Coordinate Geometry

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 7.1

Q. 1. *Find the distance between the following pairs of points:*

(i) (2, 3), (4, 1) *(ii) (– 5, 7), (– 1, 3)* *(iii) (a, b), (– a, – b)*

Sol. (*i*) Here $x_1 = 2,\ y_1 = 3,\ x_2 = 4$ and $y_2 = 1$

∴ The required distance

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-2)^2 + (1-3)^2}$$

$$= \sqrt{2^2 + (-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8}$$

$$= \sqrt{2 \times 4} = \mathbf{2\sqrt{2}}$$

(*ii*) Here, $x_1 = -5, \quad y_1 = 7$
$x_2 = -1, \quad y_2 = 3$

∴ The required distance

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[-1-(-5)]^2 + (3-7)^2}$$

$$= \sqrt{(-1+5)^2 + (-4)^2}$$

$$= \sqrt{16+16}$$

$$= \sqrt{32} = \sqrt{2 \times 16} = \mathbf{4\sqrt{2}}$$

(*iii*) Here, $x_1 = a, \quad y_1 = b$
$x_2 = -a, \quad y_2 = -b$

∴ The required distance

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-a-a)^2 + (-b-b)^2}$$

$$= \sqrt{(-2a)^2 + (-2b)^2}$$

$$= \sqrt{4a^2 + 4b^2}$$

$$= \sqrt{4\left(a^2 + b^2\right)} = \mathbf{2\sqrt{\left(a^2 + b^2\right)}}$$

Q. 2. *Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2 of the NCERT textbook?*

Sol. Part-I

Let the points be P (0, 0) and Q (36, 15).

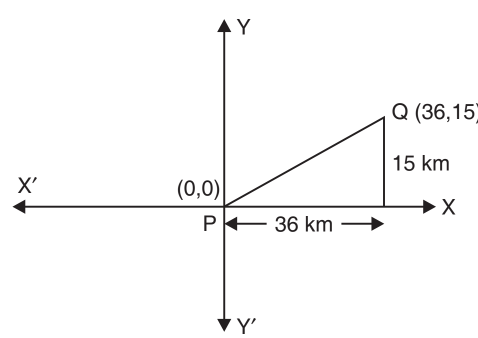
$$\therefore \quad PQ = \sqrt{(36-0)^2 + (15-0)^2}$$

$$= \sqrt{(36)^2 + (15)^2}$$

$$= \sqrt{1296 + 225}$$

$$= \sqrt{1521}$$

$$= \sqrt{39^2} = \mathbf{39}$$



Part-II

We have P (0, 0) and Q (36, 15) as the positions of two towns.

∵ Here $x_1 = 0, \quad x_2 = 36$
$y_1 = 0, \quad y_2 = 15$

$$\therefore \quad PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(36-0)^2 + (15-0)^2} = \textbf{39 km}.$$

Q. 3. *Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.*

Sol. Let the points be A (1, 5), B (2, 3) and C (– 2, – 11)

A, B and C are collinear, if

$$AB + BC = AC$$
$$AC + CB = AB$$
$$BA + AC = BC$$

$$\because \quad AB = \sqrt{(2-1)^2 + (3-5)^2}$$

$$= \sqrt{1^2 + (-2)^2}$$

$$= \sqrt{1+4} = \sqrt{\mathbf{5}}$$

$$BC = \sqrt{(-2-2)^2 + (-11-3)^2}$$

$$= \sqrt{(-4)^2 + (-14)^2}$$

$$= \sqrt{16+196} = \sqrt{\mathbf{212}}$$

$$AC = \sqrt{(-2-1)^2 + (-11-5)^2}$$

$$= \sqrt{(-3)^2 + (-16)^2}$$

$$= \sqrt{9+256} = \sqrt{\mathbf{265}}$$

But
$$AB + BC \neq AC$$
$$AC + CB \neq AB$$
$$BA + AC \neq BC$$

$\therefore$ A, B and C are **not collinear.**

Q. 4. *Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.*

Sol. Let the points be A (5, – 2), B (6, 4) and C (7, – 2). (CBSE 2012)

$$\therefore \quad AB = \sqrt{(6-5)^2 + [4-(-2)]^2}$$

$$= \sqrt{(1)^2 + (6)^2}$$

$$= \sqrt{1+36} = \sqrt{\mathbf{37}}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2}$$

$$= \sqrt{(1)^2 + (-6)^2}$$

$$= \sqrt{1+36} = \sqrt{\mathbf{37}}$$

$$AC = \sqrt{(5-7)^2 + (-2-(-2))^2}$$

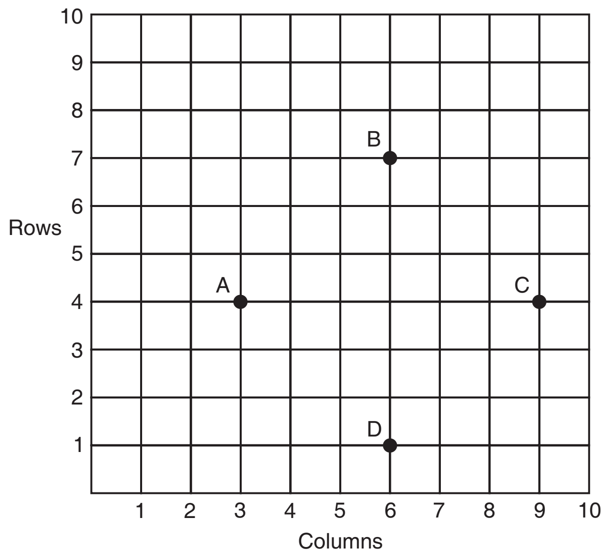
$$= \sqrt{(-2)^2 + (0)^2}$$

$$= \sqrt{4+0} = \mathbf{2}$$

We have $AB = BC \neq AC$

$\therefore$ Δ ABC is **an isosceles triangle.**

Q. 5. *In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, ''Don't you think ABCD is a square?'' Chameli disagrees. Using distance single formula, find which of them is correct.*



Sol. Let the number of horizontal columns represent the x-coordinates whereas the vertical rows represent the y-coordinates.

$\therefore$ The points are:

A (3, 4), B (6, 7), C (9, 4) and D (6, 1)

$$\therefore \quad AB = \sqrt{(6-3)^2 + (7-4)^2}$$

$$= \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = \mathbf{3\sqrt{2}}$$

$$BC = \sqrt{(9-6)^2 + (4-7)^2}$$

$$= \sqrt{3^2 + (-3)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = \mathbf{3\sqrt{2}}$$

$$CD = \sqrt{(6-9)^2 + (1-4)^2}$$

$$= \sqrt{(-3)^2 + (-3)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = \mathbf{3\sqrt{2}}$$

$$AD = \sqrt{(6-3)^2 + (1-4)^2}$$

$$= \sqrt{(3)^2 + (-3)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = \mathbf{3\sqrt{2}}$$

Since, $AB = BC = CD = AD$

i.e., All the four sides are equal.

Also $$AC = \sqrt{(9-3)^2 + (4-4)^2}$$

$$= \sqrt{(-6)^2 + (0)^2} = 6$$

and $$BD = \sqrt{(6-6)^2 + (1-7)^2}$$

$$= \sqrt{(0)^2 + (-6)^2} = 6$$

i.e., $BD = AC \Rightarrow$ Both the diagonals are also equal.

$\therefore$ *ABCD* is a square.

Thus, Champa is correct.

Q. 6. *Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:*

(i) $(-1, -2), (1, 0), (-1, 2), (-3, 0)$

(ii) $(-3, 5), (3, 1), (0, 3), (-1, -4)$

(iii) $(4, 5), (7, 6), (4, 3), (1, 2)$

Sol. (*i*) Let the points be: $A(-1, -2)$, $B(1, 0)$, $C(-1, 2)$ and $D(-3, 0)$.

$\therefore$ $$AB = \sqrt{(1+1)^2 + (0+2)^2}$$

$$= \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$$

$$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8}$$

$$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8}$$

$$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8}$$

$$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+4^2} = 4$$

$$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{(4)^2} = 4$$

$\Rightarrow$ $AB = BC = CD = AD$

i.e., All the sides are equal.

And $AC = BD$

Also, *AC* and *BD* (the diagonals) are equal.

$\therefore$ *ABCD* **is a square.**

(*ii*) Let the points be $A(-3, 5)$, $B(3, 1)$, $C(0, 3)$ and $D(-1, -4)$.

$\therefore$ $$AB = \sqrt{[3-(-3)]^2 + (1-5)^2}$$

$$= \sqrt{6^2 + (-4)^2}$$

$$= \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(0-3)^2 + (3-1)^2}$$

$$= \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{(-1-0)^2 + (-4-3)^2}$$

$$= \sqrt{(-1)^2 + (-7)^2} = \sqrt{1+49} = \sqrt{50}$$

$$DA = \sqrt{[-3-(-1)]^2 + [5-(-4)]^2}$$

$$= \sqrt{(2)^2 + (9)^2}$$

$$= \sqrt{4+81} = \sqrt{85}$$

$$AC = \sqrt{[0-(-3)]^2 + (3-5)^2}$$

$$= \sqrt{(3)^2 + (-2)^2}$$

$$= \sqrt{9+4} = \sqrt{13}$$

$$BD = \sqrt{(-1-3)^2 + (-4-1)^2} = \sqrt{(-4)^2 + (-5)^2}$$

$$= \sqrt{16+25} = \sqrt{41}$$

We see that:

$$\sqrt{13} + \sqrt{13} = 2\sqrt{13}$$

i.e., $AC + BC = AB$

⇒ *A, B, C* and *D* are collinear. Thus, *ABCD* is **not a quadrilateral**.

(*iii*) Let the points be A (4, 5), B (7, 6), C (4, 3) and D (1, 2).

∴ $$AB = \sqrt{(7-4)^2 + (6-5)^2}$$

$$= \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$BC = \sqrt{(4-7)^2 + (3-6)^2}$$

$$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$$

$$CD = \sqrt{(1-4)^2 + (2-3)^2}$$

$$= \sqrt{(-3)^2 + (-1)^2} = \sqrt{10}$$

$$DA = \sqrt{(4-1)^2 + (5-2)^2}$$

$$= \sqrt{9+9} = \sqrt{18}$$

$$AC = \sqrt{(4-4)^2 + (3-5)^2}$$

$$= \sqrt{0 + (-2)^2} = 2$$

$$BD = \sqrt{(1-7)^2 + (2-6)^2}$$

$$= \sqrt{36+16} = \sqrt{52}$$

Since, $AB = CD$ [opposite sides of the quadrilateral are equal]

$BC = DA$

And $AC \neq BD \Rightarrow$ Diagonals are unequal

$\therefore$ $ABCD$ is **a parallelogram.**

Q. 7. *Find the point on the x-axis which is equidistant from (2, – 5) and (– 2, 9).* (CBSE 2012)

Sol. We know that any point on x-axis has its ordinate = 0.

Let the required point be P $(x, 0)$.

Let the given points be A (2, – 5) and B (– 2, 9).

$$\therefore \quad PA = \sqrt{(x-2)^2 + [0-(-5)]^2}$$

$$= \sqrt{(x-2)^2 + 5^2} = \sqrt{x^2 - 4x + 4 + 25} = \sqrt{x^2 - 4x + 29}$$

$$PB = \sqrt{[x-(-2)]^2 + (0-9)^2}$$

$$= \sqrt{(x+2)^2 + (-9)^2} = \sqrt{x^2 + 4x + 4 + 81} = \sqrt{x^2 + 4x + 85}$$

Since, A and B are equidistant from P,

$$\therefore \quad PA = PB$$

$$\Rightarrow \quad \sqrt{x^2 - 4x + 29} = \sqrt{x^2 + 4x + 85}$$

$$\Rightarrow \quad x^2 - 4x + 29 = x^2 + 4x + 85$$

$$\Rightarrow \quad x^2 - 4x - x^2 - 4x = 85 - 29$$

$$\Rightarrow \quad -8x = 56$$

$$\Rightarrow \quad x = \frac{56}{-8} = -7$$

$\therefore$ The required point is **(– 7, 0).**

Q. 8. *Find the values of y for which the distance between the points P (2, – 3) and Q (10, y) is 10 units.*

Sol. The given points are P (2, – 3) and Q (10, y).

$$\therefore \quad PQ = \sqrt{(10-2)^2 + [y-(-3)]^2}$$

$$= \sqrt{8 + (y+3)^2}$$

$$= \sqrt{64 + y^2 + 6y + 9}$$

$$= \sqrt{y^2 + 6y + 73}$$

But $PQ = 10$

$$\therefore \quad \sqrt{y^2 + 6y + 73} = 10$$

Squaring both sides,

$$y^2 + 6y + 73 = 100$$

$$\Rightarrow \quad y^2 + 6y - 27 = 0$$

$$\Rightarrow \quad y^2 - 3y + 9y - 27 = 0$$

$$\Rightarrow \quad (y-3)(y+9) = 0$$

$\Rightarrow$ Either $y - 3 = 0 \Rightarrow y = 3$

or $y + 9 = 0 \Rightarrow y = -9$

$\therefore$ The required value of y is **3 or – 9.**

Q. 9. *If Q (0, 1) is equidistant from P (5, – 3) and R (x, 6), find the values of x. Also find the distances QR and PR.*

Sol. Here,

$$QP = \sqrt{(5-0)^2 + [(-3)-1]^2}$$
$$= \sqrt{5^2 + (-4)^2}$$
$$= \sqrt{25+16} = \sqrt{41}$$
$$QR = \sqrt{(x-0)^2 + (6-1)^2}$$
$$= \sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}$$

$\because$ $QP = QR$

$\therefore$ $\sqrt{41} = \sqrt{x^2 + 25}$

Squaring both sides, we have:

$$x^2 + 25 = 41$$
$$\Rightarrow \quad x^2 + 25 - 41 = 0$$
$$\Rightarrow \quad x^2 - 16 = 0 \quad \Rightarrow \quad x = \pm 4$$

Thus, the point R is **(4, 6) or (– 4, 6)**

Now,
$$QR = \sqrt{[(\pm 4)-(0)]^2 + (6-1)^2}$$
$$= \sqrt{16+25} = \mathbf{\sqrt{41}}$$

and
$$PR = \sqrt{(\pm 4 - 5)^2 + (6+3)^2}$$
$$\Rightarrow \quad PR = \sqrt{(4-5)^2 + (6+3)^2} \text{ or } \sqrt{(-4-5)^2 + (6+3)^2}$$
$$\Rightarrow \quad PR = \sqrt{1+81} \text{ or } \sqrt{(-9)^2 + 9^2}$$
$$\Rightarrow \quad PR = \sqrt{82} \text{ or } \sqrt{2 \times 9^2}$$
$$\Rightarrow \quad PR = \mathbf{\sqrt{82}} \textit{ or } \mathbf{9\sqrt{2}}$$

Q. 10. *Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).*

Sol. Let the points be A (x, y), B (3, 6) and C (– 3, 4).

$$\therefore \quad AB = \sqrt{(3-x)^2 + (6-y)^2}$$

And
$$AC = \sqrt{[(-3)-x]^2 + (4-y)^2}$$

Since, the point (x, y) is equidistant from (3, 6) and (– 3, 4).

$$\therefore \quad AB = AC$$
$$\Rightarrow \quad \sqrt{(3-x)^2 + (6-y)^2} = \sqrt{(-3-x)^2 + (4-y)^2}$$

Squaring both sides,

$$(3-x)^2 + (6-y)^2 = (-3-x)^2 + (4-y)^2$$
$$\Rightarrow (9 + x^2 - 6x) + (36 + y^2 - 12y) = (9 + x^2 + 6x) + (16 + y^2 - 8y)$$
$$\Rightarrow 9 + x^2 - 6x + 36 + y^2 - 12y - 9 - x^2 - 6x - 16 - y^2 + 8y$$
$$\Rightarrow -6x - 6x + 36 - 12y - 16 + 8y = 0$$

$$\Rightarrow \quad -12x - 4y + 20 = 0$$

$$\Rightarrow \quad -3x - y + 5 = 0 \qquad \text{[Dividing by 4]}$$

$$\Rightarrow \quad 3x + y - 5 = 0$$

which is the required relation between x and y.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 7.2

Q. 1. *Find the coordinates of the point which divides the join of (– 1, 7) and (4, – 3) in the ratio 2 : 3.*

Sol. Let the required point be $P(x, y)$.

