

Chapter 10

RATIO

Every morning Mohan and Rama take milk in cups. Mohan uses three spoons of sugar for two cups of milk while Rama uses two spoons of sugar for one cup of milk. How do we compare the quantity of sugar in the milk both the children take?

In our everyday life, when we have to buy things, play game or choose the more appropriate option out of two choices, we need to compare the situations. We often need to decide, which vegetable is better and how different their rates or prices are. Let us take an example. Shyam goes to the market to buy potatoes. One shopkeeper prices potatoes at Rs. 20 for 3 kg. The other shopkeeper prices them at Rs. 30 for 5 kg. Shyam is confused. Which is a better option? In such situations we require to think about ratios. Have you ever faced any such situation when you needed to decide about a better choice between almost similar options. Think of some such situations & write them down.

Ratios are indicated in different ways. We indicate them by the “:” symbol.

For example, a shopkeeper says, “This year the sale was twice compared to last year.” By this he means that the sale this year is two times that of last year that is the sale ratio between this year and last year is 2:1.

Take another example, a school has one teacher for 45 students. This means the ratio of students and teacher for that school is 45:1. Now if we say that the school has 90 students, then it means that the school has 2 teachers. This is because the pupil teacher ratio is 45:1 or 90:2.

Reeta says, “The ratio of the number of teachers to the number of students in the school is 1:45. Is she right? In talking about ratios we need to remember which quantity is being compared to which quantity? For example, if we compare the number of teachers with the number of pupils/ students, then the ratio would be 1:45, whereas if we compare the number of pupils to that of teachers the ratio would be 45:1. Now, if there are five teachers in that school in all, what would be the number of students in that school?

ACTIVITY 1

- (1) Write the following statements in ratios :
 - (i) The number of men sitting in a hall are 150 and the number of women are 100.
Write down the ratio of the number of men & women. 150:100
 - (ii) Mr.Sharma is 40 years old & his wife is 35 years old. Write the ratio between their ages.
- (2) A class has 20 boys & 25 girls. State the ratios between:
 - (i) Girls and boys: _____
 - (ii) Boys and girls: _____
 - (iii) Girls and total no. of students : _____
 - (iv) Boys & total no. of students: _____

- (3) Ramesh walks 6km in one hour. Tara walks 4 km in one hour. What is the ratio between the speeds of Ramesh and Tara?

- (4) Ram is 30 years old and Shyam is 20 years old. What would be the ratio between the age of Ram & Shyam?

What would be the ratio between the age of Shyam & Ram?

Think

One year has 20 weeks of rains and 120 days of raining weather in the next year. What would be the ratio of rainy days in both the years?

Can you show it as 20:120? If not, why?

Let us take another example

Rani takes 50 minutes to reach school from her house. Uma takes 1 hour to cover the same distance. What would be the ratio of time taken by Rani and Uma to travel to school? Can we indicate this as 50:___? Think about your response.

Now frame such questions and ask your friends to solve them.

In your every day life, we face many such problems when we compare different measures with one another. Such situations where direct comparisons are not possible, we need to change the comparable quantities into similar units. Look at the example about rainy days we talked about-

In one year the rains are indicated by 20 weeks and in the following year by 120 days. To change this into similar units, we will change 20 weeks into days.

$$\begin{aligned}\text{We know } 1 \text{ week} &= 7 \text{ days} \\ \therefore 20 \text{ weeks} &= 7 \times 20 \\ &= 140 \text{ days}\end{aligned}$$

Now it is easy to compare the ratio of rain in the two years because we have the same units of days i.e. 140 days and 120 days. So the ratio of rainy days in the first and second year would be 140:120 and it can be simplified as 7:6. Can you now find out the ratio between the time taken by Rani and Uma to reach school from their house?

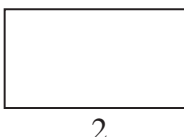
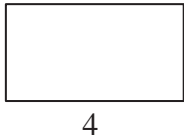
ACTIVITY 2

Write the ratios for the given statements:

- The length of a tree in the picture is 25 cm and the tree is 13 m high (long).
- Ram takes 40 minutes to complete his homework and Shyam takes 1 hour to do his homework.
- Anand came to Raipur after 15 months and Amina came after 2 years.

