Lecture-13

LC N/w Foster-II form :- (Parallel)

$$Y(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} + Hs$$

$B_L = \frac{1}{Ls}$ $\quad$ $B_C = sC$

Ques:- obtain foster-I & II from Z(s)

$$Z(s) = \frac{(s^2+2)(s^2+4)}{s(s^2+3)}$$

 HG

Soln:- **Foster - I :-**

Pole $\to \infty \to Hs$

Pole $\to$ origin $\to k_0/s$

Pole $\to$ conjugate pair $\to \dfrac{2ks}{s^2+\omega^2}$

$$Z(s) = \frac{k_0}{s} + \frac{2ks}{s^2+\omega^2} + Hs$$

$$\frac{(s^2+2)(s^2+4)}{s(s^2+3)} = \frac{k_0}{s} + \frac{2ks}{s^2+3} + Hs \quad \text{(Partial Fraction)}$$

$$k_0 = \frac{8}{3} \qquad 2k \to \frac{1}{3} \qquad H \to 1$$

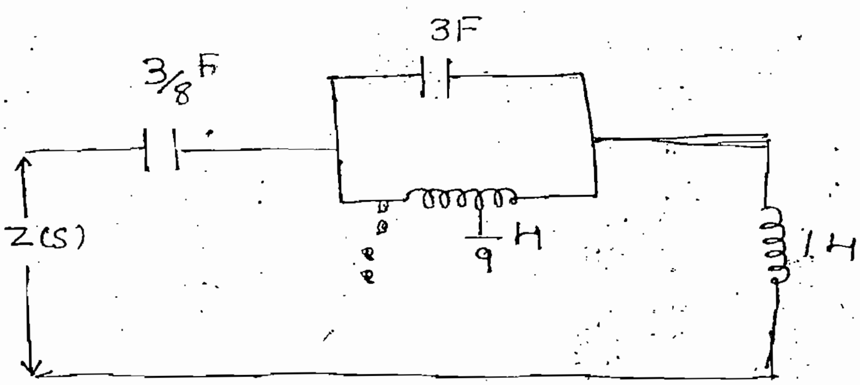
$$Z(s) = \frac{8/3}{s} + \frac{s/3}{s^2+3} + s$$

$$\Rightarrow Z(s) = \frac{1}{\frac{3}{8}s} + \frac{1}{\frac{s^2}{\frac{s}{3}} + \frac{3}{\frac{s}{3}}} + s$$

$$\Rightarrow Z(s) = \frac{1}{\frac{3}{8}s} + \frac{1}{3s + \frac{9}{s}} + s$$

$X_c = \dfrac{1}{Cs}$ $\qquad B_c = sC$ $\qquad B_L = \dfrac{1}{Ls}$

$$\frac{1}{Y_a + Y_b}$$



Foster - II :-

Note :- If $F(s)$ is given by using same function foster - I and II form are obtained.

$\to$ If $Z(s)$ is given by using $\dfrac{1}{Z(s)}$ function foster-II form is obtained

$\to$ If $Y(s)$ is given by using $\dfrac{1}{Y(s)}$ function foster-I form is obtained.

$$Y(s) = \frac{s(s^2+3)}{(s^2+2)(s^2+4)}$$

$P \to \infty$ ✗

$P \to$ origin ✗

$P \to$ conjugate pair ✓

$$Y(s) = \frac{2k_1 s}{s^2+\omega_1^2} + \frac{2k_2 s}{s^2+\omega_2^2}$$

$$\frac{s(s^2+3)}{(s^2+2)(s^2+4)} = \frac{2k_1 s}{s^2+2} + \frac{2k_2 s}{s^2+4}$$

$$2k_1 \to \frac{1}{2} \quad \$ \quad 2k_2 \to \frac{1}{2}$$

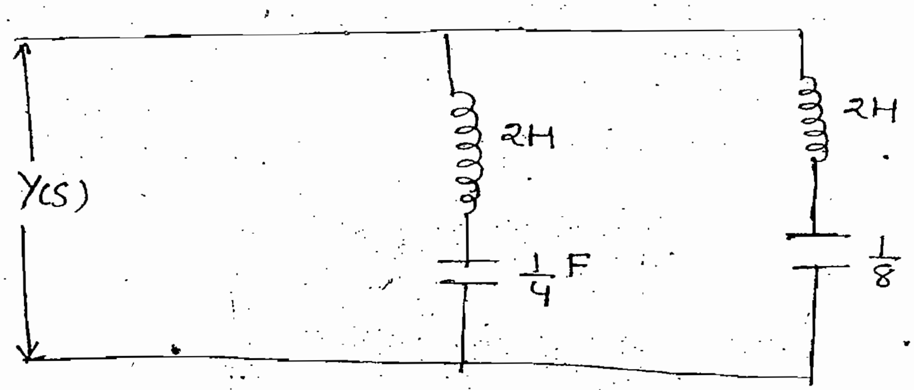
$$Y(s) = \frac{s/2}{s^2+2} + \frac{s/2}{s^2+4}$$