Here, the end points are:

(– 1, 7) and (4, – 3)

$\because$ Ratio = 2 : 3 = $m_1 : m_2$

$$\therefore \quad x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$= \frac{(2 \times 4) + 3 \times (-1)}{2 + 3}$$

$$= \frac{8 - 3}{5} = \frac{5}{5} = 1$$

And

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

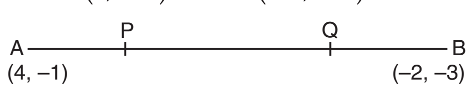
$$= \frac{2 \times (-3) + (3 \times 7)}{2 + 3}$$

$$= \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Thus, the required point is **(1, 3).**

Q. 2. *Find the coordinates of the points of trisection of the line segment joining (4, – 1) and (– 2, – 3).*

Sol. Let the given points be $A(4, -1)$ and $B(-2, -3)$.



Let the points P and Q trisect AB.

i.e., $AP = PQ = QB$

i.e., P divides AB in the ratio of 1 : 2

Q divides AB in the ratio of 2 : 1

Let the coordinates of P be (x, y).

$$\therefore \quad x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$= \frac{1(-2) + 2(4)}{1 + 2} = \frac{-2 + 8}{3} = 2$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = 1$$

$$= \frac{1(-3)+2\times(-1)}{1+2} = \frac{-3-2}{3} = \frac{-5}{3}$$

∴ The required co-ordinates of P are $\left(\mathbf{2}, \frac{\mathbf{-5}}{\mathbf{3}}\right)$

Let the co-ordinates of Q be (X, Y).

∴ $$X = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{2(-2)+1(4)}{2+1} = \frac{-4+4}{3} = 0$$

$$Y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$= \frac{2(-3)+1(-1)}{2+1} = \frac{-6+-1}{3} = \frac{-7}{3}$$

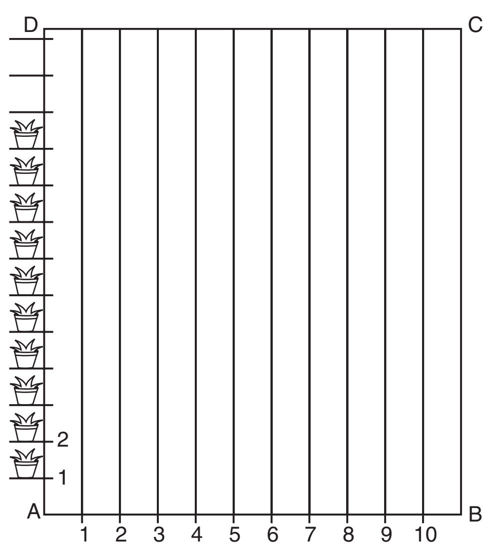
∴ The required coordinates of Q are $\left(\mathbf{0}, \frac{\mathbf{-7}}{\mathbf{3}}\right)$.

Q. 3. *To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the figure. Niharika runs* $\frac{1}{4}$ *th the distance AD on the 2nd line and posts a green flag. Preet runs* $\frac{1}{5}$ *th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?*

Sol. Let us consider 'A' as origin, then AB is the x-axis. AD is the y-axis.

Now, the position of green flag-post is $\left(2, \frac{100}{4}\right)$ or (2, 25)

And the position of red flag-post is $\left(8, \frac{100}{5}\right)$ or (8, 20)

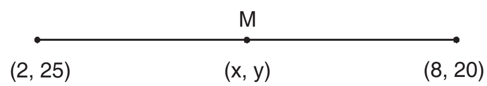


⇒ Distance between both the flags

$$= \sqrt{(8-2)^2 + (20-25)^2}$$

$$= \sqrt{6^2 + (-5)^2} = \sqrt{36+25} = \sqrt{61}$$

Let the mid-point of the line segment joining the two flags be M (x, y).



$$\therefore \quad x = \frac{2+8}{2} \quad \text{and} \quad y = \frac{25+20}{2}$$

or $\quad x = 5 \quad$ and $\quad y = (22.5)$.

Thus, the blue flag is on the 5th line at a distance **22.5 m** above AB.

Q. 4. *Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).*

Sol. Let the given points are: A (– 3, 10) and B (6, – 8).

Let the point P (– 1, 6) divides AB in the ratio $m_1 : m_2$.

∴ Using the section formula, we have:

$$(-1, 6) = \left(\frac{x_2\, m_1 + m_2\, x_1}{m_1 + m_2}, \frac{m_1\, y_2 + m_2\, y_1}{m_1 + m_2}\right)$$

$$\Rightarrow \quad (-1, 6) = \left(\frac{(m_1 \times 6) + [m_2 \times (-3)]}{m_1 + m_2}, \frac{[m_1(-8)] + (m_2 \times 10)}{m_1 + m_2}\right)$$

$$\Rightarrow \quad (-1, 6) = \frac{6m_1 + (-3m_2)}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}$$

$$\Rightarrow \quad -1 = \frac{6m_1 - 3m_2}{m_1 + m_2} \quad \text{and} \quad 6 = \frac{-8m_1 + 10m_2}{m_1 + m_2}$$

$$\Rightarrow \quad -1\,(m_1 + m_2) = 6m_1 - 3m_2 \quad \text{and} \quad 6\,(m_1 + m_2) = -8m_1 + 10m_2$$

$$\Rightarrow -m_1 - m_2 - 6m_1 + 3m_2 = 0 \quad \text{and} \quad 6m_1 + 6m_2 + 8m_1 - 10m_2 = 0$$

$$\Rightarrow \quad -7m_1 + 2m_2 = 0 \quad \text{and} \quad 14m_1 - 4m_2 = 0 \quad \text{or} \quad 7m_1 - 2m_2 = 0$$

$$\Rightarrow \quad 2m_2 = 7m_1 \quad \text{and} \quad 7m_1 = 2m_2$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{2}{7} \quad \text{and} \quad \frac{m_1}{m_2} = \frac{2}{7}$$

$$\Rightarrow \quad m_1 : m_2 = 2 : 7 \quad \text{and} \quad m_1 : m_2 = 2 : 7$$

Thus, the required ratio is **2 : 7.**

Q. 5. *Find the ratio in which the line segment joining A (1, – 5) and B (– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.*

Sol. The given points are: A (1, – 5) and B (– 4, 5).

Let the required ratio = k : 1 and the required point be P (x, y).

Part-I: To find the ratio

Since the point P lies on x-axis,

∴ Its y-coordinate is 0.

$$x = \frac{m_1\, x_2 + m_2\, x_1}{m_1 + m_2} \quad \text{and} \quad 0 = \frac{m_1\, y_2 + m_2\, y_1}{m_1 + m_2}$$

$$\Rightarrow \quad x = \frac{k(-4)+1(1)}{k+1} \quad \text{and} \quad 0 = \frac{k(5)+1(-5)}{k+1}$$

$$\Rightarrow \quad x = \frac{-4k+1}{k+1} \quad \text{and} \quad 0 = \frac{5k-5}{k+1}$$

$$\Rightarrow \quad x(k+1) = -4k+1 \quad \text{and} \quad 5k-5=0 \quad \Rightarrow \quad k = 1$$

Part II : To find coordinates of P :

$$\Rightarrow \quad x(k+1) = -4k+1 \quad \Rightarrow \quad x(1+1) = -4+1 \qquad [\because k = 1]$$

$$\Rightarrow \quad 2x = -3$$

$$\Rightarrow \quad x = \frac{-3}{2}$$

$\therefore$ The required ratio $k : 1 = \mathbf{1 : 1}$

Coordinates of P are $(x, 0) = \left(\mathbf{\frac{-3}{2}, 0}\right)$.

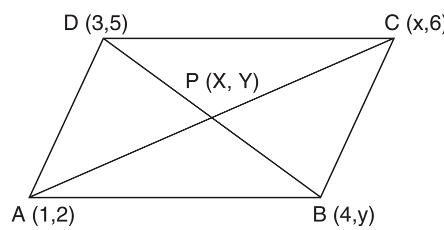
Q. 6. *If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.*

Sol. We have the parallelogram vertices

A (1, 2), B (4, y), C (x, 6) and D (3, 5).

Since, the diagonals of a parallelogram bisect each other.

$\therefore$ The coordinates of P are:



$$X = \frac{x+1}{2} = \frac{3+4}{2}$$

$$\Rightarrow \quad x + 1 = 7 \quad \Rightarrow \quad x = 6$$

$$Y = \frac{5+y}{2} = \frac{6+2}{2}$$

$$\Rightarrow \quad 5 + y = 8 \quad \Rightarrow \quad y = 3$$

$\therefore$ The required values of x and y are:

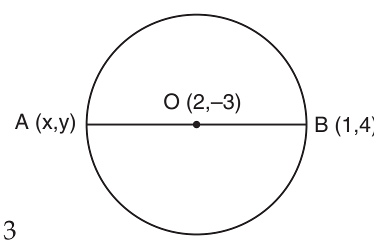
$$\mathbf{x = 6, \quad y = 3}$$

Q. 7. *Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).*

Sol. Here, centre of the circle is O (2, – 3).

Let the end points of the diameter be

A (X, Y) and B (1, 4)



The centre of a circle bisects the diameter.

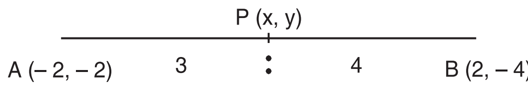
$$\therefore \quad 2 = \frac{x+1}{2} \quad \Rightarrow \quad x + 1 = 4 \quad \text{or} \quad x = 3$$

$$\text{And} \quad -3 = \frac{y+4}{2} \quad \Rightarrow \quad y + 4 = -6 \quad \text{or} \quad y = -10$$

Hence the coordinates of A are (3, – 10).

Q. 8. *If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that* $AP = \frac{3}{7}$ *AB and P lies on the line segment AB.*

Sol.



Here, the given points are
$A\ (-2, -2)$ and $B\ (2, -4)$
Let the coordinates of P are (x, y).
Since, the point P divides AB such that

$$AP = \frac{3}{7}AB \quad \text{or} \quad \frac{AP}{AB} = \frac{3}{7}$$

$\Rightarrow$ Since $\quad AB = AP + BP$

$$\therefore \quad \frac{AP}{AB} = \frac{3}{7} \Rightarrow \frac{AP}{AP + AB} = \frac{3}{7}$$

$$\Rightarrow \quad \frac{AP + BP}{AP} = \frac{7}{3}$$

$$\Rightarrow \quad 1 + \frac{BP}{AP} = \frac{3+4}{3} = 1 + \frac{4}{3}$$

$$\Rightarrow \quad \frac{BP}{AP} = \frac{4}{3}$$

$$\Rightarrow \quad AP : PB = 3 : 4$$

i.e., $P\ (x, y)$ divides AB in the ratio 3 : 4.

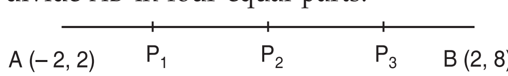
$$\therefore \quad x = \frac{3 \times 2 + 4 \times (-2)}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$$

$$y = \frac{3 \times (-4) + 4 \times (-2)}{3+4} = \frac{-12-8}{7} = \frac{-20}{7}$$

Thus, the coordinates of P are $\left(\mathbf{\frac{-2}{7}, \frac{-20}{7}}\right)$.

Q. 9. *Find the coordinates of the points which divide the line segment joining A (– 2, 2) and B (2, 8) into four equal parts.*

Sol. Here, the given points are:
$A\ (-2, 2)$ and $B\ (2, 8)$
Let P_1, P_2 and P_3 divide AB in four equal parts.



$$\because \quad AP_1 = P_1P_2 = P_2P_3 = P_3B$$

Obviously, P_2 is the mid point of AB
$\therefore$ Coordinates of P_2 are:

$$\left(\frac{-2+2}{2}, \frac{2+8}{2}\right) \text{ or } (0, 5)$$

Again, P_1 is the mid point of AP_2.
$\therefore$ Coordinates of P_1 are:

$$\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) \text{ or } \left(-1, \frac{7}{2}\right)$$

Also P_3 is the mid point of $P_2 B$.

