way, we obtain the graph as shown in Fig. 14.3:

Fig. 14.3

Observe that since there are no gaps in between consecutive rectangles, the resultant graph appears like a solid figure. This is called a *histogram*, which is a graphical representation of a grouped frequency distribution with continuous classes. Also, unlike a bar graph, the width of the bar plays a significant role in its construction.

Here, in fact, areas of the rectangles erected are proportional to the corresponding frequencies. However, since the widths of the rectangles are all equal, the lengths of the rectangles are proportional to the frequencies. That is why, we draw the lengths according to (iii) above.

Now, consider a situation different from the one above.

Example 7 : A teacher wanted to analyse the performance of two sections of students in a mathematics test of 100 marks. Looking at their performances, she found that a few students got under 20 marks and a few got 70 marks or above. So she decided to group them into intervals of varying sizes as follows: 0 - 20, 20 - 30, . . ., 60 - 70, 70 - 100. Then she formed the following table:

Table 14.7

Marks	Number of students
0 - 20	7
20 - 30	10
30 - 40	10
40 - 50	20
50 - 60	20
60 - 70	15
70 - above	8
Total	90

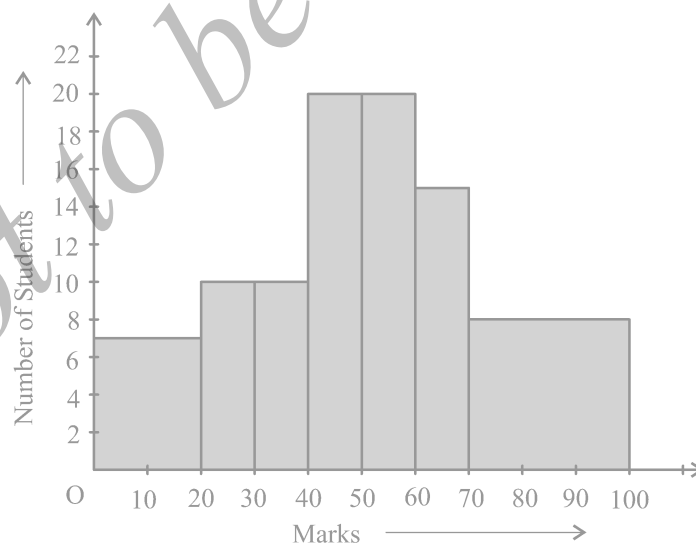A histogram for this table was prepared by a student as shown in Fig. 14.4.

Fig. 14.4

Carefully examine this graphical representation. Do you think that it correctly represents the data? No, the graph is giving us a misleading picture. As we have mentioned earlier, the areas of the rectangles are proportional to the frequencies in a histogram. Earlier this problem did not arise, because the widths of all the rectangles were equal. But here, since the widths of the rectangles are varying, the histogram above does not give a correct picture. For example, it shows a greater frequency in the interval 70 - 100, than in 60 - 70, which is not the case.

So, we need to make certain modifications in the lengths of the rectangles so that the areas are again proportional to the frequencies.

The steps to be followed are as given below:

1. Select a class interval with the minimum class size. In the example above, the minimum class-size is 10.

2. The lengths of the rectangles are then modified to be proportionate to the class-size 10.

For instance, when the class-size is 20, the length of the rectangle is 7. So when the class-size is 10, the length of the rectangle will be $\dfrac{7}{20} \times 10 = 3.5$.

Similarly, proceeding in this manner, we get the following table:

Table 14.8

Marks	Frequency	Width of the class	Length of the rectangle
0 - 20	7	20	$\dfrac{7}{20} \times 10 = 3.5$
20 - 30	10	10	$\dfrac{10}{10} \times 10 = 10$
30 - 40	10	10	$\dfrac{10}{10} \times 10 = 10$
40 - 50	20	10	$\dfrac{20}{10} \times 10 = 20$
50 - 60	20	10	$\dfrac{20}{10} \times 10 = 20$
60 - 70	15	10	$\dfrac{15}{10} \times 10 = 15$
70 - 100	8	30	$\dfrac{8}{30} \times 10 = 2.67$

Since we have calculated these lengths for an interval of 10 marks in each case, we may call these lengths as "proportion of students per 10 marks interval".