$$\Rightarrow Y(s) = \frac{1}{\frac{s^2}{s/2} + \frac{2}{s/2}} + \frac{1}{\frac{s^2}{\frac{s}{2}} + \frac{4}{\frac{s}{2}}}$$

$$\Rightarrow Y(s) = \frac{1}{2s + \frac{4}{s}} + \frac{1}{2s + \frac{8}{s}}$$

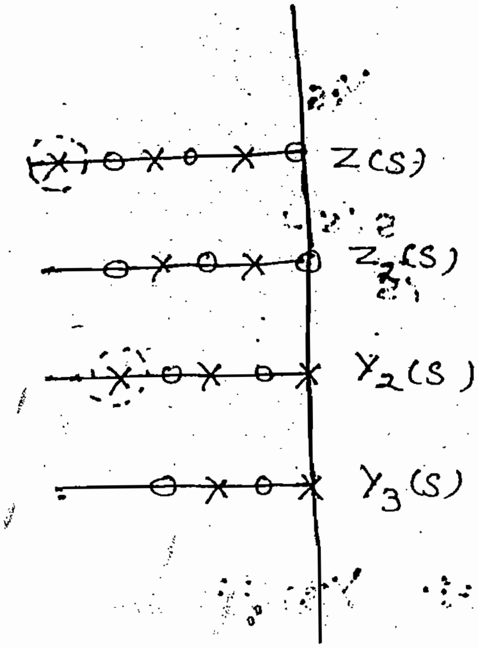
$X_L = Ls \quad X_C = \frac{1}{Cs} \qquad\qquad X_L \quad X_C$

$$\frac{1}{Z_a+Z_b} \qquad\qquad \frac{1}{Z_c+Z_d}$$



LC N/w Cauer-I form :- $\longrightarrow$ Ladder

(1) Removal of Pole at ∞ :-



$$Z_2(s) = Z(s) - H_1 s \quad (X_L = Ls)$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

$$Y_3(s) = Y_2(s) - H_2 s \quad (B_C = sC)$$

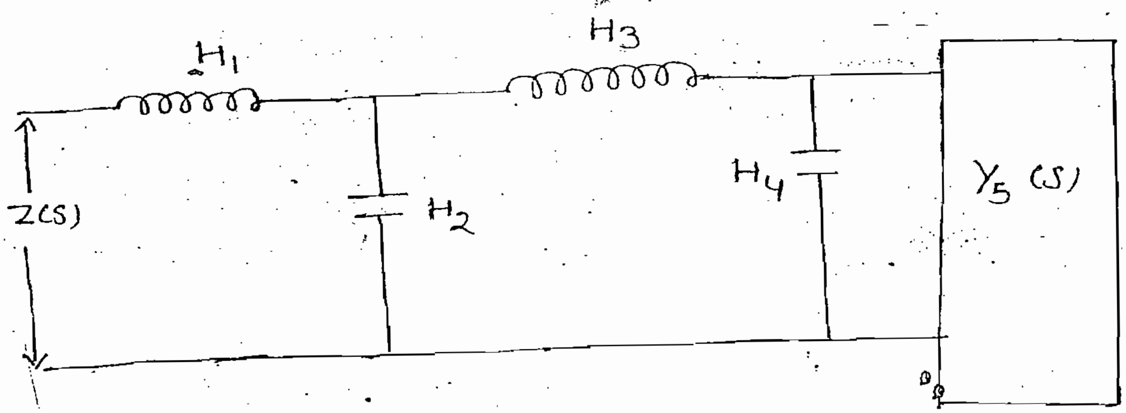
$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$Z_4(s) = Z_3(s) - H_3 s$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$