$\therefore$ Coordinates of P_3 are:

$\left(\frac{0+2}{2}, \frac{5+8}{2}\right)$ or $\left(1, \frac{13}{2}\right)$

Thus, the coordinates of P_1, P_2 and P_3 are:

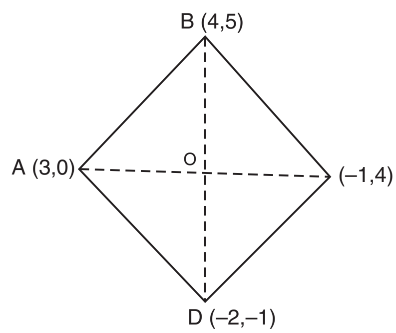
(0, 5), $\left(-1, \frac{7}{2}\right)$ and $\left(1, \frac{13}{2}\right)$ respectively.

Q. 10. *Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order.*

*[**Hint:** Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)]*

Sol. Let the vertices of the given rhombus are:

A (3, 0), B (4, 5), C (– 1, 4) and D (– 2, – 1)



$\because$ AC and BD are the diagonals of rhombus $ABCD$.

$$\therefore \text{ Diagonal } AC = \sqrt{(-1-3)^2 + (4-0)^2} = \sqrt{(-4)^2 + (4)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

$$\text{Diagonal } BD = \sqrt{(-2-4)^2 + (-1-5)^2}$$

$$= \sqrt{(-6)^2 + (-6)^2} = \sqrt{36+36} = 2\sqrt{2}$$

$\because$ For a rhombus,

$$\text{Area} = \frac{1}{2} \times \text{(Product of diagonals)}$$

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$

$$= \frac{1}{2} \times 2 \times 4 \times 6 \text{ Square units.}$$

$= 4 \times 6 =$ **24 Square units.**

- **Area of Triangle**

I. If A (x_1, y_1); B (x_2, y_2) and C (x_3, y_3) are the vertices of Δ ABC, then the area of Δ ABC = $\frac{1}{2}$ $[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$.

II. The three points A, B and C are collinear if and only if area of Δ ABC = 0.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 7.3

Q. 1. *Find the area of the triangle whose vertices are:*

(i) (2, 3), (– 1, 0), (2, – 4) *(ii) (– 5, – 1), (3, – 5), (5, 2)*

Sol. (*i*) Let the vertices of the triangle be

A (2, 3), B (– 1, 0) and C (2, – 4)

Here, $x_1 = 2, \quad y_1 = 3$

$x_2 = -1, \quad y_2 = 0$

$x_3 = 2, \quad y_3 = -4$

$\because$ Area of a $\Delta = \frac{1}{2}\ [x_1\ (y_2 - y_3) + x_2\ (y_3 - y_1) + x_3\ (y_1 - y_2)]$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\ [2\ \{0 - (-4)\} + (-1)\ \{-4 - (3)\} + 2\ \{3 - 0\}]$

$= \frac{1}{2}\ [2\ (0 + 4) + (-1)\ (-4 - 3) + 2\ (3)]$

$= \frac{1}{2}\ [8 + 7 + 6]$

$= \frac{1}{2}[21] = \frac{21}{2}$ sq. units.

(*ii*) Let the vertices of the triangle be

A (– 5, – 1), B (3, – 5) and C (5, 2)

i.e., $x_1 = -5, \quad y_1 = -1$

$x_2 = 3, \quad y_2 = -5$

$x_3 = 5, \quad y_3 = 2$

$\because$ Area of a $\Delta = \frac{1}{2}\ [x_1\ (y_2 - y_3) + x_2\ (y_3 - y_1) + x_3\ (y_1 - y_2)]$

$\therefore$ Area of ΔABC

$= \frac{1}{2}\ [-5\ \{-5 - 2\} + 3\ \{2 - (-1)\} + 5\ \{-1 - (-5)\}]$

$= \frac{1}{2}\ [-5\ \{-7\} + 3\ \{2 + 1\} + 5\ \{-1 + 5\}]$

$= \frac{1}{2}\ [(-5)\ (-7) + 3\ (3) + 5\ (4)]$

$= \frac{1}{2}\ [35 + 9 + 20]$

$= \frac{1}{2} \times 64$ = **32 sq. units.**

Q. 2. *In each of the following find the value of 'k', for which the points are collinear.*

(i) (7, – 2), (5, 1), (3, k) *(ii) (8, 1), (k, – 4), (2, – 5)*

Sol. The given three points will be collinear if the Δ formed by them has zero area.

(*i*) Let A (7, – 2), B (5, 1) and C (3, k) be the vertices of a triangle.

$\therefore$ The given points will be collinear, if ar ($\Delta\ ABC$) = 0

or $7\ (1 - k) + 5\ (k + 2) + 3\ (-2 - 1) = 0$

$\Rightarrow \quad 7 - 7k + 5k + 10 + (-6) - 3 = 0$

$\Rightarrow \quad 17 - 9 + 5k - 7k = 0$

$\Rightarrow \quad 8 - 2k = 0$

$\Rightarrow \quad 2k = 8$

$\Rightarrow \quad k = \frac{8}{2} = 4$

The required value of $k = 4$.

(*ii*) Let (8, 1), (k, – 4) and (2, – 5) be the verticles of a triangle.

∴ For the above points being collinear, ar (Δ *ABC*) = 0

i.e., $8(-4+5) + k(-5-1) + 2[1-(-4)] = 0$

$\Rightarrow \quad 8(+1) + k(-6) + 2(5) = 0$

$\Rightarrow \quad 8 + (-6k) + 10 = 0$

$\Rightarrow \quad -6k + 18 = 0$

$\Rightarrow \quad k = (-18) \div (-6) = 3$

Thus, $k = 3$.

Q. 3. *Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, – 1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.*

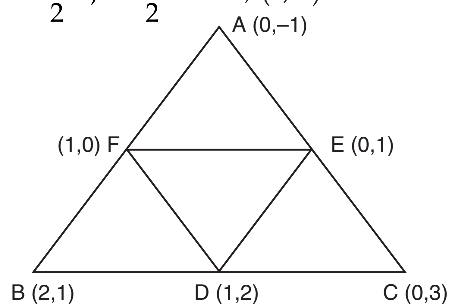
Sol. Let the vertices of the triangle be *A* (0, – 1), *B* (2, 1) and *C* (0, 3).

Let *D, E* and *F* be the mid-points of the sides *BC, CA* and *AB* respectively. Then:

Coordinates of *D* are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ *i.e.,* $\left(\frac{2}{2}, \frac{4}{2}\right)$ or (1, 2)

Coordinates of *E* are: $\frac{0+0}{2}, \frac{3+(-1)}{2}$ *i.e.,* (0, 1)

Coordinates of *F* are: $\frac{2+0}{2}, \frac{1+(-1)}{2}$ *i.e.,* (1, 0)



Now, ar (Δ *ABC*) $= \frac{1}{2}[0(1-3) + 2\{3-(-1)\} + 0(-1-1)]$

$= \frac{1}{2}[0(-2) + 8 + 0(-2)]$

$= \frac{1}{2}[0 + 8 + 0] = \frac{1}{2} \times 8 = 4$ sq. units

And ar Δ (*DEF*) $= \frac{1}{2}[1(1-0) + 0(0-2) + 1(2-1)]$

$= \frac{1}{2}[1(1) + 0 + 1(1)]$

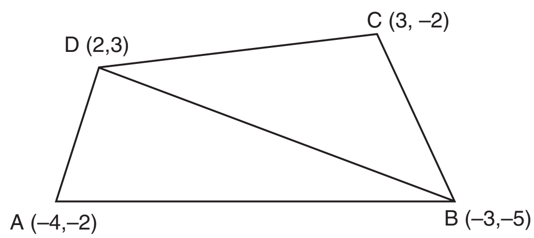
$= \frac{1}{2}[1 + 0 + 1] = \frac{1}{2} \times 2 = 1$ sq. unit

$$\therefore \quad \frac{\text{ar}(\Delta\, DEF)}{\text{ar}(\Delta\, ABC)} = \frac{1}{4}$$

$\Rightarrow$ ar (Δ DEF) : ar (Δ ABC) = **1 : 4.**

Q. 4. *Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2), (– 3, – 5), (3, – 2) and (2, 3).*

Sol. Let A (– 4, – 2), B (– 3, – 5), C (3, – 2) and D (2, 3) be the vertices of the quadrilateral. Let us join diagonal BD.



Now, **ar (Δ *ABD*)** $= \frac{1}{2}$ [(– 4) {– 5 – 3} + (– 3) {3 – (– 2)} + 2 {(– 2) – (– 5)}]

$= \frac{1}{2}$ [(– 4) (– 8) + (– 3) (5) + 2 (– 2 + 5)]

$= \frac{1}{2}$ [32 + (– 15) + 6]

$= \frac{1}{2}[23] = \frac{23}{2}$ sq. units

ar (Δ *CBD*) $= \frac{1}{2}$ [3 (– 5 – 3) + (– 3) {3 – (– 2)} + 2 {(– 2) – (– 5)}]

$= \frac{1}{2}$ [3 (– 8) + – 3 (5) + 2 (3)]

$= \frac{1}{2}$ [– 24 – 15 + 6]

$= \frac{1}{2}[-33] = \frac{33}{2}$ sq. units, (numerically)

Since, ar (quad $ABCD$) = ar (Δ ABD) + ar (Δ CBD)

$\therefore$ ar (quad $ABCD$) $= \left(\frac{23}{2} + \frac{33}{2}\right)$ sq. units

$= \frac{56}{2}$ sq. units = 28 sq. units.

Q. 5. *You have studied in class IX (Chapter 9, Example-3) that, a median of a triangle divides it into two triangles of equal areas. Verify this result for Δ ABC whose vertices are A (4, – 6), B (3, – 2) and C (5, 2).*

Sol. Here, the vertices of the triangle are A (4, – 6), B = (3, – 2) and C (5, 2).

Let D be the mid-point of BC.

$\therefore$ The coordinates of the mid point D are:

$$\left\{\frac{3+5}{2}, \frac{-2+2}{2}\right\} \text{ or } (4, 0).$$

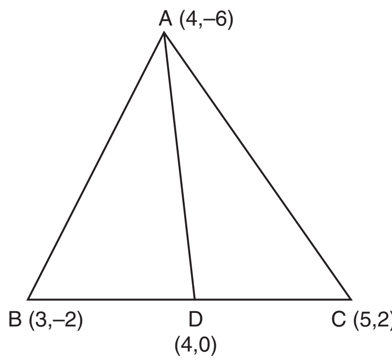
Since AD divides the triangle ABC into two parts *i.e.,* $\Delta\ ABD$ and $\Delta\ ACD$,

Now, **ar (Δ *ABD*)** $= \frac{1}{2}$ [4 {(– 2) – 0} + 3 (0 + 6) + 4 (– 6 + 2)]

$= \frac{1}{2}$ [(– 8) + 18 + (– 16)]

$= \frac{1}{2}(-6)$ = – 3 sq. units.

= 3 sq. units (numerically) ...(1)



ar (Δ *ACD*) $= \frac{1}{2}$ [4 (0 – 2) + 4 (2 + 6) + 5 (– 6 – 0)]

$= \frac{1}{2}$ [– 8 + 32 – 30]

$= \frac{1}{2}[-6]$ = – 3 sq. units

= 3 sq. units (numerically) ...(2)

From (1) and (2)

ar (Δ *ABD*) = ar (Δ *ACD*)

i.e. A median divides the triangle into two triangles of equal areas.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 7.4

Q. 1. *Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A (2, – 2) and B (3, 7).*

Sol. Let the required ratio be k : 1 and the point C divides them in the above ratio.

$\therefore$ Coordinates of C are:

$$\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$$

Since the point C lies on the given line $2x + y - 4 = 0$,

$\therefore$ We have:

$$2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$$

$\Rightarrow \quad 2(3k + 2) + (7k - 2) = 4 \times (k + 1)$

$\Rightarrow \quad 6k + 4 + 7k - 2 - 4k - 4 = 0$

$\Rightarrow \quad (6 + 7 - 4)k + (4 - 2 - 4) = 0$

$\Rightarrow \quad 9k + (-2) = 0$

$\Rightarrow \quad 9k - 2 = 0$

$\Rightarrow \quad k = \dfrac{2}{9}$

$\therefore$ The required ratio

$= k : 1$

$= \dfrac{2}{9} : 1$

$= 2 : 9$

Q. 2. *Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.*

Sol. The given points are:

$A(x, y)$, $B(1, 2)$ and $C(7, 0)$

The points A, B and C will be collinear if

$x(2 - 0) + 1(0 - y) + 7(y - 2) = 0$

or if $\quad 2x - y + 7y - 14 = 0$

or if $\quad 2x + 6y - 14 = 0$

or if $\quad x + 3y - 7 = 0$

which is the required relation between x and y.

Q. 3. *Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3).*

Sol. Let $P(x, y)$ be the centre of the circle passing through

$A(6, -6)$, $B(3, -7)$ and $C(3, 3)$

$\therefore \quad AP = BP = CP$

Taking *AP* = *BP*, we have $AP^2 = BP^2$

$\Rightarrow \quad (x - 6)^2 + (y + 6)^2 = (x - 3)^2 + (y + 7)^2$

$\Rightarrow \quad x^2 - 12x + 36 + y^2 + 12y + 36 = x^2 - 6x + 9 + y^2 + 14y + 49$

$\Rightarrow \quad -12x + 6x + 12y - 14y + 72 - 58 = 0$

$\Rightarrow \quad -6x - 2y + 14 = 0$

$\Rightarrow \quad 3x + y - 7 = 0 \qquad$ ...(1) [Dividing by (– 2)]

Taking *BP* = *CP*, we have $BP^2 = CP^2$

$\Rightarrow \quad (x - 3)^2 + (y + 7)^2 = (x - 3)^2 + (y - 3)^2$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 + 14y + 49 = x^2 - 6x + 9 + y^2 - 6y + 9$

$\Rightarrow \quad -6x + 6x + 14y + 6y + 58 - 18 = 0$

$\Rightarrow \quad 20y + 40 = 0$

$\Rightarrow \quad y = \dfrac{-40}{20} = -2 \qquad$ ...(2)

From (1) and (2),

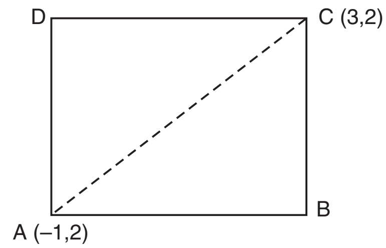
$3x - 2 - 7 = 0$

$\Rightarrow \quad 3x = 9 \quad \Rightarrow \quad x = 3$

i.e., $x = 3$ and $y = -2$

$\therefore$ The required centre is **(3, – 2).**

Q. 4. *The two opposite vertices of a square are (– 1, 2) and (3, 2). Find the coordinates of the other two vertices.*



Sol. Let us have a square *ABCD* such that *A* (– 1, 2) and *C* (3, 2) are the opposite vertices.