So, the correct histogram with varying width is given in Fig. 14.5.
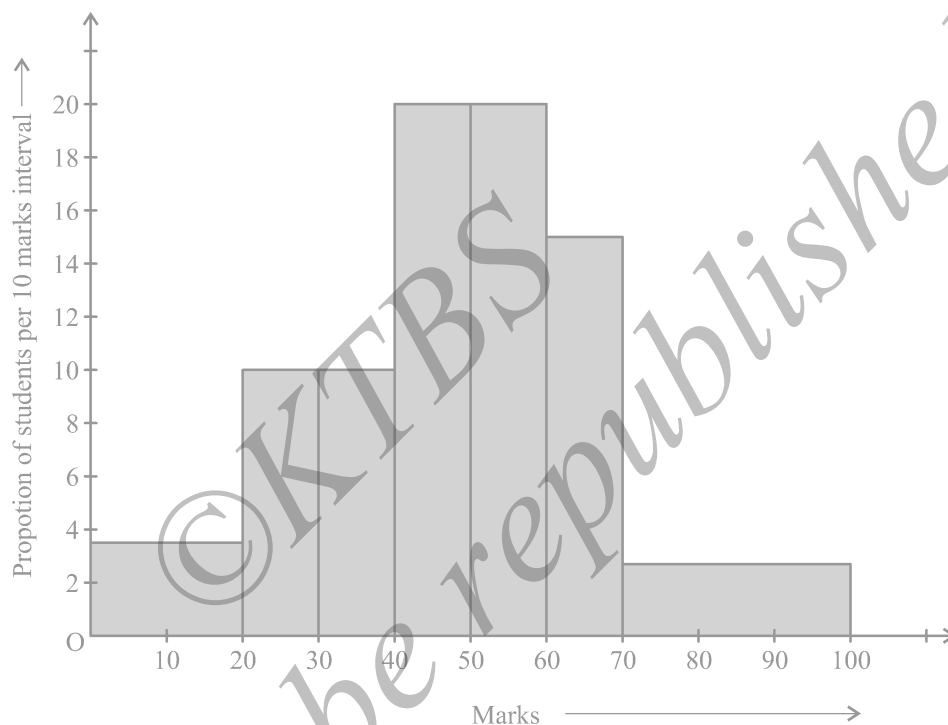
Fig. 14.5

(C) Frequency Polygon

There is yet another visual way of representing quantitative data and its frequencies. This is a polygon. To see what we mean, consider the histogram represented by Fig. 14.3. Let us join the mid-points of the upper sides of the adjacent rectangles of this histogram by means of line segments. Let us call these mid-points B, C, D, E, F and G. When joined by line segments, we obtain the figure BCDEFG (see Fig. 14.6). To complete the polygon, we assume that there is a class interval with frequency zero before 30.5 - 35.5, and one after 55.5 - 60.5, and their mid-points are A and H, respectively. ABCDEFGH is the frequency polygon corresponding to the data shown in Fig. 14.3. We have shown this in Fig. 14.6.
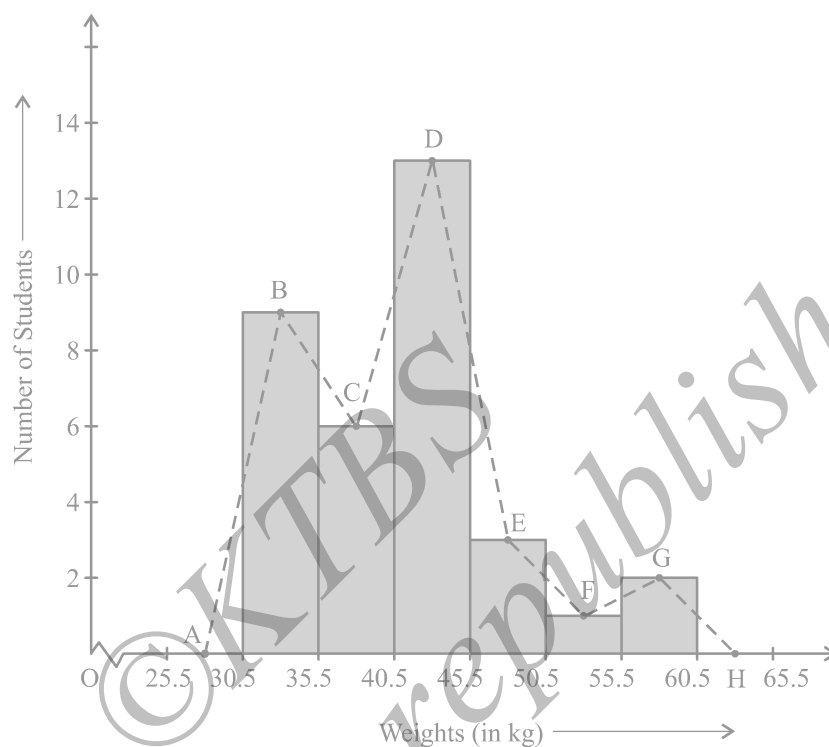
Fig. 14.6

Although, there exists no class preceding the lowest class and no class succeeding the highest class, addition of the two class intervals with zero frequency enables us to make the area of the frequency polygon the same as the area of the histogram. Why is this so? (**Hint :** Use the properties of congruent triangles.)

Now, the question arises: how do we complete the polygon when there is no class preceding the first class? Let us consider such a situation.

Example 8 : Consider the marks, out of 100, obtained by 51 students of a class in a test, given in Table 14.9.