Can you now find out some problems of unlike or dissimilar units and write them in ratios?

The two quantities that compare in a ratio are known as terms. First term and second term. For example if we are comparing quantity 'a' with quantity 'b', then we will have the first term as 'a' and the second term as 'b' when written as ratio a:b. Whereas when 'b' is compared to 'a', the first term would be 'b' and the second term would be 'a'. In another example given below, the lengths of two rectangles have been shown b:a. Let us compare them.

S. No.	Figures	Sides	Area
1.	1 	Length = 2 units, breadth = 1 unit, ratio of length & breadth = 2:1	$A = 2 \times 1$ = 2 square units
2.	2 	Length = 4 units, breadth = 2 units, ratio of length & breadth = 4:2	$A = 4 \times 2$ = 8 square units

In the figure above, rectangle 1, shows a ratio of 2:1 for their length and breadth.

In fig 2, the ratio of length and breadth = 4:2

The ratio of area in the two cases would be 2:8 or 1:4.

You must have seen the map of India. How is it possible to depict a big country like India on a small map? Think about this.

Below every map, we can see the scale for the map indicated e.g. 1 cm = 100km; which means the distance 100 km has been depicted in the map by 1 cm. Thus, a map is a figure of proportion.

Before you use a ratio, remember to check that the quantities are in the same units.

ACTIVITY 3

1. All the students of the class find out their heights one by one.
2. Students stretch their arms and find the distance between the ends of both the hands.
3. What relationship do you observe between the height of each student and the distance between the ends of their arms.

Complete the table.

S. No.	Name of the student	Height	Distance between the ends of arms stretched	Ratio	Simplified form
1.					
2.					
3.					
4.					

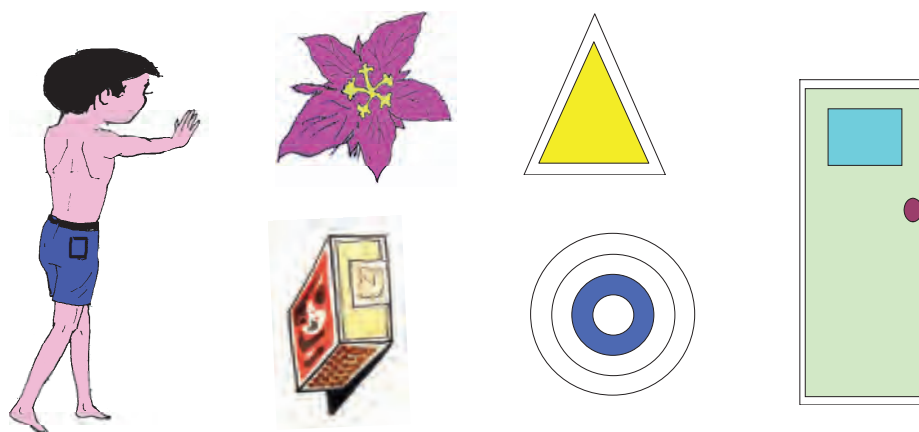
Draw the conclusions.

Aesthetics of Proportionate Figures

Ratio does not take place only in number but in many other examples around us. For example, if the legs of a man shown in a picture are very long or the head is larger than the body.

Then how does the man in the picture look? Naturally, it looks awkward. Similarly, suppose your class has a wall which has a short length & breadth and on that small wall a very big picture is put up or the wall is very big and a very small picture is put up at one end or in the middle of that wall, or a big frame has a very small picture in it, then most of the time, it would seem very awkward or unusual.

This is because all the above descriptions lack proportion. Our eyes are accustomed to see things in a definite proportion. Given below are a few proportionate & unproportionate figures. Identify the pictures in which you find suitable proportion or ratio and try to think about the reasons that made you leave the pictures that you didn't select.



Comparing two Quantities

Together a picture is proportionate or not is decided by comparing the shapes of different parts of the pictures. For example, the plank of a door can be very long, a match box wouldn't look like a match box if one of its sides become longer in length than the other sides. We can compare such situations in several ways and base the comparisons on different aspects.

The easiest way is to find out how big or small is the first quantity is from the second one. This information can be obtained in many different ways.