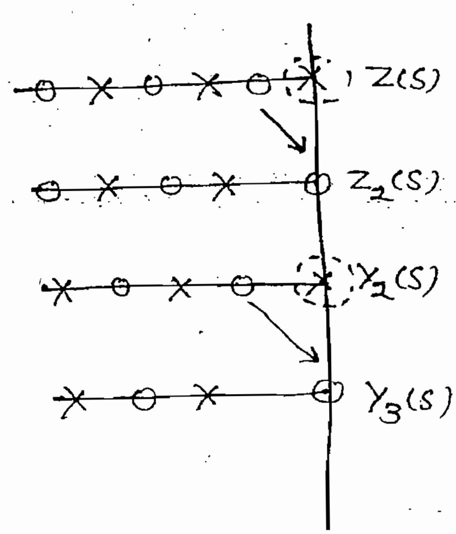
$$Y_5(s) = Y_5(s) - H_4 s$$

$$Y_4(s) = Y_5(s) + H_4 s$$



LC N/w Cauer-II form :-

Removal of Pole at origin :-



$$Z_2(s) = Z(s) - \frac{k_{01}}{s} \rightarrow \left(X_C = \frac{1}{Cs}\right)$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

$$Y_3(s) = Y_2(s) - \frac{k_{02}}{s} \quad \left(B_L = \frac{1}{Ls}\right)$$

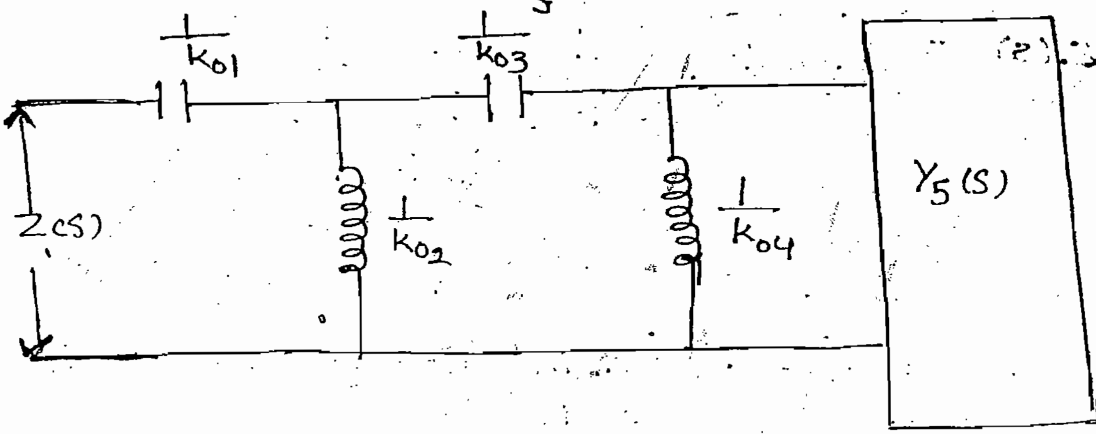
$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$\therefore Z_4(s) = Z_3(s) - \frac{k_{03}}{s}$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$

$$Y_5(s) = Y_4(s) - \frac{k_{04}}{s}$$

$$Y_4(s) = Y_5(s) + \frac{k_{04}}{s}$$



Ques:- Obtain Cauer - I and II form

$$Z(s) = \frac{s^3+2s}{s^4+5s^2+4}$$

Soln:- Cauer - I :-

Pole at ∞ $\longrightarrow$ not present

then reciprocal

$$Y(s) = \frac{s^4+5s^2+4}{s^3+2s}$$

$s^3+2s \,\big)\, s^4+5s^2+4 \,\big(\, s \rightarrow B_C = sC \rightarrow Y$

$s^4 + 2s^2$

$3s^2+4 \,\big)\, s^3+2s \,\big(\, \frac{s}{3} \rightarrow X_L = Ls \rightarrow Z$