Table 14.9

Marks	Number of students
0 - 10	5
10 - 20	10
20 - 30	4
30 - 40	6
40 - 50	7
50 - 60	3
60 - 70	2
70 - 80	2
80 - 90	3
90 - 100	9
Total	51

Draw a frequency polygon corresponding to this frequency distribution table.

Solution : Let us first draw a histogram for this data and mark the mid-points of the tops of the rectangles as B, C, D, E, F, G, H, I, J, K, respectively. Here, the first class is 0-10. So, to find the class preceeding 0-10, we extend the horizontal axis in the negative direction and find the mid-point of the imaginary class-interval (–10) - 0. The first end point, i.e., B is joined to this mid-point with zero frequency on the negative direction of the horizontal axis. The point where this line segment meets the vertical axis is marked as A. Let L be the mid-point of the class succeeding the last class of the given data. Then OABCDEFGHIJKL is the frequency polygon, which is shown in Fig. 14.7.
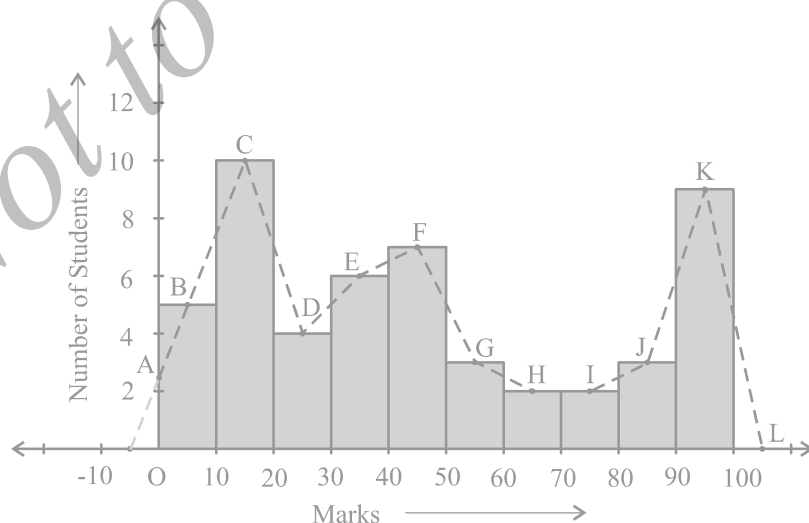

Fig. 14.7

Frequency polygons can also be drawn independently without drawing histograms. For this, we require the mid-points of the class-intervals used in the data. These mid-points of the class-intervals are called **class-marks**.

To find the class-mark of a class interval, we find the sum of the upper limit and lower limit of a class and divide it by 2. Thus,

$$\textbf{Class-mark} = \frac{\textbf{Upper limit + Lower limit}}{\textbf{2}}$$

Let us consider an example.

Example 9 : In a city, the weekly observations made in a study on the cost of living index are given in the following table:

Table 14.10

Cost of living index	Number of weeks
140 - 150	5
150 - 160	10
160 - 170	20
170 - 180	9
180 - 190	6
190 - 200	2
Total	52

Draw a frequency polygon for the data above (without constructing a histogram).

Solution : Since we want to draw a frequency polygon without a histogram, let us find the class-marks of the classes given above, that is of 140 - 150, 150 - 160,....

For 140 - 150, the upper limit = 150, and the lower limit = 140

So, the class-mark = $\frac{150 + 140}{2} = \frac{290}{2} = 145$.

Continuing in the same manner, we find the class-marks of the other classes as well.

So, the new table obtained is as shown in the following table:

Table 14.11

Classes	Class-marks	Frequency
140 - 150	145	5
150 - 160	155	10
160 - 170	165	20
170 - 180	175	9
180 - 190	185	6
190 - 200	195	2
Total		52

We can now draw a frequency polygon by plotting the class-marks along the horizontal axis, the frequencies along the vertical-axis, and then plotting and joining the points B(145, 5), C(155, 10), D(165, 20), E(175, 9), F(185, 6) and G(195, 2) by line segments. We should not forget to plot the point corresponding to the class-mark of the class 130 - 140 (just before the lowest class 140 - 150) with zero frequency, that is, A(135, 0), and the point H (205, 0) occurs immediately after G(195, 2). So, the resultant frequency polygon will be ABCDEFGH (see Fig. 14.8).
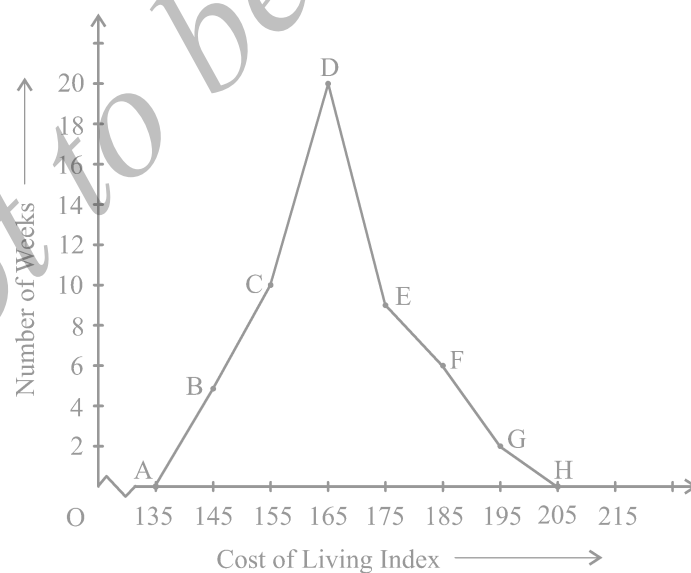