1. Bhawna get 40 marks & Renu get 20 marks in a test. Which means Renu get 20 marks less than Bhawna.
2. Out of 600 students, 200 students did not come to school on a day, it means only 400 students came to the school that day.
3. If two line segments measure 8 cm and 4 cm respectively, then the first line segment is 4 cm longer than the second line segment.
4. The height of a door is 8 feet and its width is 2 feet means that its length is 6 feet more than its width. Saying that the difference between the length & breadth of the door is 6 feet does not indicate whether the measurement is proportionate because if a door is 18 feet high & 12 feet wide, the difference is yet 6 feet but the two size are very different from each other. There can be another basis of comparing the quantities. We can also see how many times the second quantity is the first or how many times is the first quantity of the second. Thus the height of the door in the first case is 4 times the width, whereas in the case of the second door, the height of the door is $1\frac{1}{2}$ times its width.

We can find many such examples to see how many times is one quantity of the other.

1 st quantity	2 nd quantity	How many times of the first quantity is the second quantity	How many times of the second quantity is the first quantity
2 cm	6 cm	3 times	$\frac{1}{3}$ times
500 g	1000 g	2 times	$\frac{1}{2}$ times
200 rupees	1000 rupees	5 times	$\frac{1}{5}$ times
5 litre	20 litre	4 times	$\frac{1}{4}$ times
4 metre	32 metre	8 times	$\frac{1}{8}$ times
3 metre	5 metre	$\frac{5}{3}$ times or 5:3	$\frac{3}{5}$ times or 3:5

Now tell about some examples from your everyday experience where we need to know how many times of one quantity is the other quantity. The length of your room is 30 feet and its width is 15 feet. This means the length of the room is twice its width. We can write this as :

$$\frac{\text{Length of the room}}{\text{Width of the room}} = \frac{30 \text{ feet}}{15 \text{ feet}} = \frac{2}{1} = 2 \text{ times}$$

This means length is twice the width and the ratio of length & breadth is 2:1. We can also say that the ratio of breadth & length is 1:2.

Therefore, ratio is a relationship based on quantity.

1. The ratio between 50 books and 10 books = 50:10 = 5:1.
2. Ram is 20 yrs old and Shyam is 30 yrs old. The ratio between their age is 20:30 = 2:3.
3. The ratio between 400 kg wheat & 100 kg wheat = 400:100 = 4:1.
4. The ratio between quantity a & quantity b = a:b.

Practice

S. No.	Name of the school	No. of teachers	No. of pupils	Ratio	Simplified form
1.	Govt. Model Middle School	06	150		
2.	Bhagat Singh M.S.	10	350		
3.	Paramount M. S.	15	600		
4.	Lakshmibai Girls School	08	264		

Some Points to Remember About Ratios

1. If the ratio between two quantities 'a' and 'b' is shown by a:b, then 'a' is the first term and 'b' the second term (where 'a' & 'b' are whole numbers).
2. Like quantities in a ratio are indicated in the same units.

ACTIVITY 4

Shally and her family members were standing near a tree in an open pasture. Shally saw the shadows on the ground & began to measure them. The heights of the family members and the lengths of their shadows have been given in the table below.

S. No.	Members	Length of the shadow	Length of the Member
1.	Father	92 cm	184 cm
2.	Mother	80 cm	160 cm
3.	Brother	45 cm	90 cm
4.	Self	75 cm	150 cm
5.	Tree	215 cm	-----

Now Shally had a problem. She knew the heights of her family members & could measure the lengths of their shadow, but she could measure only the length of the shadow of the tree because it was difficult for her to measure its height.

Can she find out the height of the tree with the help of the length of its shadow? Let us see how Shally solved her problem?

She noticed a relationship between the numbers in the table. She found that the height of every person is twice the length of the shadow. So, she thought that if the ratio of the height of every person & the length of his/her shadow is 2:1, then height of the tree also should be twice the length of its shadow. Thus she could find out that the height of the tree was 430cm.

Example 1.

A person gave 25 rupees to his son and 36 rupees to his daughter. Find out the ratio of money given by the person ?