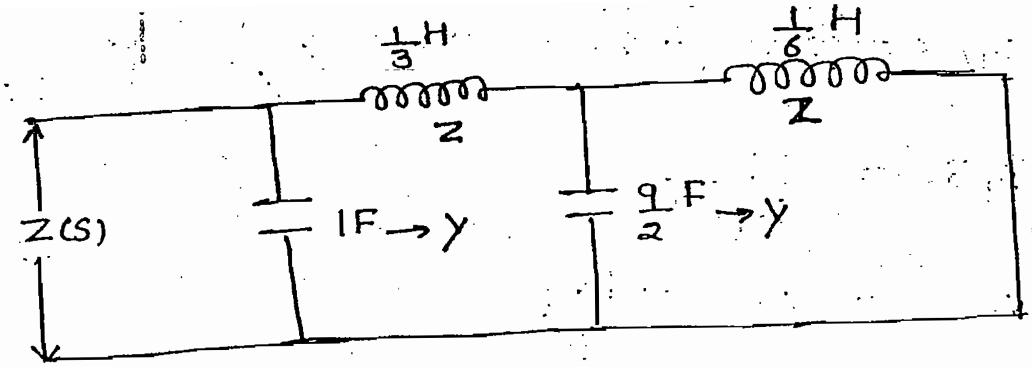
$s^3 + \frac{4s}{3}$

$\frac{2s}{3} \,\big)\, 3s^2+4 \,\big(\, \frac{9}{2}s \rightarrow B_C = SC \rightarrow Y$

$3s^2$

$4 \,\big)\, \frac{2s}{3} \,\big(\, \frac{s}{6} \rightarrow X_L = \rightarrow Z$

$\frac{2s}{3}$



$$Z(s) = \cfrac{1}{s + \cfrac{1}{\frac{s}{3} + \cfrac{1}{\frac{9s}{2} + \cfrac{1}{\frac{s}{6}}}}}$$

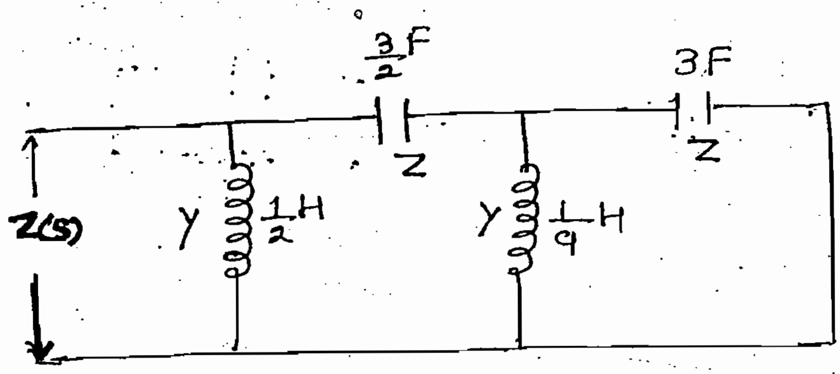
Cauer – II Form :-

Pole → origin not present then take reciprocal

$$Z(s) = \frac{s(s^2+2)}{s^4+5s^2+4}$$

$$Y(s) = \frac{s^4+5s^2+4}{s^3+2s}$$

$$\begin{array}{l}
2s+s^3 \,\big)\, s^4+5s^2+4 \,\big(\, \frac{2}{s} \to y \quad B_L = \frac{1}{Ls} \\
\qquad\quad 2s^2+4 \\
\qquad\quad \overline{3s^2+s^4} \,\big)\, 2s+s^3 \,\big(\, {}^{2}/_{3s} \to Z \quad X_c = \frac{1}{cs} \\
\qquad\qquad\qquad 2s+\frac{2s^3}{3} \\
\qquad\qquad\qquad \overline{\frac{s^3}{3}} \,\big)\, 3s^2+s^4 \,\big(\, \frac{9}{s} \to y \quad B_L \\
\qquad\qquad\qquad\qquad 3s^2 \\
\qquad\qquad\qquad\qquad \overline{s^4} \,\big)\, \frac{s^3}{3} \,\big(\, \frac{1}{3s} \to Z \quad X_c \\
\qquad\qquad\qquad\qquad\qquad \frac{s^3}{3} \\
\qquad\qquad\qquad\qquad\qquad \overline{\times}
\end{array}$$



$$Z(s) = \cfrac{1}{\cfrac{2}{s} + \cfrac{1}{\cfrac{2}{3s} + \cfrac{1}{\cfrac{9}{s} + \cfrac{1}{\cfrac{1}{3s}}}}}$$