Fig. 14.8

Frequency polygons are used when the data is continuous and very large. It is very useful for comparing two different sets of data of the same nature, for example, comparing the performance of two different sections of the same class.

EXERCISE 14.3

1. A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 - 44 (in years) worldwide, found the following figures (in %):

S.No.	Causes	Female fatality rate (%)
1.	Reproductive health conditions	31.8
2.	Neuropsychiatric conditions	25.4
3.	Injuries	12.4
4.	Cardiovascular conditions	4.3
5.	Respiratory conditions	4.1
6.	Other causes	22.0

(i) Represent the information given above graphically.

(ii) Which condition is the major cause of women's ill health and death worldwide?

(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

2. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below.

Section	Number of girls per thousand boys
Scheduled Caste (SC)	940
Scheduled Tribe (ST)	970
Non SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.

(ii) In the classroom discuss what conclusions can be arrived at from the graph.

3. Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

(i) Draw a bar graph to represent the polling results.

(ii) Which political party won the maximum number of seats?

4. The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following table:

Length (in mm)	Number of leaves
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

(i) Draw a histogram to represent the given data. [Hint: First make the class intervals continuous]

(ii) Is there any other suitable graphical representation for the same data?

(iii) Is it correct to conclude that the maximum number of leaves are 153 mm long? Why?

5. The following table gives the life times of 400 neon lamps:

Life time (in hours)	Number of lamps
300 - 400	14
400 - 500	56
500 - 600	60
600 - 700	86
700 - 800	74
800 - 900	62
900 - 1000	48

(i) Represent the given information with the help of a histogram.

(ii) How many lamps have a life time of more than 700 hours?

6. The following table gives the distribution of students of two sections according to the marks obtained by them:

Section A		Section B	
Marks	**Frequency**	**Marks**	**Frequency**
0 - 10	3	0 - 10	5
10 - 20	9	10 - 20	19
20 - 30	17	20 - 30	15
30 - 40	12	30 - 40	10
40 - 50	9	40 - 50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

7. The runs scored by two teams A and B on the first 60 balls in a cricket match are given below:

Number of balls	Team A	Team B
1 - 6	2	5
7 - 12	1	6
13 - 18	8	2
19 - 24	9	10
25 - 30	4	5
31 - 36	5	6
37 - 42	6	3
43 - 48	10	4
49 - 54	6	8
55 - 60	2	10

Represent the data of both the teams on the same graph by frequency polygons.

[**Hint :** First make the class intervals continuous.]

8. A random survey of the number of children of various age groups playing in a park was found as follows:

Age (in years)	Number of children
1 - 2	5
2 - 3	3
3 - 5	6
5 - 7	12
7 - 10	9
10 - 15	10
15 - 17	4

Draw a histogram to represent the data above.

9. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:

Number of letters	Number of surnames
1 - 4	6
4 - 6	30
6 - 8	44
8 - 12	16
12 - 20	4

(i) Draw a histogram to depict the given information.

(ii) Write the class interval in which the maximum number of surnames lie.

14.5 Measures of Central Tendency

Earlier in this chapter, we represented the data in various forms through frequency distribution tables, bar graphs, histograms and frequency polygons. Now, the question arises if we always need to study all the data to 'make sense' of it, or if we can make out some important features of it by considering only certain representatives of the data. This is possible, by using measures of central tendency or averages.

Consider a situation when two students Mary and Hari received their test copies. The test had five questions, each carrying ten marks. Their scores were as follows:

Question Numbers	1	2	3	4	5
Mary's score	10	8	9	8	7
Hari's score	4	7	10	10	10

Upon getting the test copies, both of them found their average scores as follows:

$$\text{Mary's average score} = \frac{42}{5} = 8.4$$

$$\text{Hari's average score} = \frac{41}{5} = 8.2$$

Since Mary's average score was more than Hari's, Mary claimed to have performed better than Hari, but Hari did not agree. He arranged both their scores in ascending order and found out the middle score as given below:

Mary's Score	7	8	⑧	9	10
Hari's Score	4	7	⑩	10	10

Hari said that since his middle-most score was 10, which was higher than Mary's middle-most score, that is 8, his performance should be rated better.

But Mary was not convinced. To convince Mary, Hari tried out another strategy. He said he had scored 10 marks more often (3 times) as compared to Mary who scored 10 marks only once. So, his performance was better.