Solution : Son's share = 25 rupees
 Daughter's share = 36 rupees
 Son's share : Daughter's share
 \Rightarrow 25:36.

Example 2.

A stick is 90 cm long and a bamboo is 4 m and 50cm long. Find out the ratio between the stick and the bamboo ?

Solution: Length of the stick = 90cm
 Length of the bamboo = 4m 50cm
 $= 400\text{cm} + 50\text{cm}$ (\because 1 meter = 100cm)
 $= 450\text{cm}$

[We have converted the length of the stick and the bamboo into the same units (cm)]

Therefore, the length of the stick : length of the bamboo.

$$= 90 : 450$$

$$= 1 : 5 \text{ (simplified form)}$$

Example 3.

Rajesh earns Rs. 12500/- per month. Out of this, he saves Rs. 2500. Find out (i) the ratio of Rajesh's earning & expenditure. (ii) Rajesh's earning and savings.

Solution:

Rajesh's monthly income	= 12500 Rs.
Rajesh's monthly savings	= 2500 Rs.
Rajesh's monthly expenditure	= 12500 - 2500 rupees
	= 10,000 rupees

Therefore, the ratio of Rajesh's income and his expenditure = 12500 : 10000

$$\begin{aligned} \text{The ratio of Rajesh's income and his savings} &= 12500 : 2500 \\ &= 5 : 1 \end{aligned}$$

Example 4.

Find the ratio of the cost of a pen and pencil. When the price of pens are Rs. 144 per dozen and Rs. 90 for 10 pencil ?

Solution: Here, we will have to calculate the cost of one pen and one pencil first.

$$\text{One dozen or 12 pens cost} = \text{Rs. 144}$$

$$\therefore \quad 1 \text{ pen will cost } \frac{144}{12} = \text{Rs. 12}$$

$$10 \text{ pencil cost} = \text{Rs. 90}$$

$$\therefore \quad 1 \text{ pencil will cost } \frac{90}{10} = \text{Rs. 9}$$

Thus the ratio -

cost of 1 pen : cost of pencil

$$= \frac{12}{9} = \frac{4}{3}$$

$$= 4 : 3$$

Example 5.

40 toffees have to distribute between Chhotu and Mintoo in the ratio 4:1.

Solution: Total number of divisions = 4 + 1 = 5 or 40 toffees are divided into 5 parts, Chhotu will get 4 parts and Mintoo will get 1 part of the total number of toffees.

$$\begin{aligned} \text{Therefore, Chhotu's share} &= \frac{40}{5} \times 4 \\ &= 32 \text{ toffees} \end{aligned}$$

$$\begin{aligned} \text{Mintoo's share} &= \frac{40}{5} \times 1 \\ &= 8 \text{ toffees.} \end{aligned}$$

Example 6.

Find out ratio of the following measures:

- (a) Rs. 5 and 50 paise.
- (b) 500 cm and 10 meter.
- (c) 8 kilogram and 640 gram.

Solution:

- (a) The ratio of Rs. 5 and 50 paise.

$$\begin{aligned} 5 \text{ Rs.} &= 5 \times 100 \text{ paise} \\ &= 500 \text{ paise} \end{aligned}$$

$$\therefore \text{Ratio} = \frac{500}{50} = \frac{10}{1} = 10:1$$

- (b) The ratio of 500 cm and 10 meter.

$$\begin{aligned} 10 \text{ meter} &= 10 \times 100 \text{ cm} = 1000 \text{ cm} \\ 500 \text{ cm} : 1000 \text{ cm} \\ &1 : 2 \end{aligned}$$

- (c) The ratio of 8 kg and 600 g

$$\begin{aligned} 8 \text{ kilogram} &= 8 \times 1000 \text{ gram} \\ &= 8000 \text{ gram} \end{aligned}$$

$$\therefore \text{Ratio} = \frac{8000}{640} = \frac{25}{2} = 25:2$$

EXERCISE 10.1

1. Collect things around you. Find out the length of the objects and find the ratio of lengths between the different objects. Write them in simplified form.