RC N/w Foster-I form :-

$$Z_{LC}(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} + Hs$$

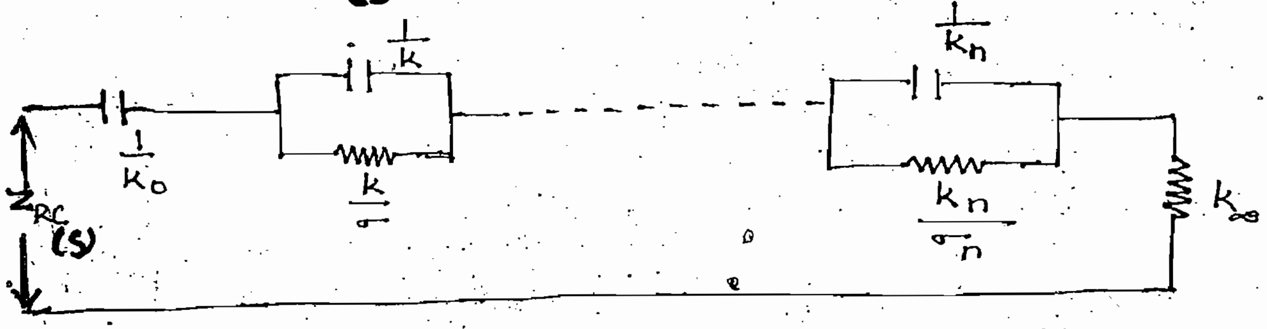
$$Z_{RC}(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{k_i}{s+\sigma_i} + k_\infty$$

$$x_c = \frac{1}{cs}$$

$$\frac{k}{s+\sigma}$$

$$\frac{1}{\frac{s}{K}+\frac{\sigma}{K}} = \frac{1}{Y_a+Y_b}$$

$B_c = SC$ $\quad$ G

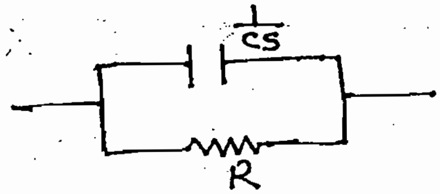


→ Series N/w is obtained

RC N/w Foster-II form :-

$$Z_{RC}(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{k_i}{s+\sigma_i}$$

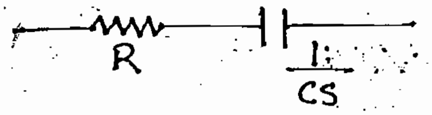
$$\rightarrow Z_{RC}(s) = \frac{R\frac{1}{cs}}{R+\frac{1}{cs}}$$



$$Z_{RC}(s) = \frac{R}{Rcs+1}$$

$$Z_{RC}(s) = \frac{R}{RC\left(s+\frac{1}{RC}\right)}$$

$$\rightarrow Y_{RC}(s) = \frac{1}{R+\frac{1}{cs}}$$



$$Y_{RC}(s) = \frac{cs}{Rcs+1}$$

$$Y_{RC}(s) = \frac{cs}{RC\left(s+\frac{1}{RC}\right)}$$

$$\frac{Y_{RC}(s)}{s} = \frac{1}{R\left(s+\frac{1}{RC}\right)}$$

Note:-

→ Pole-zero pattern of Z_{RC} and Y_{RC} $Y(s)$ are identical. Thereby both functions can be represented by same mathematical equation.

$$\frac{Y_{RC}(s)}{s} = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{k_i}{s+\sigma_i} + k_\infty$$

$$Y_{RC}(s) = k_0 + \sum_{i=1}^{n} \frac{k_i s}{s+\sigma_i} + k_\infty s$$

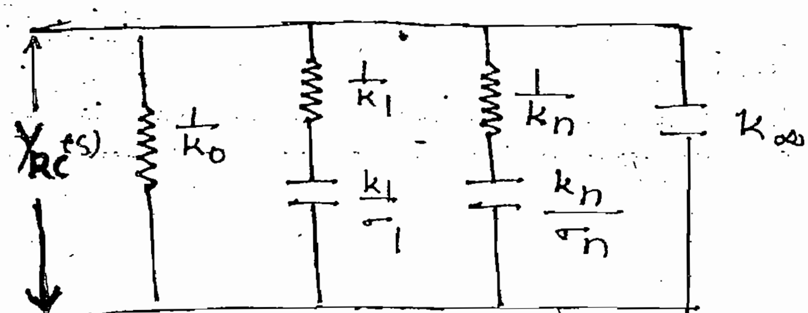
$$\frac{ks}{s+\sigma}$$

$$\frac{1}{\frac{s}{ks}+\frac{\sigma}{ks}}$$

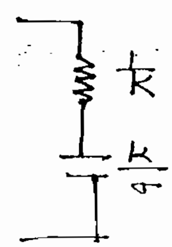
$$Y_c = \frac{1}{cs}$$

R

$$\frac{1}{Z_a+Z_b}$$



Dotted upto n



RC N/w Cauer-I form:-

LC $\rightarrow$ $Z_2(S) = Z(S) - H_1 S$ $\quad (X_L = LS)$ $\qquad Y_2(S) = \frac{1}{Z_2(S)}$