Now, to settle the dispute between Hari and Mary, let us see the three measures they adopted to make their point.

The average score that Mary found in the first case is the *mean*. The 'middle' score that Hari was using for his argument is the *median*. The most often scored mark that Hari used in his second strategy is the *mode*.

Now, let us first look at the mean in detail.

The **mean** (or **average**) of a number of observations is the sum of the values of all the observations divided by the total number of observations.

It is denoted by the symbol $\overline{x}$, read as 'x bar'.

Let us consider an example.

Example 10 : 5 people were asked about the time in a week they spend in doing social work in their community. They said 10, 7, 13, 20 and 15 hours, respectively.

Find the mean (or average) time in a week devoted by them for social work.

Solution : We have already studied in our earlier classes that the mean of a certain number of observations is equal to $\dfrac{\text{Sum of all the observations}}{\text{Total number of observations}}$. To simplify our

working of finding the mean, let us use a variable x_i to denote the ith observation. In this case, i can take the values from 1 to 5. So our first observation is x_1, second observation is x_2, and so on till x_5.

Also $x_1 = 10$ means that the value of the first observation, denoted by x_1, is 10. Similarly, $x_2 = 7$, $x_3 = 13$, $x_4 = 20$ and $x_5 = 15$.

Therefore, the mean $\bar{x} = \dfrac{\text{Sum of all the observations}}{\text{Total number of observations}}$

$$= \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$= \frac{10 + 7 + 13 + 20 + 15}{5} = \frac{65}{5} = 13$$

So, the mean time spent by these 5 people in doing social work is 13 hours in a week.

Now, in case we are finding the mean time spent by 30 people in doing social work, writing $x_1 + x_2 + x_3 + \ldots + x_{30}$ would be a tedious job. We use the Greek symbol Σ (for the letter Sigma) for *summation*. Instead of writing $x_1 + x_2 + x_3 + \ldots + x_{30}$, we write $\displaystyle\sum_{i=1}^{30} x_i$, which is read as 'the sum of x_i as i varies from 1 to 30'.

So, $\bar{x} = \dfrac{\displaystyle\sum_{i=1}^{30} x_i}{30}$

Similarly, for n observations $\bar{x} = \dfrac{\displaystyle\sum_{i=1}^{n} x_i}{n}$

Example 11 : Find the mean of the marks obtained by 30 students of Class IX of a school, given in Example 2.

Solution : Now, $\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_{30}}{30}$

$\displaystyle\sum_{i=1}^{30} x_i = 10 + 20 + 36 + 92 + 95 + 40 + 50 + 56 + 60 + 70 + 92 + 88$
$\qquad\qquad 80 + 70 + 72 + 70 + 36 + 40 + 36 + 40 + 92 + 40 + 50 + 50$
$\qquad\qquad 56 + 60 + 70 + 60 + 60 + 88 = 1779$

So, $\bar{x} = \dfrac{1779}{30} = 59.3$

Is the process not time consuming? Can we simplify it? Note that we have formed a frequency table for this data (see Table 14.1).

The table shows that 1 student obtained 10 marks, 1 student obtained 20 marks, 3 students obtained 36 marks, 4 students obtained 40 marks, 3 students obtained 50 marks, 2 students obtained 56 marks, 4 students obtained 60 marks, 4 students obtained 70 marks, 1 student obtained 72 marks, 1 student obtained 80 marks, 2 students obtained 88 marks, 3 students obtained 92 marks and 1 student obtained 95 marks.

So, the total marks obtained = $(1 \times 10) + (1 \times 20) + (3 \times 36) + (4 \times 40) + (3 \times 50)$
$+ (2 \times 56) + (4 \times 60) + (4 \times 70) + (1 \times 72) + (1 \times 80)$
$+ (2 \times 88) + (3 \times 92) + (1 \times 95)$

$= f_1 x_1 + \ldots + f_{13} x_{13}$, where f_i is the frequency of the ith entry in Table 14.1.

In brief, we write this as $\sum_{i=1}^{13} f_i x_i$.

So, the total marks obtained $= \sum_{i=1}^{13} f_i x_i$
$= 10 + 20 + 108 + 160 + 150 + 112 + 240 + 280 + 72 + 80 + 176 + 276 + 95$
$= 1779$

Now, the total number of observations
$= \sum_{i=1}^{13} f_i$
$= f_1 + f_2 + \ldots + f_{13}$
$= 1 + 1 + 3 + 4 + 3 + 2 + 4 + 4 + 1 + 1 + 2 + 3 + 1$
$= 30$

So, the mean $\bar{x} = \dfrac{\text{Sum of all the observations}}{\text{Total number of observations}} = \left(\dfrac{\sum_{i=1}^{13} f_i x_i}{\sum_{i=1}^{13} f_i} \right)$

$= \dfrac{1779}{30} = 59.3$

This process can be displayed in the following table, which is a modified form of Table 14.1.