New pencil _____

Pen _____

Nail _____

Alpin _____



Find the ratio of lengths of :

- | | |
|---------------------|-------------------------|
| (i) alpin and nail | (ii) nail and pencil |
| (iii) pen and nail | (iv) nail and alpin |
| (v) alpin and pen | (vi) pen and pencil |
| (vii) pen and alpin | (viii) pencil and alpin |

2. Find out the ratio of the following:

- | | |
|------------------------------------|-----------------------------|
| (i) 15 minutes and 1 hour | (ii) 250 gms and 1 kilogram |
| (iii) 15 paise and 1 rupee | (iv) 12 paise and rupees 5 |
| (v) $2\frac{1}{2}$ cms and 1 meter | (vi) 10 meters and 25 cms |
| (vii) 40 cm and 2.5 meters. | |

3. Write each of the ratios in simplified forms:

- | | | |
|-----------------|------------------|-------------------------|
| (i) 50 : 400 | (ii) 85 : 255 | (iii) 1 dozen : 1 score |
| (iv) 27 : 57 | (v) 24 : 68 | (vi) 250 : 375 |
| (vii) 65 : 91 | (viii) 2.5 : 7.5 | (ix) 50 : 255 |
| (x) 500 : 10000 | | |

4. Vishakha's annual income is Rs. 80,000; out of which she gave Rs. 5000 as income tax. Find the ratios of :

- | | |
|-------------------------|--------------------------|
| (i) income tax : income | (ii) income : income tax |
|-------------------------|--------------------------|

5. Munnu and Bunnu participated in race. In the given time Munnu covered 210 meter and Bunnu covered 180 meter in the same time. What will be the ratio of the distance covered by Munnu and Bunnu in the race?
6. Satish is a scientist and earns Rs. 20000 per month. His wife Anita is a doctor and earns rupees Rs. 15000 per month. Find put the ratios of :
- (i) Satish's income : Anita's income
- (ii) Satish's income : Their total income
7. The number of students in a school is 1500. Out of this 600 are girls. Find out the ratio of the number of boys and girls in the school.
8. Divide 20 balloons in the ratio of 2:3 between Bhanu and Bangaroo. How many balloons will Bhanu and Bangaroo get?
9. Rajesh and Javed together opened a shop. In this shop Rajesh's share was Rs. 45000 while Javed gave Rs. 36000. Find out the investment ratio of Rajesh and Javed.
10. In an examination out of 117 candidates 65 failed. Find out the ratio of successful and unsuccessful candidates.
11. Ratna and Sheela picked 18 mangoes. Both now wish to divide the mangoes between each other. Ratna wants the mangoes to be distributed in the ratio of their ages. Find out how many mangoes would Ratna & Sheela get if it is divided in this manner, when Ratna is 15 years old and Sheela is 12 years old?
12. The present age of a father is 50 years & that of a son is 20 years. Find the ratio of :
- (i) present age of the father and the son.
- (ii) both their ages when the son's age was 10 years.
- (iii) both their ages when the father's age was 35 years.
- (iv) both their ages when the son's age is 40 years.
- (v) both their ages when the father's age was 75 years.

13. The ratio of income of Ram & Shyam is 3:4. If their total income is Rs. 21000. Find out the earnings of Ram & Shyam individually.
14. Point B is placed in between A & C in such a way that AB:BC is equal to 7:3. If AC = 40 km. Find out the values of AB and BC.
15. I have 6 samosas which I wish to divide with my friends. If
 - i) I divide it in the ratio 1:1 between me and my friend, how many samosas will each one of us get?
 - ii) I divide it in the ratio 2:1:3 with my two friends, how many samosas will each one of us get?

UNITARY METHOD

Look at some situations given below :-

1. You go to the market and buy two copies for Rs. 20. Now, if you need 5 more copies, how much money should you have?
2. Shyam has 2 litre of petrol in his scooter. He thinks that he can easily travel 50 kilometre with that amount of petrol. He has to travel 100 kilometre, what is the minimum amount of petrol that he needs to have in his vehicle to travel that distance?
3. On your birthday, you wish to give a small token gift to your friends. You choose a small car in the toy shop for a gift. If 3 toys cost Rs.75 and you need to buy 15 such toy cars how much money should you have?