Let *B* (*x*, *y*) be an unknown vertex.

Since all sides of a square are equal,

$\therefore \quad AB = BC$

$\Rightarrow \quad AB^2 = BC^2$

$\Rightarrow \quad (x + 1)^2 + (y - 2)^2 = (x - 3)^2 + (y - 2)^2$

$\Rightarrow \quad 2x + 1 = -6x + 9$

$\Rightarrow \quad 8x = 8 \quad \Rightarrow \quad x = 1$ ...(1)

Since each angle of a square = 90°,

$\therefore$ *ABC* is a right angled triangle.

$\therefore$ Using Pythagoras theorem, we have:

$$AB^2 + BC^2 = AC^2$$

$\Rightarrow [(x + 1)^2 + (y - 2)^2] + [(x - 3)^2 + (y - 2)^2] = [(3 + 1)^2 + (2 - 2)^2]$

$\Rightarrow 2x^2 + 2y^2 + 2x - 4y - 6x - 4y + 1 + 4 + 9 + 4 = 16$

$\Rightarrow 2x^2 + 2y^2 - 4x - 8y + 2 = 0$

$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$ ...(2)

Substituting the value of *x* from (1) into (2) we have:

$1 + y^2 - 2 - 4y + 1 = 0$

$\Rightarrow \quad y^2 - 4y + 2 - 2 = 0$

$\Rightarrow \quad y^2 - 4y = 0$

$\Rightarrow \quad y(y - 4) = 0$

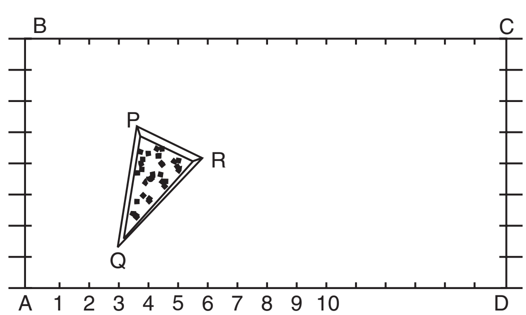
$\Rightarrow \quad y = 0 \quad \text{or} \quad y = 4$

Hence, the required other two vertices are: (1, 0) and (1, 4).

Q. 5. *The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the Fig. The students are to sow seeds of flowering plants on the remaining area of the plot.*

(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?
Also calculate the areas of the triangles in these cases. What do you observe?



Sol. (*i*) By taking *A* as the origin and *AD* and *AB* as the coordinate axes, we have ***P* (4, 6), *Q* (3, 2)** and ***R* (6, 5)** as the vertices of Δ *PQR*.

(*ii*) By taking *C* as the origin and *CB* and *CD* as the coordinate axes, then the vertices of Δ *PQR* are

***P* (12, 2), *Q* (13, 6)** and ***R* (10, 3)**

Now, ar (Δ *PQR*) [*when P (4, 6), Q (3, 2) and R (6, 5) are the vertices*]

$= \frac{1}{2}$ [4 (2 – 5) + 3 (5 – 6) + 6 (6 – 2)]

$= \frac{1}{2}$ [– 12 – 3 + 24]

$= \frac{9}{2}$ sq. units. [*taking numerical value*]

ar (Δ *PQR*) [*when P (12, 2), Q (13, 6) and R (10, 3) are the vertices.*]

$= \frac{1}{2}$ [12 (6 – 3) + 13 (3 – 2) + 10 (2 – 6)]

$= \frac{1}{2}$ [36 + 13 – 40]

$= \frac{9}{2}$ sq. units.

Thus, in both cases, the area of Δ *PQR* is the same.

Q. 6. *The vertices of a Δ ABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that* $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. *Calculate the area of the Δ ADE and compare it with the area of Δ ABC. (Recall Theorem 6.2 and Theorem 6.6).*

Sol. We have $\frac{AD}{AB} = \frac{1}{4}$

$\Rightarrow \frac{AB}{AD} = \frac{4}{1}$

$\Rightarrow \frac{AD + DE}{AD} = \frac{4}{1}$

$\Rightarrow \frac{AD}{AD} + \frac{DE}{AD} = \frac{4}{1} = 1 + \frac{3}{1}$

$\Rightarrow 1 + \frac{DE}{AD} = 1 + \frac{3}{1} \Rightarrow \frac{DE}{AD} = \frac{3}{1}$

$\Rightarrow AD : DE = 1 : 3$

Thus, the point *D* divides *AB* in the ratio 1 : 3

∴ **The coordinates of *D* are:**

$$\left[\frac{(1 \times 1) + (3 \times 4)}{1 + 3}, \frac{(1 \times 5) + (3 \times 6)}{1 + 3}\right]$$

or $\left[\frac{1 + 12}{4}, \frac{5 + 18}{4}\right]$

or $\left(\frac{13}{4}, \frac{23}{4}\right)$

Similarly, *AE* : *EC* = 1 : 3

i.e., *E* divides *AC* in the ratio 1 : 3

⇒ **Coordinates of *E* are:**

$$\left[\frac{(1\times 7)+(3\times 4)}{1+3}, \frac{1\times 2+3\times 6}{1+3}\right]$$

or $\left[\frac{7+12}{4}, \frac{2+18}{4}\right]$

or $\left[\frac{19}{4}, 5\right]$

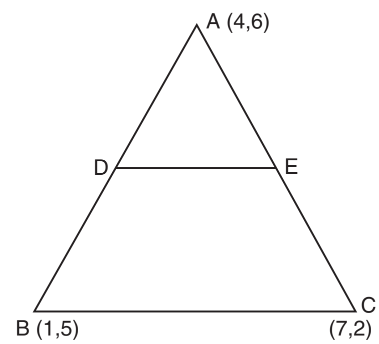
Now, **ar (Δ *ADE*)**

$$= \frac{1}{2}\left[4\left(\frac{23}{4}-5\right)+\frac{13}{4}(5-6)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$$

$$= \frac{1}{2}\left[(23-20)+\frac{13}{4}(1)+\frac{19}{4}\left(\frac{24-23}{4}\right)\right]$$

$$= \frac{1}{2}\left(3-\frac{13}{4}+\frac{19}{16}\right)$$

$$= \frac{1}{2}\left[\frac{48+52+19}{16}\right] = \frac{15}{32} \text{ sq. units.}$$



Area **of Δ *ABC***

$$= \frac{1}{2}\ [4\ (5-2) + 1\ (2-6) + 7\ (6-5)]$$

$$= \frac{1}{2}\ [(4\times 3) + 1\times(-4) + 7\times 1]$$

$$= \frac{1}{2}\ [12 + (-4) + 7]$$

$$= \frac{1}{2}(15) = \frac{15}{2} \text{ sq. units.}$$

Now, $\frac{\text{ar}(\Delta\, ADE)}{\text{ar}(\Delta\, ABC)} = \frac{\frac{15}{32}}{\frac{15}{2}} = \frac{15}{32}\times\frac{2}{15} = \frac{1}{16}$

⇒ ar (Δ *ADE*) : ar (Δ *ABC*) **= 1 : 16.**

Q. 7. *Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of Δ ABC.*

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1.

(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

(iv) What do you observe?

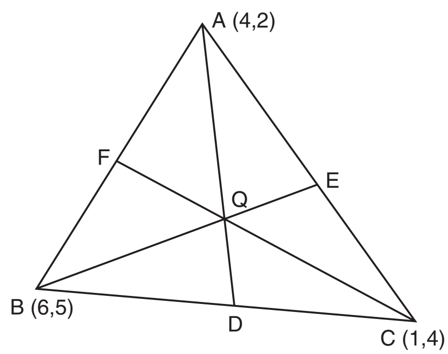
*[**Note:** The point which is common to all the three medians is called the centroid and this point divides each median in the ratio 2 : 1.]*

(v) If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of Δ ABC, find the coordinates of the centroid of the triangle.

Sol. We have the vertices of Δ *ABC* as *A* (4, 2), *B* (6, 5) and *C* (1, 4).

(*i*) Since *AD* is a median

$\therefore$ Coordinates of D are: $\left(\frac{6+1}{2}, \frac{5+4}{2}\right)$ or $\left(\frac{7}{2}, \frac{9}{2}\right)$



(*ii*) Since $AP : PD = 2 : 1$ *i.e., P* divides AD in the ratio 2 : 1.

$\therefore$ Coordinates of P are:

$$\left[\frac{2\left(\frac{7}{2}\right)+(1\times 4)}{2+1}, \frac{2\left(\frac{9}{2}\right)+1\times 2}{2+1}\right] \text{ or } \left(\frac{11}{3}, \frac{11}{3}\right)$$

(*iii*) $BQ : QE = 2 : 1 \Rightarrow$ [*The point Q divides BE in the radio 2 : 1*]

$\therefore$ Coordinates of Q are:

$$\left[\frac{2\left(\frac{5}{2}\right)+1\times 6}{2+1}, \frac{(2\times 3)+(1\times 5)}{2+1}\right]$$

or $\left[\frac{5+6}{3}, \frac{6+5}{3}\right]$

or $\left[\frac{11}{3}, \frac{11}{3}\right]$

Coordinates of Q are:

$\left(\frac{4+6}{2}, \frac{2+5}{2}\right)$ or $\left(5, \frac{7}{2}\right)$

Coordinates of R are:

$$\left[\frac{2\times 5+1\times 1}{2-1}, \frac{2\times\frac{7}{2}+1\times 4}{2+1}\right]$$

or $\left[\frac{10+1}{3}, \frac{7+4}{3}\right]$

or $\left[\frac{11}{3}, \frac{11}{3}\right]$

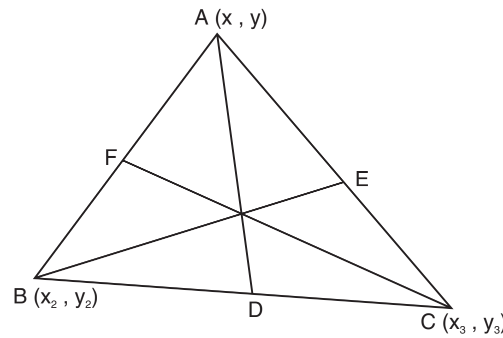
(*iv*) We observe that *P, Q* and *R* represent the same point.

(*v*) Here, we have

$A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ as the vertices of ΔABC. Also *AD*, *BE* and *CF* are its medians.

$\therefore$ *D*, *E* and *F* are the mid points of *BC*, *CA* and *AB* respectively.

We know, the centroid is a point on a median, dividing it in the ratio 2 : 1.



Concidering the median *AD*,

Coordinates of *AD* are:

$$q\left[\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right]$$

Let *G* be the centroid.

$\therefore$ Coordinates of the centroid are:

$$\left[\frac{(1 \times x_1) + 2\left(\frac{x_2 + x_3}{2}\right)}{1+2}, \frac{(1 \times y_1) + 2\left(\frac{y_2 - y_3}{2}\right)}{1+2}\right]$$

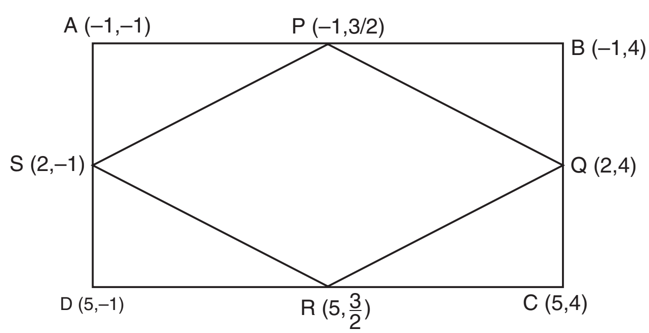
$$= \left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$$

Similarly, considering the other medians we find that in each the coordinates of *G* are $\frac{x_1 + x_3 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$.

i.e., The coordinates of the centroid are $\left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$.

Q. 8. *ABCD is a rectangle formed by the points A (– 1, – 1), B (– 1, 4), C (5, 4) and D (5, – 1). P, Q, R and S are the mid points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.*

Sol. We have a rectangle whose vertices are *A* (– 1, – 1), *B* (– 1, 4), *C* (5, 4) and *D* (5, – 1).



$\because$ *P* is mid-point of *AB*

$\therefore$ Coordinates of *P* are:

$$\left[\frac{-1-1}{2}, \frac{-1+4}{2}\right] \text{ or } \left(-1, \frac{3}{2}\right)$$

Similarly, the coordinates of *Q* are:

$$\left(\frac{-1+5}{2}, \frac{4+4}{2}\right) \text{ or } (2, 4)$$

Coordinates of R are:

$$\left(\frac{5+5}{2}, \frac{-1+4}{2}\right) \text{ or } \left(5, \frac{3}{2}\right)$$

Coordinates of S are:

$$\left(\frac{-1+5}{2}, \frac{-1-1}{2}\right) \text{ or } (2, -1)$$

Now,
$$PQ = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$SR = \sqrt{(5-2)^2 + \left(\frac{3}{2} - 4\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$RS = \sqrt{(2-5)^2 + \left\{-1 + \left(-\frac{3}{2}\right)\right\}^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$SP = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$SR = \sqrt{(5+1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} = \sqrt{6^2 + 0} = 6$$

$$QS = \sqrt{(2-2) + (4+1)^2} = \sqrt{0 + 5^2} = 5$$

We see that:

$$PQ = QR = RS = SP$$

i.e., all sides of $PQRS$ are equal.

∴ It can be a square or a rhombus.