Table 14.12

Marks (x_i)	Number of students (f_i)	$f_i x_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
	$\sum_{i=1}^{13} f_i = 30$	$\sum_{i=1}^{13} f_i x_i = 1779$

Thus, in the case of an ungrouped frequency distribution, you can use the formula

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

for calculating the mean.

Let us now move back to the situation of the argument between Hari and Mary, and consider the second case where Hari found his performance better by finding the middle-most score. As already stated, this measure of central tendency is called the *median*.

The **median** is that value of the given number of observations, which divides it into exactly two parts. So, when the data is arranged in ascending (or descending) order the median of ungrouped data is calculated as follows:

(i)   When the number of observations (n) is odd, the median is the value of the $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ observation. For example, if $n = 13$, the value of the $\left(\dfrac{13+1}{2}\right)^{\text{th}}$, i.e., the 7th observation will be the median [see Fig. 14.9 (i)].

(ii) When the number of observations (n) is even, the median is the mean of the $\left(\dfrac{n}{2}\right)^{\text{th}}$ and the $\left(\dfrac{n}{2}+1\right)^{\text{th}}$ observations. For example, if $n = 16$, the mean of the values of the $\left(\dfrac{16}{2}\right)^{\text{th}}$ and the $\left(\dfrac{16}{2}+1\right)^{\text{th}}$ observations, i.e., the mean of the values of the 8th and 9th observations will be the median [see Fig. 14.9 (ii)].
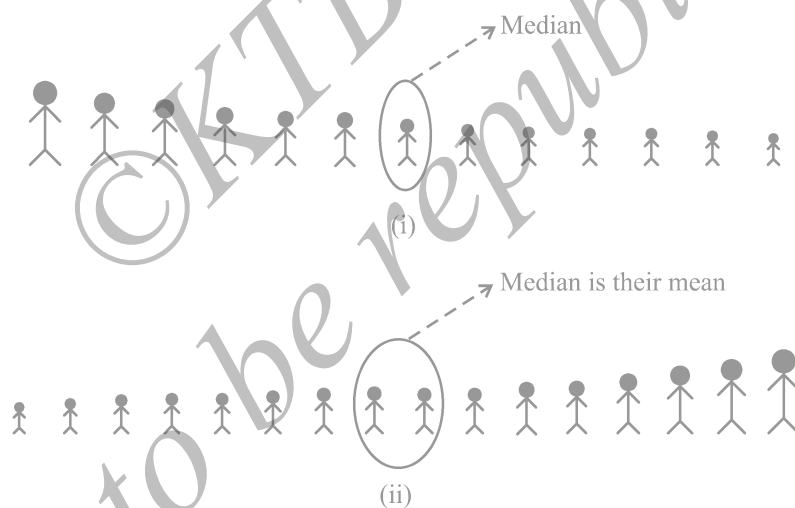

Fig. 14.9

Let us illustrate this with the help of some examples.

Example 12 : The heights (in cm) of 9 students of a class are as follows:

    155   160   145   149   150   147   152   144   148

Find the median of this data.

Solution : First of all we arrange the data in ascending order, as follows:

    144   145   147   148   149   150   152   155   160

Since the number of students is 9, an odd number, we find out the median by finding

the height of the $\left(\dfrac{n+1}{2}\right)$th $=\left(\dfrac{9+1}{2}\right)$th = the 5th student, which is 149 cm.

So, the median, i.e., the medial height is 149 cm.

Example 13 : The points scored by a Kabaddi team in a series of matches are as follows:

17,　2,　7,　27,　15,　5,　14,　8,　10,　24,　48,　10,　8,　7,　18,　28

Find the median of the points scored by the team.

Solution : Arranging the points scored by the team in ascending order, we get

2,　5,　7,　7,　8,　8,　10,　10,　14,　15,　17,　18,　24,　27,　28,　48.

There are 16 terms. So there are two middle terms, i.e. the $\dfrac{16}{2}$th and $\left(\dfrac{16}{2}+1\right)$th, i.e., the 8th and 9th terms.

So, the median is the mean of the values of the 8th and 9th terms.

i.e, the median $=\dfrac{10+14}{2}=12$

So, the medial point scored by the Kabaddi team is 12.

Let us again go back to the unsorted dispute of Hari and Mary.

The third measure used by Hari to find the average was the *mode*.

The **mode** is that value of the observation which occurs most frequently, i.e., an observation with the maximum frequency is called the mode.

The readymade garment and shoe industries make great use of this measure of central tendency. Using the knowledge of mode, these industries decide which size of the product should be produced in large numbers.

Let us illustrate this with the help of an example.

Example 14 : Find the mode of the following marks (out of 10) obtained by 20 students:

4, 6, 5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 10, 10, 3, 4, 7, 6, 9, 9

Solution : We arrange this data in the following form :

2, 3, 3, 4, 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 9, 9, 9, 9, 10, 10

Here 9 occurs most frequently, i.e., four times. So, the mode is 9.