Let us now think about the solutions to the above situations :

If two copies costs Rs. 20

Then 1 copy costs $\frac{20}{2} = \text{Rs.}10$

Now if 1 copy costs Rs. 10

5 copy would cost = $10 \times 5 = \text{Rs. } 50$

Now are these methods to solve the other two situations also.

You can try several such problems with your friends.

In such conditions where the cost of many objects is known and the cost of one object is found in order to get the cost of the number of objects asked for, unitary method or unitary law is used.

ACTIVITY 5

Observe the given table & fill in the blanks.

Time taken	Distance covered on foot (km)	Distance covered by bicycle (km)	Distance covered by car (km)	Distance covered by train (km)
2 hours	8	20	70	120
1 hours	4	—	—	—
5 hours	20	—	—	—

We find that :

Distance travelled in 2 hours = 8 km

Distance travelled in 1 hour = $\frac{8}{2} = 4$ km

Distance travelled in 5 hours = $4 \times 5 = 20$ km

Here to find out the distance covered in 5 hours by cycle, car, train or on foot, first we will have to find out the distance travelled in 1 hour.

Example 7

In a hostel the consumption of rice for 8 students is 4 kilogram, how much rice would be consumed by 30 students ?

Solution :

We shall solve this problem in 2 steps :

Step 1: First, we shall find out how much rice is consumed by 1 student .

When 8 students consume 4 kg of rice

1 student consumes $\frac{4}{8} = \frac{1}{2}$ kg of rice.

Step 2: Now from this, we shall find out the consumption of rice by 30 students.

\therefore 30 students would consume $\frac{1}{2} \times 30 = 15$ kilogram

Thus, 30 students would consume 15 kg of rice.

Example 8

An aeroplane flies 4000 km in 5 hours. How much distance does it fly in 3 hours?

Solution :

We shall find out the distance covered by the aeroplane in one hour and in 2nd step, the distance covered in the asked period of time (3 hours) can be determined.

Step 1: Distance covered in 5 hours = 4000 km

Distance covered in 1 hour = $\frac{4000}{5}$ km
= 800 km

$$\begin{aligned}
 \text{Step 2: Distance covered in 1 hours} &= 800 \text{ km} \\
 \text{Distance covered in 3 hours} &= 800 \times 3 \text{ km} \\
 &= 2400 \text{ km}
 \end{aligned}$$

Therefore, the aeroplane would fly 2400 km in 3 hours.

Example 9

A woman saves Rs. 18000 in 15 months.

- (i) What would be her savings in 7 months?
- (ii) In how many months will she save Rs. 30,000?

Solution :

$$\text{Step 1: Savings in 15 months} = \text{Rs}18,000$$

$$\begin{aligned}
 \therefore \text{Savings in 1 month} &= \frac{18000}{15} \\
 &= \text{Rs}1200
 \end{aligned}$$

$$\text{Step 2: Savings in One month} = \text{Rs}1200$$

$$\therefore \text{Savings in 7 months} = 1200 \times 7 = \text{Rs}.8400$$

Again Rs.1200 are saved in 1 month.

$$\text{Rs}.30,000 \text{ are saved in } \frac{1}{1200} \times 30,000 = 25 \text{ months.}$$

EXERCISE 10.2

1. Three copies cost Rs.16.5 What will be the cost of 7 copies?
2. A car moves 165 kms in 3 hours. Then find out
 - (i) How much time would it need to move 440 kms?
 - (ii) How much distance will be covered by the car in $6\frac{1}{2}$ hours?
3. 72 books weight 9 kilograms ?
 - (i) Find the weigh of 80 books.
 - (ii) How many books would weight 6 kgs?
4. A worker earns Rs1500 in 25 days. Find out his income in 30 days ?
5. If 22 metres of cloth cost Rs704 , what would be the cost of 20 metres of cloth?
6. Complete the given table :

Number of books	Price (in Rupees)
50	2500
75	-----
-----	100
-----	3000

What Have We Learnt ?

- (1) Ratio of two similar quantities shows how many times is one quantity of the other.
- (2) The ratio of two quantities is generally written in its simplified form. For example $na : nb$ is written as $a : b$.
- (3) When the price or value of a unit quantity is found out from the given number of the quantity and then the value/ price for the asked number of quantity is determined, the method is known as the unitary method.