But its diagonals are not equal.

i.e., $PR \neq QS$

∴ $PQRS$ is a **rhombus.**

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *Find a point on the y-axis equidistant from (– 5, 2) and (9, – 2).* (CBSE 2012)

Sol. Let the required point on the y-axis be $P(0, y)$

$$\therefore \quad PA = PB$$

$$\Rightarrow \sqrt{(0+5)^2 + (y-2)^2} = \sqrt{(0-9)^2 + (y+2)^2}$$

$$\Rightarrow \sqrt{5^2 + y^2 + 4 - 4y} = \sqrt{(-9)^2 + y^2 + 4 + 4y}$$

$$\Rightarrow 25 + y^2 + 4 - 4y = 81 + y^2 + 4 + 4y$$

$$\Rightarrow y^2 - y^2 - 4y - 4y = 81 + 4 - 4 - 25$$

$$\Rightarrow -8y = 85 - 29$$

$$\Rightarrow -8y = 56$$

$$\Rightarrow \quad y = \frac{56}{-8} = -7$$

∴ The required point is **(0, – 7).**

Q. 2. *Find a point on x-axis at a distance of 4 units from the point A (2, 1).*

Sol. Let the required point on x-axis be P (x, 0).

$$\therefore \quad PA = 4$$

$$\Rightarrow \quad \sqrt{(x-2)^2+(0-1)^2} = 4$$

$$\Rightarrow \quad x^2 - 4x + 4 + 1 = 4^2 = 16$$

$$\Rightarrow \quad x^2 - 4x + 1 + 4 - 16 = 0$$

$$\Rightarrow \quad x^2 - 4x - 11 = 0$$

$$\Rightarrow \quad x = 2 \pm \sqrt{15}$$

Thus, the coordinates of P are: $\left(\mathbf{2 \pm \sqrt{15}, 0}\right)$.

Q. 3. *Find the distance of the point (3, – 4) from the origin.*

Sol. The coordinates of origin (0, 0).

∴ Distance of (3, – 4) from the origin

$$= \sqrt{(3-0)^2+(-4-0)^2}$$

$$= \sqrt{(3)^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = \mathbf{5}.$$

Q. 4. *For what value of x is the distance between the points A (– 3, 2) and B (x, 10) 10 units?*

Sol. The distance between A (– 3, 2) and B (x, 10)

$$= \sqrt{(x+3)^2+(10-2)^2}$$

$$\therefore \quad \sqrt{(x+3)^2+(10-2)^2} = 10$$

$$\Rightarrow \quad (x + 3)^2 + (8)^2 = 10^2$$

$$\Rightarrow \quad (x + 3)^2 = 10^2 - 8^2$$

$$\Rightarrow \quad (x + 3)^2 = (10 - 2)\ (10 + 8) = 36$$

$$\Rightarrow \quad x + 3 = \pm\sqrt{36} = \pm 6$$

For +ve sign, $x = 6 - 3 = 3$

For –ve sign, $x = -6 - 3 = -9$

Q. 5. *Find a point on the x-axis which is equidistant from the points A (5, 2) and B (1, – 2).*

Sol. The given points are:

A (5, 2) and B (1, – 2)

Let the required point on the x-axis be C (x, 0).

Since, C is equidistant from A and B.

$$\therefore \quad AC = BC$$

$$\therefore \quad \sqrt{(x-5)^2+(0-2)^2} = \sqrt{(x-1)^2+(0+2)^2}$$

$$\Rightarrow \quad (x - 5)^2 + (-2)^2 = (x - 1)^2 + (2)^2$$

$$\Rightarrow \quad x^2 + 25 - 10x + 4 = x^2 + 1 - 2x + 4$$

$$\Rightarrow \quad -10x + 2x = 5 - 29$$

$$\Rightarrow \quad -8x = -24$$

$$\Rightarrow \quad x = \frac{-24}{-8} = 3$$

∴ The required point is **(0, 3).**

Q. 6. *Establish the relation between x and y when P (x, y) is equidistant from the points A (– 1, 2) and B (2, – 1).*

Sol. ∵ P is equidistant from A and B

$$\therefore \quad PA = PB$$

$$\sqrt{(x+1)^2 + (y-2)^2} = \sqrt{(x-2)^2 + (y+1)^2}$$

$$\Rightarrow \quad (x+1)^2 + (y-2)^2 = (x-2)^2 + (y+1)^2$$

$$\Rightarrow \quad x^2 + 1 + 2x + y^2 - 4y + 4 = x^2 + 4 - 4x + y^2 + 1 + 2y$$

$$\Rightarrow \quad 2x - 4x + 5 = -4x + 2y + 5$$

$$\Rightarrow \quad 2x + 4x + 5 = 2y + 4y + 5$$

$$\Rightarrow \quad 6x = 6y$$

$$\Rightarrow \quad \boldsymbol{x = y}$$

which is the required relation.

Q. 7. *Show that the points (7, – 2), (2, 3) and (– 1, 6) are collinear.*

Sol. Here, the vertices of a triangle are (7, – 2), (2, 3) and (– 1, 6)

∴ Area of the triangle

$$= \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\,[7(3-6) + 2(6+2) + (-1)(-2-3)]$$

$$= \frac{1}{2}\,[7 \times (-3) + 2 \times 8 + (-1)(-5)]$$

$$= \frac{1}{2}\,[-21 + 16 + 5]$$

$$= \frac{1}{2}[0] = 0$$

Since area of triangle = 0

∴ The vertices of the triangle are collinear.

Thus, the given points **are collinear.**

Q. 8. *Find the distance between the points*

$\left(\frac{-8}{5}, 2\right)$ *and* $\left(\frac{2}{5}, 2\right)$ (CBSE 2009)

Sol. Distance between $\left(\frac{-8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$ is given by

$$\sqrt{\left(\frac{2}{5} + \frac{8}{5}\right)^2 + (2-2)^2} = \sqrt{2^2 - 0^2} = \mathbf{2\ units.}$$

Q. 9. *If the mid point of the line joining the points P (6, b – 2) and Q (– 2, 4) is (2, – 3), find the value of b.* (CBSE 2009 F)

Sol. Here, P (6, b – 2) and Q (– 2, 4) are the given points.

∴ Mid point of PQ is given by:

$$\left[\frac{6+(-2)}{2}, \frac{4+b-2}{2}\right]$$

or $\left[\frac{6-2}{2}, \frac{4-2+b}{2}\right]$

or $\left[2, \frac{2+b}{2}\right]$

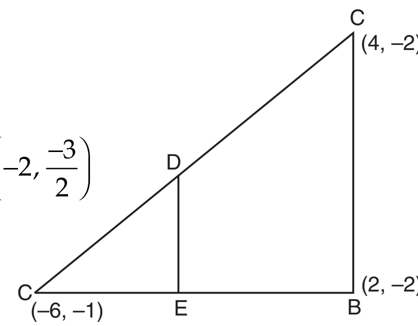
$\therefore \quad \frac{2+b}{2} = -3 \Rightarrow 2 + b = -6$

$\Rightarrow \quad b = -6 - 2$

$\Rightarrow \quad b = -8$

Q. 10. *In the given figure, ABC is a triangle. D and E are the mid points of the sides BC and AC respectively. Find the length of DE. Prove that DE =* $\frac{1}{2}$*AB* (CBSE 2011)

Sol. Co-ordinates of the mid point of BC are:



$$= \left(\frac{-6+2}{2}, \frac{-1+(-2)}{2}\right)$$

$$= \left(-2, \frac{-3}{2}\right) \Rightarrow E\left(-2, \frac{-3}{2}\right)$$

Co-ordinates of the mid point of AC are:

$$= \left(\frac{-6+4}{2}, \frac{-1+(-2)}{2}\right)$$

$$= \left(-1, \frac{-3}{2}\right) \Rightarrow D\left(-1, \frac{-3}{2}\right)$$

Now,

$$\left.\begin{aligned} DE &= \sqrt{(-2+1)^2 + \left(-3/2 + \frac{3}{2}\right)^2} \\ &= \sqrt{(-1)^2 + 0} = 1 \\ AB &= \sqrt{(4-2)^2 + (-2+2)^2} \\ &= \sqrt{2^2 + 0} = 2 \end{aligned}\right\} \Rightarrow DE = \frac{1}{2}AB \text{ Hence proved.}$$

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *Points P (5, – 3) is one of the two points of trisection of the line segment joining the points A (7, – 2) and B (1, – 5) near to A. Find the coordinates of the other point of trisection.*

(AI CBSE 2010)

Sol.

Since P is near to A

∴ other point Q is the mid point of PB

$$\Rightarrow \quad x = \frac{5+1}{2} = 3$$

$$\Rightarrow \quad y = \frac{-3-5}{2} = -\frac{8}{2} = -4$$

Thus, the point Q is (3, – 4)

Q. 2. *Find the area of the quadrilateral ABCD whose vertices are A (1, 0), B (5, 3), C (2, 7) and D (– 2, 4).* [AI CBSE 2009]

Sol. Area of **Δ *ABC***

$$= \frac{1}{2}[1(3-7) + 5(7-0) + 2(0-3)]$$

$$= \frac{1}{2}[-4 + 35 - 6] = \frac{1}{2} \times 25$$

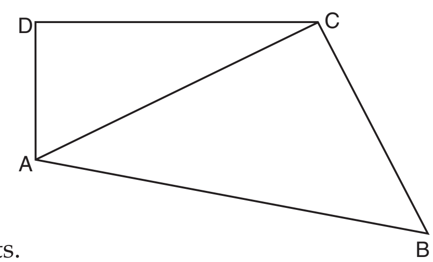
$$= \frac{25}{2} \text{ sq. units.}$$

Area of **Δ *ACD***

$$= \frac{1}{2}[1(7-4) + 2(4-0) + (-2)(0-7)]$$

$$= \frac{1}{2}[3 + 8 + 14] = \frac{1}{2} \times 25$$

$$= \frac{25}{2} \text{ sq. units.}$$



∴ Area of the quad. *ABCD*

$$= \frac{25}{2} \text{ sq. units} + \frac{25}{2} \text{ sq. units.}$$

= **25 sq. units.**

Q. 3. *Points P, Q, R and S, in this order, divide a line segment joining A (2, 6), B (7, – 4) in five equal parts. Find the coordinates of P and R.* [AI CBSE 2009 Comptt.]

Sol.

∵ *P*, *Q*, *R* and *S* divide *AB* in five equal parts.

∴ *AP* = *PQ* = *QR* = *RS* = *SB*

Now, *P* divides *AB* in the ratio 1 : 4

∴ Coordinates of *P* are:

$$\left[\frac{1 \times 7 + 4 \times 2}{1+4}, \frac{1 \times (-4) + 4 \times 6}{1+4}\right]$$

or $\left[\frac{7+8}{5}, \frac{-4+24}{5}\right]$ or **(3, 4)**

Again, *R* divides *AB* in the ratio 3 : 2

∴ Coordinates of *R* are:

$$\left[\frac{2 \times 2 + 3 \times 7}{2+3}, \frac{2 \times 6 + 3 \times (-4)}{2+3}\right] \text{ or } \left[\frac{4+21}{5}, \frac{0}{5}\right] \text{ or } \mathbf{(5, 0)}$$

Q. 4. *A (– 4, – 2), B (– 3, – 5), C (3, – 2) and D (2, k) are the vertices of a quad. ABCD. Find the value of k, if the area of the quad is 28 sq. units.*

Sol. Area of quad $ABCD$ = 28 sq. units

$\therefore$ [ar (Δ ABD)] + [ar (Δ BCD)] = 28 sq. units

$$\Rightarrow \frac{1}{2} [-4(-5-k) - 3(k+2) + 2(-2+5)]$$

$$+ \frac{1}{2} [-3(-2-k) + 3(k+5) + 2(-5+2)] = 28$$

$$\Rightarrow \frac{1}{2} [20 + 4k - 3k - 6 + 6] + \frac{1}{2} [6 + 3k + 3k + 15 - 6] = 28$$

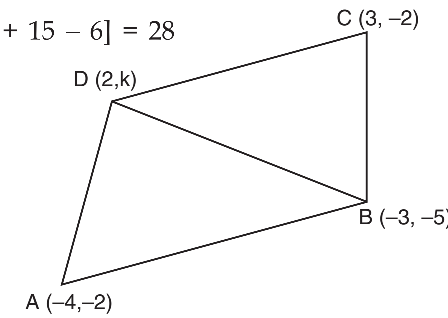
$$\Rightarrow \frac{1}{2} [k + 20] + \frac{1}{2} [6k + 15] = 28$$

$$\Rightarrow k + 20 + 6k + 15 = 56$$

$$\Rightarrow 7k + 35 = 56$$

$$\Rightarrow 7k = 56 - 35 = 21$$

$$\Rightarrow k = \frac{21}{7} = \mathbf{3}$$



Q. 5. *Find the point on y-axis which is equidistant from the points (5, – 2) and (– 3, 2).*

Sol. Let the required point be P (0, y)

$\because$ The given points are A (5, – 2) and B (– 3, 2)

$$\therefore \quad PA = PB$$

$$\Rightarrow \quad PA^2 = PB^2$$

$$\therefore \quad (5-0)^2 + (-2-y)^2 = (-3-0)^2 + (2-y)^2$$

$$\Rightarrow \quad 5^2 + (-2-y)^2 = (-3)^2 + (2-y)^2$$

$$\Rightarrow \quad 25 + 4 + y^2 + 4y = 9 + 4 + y^2 - 4y$$

$$\Rightarrow \quad 25 + 4y = 9 - 4y$$

$$\Rightarrow \quad 8y = -16 \Rightarrow y = -2$$

Thus, the required point is **(0, – 2)**

Q. 6. *Find the point on y-axis which is equidistant from (– 5, 2) and (9, – 2).* (CBSE 2009 C)

Sol. Let the required point on Y-axis be P (0, y).