Example 15 : Consider a small unit of a factory where there are 5 employees : a supervisor and four labourers. The labourers draw a salary of ₹ 5,000 per month each while the supervisor gets ₹ 15,000 per month. Calculate the mean, median and mode of the salaries of this unit of the factory.

Solution : Mean $= \dfrac{5000 + 5000 + 5000 + 5000 + 15000}{5} = \dfrac{35000}{5} = 7000$

So, the mean salary is ₹ 7000 per month.

To obtain the median, we arrange the salaries in ascending order:

5000,   5000,   5000,   5000,   15000

Since the number of employees in the factory is 5, the median is given by the $\left(\dfrac{5+1}{2}\right)$th $= \dfrac{6}{2}$th = 3rd observation. Therefore, the median is ₹ 5000 per month.

To find the mode of the salaries, i.e., the modal salary, we see that 5000 occurs the maximum number of times in the data 5000, 5000, 5000, 5000, 15000. So, the modal salary is ₹ 5000 per month.

Now compare the three measures of central tendency for the given data in the example above. You can see that the mean salary of ₹ 7000 does not give even an approximate estimate of any one of their wages, while the medial and modal salaries of ₹ 5000 represents the data more effectively.

Extreme values in the data affect the mean. This is one of the weaknesses of the mean. So, if the data has a few points which are very far from most of the other points, (like 1,7,8,9,9) then the mean is not a good representative of this data. Since the median and mode are not affected by extreme values present in the data, they give a better estimate of the average in such a situation.

Again let us go back to the situation of Hari and Mary, and compare the three measures of central tendency.

Measures of central tendency	Hari	Mary
Mean	8.2	8.4
Median	10	8
Mode	10	8

This comparison helps us in stating that these measures of central tendency are not sufficient for concluding which student is better. We require some more information to conclude this, which you will study about in the higher classes.

<div align="center">**EXERCISE 14.4**</div>

1. The following number of goals were scored by a team in a series of 10 matches:

    <div align="center">2, 3, 4, 5, 0, 1, 3, 3, 4, 3</div>

    Find the mean, median and mode of these scores.

2. In a mathematics test given to 15 students, the following marks (out of 100) are recorded:

    <div align="center">41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60</div>

    Find the mean, median and mode of this data.

3. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x.

    <div align="center">29, 32, 48, 50, x, $x+2$, 72, 78, 84, 95</div>

4. Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

5. Find the mean salary of 60 workers of a factory from the following table:

Salary (in ₹)	Number of workers
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1
Total	60

6. Give one example of a situation in which

    (i) the mean is an appropriate measure of central tendency.

    (ii) the mean is not an appropriate measure of central tendency but the median is an appropriate measure of central tendency.

14.6 Summary

In this chapter, you have studied the following points:

1. Facts or figures, collected with a definite purpose, are called data.

2. Statistics is the area of study dealing with the presentation, analysis and interpretation of data.

3. How data can be presented graphically in the form of bar graphs, histograms and frequency polygons.

4. The three measures of central tendency for **ungrouped data** are:

   (i) Mean : It is found by adding all the values of the observations and dividing it by the total number of observations. It is denoted by $\bar{x}$.

   So, $\bar{x} = \dfrac{\displaystyle\sum_{i=1}^{n} x_i}{n}$. For an ungrouped frequency distribution, it is $\bar{x} = \dfrac{\displaystyle\sum_{i=1}^{n} f_i x_i}{\displaystyle\sum_{i=1}^{n} f_i}$.

   (ii) Median : It is the value of the middle-most observation (s).

   If n is an odd number, the median = value of the $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ observation.

   If n is an even number, median = Mean of the values of the $\left(\dfrac{n}{2}\right)^{\text{th}}$ and $\left(\dfrac{n}{2}+1\right)^{\text{th}}$ observations.

   (iii) Mode : The mode is the most frequently occurring observation.

PROBABILITY

It is remarkable that a science, which began with the consideration of games of chance, should be elevated to the rank of the most important subject of human knowledge.   —*Pierre Simon Laplace*

15.1 Introduction

In everyday life, we come across statements such as

(1) It will **probably** rain today.

(2) I **doubt** that he will pass the test.

(3) **Most probably**, Kavita will stand first in the annual examination.

(4) **Chances** are high that the prices of diesel will go up.

(5) There is a 50-50 **chance** of India winning a toss in today's match.

The words 'probably', 'doubt', 'most probably', 'chances', etc., used in the statements above involve an element of uncertainty. For example, in (1), 'probably rain' will mean it may rain or may not rain today. We are predicting rain today based on our past experience when it rained under similar conditions. Similar predictions are also made in other cases listed in (2) to (5).

The uncertainty of 'probably' etc can be measured numerically by means of 'probability' in many cases.