$Y_3(S) - H_2 S$

RC$\rightarrow$ Step-I:-

Removal of constant from $Z(S)$

$$Z_2(S) = Z(S) - K_1$$

$$Y_2(S) = \frac{1}{Z_2(S)}$$

Step-II:-

Removal of Pole at ∞ from $Y(S)$

$$Y_3(S) = Y_2(S) - H_1 S$$

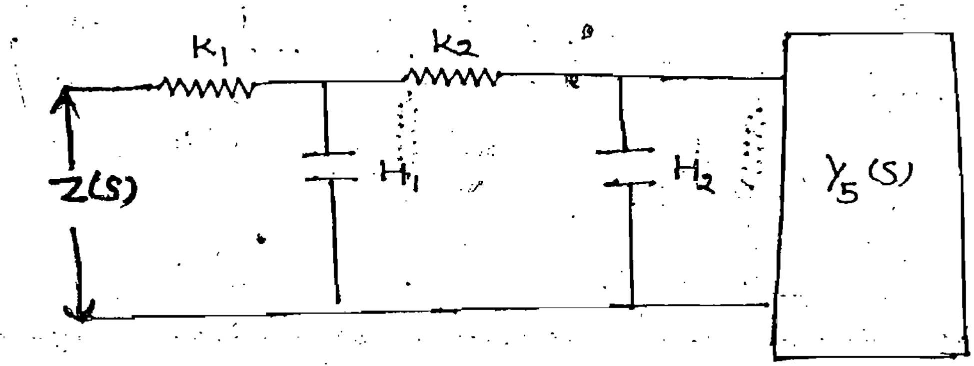
$$Z_3(S) = \frac{1}{Y_3(S)}$$

$$Z_4(S) = Z_3(S) - K_2$$

$$Y_4(S) = \frac{1}{Z_4(S)}$$

$$Y_5(S) = Y_4(S) - H_2 S$$

$$Y_4(S) = Y_5(S) + H_2 S$$



$\rightarrow$ Ladder N/w is obtained

Note:-

Step-I and Step-II are alternatively repeated until the total function is realised

111

RC N/w Cauer-II form!-

LC → $Z_2(s) = Z(s) - \frac{K_{01}}{s}$ $\left(X_c = \frac{1}{Cs}\right)$ $Y_2(s) = \frac{1}{Z_2(s)}$, $Y_3(s) - \frac{K_{02}}{s}$ $\left(B_L = \frac{1}{Ls}\right)$

RC → Step-I:-

Removal of pole at origin from Z(s)

$$Z_2(s) = Z(s) - \frac{K_{01}}{s}$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

Step-II:-

Removal of constant from Y(s)

$$Y_3(s) = Y_2(s) - k_1$$

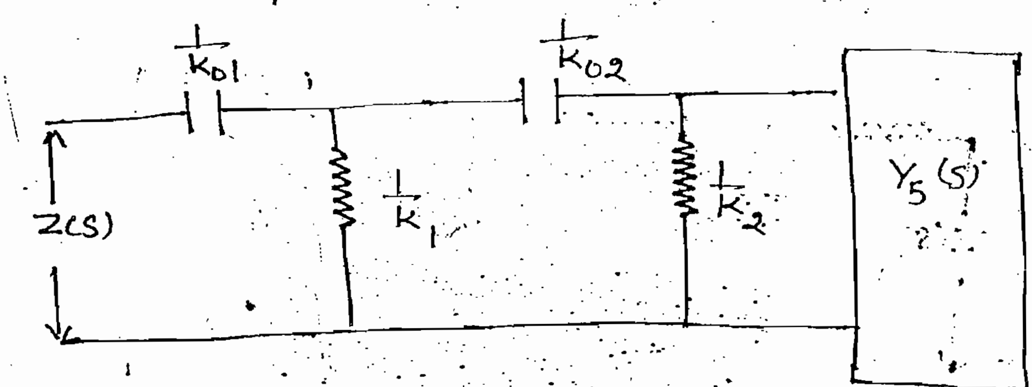
$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$Z_4(s) = Z_3(s) - \frac{K_{02}}{s}$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$

$$Y_5(s) = Y_4(s) - k_2$$

$$Y_4(s) = Y_5(s) + k_2$$



→ Ladder N/w is obtained.

Note!-

Step-I and II are alternatively repeated until the total function is realised.