The given points are A (– 5, 2) and B (9, – 2)

$$\because \quad AP = BP$$

$$\therefore \quad \sqrt{(0+5)^2 + (y-2)^2} = \sqrt{(0-9)^2 + (y+2)^2}$$

$$\Rightarrow \quad 5^2 + (y-2)^2 = 9^2 + (y+2)^2$$

$$\Rightarrow \quad 25 + y^2 - 4y + 4 = 81 + y^2 + 4 + 4y$$

$$\Rightarrow \quad -4y - 4y = 81 + 4 - 4 - 25$$

$$\Rightarrow \quad -8y = 56$$

$$\Rightarrow \quad y = \frac{56}{-8} = -7$$

$\therefore$ The required point = **(0, –7)**

Q. 7. *Find the value of x for which the distance between the points P (4, – 5) and Q (12, x) is 10 units.* (CBSE 2009 C)

Sol. The given points are P (4 – 5) and Q (12, x) such that PQ = 10

$\therefore \quad \sqrt{(12-4)^2+(x+5)^2} = 10$

$\Rightarrow \quad (12-4)^2 + (x+5)^2 = 10^2$

$\Rightarrow \quad 8^2 + (x+5)^2 = 100$

$\Rightarrow \quad 64 + x^2 + 25 + 10x = 100$

$\Rightarrow \quad x^2 + 10x - 11 = 0$

$\Rightarrow \quad (x-1)(x+11) = 0$

$\Rightarrow$ $\mathbf{x = 1}$ or $\mathbf{x = -11}$

Q. 8. *Find the relation between x and y if the points (2, 1), (x, y) and (7, 5) are collinear.* (AI CBSE 2009)

Sol. Here, $x_1 = 2, \; y_1 = 1$

$x_2 = x, \; y_2 = y$

$x_3 = 7, \; y_3 = 5$

$\therefore \quad \text{Area of triangle} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[2(y-5) + x(5-1) + 7(1-y)]$

$= \frac{1}{2}[2y - 10 + 5x - x + 7 - 7y]$

$= \frac{1}{2}[-5y + 4x - 3]$

$\because \quad \text{ar}(\Delta) = 0$

$\therefore \quad \frac{1}{2}[-5y + 4x - 3] = 0$

$\Rightarrow \quad 4x - 5y - 3 = 0$

which is the required relation.

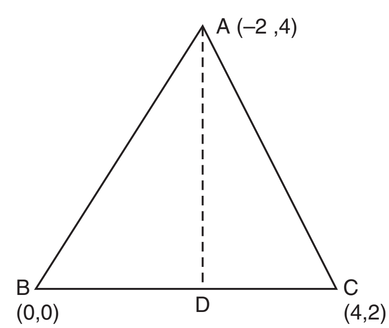
Q. 9. *If A (– 2, 4), B (0, 0) and C (4, 2) are the vertices of Δ ABC, then find the length of the median through the vertex A.* (CBSE 2009 C)

Sol. $\because$ AD is the median on BC

$\therefore$ D is the mid-point of BC.

$\Rightarrow$ Coordinates of D are:

$\left(\frac{0+4}{2}, \frac{0+2}{2}\right)$ *i.e.,* (2, 1)



Now, the length of the median

$AD = \sqrt{(2+2)^2+(1-4)^2}$

$= \sqrt{(4)^2+(-3)^2}$

$= \sqrt{16+9} = \sqrt{25}$ = **5 units.**

Q. 10. *If the points A (4, 3) and B (x, 5) are on the circle with the centre O (2, 3), find the value of x.* (AI CBSE 2009)

Sol. Let O (2, 3) be the centre of the circle.

$\therefore \quad OA = OB \Rightarrow OA^2 = OB^2$

$\Rightarrow \quad (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$

$\Rightarrow \quad 2^2 = (x-2)^2 + 2^2$

$\Rightarrow \quad (x-2)^2 = 0$

$\Rightarrow \quad x - 2 = 0$

$\Rightarrow \quad x = 2$

Thus, the required value of x is **2.**

Q. 11. *If the vertices of a Δ are (2, 4), (5, k) and (3, 10) and its area is 15 sq. units, then find the value of 'k'.* (AI CBSE 2008)

Sol. The area of the given Δ

$$= \frac{1}{2}[2(k-10) + 5(10-4) + 3(4-k)]$$

$$= \frac{1}{2}[2k - 20 + 50 - 20 + 12 - 3k]$$

$$= \frac{1}{2}[-k + 22]$$

But ar (Δ) = 15 [given]

$\therefore \quad \frac{1}{2}[-k + 22] = 15$

$\Rightarrow \quad -k + 22 = 30$

$\Rightarrow \quad -k = 30 - 22 = 8$

$\Rightarrow \quad k = \mathbf{-8}$

Q. 12. *The vertices of a triangle are:*
(1, k), (4, – 3), (– 9, 7) and its area is 15 sq. units. Find the value of k. (AI CBSE 2008)

Sol. Area of the given triangle

$$= \frac{1}{2}[1(-3-7) + 4(7-k) - 9(k+3)] = 15$$

$\Rightarrow \quad \frac{1}{2}[-10 + 28 - 4k - 9k - 27] = 15$

$\Rightarrow \quad -13k - 9 = 30$

$\Rightarrow \quad -13k = 39$

$\Rightarrow \quad k = \frac{39}{-13} \quad \Rightarrow \quad k = -3$

Q. 13. *Find the area of a Δ ABC whose vertices are A (– 5, 7), B (– 4, – 5) and C (4, 5).* (AI CBSE 2008)

Sol. Here, $x_1 = -5, \quad y_1 = 7$

$x_2 = -4, \quad y_2 = -5$

$x_3 = 4, \quad y_3 = 5$

Now, $\text{ar}(\Delta) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}[(-5)(-5-5) + (-4)(5-7) + 4(7+5)]$$

$$= \frac{1}{2}[50 + 8 + 48]$$

$$= \frac{1}{2} \times 106 = 53$$

∴ The required ar (Δ ABC) = **53 sq. units.**

Q. 14. *Find the value of k such that the points (1, 1), (3, k) and (– 1, 4) are collinear.*

(AI CBSE 2008)

Sol. For the three points, to be collinear, the area of triangle formed by them must be zero.

$\therefore \quad$ Area of triangle $= 0$

$$\Rightarrow \frac{1}{2} [1 (k - 4) + (3) (4 - 1) + (-1) (1 - k)] = 0$$

$$\Rightarrow \frac{1}{2} [k - 4 + 9 - 1 + k] = 0$$

$$\Rightarrow \frac{1}{2} [2k + 4] = 0$$

$$\Rightarrow k + 2 = 0$$

$$\Rightarrow k = \mathbf{-2}$$

Q. 15. *For what value of p, the points (– 5, 1), (1, p) and (4, – 2) are collinear?* (CBSE 2008)

Sol. Since the points are collinear,

$\therefore$ The area of the Δ formed by these points must be zero.

i.e., $\frac{1}{2} [-5 (p + 2) + 1 (-2 - 1) + 4 (1 - p)] = 0$

$$\Rightarrow -5p - 10 - 3 + 4 - 4p = 0$$

$$\Rightarrow -9p - 9 = 0$$

$$\Rightarrow -9p = 9$$

$$\Rightarrow p = \frac{9}{-9} = -\mathbf{1}$$

Q. 16. *For what value of p, are the points (2, 1), (p, – 1) and (– 1, 3) collinear?* (CBSE 2008)

Sol. $\because$ The given points are collinear.

$\therefore$ The area of a triangle formed by these points must be zero.

i.e., Area of triangle $= 0$

$$\Rightarrow \frac{1}{2} [2 (-1 - 3) + p (3 - 1) + (-1) (1 + 1)] = 0$$

$$\Rightarrow \frac{1}{2} [-8 + 2p - 2] = 0$$

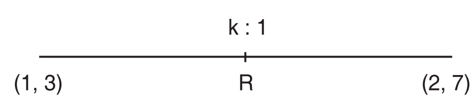
$$\Rightarrow \frac{1}{2} [-10 + 2p] = 0$$

$$\Rightarrow -5 + p = 0$$

$$\Rightarrow p = \mathbf{5}$$

Q. 17. *Find the ratio in which the line 3x + 4y – 9 = 0 divides the line segment joining the points (1, 3) and (2, 7).* (CBSE 2008)

Sol. Let the ratio be $k : 1$.



$\therefore$ Coordinates of R are:

$$\left(\frac{2k + 1}{k + 1}, \frac{7k + 3}{k + 1}\right)$$

Since, R lies on the line $3x + 4y - 9 = 0$

$$\therefore \quad 3\left(\frac{2k+1}{k+1}\right) + 4\left(\frac{7k+3}{k+1}\right) - 9 = 0$$

$$\Rightarrow \quad 6k + 3 + 28k + 12 - 9k + 9 = 0$$

$$\Rightarrow \quad (6k + 28k - 9k) + (3 + 12 - 9) = 0$$

$$\Rightarrow \quad 25k + 6 = 0$$

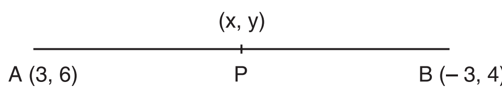
$$\Rightarrow \quad k = \frac{-6}{25}$$

$\therefore$ The required ratio is

$-6 : 25$ or **6 : 25**

Q. 18. *If the point P (x, y) is equidistant from the points A (3, 6) and B (– 3, 4), prove that* $3x + y - 5 = 0$. (AI CBSE 2008)

Sol.



$\because$ P is equidistant from A and B.

$$\therefore \quad AP = BP$$

$$\Rightarrow \quad AP^2 = BP^2$$

$$\Rightarrow \quad (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$\Rightarrow x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$$

$$\Rightarrow \quad x^2 + y^2 - 6x + 45 = x^2 + y^2 + 6x - 8y + 25$$

$$\Rightarrow (-6x - 6x) + (-12y + 8y) + 45 - 25 = 0$$

$$\Rightarrow \quad -12x + 20 - 4y = 0$$

$$\Rightarrow \quad -3x - y + 5 = 0$$

or $\quad \mathbf{3x + y - 5 = 0}$

Q. 19. *The coordinates of A and B are (1, 2) and (2, 3). If P lies on AB, then find the coordinates of P such that:*

$$\frac{AP}{PB} = \frac{4}{3}$$

(AI CBSE 2008)

Sol. $\because \quad \frac{AP}{PB} = \frac{4}{3}$

$$\therefore \quad AP : PB = 4 : 3$$

Here, P divides AB internally in the ratio 4 : 3.

$\therefore$ P has coordinates as:

$$\left[\frac{4 \times 2 + 3 \times 1}{4+3}, \frac{4 \times 3 + 3 \times 2}{4+3}\right]$$

or $\left[\frac{8+3}{7}, \frac{12+6}{7}\right]$

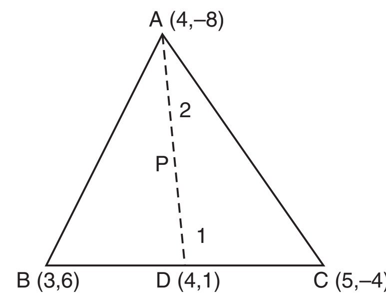
or $\left[\mathbf{\frac{11}{7}}, \mathbf{\frac{18}{7}}\right]$

Q. 20. *If A (4, – 8), B (3, 6) and C (5, – 4) are the vertices of a Δ ABC, D(4, 1) is the mid-point of BC and P is a point on AD joined such that* $\frac{AP}{PD} = 2$, *find the coordinates of P.*

(AI CBSE 2008)

Sol. ∵ D is the mid-point of B

∴ We have $D\left[\frac{3+5}{2}, \frac{6-4}{2}\right]$ or D [4, 1]



Since, $\frac{AP}{PD} = \frac{2}{1}$

⇒ $AP : PD = 2 : 1$

∴ Coordinates of P are:

$$\left[\frac{2\times4+1\times4}{2+1}, \frac{2\times1+1\times(-8)}{2+1}\right]$$

or $\left[\frac{8+4}{3}, \frac{2-8}{3}\right]$

or $\left[\frac{12}{3}, \frac{-6}{3}\right]$ or **[4, – 2]**

Q. 21. *Show that the triangle PQR formed by the points* $P(\sqrt{2}, \sqrt{2})$, $Q(-\sqrt{2}, -\sqrt{2})$ *and* $R(-\sqrt{6}, -\sqrt{6})$ *is an equilateral triangle.*

OR

Name the type of triangle PQR formed by the points $P(\sqrt{2}, \sqrt{2}), Q(-\sqrt{2}, -\sqrt{2})$ *and* $R(-\sqrt{6}, -\sqrt{6})$.

[NCERT Exemplar]

Sol. We have, $P(\sqrt{2}, \sqrt{2})$

$Q(-\sqrt{2}, -\sqrt{2})$

and $R(-\sqrt{6}, -\sqrt{6})$

∴ $PQ = \sqrt{(\sqrt{2}+\sqrt{2})^2 + (\sqrt{2}+\sqrt{2})^2} = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2}$

$= \sqrt{4\times2+4\times2} = \sqrt{8+8} = \sqrt{16} = 4$

$PR = \sqrt{(\sqrt{2}+\sqrt{6})^2 + (\sqrt{2}-\sqrt{6})^2}$

$= \sqrt{2+6+2\sqrt{12}+2+6-2\sqrt{12}} = \sqrt{2+6+2+6} = \sqrt{16} = 4$

$RQ = \sqrt{\left[(-\sqrt{2})+\sqrt{6}\right]^2 + (-\sqrt{2}-\sqrt{6})^2}$

$= \sqrt{2+6-2\sqrt{12}+2+6+2\sqrt{12}} = \sqrt{2+6+2+6} = \sqrt{16} = 4$

Since, $PQ = PR = RQ$ = each (= 4)

∴ PQR is an equilateral triangle.

Q. 22. *The line joining the points (2, – 1) and (5, – 6) is bisected at P. If P lies on the line 2x + 4y + k = 0, find the value of k.*

(AI CBSE 2008)

Sol. We have A (2, – 1) and B (5, – 6).

$\because$ P is the mid point of AB,

$\therefore$ Coordinates of P are: $\left[\frac{2+5}{2}, \frac{-1-6}{2}\right]$ or $\frac{7}{2}, \frac{-7}{2}$

Since P lies on the line $2x + 4y + k = 0$

$\therefore$ We have:

$$2x + 4y + k = 0 \Rightarrow 2\left(\frac{7}{2}\right) + 4\left(\frac{-7}{2}\right) + k = 0$$

$$\Rightarrow \quad 7 - 14 + k = 0$$

$$\Rightarrow \quad -7 + k = 0 \Rightarrow \boldsymbol{k = 7}$$

Q. 23. *Find the point on y-axis which is equidistant from the points (5, – 2) and (– 3, 2).* (CBSE 2009)

Sol. $\because$ Let P is on the y-axis

$\therefore$ Coordinates of P are: $(0, y)$

Since, $PA = PB$

$\therefore$ $PA^2 = PB^2$

$$\Rightarrow \quad (5 - 0)^2 + (-2 - y)^2 = (-3 - 0)^2 + (2 - y)^2$$

$$\Rightarrow \quad 25 + 4 + 4y + y^2 = 9 + 4 - 4y + y^2$$

$$\Rightarrow \quad 25 + 4y = 9 - 4y$$

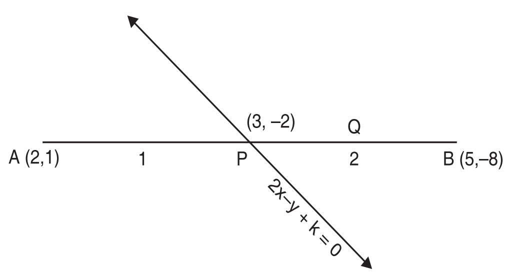
$$\Rightarrow \quad 8y = -16$$

$$\Rightarrow \quad y = \frac{-16}{8} = -2$$

$\therefore$ The required point is **(0, – 2).**

Q. 24. *The line joining the points (2, 1) and (5, – 8) is trisected at the points P and Q. If point P lies on the line 2x – y + k = 0, find the value of k.* (CBSE 2009)

Sol.