Though probability started with gambling, it has been used extensively in the fields of Physical Sciences, Commerce, Biological Sciences, Medical Sciences, Weather Forecasting, etc.

15.2 Probability – an Experimental Approach

In earlier classes, you have had a glimpse of probability when you performed experiments like tossing of coins, throwing of dice, etc., and observed their *outcomes*. You will now learn to measure the chance of occurrence of a particular *outcome* in an experiment.

The concept of probability developed in a very strange manner. In 1654, a gambler Chevalier de Mere, approached the well-known 17th century French philosopher and mathematician Blaise Pascal regarding certain dice problems. Pascal became interested in these problems, studied them and discussed them with another French mathematician, Pierre de Fermat. Both Pascal and Fermat solved the problems independently. This work was the beginning of Probability Theory.

Blaise Pascal (1623–1662)
Fig. 15.1

Pierre de Fermat (1601–1665)
Fig. 15.2

The first book on the subject was written by the Italian mathematician, J.Cardan (1501–1576). The title of the book was 'Book on Games of Chance' (Liber de Ludo Aleae), published in 1663. Notable contributions were also made by mathematicians J. Bernoulli (1654–1705), P. Laplace (1749–1827), A.A. Markov (1856–1922) and A.N. Kolmogorov (born 1903).

Activity 1 : (i) Take any coin, toss it ten times and note down the number of times a head and a tail come up. Record your observations in the form of the following table

Table 15.1

Number of times the coin is tossed	Number of times head comes up	Number of times tail comes up
10	—	—

Write down the values of the following fractions:

$$\frac{\text{Number of times a head comes up}}{\text{Total number of times the coin is tossed}}$$

and

$$\frac{\text{Number of times a tail comes up}}{\text{Total number of times the coin is tossed}}$$

    (ii) Toss the coin twenty times and in the same way record your observations as above. Again find the values of the fractions given above for this collection of observations.

    (iii) Repeat the same experiment by increasing the number of tosses and record the number of heads and tails. Then find the values of the corresponding fractions.

You will find that as the number of tosses gets larger, the values of the fractions come closer to 0.5. To record what happens in more and more tosses, the following group activity can also be performed:

Acitivity 2 : Divide the class into groups of 2 or 3 students. Let a student in each group toss a coin 15 times. Another student in each group should record the observations regarding heads and tails. [Note that coins of the same denomination should be used in all the groups. It will be treated as if only one coin has been tossed by all the groups.]

Now, on the blackboard, make a table like Table 15.2. First, Group 1 can write down its observations and calculate the resulting fractions. Then Group 2 can write down its observations, but will calculate the fractions for the combined data of Groups 1 and 2, and so on. (We may call these fractions as *cumulative fractions*.) We have noted the first three rows based on the observations given by one class of students.

Table 15.2

Group (1)	Number of heads (2)	Number of tails (3)	Cumulative number of heads — Total number of times the coin is tossed (4)	Cumulative number of tails — Total number of times the coin is tossed (5)
1	3	12	$\dfrac{3}{15}$	$\dfrac{12}{15}$
2	7	8	$\dfrac{7 + 3}{15 + 15} = \dfrac{10}{30}$	$\dfrac{8 + 12}{15 + 15} = \dfrac{20}{30}$
3	7	8	$\dfrac{7 + 10}{15 + 30} = \dfrac{17}{45}$	$\dfrac{8 + 20}{15 + 30} = \dfrac{28}{45}$
4	⋮	⋮	⋮	⋮

What do you observe in the table? You will find that as the total number of tosses of the coin increases, the values of the fractions in Columns (4) and (5) come nearer and nearer to 0.5.

Activity 3 : (i) Throw a die* 20 times and note down the number of times the numbers

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*A die is a well balanced cube with its six faces marked with numbers from 1 to 6, one number on one face. Sometimes dots appear in place of numbers.

1, 2, 3, 4, 5, 6 come up. Record your observations in the form of a table, as in Table 15.3:

Table 15.3

Number of times a die is thrown	Number of times these scores turn up					
	1	2	3	4	5	6
20						

Find the values of the following fractions:

$$\frac{\text{Number of times 1 turned up}}{\text{Total number of times the die is thrown}}$$

$$\frac{\text{Number of times 2 turned up}}{\text{Total number of times the die is thrown}}$$

$$\vdots$$

$$\frac{\text{Number of times 6 turned up}}{\text{Total number of times the die is thrown}}$$

(ii) Now throw the die 40 times, record the observations and calculate the fractions as done in (i).

As the number of throws of the die increases, you will find that the value of each fraction calculated in (i) and (ii) comes closer and closer to $\frac{1}{6}$.

To see this, you could perform a group activity, as done in Activity 2. Divide the students in your class, into small groups. One student in each group should throw a die ten times. Observations should be noted and cumulative fractions should be calculated.

The values of the fractions for the number 1 can be recorded in Table 15.4. This table can be extended to write down fractions for the other numbers also or other tables of the same kind can be created for the other numbers.