$\because$ AB is trisected at P and Q

$\therefore$ Coordinates of P are:

$$\left[\frac{1 \times 5 + 2 \times 2}{1+2}, \frac{1 \times (-8) + 2 \times 1}{1+2}\right]$$

or $\left(\frac{9}{3}, \frac{-6}{3}\right)$ or $(3, -2)$

Since, P $(3, -2)$ lies on $2x - y + k = 0$

$\therefore$ We have:

$$2(3) - (-2) + k = 0$$

$$\Rightarrow \quad 6 + 2 + k = 0$$

$$\Rightarrow \quad 8 + k = 0 \Rightarrow \boldsymbol{k = -8}$$

Q. 25. *If P (x, y) is any point on the line joining the points A (a, 0) and B (0, b), then show that:*

$$\frac{x}{a} + \frac{y}{b} = 1$$ (CBSE 2009)

Sol. ∵ P lies on the line joining A and B.

∴ A, B and P are collinear.

⇒ The area of a Δ formed by A $(a, 0)$, B $(0, b)$ and P (x, y) is zero.

∴ We have:

$$x_1 [y_2 - y_3] + x_2 [y_3 - y_2] + x_3 [y_1 - y_2] = 0$$

$$\Rightarrow \quad x [0 - b] + a [b - y] + 0 [y - 0] = 0$$

$$\Rightarrow \quad -bx + ab - ay = 0$$

$$\Rightarrow \quad -(bx + ay) = -ab$$

$$\Rightarrow \quad bx + ay = ab$$

$$\Rightarrow \quad \frac{bx}{ab} + \frac{ay}{ab} = \frac{ab}{ab}$$ [*Dividing by ab*]

$$\Rightarrow \quad \mathbf{\frac{x}{a} + \frac{y}{b} = 1}$$

Q. 26. *Find the point on x-axis which is equidistant from the points (2, – 5) and (– 2, 9).* (CBSE 2009)

Sol. ∵ The required point 'P' is on x-axis.

∴ Coordinates of P are $(x, 0)$.

∴ We have

$$AP = PB$$

$$\Rightarrow \quad AP^2 = PB^2$$

$$\Rightarrow \quad (2 - x)^2 + (-5 + 0)^2 = (-2 - x)^2 + (9 - 0)^2$$

$$\Rightarrow \quad 4 - 4x + x^2 + 25 = 4 + 4x + x^2 + 81$$

$$\Rightarrow \quad 4x + 25 = 4x + 81$$

$$\Rightarrow \quad -8x = 56$$

$$\Rightarrow \quad x = \frac{56}{-8} = -7$$

∴ The required point is **(– 7, 0).**

Q. 27. *The line segment joining the points P (3, 3) and Q (6, – 6) is trisected at the points A and B such that A is nearer to P. It also lies on the line given by 2x + y + k = 0. Find the value of k.* (CBSE 2009)

Sol. ∵ PQ is trisected by A such that

$$PA : AQ = 1 : 2$$

∴ The coordinates of A are:

$$\left[\frac{1 \times 6 + 2 \times 3}{1 + 2}, \frac{1 \times (-6) + 2 \times 3}{1 + 2}\right]$$

or $$\left[\frac{6 + 6}{3}, \frac{-6 + 6}{3}\right]$$

or $$\left[\frac{12}{3}, \frac{0}{3}\right]$$ or $(4, 0)$.

Since, A (4, 0) lies on the line $2x + y + k = 0$

$\therefore \quad 2(4) + (0) + k = 0$

$\Rightarrow \quad 8 + k = 0 \quad \Rightarrow \quad \boldsymbol{k = -8}$

Q. 28. *Find the ratio in which the points (2, 4) divides the line segment joining the points A (– 2, 2) and B (3, 7). Also find the value of y.* (AI CBSE 2009)

Sol. Let P $(2, y)$ divides the join of A (– 2, 2) and B (3, 7) in the ratio k:1

$\therefore$ Coordinates of P are:

$$\frac{3k-2}{k+1}, \frac{7k+2}{k+1}$$

$$\Rightarrow \quad \frac{3k-2}{k+1} = 2 \quad \text{and} \quad \frac{7k+2}{k+1} = y$$

$$\text{Now,} \quad \frac{3k-2}{k+1} = 2 \quad \Rightarrow \quad 3k - 2 = 2k + 2 \quad \Rightarrow \quad k = 4$$

$$\text{And} \quad \frac{7k+2}{k+1} = 7 \quad \Rightarrow \quad \frac{7(4)+2}{4+1} = y$$

$$\Rightarrow \quad \frac{30}{5} = y \quad \Rightarrow \quad 6 = y$$

Thus, $\boldsymbol{y = 6}$ and $\boldsymbol{k = 4}$

Q. 29. *Find the area of the quadrilateral ABCD whose vertices are:*
A (– 4, – 2), B (– 3, – 5), C (3, – 2) and D (2, 3) (AI CBSE 2009)

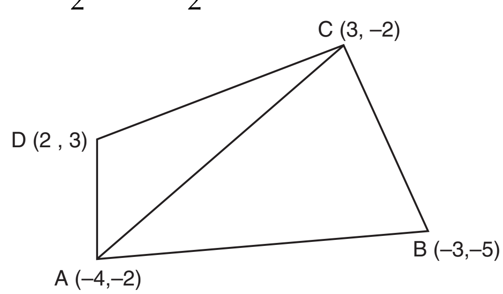
Sol. Area of (Δ ABC)

$$= \frac{1}{2}[(-4)(-5+2) + (-3)(-2+2) + 3(-2+5)]$$

$$= \frac{1}{2}[-4(-3) + (-3)(0) + 3(3)]$$

$$= \frac{1}{2}[-12 + 0 + 9]$$

$$= \frac{1}{2}[21] = \frac{21}{2} \text{ sq. units.}$$



Also, ar (Δ ACD)

$$= \frac{1}{2}[(-4)(-2-3) + 3(3+2) + 2(-2+2)]$$

$$= \frac{1}{2}[20 + 15 + 0]$$

$$= \frac{35}{2} \text{ sq. units.}$$

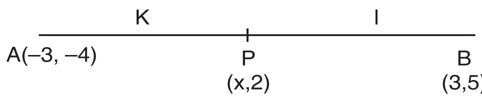
$\therefore$ ar (quad. $ABCD$) = ar (Δ ABC) + ar (Δ ACD)

$$= \frac{21}{2} + \frac{35}{2} \text{ sq. units.}$$

$$= \frac{56}{2} = \mathbf{28 \text{ sq. units.}}$$

Q. 30. *Find the ratio in which the point (x, 2) divides the line segment joining the points (– 3, – 4) and (3, 5). Also find the value of x.* (AI CBSE 2009)

Sol. Let the required ratio = $k : 1$



$\therefore$ Coordinates of the point P are:

$$\left(\frac{3k-3}{k+1}, \frac{5k-4}{k+1}\right)$$

But the coordinates of P are $(x, 2)$

$$\therefore \quad \frac{5k-4}{k+1} = 2 \Rightarrow 5k - 4 = 2k + 2$$

$$\Rightarrow \quad 3k = 6$$

$$\Rightarrow \quad k = \frac{6}{3} = \mathbf{2}$$

$\therefore$ The required ratio is 2 : 1

Now,
$$x = \frac{3k-3}{k+1}$$

$$= \frac{3(2)-3}{2+1}$$

$$= \frac{6-3}{3} = \frac{3}{3} = \mathbf{1}$$

Q. 31. *Find the area of the triangle formed by joining the mid-points of the sides of triangle whose vertices are (0, – 1), (2, 1), and (0, 3).* (AI CBSE 2009)

Sol. We have the vertices of the given triangle as A (0, – 1), B (2, 1) and C (0, 3). Let D, E and F be the mid-points of AB, BC and AC.

$\therefore$ Coordinates of D are $\left[\frac{0+2}{2}, \frac{-1+1}{2}\right]$ or (1, 0)

E are $\left[\frac{2+0}{2}, \frac{1+3}{2}\right]$ or (1, 2)

F are $\left[\frac{0+0}{2}, \frac{3+(-1)}{2}\right]$ or (0, 1)

$\therefore$ Coordinates of the vertices of Δ DEF are (1, 0), (1, 2) and (0, 1).

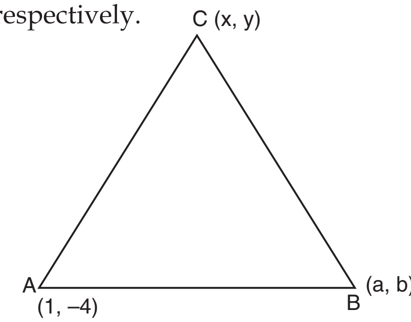
Now, area of $\Delta\, DEF$ $= \frac{1}{2}[1(2-1) + 1(1-0) + 0(0-2)]$

$$= \frac{1}{2} \times 2 = \mathbf{1 \text{ sq. units.}}$$

Q. 32. *Find the area of the ΔABC with A(1, –4), and the mid-point of sides through A being (2, –1) and (0, –1).* [NCERT Exemplar]

Sol. Let the co-ordinates of B and C are (a, b) and (x, y) respectively.

Sides through A are AB and AC



$$\therefore \quad (2, -1) = \left(\frac{1+a}{2}, \frac{-4+b}{2}\right)$$

$$= \frac{1+a}{2} = 2 \text{ and } \frac{-4+b}{2} = -1$$

$$= 1 + a = 4 \quad \text{and} \quad -4 + b = -2$$

$$= a = 3 \quad \text{and} \quad b = 2$$

Also, $$(0, -1) = \left(\frac{1+x}{2}, \frac{-4+y}{2}\right)$$

$$= \frac{1+x}{2} = 0 \text{ and } \frac{-4+y}{2} = -1$$

$$= 1 + x = 0 \text{ and } -4 + y = -2$$

$$= x = -1 \quad \text{and} \quad y = 2$$

Thus, the co-ordinates of the vertices of ΔABC are: $A(1, -4)$, $B(3, 2)$ and $C(-1, 2)$

$\therefore$ Area of ΔABC

$$= \frac{1}{2}[1(2-2)+3(2+4)-1(-4-2)]$$

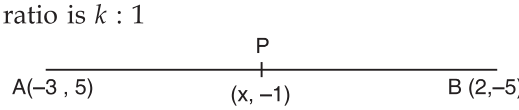
$$= \frac{1}{2}[0+18+6]$$

$$= \frac{1}{2}[24]$$

$$= \textbf{12 sq. units}$$

Q. 33. *Find the ratio in which the point (x, – 1) divides the line segment joining the points (– 3, 5) and (2, – 5). Also find the value of x.* (AI CBSE 2009)

Sol. Let the required ratio is $k : 1$



$\therefore$ The coordinates of P are:

$$\left[\frac{2k-3}{k+1}, \frac{-5k+5}{k+1}\right]$$

But the coordinates of P are $(x, -1)$

$$\therefore \quad \frac{-5k+5}{k+1} = -1 \Rightarrow -5k + 5 = -k - 1$$

$$\Rightarrow \quad 2k = 3 \quad \text{or} \quad k = \frac{3}{2}$$

Also, $$x = \frac{2k-3}{k+1} = \frac{2\left(\frac{3}{2}\right)-3}{\frac{3}{2}+1} = \frac{3-3}{\frac{5}{2}} = 0$$

$$\therefore \quad x = \mathbf{0}$$

And $$k = \mathbf{\frac{3}{2}}$$

Q. 34. *If the mid-point of the line segment joining the point A(3, 4) and B(k, 6) is P(x, y) and x + y – 10 = 0, then find the value of k.* [NCERT Exemplar]

Sol. $\because$ Mid point of the line segment joining $A(3, 4)$ and $B(k, 6)$

$$= \left(\frac{3+k}{2}, \frac{4+6}{2}\right) = \left(\frac{3+k}{2}, 5\right)$$

$$\therefore \quad \left(\frac{3+k}{2}, 5\right) = (x, y) \quad \Rightarrow \quad \frac{3+k}{2} = x \text{ and } 5 = y$$

Since, $\quad x + y - 10 = 0$

$$\Rightarrow \quad \frac{3+k}{2} + 5 - 10 = 0 \qquad \left[\because x = \frac{3+k}{2} \text{ and } y = 5\right]$$

$$\Rightarrow \quad 3 + k + 10 - 20 = 0$$

$$\Rightarrow \quad 3 + k = 10$$

$$\Rightarrow \quad k = 10 - 3 = 7$$

Thus, the required value of $k = 7$

Q. 35. *Point P, Q, R and S divide the line segment joining the points A (1, 2) and B (6, 7) in 5 equal parts. Find the co-ordinates of the points P, Q and R.* [AI. CBSE (Foreign) 2014]

Sol.

$\therefore$ P, Q, R and S, divide AB into five equal parts.

$\therefore$ AP = PQ = QR = RS = SB

Now, P divides AB in the ratio 1 : 4

Let, the co-ordinates of P be x and y.

$\therefore$ Using the section formula i.e.,

$$x = \frac{mx_2 + nx_1}{m+n}, \; y = \frac{my_2 + ny_1}{m+n}, \text{ we have}$$

$$\therefore \quad x = \frac{1(6)+4(1)}{1+4} = \frac{6+4}{5} = 2$$

$$y = \frac{1(7)+4(2)}{1+4} = \frac{7+8}{5} = 3$$

$$(x, y) = (2, 3)$$

Next, Q divides AB in the ratio 2 : 3

$\therefore$ Co-ordinates of Q are :

$$\left[\frac{2(6)+3(1)}{2+3} ; \quad \frac{2(7)+3(2)}{5}\right] \text{ or } \left[\frac{15}{5}, \frac{20}{5}\right] \text{ or } (3, 4)$$

Now, R divides AB in the ratio 3 : 2

$\Rightarrow$ Co-ordinates of R are :

$$\left[\frac{3(6)+2(1)}{3+2}, \quad \frac{3(7)+2(2)}{3+2}\right] \text{ or } \left(\frac{20}{5}, \frac{25}{5}\right) \text{ or } (4, 5)$$

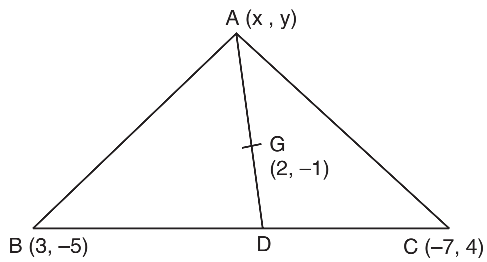
The co-ordinates of P, Q and R are respectively :

(2, 3), (3, 4) and (4, 5).

TEST YOUR SKILLS

1. The line-segment joining the points (3, – 4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, – 2) and $\left(\frac{5}{3}, q\right)$ respectively, find the values of p and q. *[CBSE 2005]*

2. In the figure, find the coordinates of A. *[AI CBSE 2005]*



3. The line joining the points (2, 1) and (5, – 8) is trisected at the points P and Q. If P lies on the line $2x - y + k = 0$, find the value of k. *[AI CBSE 2006]*

4. If the coordinates of the mid-points of the sides of a Δ are (10, 5), (8, 4) and (6, 6), then find the coordinates of its vertices. *[AI CBSE 2006]*

5. Find the coordinates of the points which divide the line segment joining the points (– 4, 0), and (0, 6) in three equal pasts. *[CBSE 2005C]*

6. Find the coordinates of the point equidistant from the points A (1, 2), B (3, – 4) and C (5, – 6). *[CBSE 2005C]*

7. Prove that the points A (– 4, – 1), B (– 2, – 4), C (4, 0) and D (2, 3) are the vertices of a rectangle. *[CBSE 2005C]*

8. Find the coordinates of the points which divide the line-segment joining the points (– 4, 0) and (0, 6) in four equal parts. *[CBSE 2005C]*

9. Find the coordinates of the points which divide the line-segment joining the points (2, –2) and (–7, 4) in three equal parts. *[CBSE 2011]*

10. Find the coordinates of the point equidistant from the points A (5, 1), B (– 3, –7) and C (7, – 1). *[AI CBSE 2005 Comptt]*

11. The vertices of a Δ ABC and given by A (2, 3) and B (– 2, 1) and its centroid is $G\left(1, \frac{2}{3}\right)$. Find the coordinates of the third vertex C of the Δ ABC. *[AI CBSE 2005C]*

12. If the points (x, y) is equidistant from the points ($a + b$, $b - a$) and ($a - b$, $a + b$). Prove that $bx = ay$. *[AI CBSE 2005 Comptt]*

13. Two vertices of a Δ ABC are given by A (6, 3) and B (–1, 7) and its centroid is G (1, 5). Find the coordinates of the third vertex of the Δ ABC. *[AI CBSE 2005C]*

14. Two of the vertices of a Δ ABC are given by A (6, 4) and B (– 2, 2) and its centroid is G (3, 4). Find the coordinates of the third vertex C of the Δ ABC. *[AI CBSE 2005C]*

15. The coordinates one end point of a diameter of a circle are (4, –1). If the coordinates of the centre be (1, –3) find the coordinates of the other end of the diameter. *[CBSE 2006]*

16. Show that the points A (1, 2), B (5, 4) C (3, 8) and D (– 1, 6) are the vertices of a square. *[CBSE 2006]*

17. Find the value of P for which the points (– 1, 3), (2, p) and (5, – 1) are collinear. *[CBSE 2006]*

18. Find the distance of the point (– 6, 8) from the origin. *[AI CBSE 2006]*

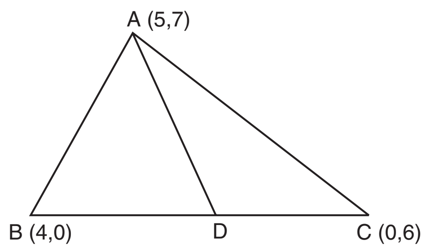
19. Find the coordinates of the point equidistant from three given points A (5, 3), B (5, – 5) and C (1, – 5). *[AI CBSE 2006]*

20. Find the value of p for which the points (– 5, 1), (1, p) and (4, – 2) are collinear. *[AI CBSE 2006]*

21. Find the coordinates of the point on the line joining P (1, – 2) and Q (4, 7) that is twice as far from P as from Q. *[AI CBSE 2006]*

22. Find the perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0). *[CBSE 2011, NCERT Exemplar Problem]*

23. In the following figure, find the length of the median AD. *[CBSE 2006C]*



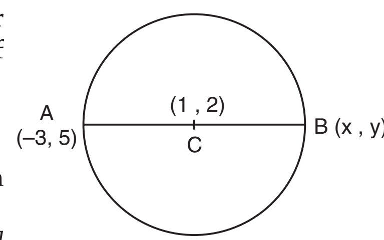
24. Prove that the points (3, 0), (6, 4) and (– 1, 3) are vertices of right angled triangle. Also prove that these are the vertices of an isosceles triangle. *[CBSE 2006C]*

25. In what ratio is the line segment joining the points (– 2, – 3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division. *[CBSE 2006C]*

26. If A (5, – 1), B (– 3, – 2) and C (– 1, 8) are the vertices of ΔABC, find the length of median through A and the coordinates of the centroid. *[CBSE 2006C]*

27. Find the value of k if the points A (2, 3), B (4, k) and C (6, – 3) are collinear. *[CBSE 2006C]*

28. In the figure, A and B are the end points of a diameter of a circle having its centre at (1, 2). If the coordinates of A are (– 3, 5) find the the coordinates of point B. *[AI CBSE 2006C]*



29. If (– 2, – 1), (a, 0), (4, b) and (1, 2) are the vertices of a parallelogram, find the value of 'a' and 'b'. *[AI CBSE 2006C]*

30. The vertices of a triangle are (– 1, 3), (1, – 1) and (5, 1). Find the lengths of medians through vertices (– 1, 3) and (5, 1). *[AI CBSE 2006C]*

31. By distance formula, show that the points (1, – 1); (5, 2) and (9, 5) are collinear. *[AI CBSE 2006C]*

32. Show that the points (7, 10), (– 2, 5) and (3, – 4) are the vertices of an isosceles right triangle. *[CBSE 2007]*

33. In what ratio the line $x - y - 2 = 0$ divides the line segment joining (3, – 1) and (8, 9)? *[CBSE 2007]*

34. Find the ratio in which the line joining the points (6, 4) and (1, – 7) is divided by x-axis. *[CBSE 2007]*

35. Find the ratio in which the point (– 3, k) divides the line segment joining the points (– 5, – 4) and (– 2, 3). Hence find the value of k. *[AI CBSE 2007]*

36. Three consecutive vertices of a parallelogram are (– 2, – 1), (1, 0) and (4, 3). Find the coordinates of the fourth vertex. *[AI CBSE 2007]*

37. For what value P, are the points (2, 1), (p, – 1) and (– 1, 3) collinear?

38. Show that the point P (– 4, 2) lies on the line segment joining the points A (– 4, 6) and B (– 4, – 6).

39. Find the value (s) of k for which the points [$(3k – 1)$, $(k – 2)$], [k, $(k – 7)$] and [$(k – 1)$, $(– k – 2)$] are collinear. *[AI CBSE (Foreign) 2014]*

Hint: *Here,* $x_1 = (3k - 1)$ $\quad x_2 = k$ $\quad x_3 = (k - 1)$

$y_1 = (k - 2)$ $\quad y_2 = (k - 7)$ $\quad y_3 = (-k - 2)$

For the given points to be collinear, we have

$$(3k - 1)[(k - 7) - (-k - 2)] + k[(-k - 2) - (k - 2)] + (k - 1)[(k - 2) - (k - 7)] = 0$$

$\Rightarrow$ $6k^2 - 17k + 5 - 2k^2 + 5k - 5 = 0$ *or* $4k^2 - 12 = 0 \Rightarrow k = 0, 3$

40. If the point A (0, 2) is equidistant from the point B(3, p) and C (p, 5), find p. *[CBSE (Delhi) 2014]*

Hint: *Here,* $AB = AC$

$\therefore$ $$\sqrt{3^2 + (p-2)^2} = \sqrt{p^2 + 3^2} \Rightarrow 9 + (p-2)^2 = p^2 + 9$$

$\Rightarrow$ $$(p - 2)^2 = p^2 \Rightarrow p^2 + 4 - 4p = p^2 \Rightarrow p = 1$$

41. Find the ratio in which the point $P(x, 2)$ divides the line segment joining the points A(12, 5) and B(4, – 3). Also find the value of x. *[CBSE (Delhi) 2014]*

Hint: *Here,* $x_1 = 12$, $x_2 = 4$, $y_1 = 5$ *and* $y_2 = -3$

Let $P(x, 2)$ *divide the given line segment in the ratio* $k : 1$

$\therefore$ co-ordinates of P are :

$$x = \frac{k(4) + 1(12)}{k+1}, \qquad 2 = \frac{k(-3) + 1(5)}{k+1}$$

$\Rightarrow$ $$[x(k + 1) = 4k + 12], \qquad [2(k + 1) = -3k + 5]$$

$$2(k + 1) = -3k + 5 \Rightarrow 2k + 2 = -3k + 5 \Rightarrow 5k = 3 \text{ or } k = \frac{3}{5}$$

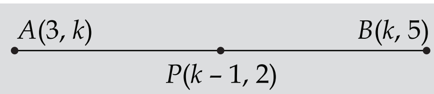
$$x(k + 1) = k(4) + 12 \Rightarrow x\left[\left(\frac{3}{5}\right) + 1\right] = \frac{3}{5}(4) + 12$$

$$\Rightarrow \frac{8}{5}x = \frac{72}{5} \quad \text{or} \quad x = \frac{72}{5} \times \frac{5}{8} = 9$$

i.e., $x = 9$ *and ratio* $= 3 : 5$

42. If the point $P(k – 1, 2)$ is equidistant from the point A (3, k) and B(k, 5) then find the values of k. *[AI. CBSE 2014]*

Hint:



$$AP = BP \Rightarrow \sqrt{(k-1)-3]^2 + (2-k)^2} = \sqrt{(k-1-k)^2 + (2-5)^2}$$

$\Rightarrow$ $$\sqrt{(k-4)^2 + (2-k)^2} = \sqrt{(-1)^2 + (-3)^2}$$

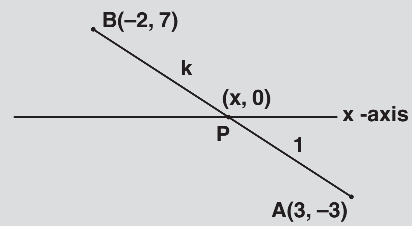
$\Rightarrow$ *Solving it we get $k = 5$*

43. Find the ratio in which the line segment joining the points A(3, –3) and B(–2, 7) is divided by x– axis. Also find the co-ordinates of the point of division. *[AI CBSE 2014]*

Hint: *Let the x–axis meets AB at P(x, O)*

Let (k : 1) is the required ratio.

$\therefore$ *Co-ordinates of P are given as*



$$x = \frac{3k-2}{k+1}$$

$$y = \frac{-3k+7}{k+1} = 0 \Rightarrow k = \frac{7}{3}$$

Now, $$x = \frac{3\left(\frac{7}{3}\right)-2}{\frac{7}{3}+1} = \frac{3}{2}$$

We get $x = \frac{3}{2}$ *and Required ratio as 7 : 3*

$\Rightarrow$ co-ordinates of the point of division $\left(\frac{3}{2}, 0\right)$

44. The mid-point P of the line segment joining the points A(–10, 4) and B(–2, 0) lies on the line segment joining the points C(–9, –4) and D(–4, y). Find the ratio in which P divides CD. Also find the value of y. *[AI. CBSE (Foreign) 2014]*

Hint: *Mid point of AB [where A (–10, 4) and B (–2, 0)] is P(–6, 2)*

Let P(–6, 2) divide CD, [where C(–9, –4) and D(–4, y)] in k : 1

$\therefore$ $$\frac{k(-4)+1(-9)}{k+1} = -6 \Rightarrow k = 3/2 \quad and \quad 2 = \frac{k(y)+1(-4)}{k+1} \Rightarrow y = 6$$

$\therefore$ *Required ratio = 3 : 2 and y = 6.*

ANSWERS

Test Your Skills

1. $p = \frac{7}{3}; \ q = 0$ **2.** A(10, –2) **3.** k = –8 or k = –13

4. (4, 5), (8, 3) and (12, 3) **5.** $\left(\frac{-8}{3}, 2\right), \left(\frac{-4}{3}, 3\right)$ **6.** (–1, –2)

7. ___ **8.** $\left(-3, \frac{3}{2}\right)$, $(2, 3)$ and $\left(-1, \frac{9}{2}\right)$ **9.** (–1, 0) and (–4, 2)

10. (2, –4)	**11.** (3, –2)	**12.** ___
13. (–2, 5)	**14.** (5, 6)	**15.** (–2, –5)
16. ___	**17.** $p = 1$	**18.** 10
19. (3, –1)	**20.** $p = -1$	**21.** (2, 1)
22. 12 units	**23.** 5 cm	**24.** ___
25. 2 : 3, (0, 1)	**26.** $\sqrt{65}; \left(\frac{1}{3}, \frac{5}{3}\right)$	**27.** $k = 0$
28. (5, –1)	**29.** $a = 1$, $b = 3$	**30.** $\left(\frac{5}{3}, 1\right)$
31. ___	**32.** ___	**33.** 2 : 3
34. 4 : 7	**35.** 2 : 1, $k = \frac{2}{3}$	**36.** (1, 2), (1, 4)
37. $p = 5$	**38.** ___	