Short Answer Type Questions – I

[2 MARKS]

Que 1. Write the coordinates of a point on $x-\text{axis}$ which is equidistant from the points $(-3, 4)$ and $(2, 5)$.

Sol. Let the required point be $(x, 0)$.

Since, $(x, 0)$ is equidistant from the points $(-3, 4)$ and $(2, 5)$

$\therefore \quad \sqrt{(-3-x)^2+(4-0)^2}=\sqrt{(2-x)^2+(5-0)^2}$

$\Rightarrow \quad \sqrt{9+x^2+6x+16}=\sqrt{4+x^2-4x+25}$

$\therefore \quad x^2+6x+25=x^2-4x+29 \quad \Rightarrow \quad 10x=4$ or $x=\frac{4}{10}=\frac{2}{5}$

$\therefore$ Required point is $\left(\frac{2}{5}, 0\right)$

Que 2. Find the values of x for which the distance between the points $P(2, 3)$ and $Q(x, 5)$ is 10.

Sol. Distance between the given points $=\sqrt{(x-2)^2+(5+3)^2}$

$\Rightarrow \quad 10=\sqrt{x^2+4-4x+64}$

$\Rightarrow \quad 100=x^2-4x+68$

$\Rightarrow \quad x^2-4x-32=0$

$\Rightarrow \quad x^2-8x+4x-32=0$

$\Rightarrow \quad (x-8)(x+4)=0 \quad \Rightarrow \quad x=8, -4$

Que 3. What is the distance between the points $(10\cos 30^0, 0)$ and $0, 10\cos 60^0)$?

Sol. Distance between the given points $=\sqrt{(0-10\cos 30^0)^2+(10\cos 60^0-0)^2}$

$=\sqrt{100cos^2 30^0+100cos^2 60^0}$

$=\sqrt{100\left[\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2\right]}=\sqrt{100\left(\frac{3}{4}+\frac{1}{4}\right)}=\sqrt{100}=10$ units.

Que 4. In Fig.6.8, if $A(-1, 3), B(1, -1)$ and $C(5, 1)$ are the vertices of a triangle ABC, what is the length of the median through vertex A?

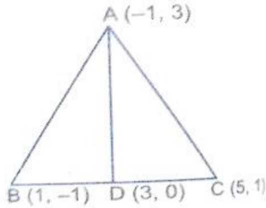

Fig. 6.8

Sol. Coordinates of the mid-point of $BC = \left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = (3,0)$

∴ Length of the median through $A = \sqrt{(3+1)^2 + (0-3)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ units.

Que 5. Find the ratio in which the line segment joining the points $P(3,-6)$ and $Q(5,3)$ is divided by the x −axis.

Sol. Let the required ratio be $\lambda : 1$

Then, the point of division is $\left(\frac{5\lambda+3}{\lambda+1}, \frac{3\lambda+6}{\lambda+1}\right)$

Given that this point lies on the x-axis

∴ $\frac{3\lambda-6}{\lambda+1} = 0$ or $3\lambda = 6$ or $\lambda = 2$

Thus, the required ratio is 2: 1.

Que 6. Point $P(5,-3)$ is one of the two points of trisection of the line segment joining the points $A(7,-2)$ and $B(1,-5)$. State true or false and justify your answer.

Sol. Points of trisection of line segment AB are given by

$= \left(\frac{2\times1+1\times7}{3}, \frac{2\times-5+1\times-2}{3}\right)$ and $\left(\frac{1\times1+2\times7}{3}, \frac{1\times-5+2\times-2}{3}\right)$

$= \left(\frac{9}{3}, \frac{-12}{3}\right)$ and $\left(\frac{15}{3}, \frac{-9}{3}\right)$ or $(3,-4)$ and $(5,-3)$

∴ Given statement is true.

Que 7. ΔABC with vertices $A(-2,0), B(2,0)$ and $C(0,2)$ is similar to ΔDEF with vertices $D(-4,0), E(4,0)$ and $F(0,4)$. State true or false and justify your answer.

Sol. $AB = \sqrt{(2+2)^2 + 0} = \sqrt{16} = 4$

$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{8} = 2\sqrt{2}$

$$CA = \sqrt{(-2-0)^2 + (0-2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$DE = \sqrt{(4+4)^2 + 0} = \sqrt{64} = 8$$

$$EF = \sqrt{(0-4)^2 + (4-0)^2} = \sqrt{32} = 4\sqrt{2}$$

$$FD = \sqrt{(-4-0)^2 + (0-4)^2} = \sqrt{32} = 4\sqrt{2}$$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{1}{2} \quad \Rightarrow \quad \Delta ABC \sim \Delta DEF$

Que 8. Point $P(0,2)$ is the point of intersection of y-axis and perpendicular bisector of line segment joining the points, $A(-1,1)$ and $B(3,3)$. State true or false and justify your answer.

Sol. The point $P(0,2)$ lies on y-axis

Also, $AP = \sqrt{(0+1)^2 + (2-1)^2} = \sqrt{2}$

$BP = \sqrt{(0-3)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$\because \quad AP \neq BP$

$\therefore$ $P(0,2)$ Does not lie on the perpendicular bisector of AB. So, given statement is false.

Que 9. Check whether $(5,2), B\,(6,4)$ and $(7,-2)$ are the vertices of an isosceles triangle.

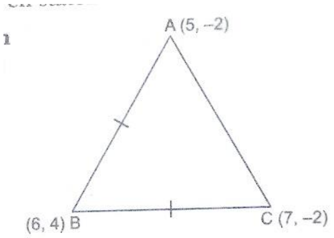

Fig. 6.9

Sol. Let $A\,(5,-2), B(6,4)$ and $C(7,-2)$ be the vertices of a triangle

Then we have,

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$$

$$AC = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{4} = 2$$

Here, $AB = BC$

$\therefore \quad \Delta ABC$ is an isosceles triangle.

Que 10. If $(1,2), (4,y), (x,6)$ and $(3,5)$ are the vertices of a parallelogram taken in order, find x and y.

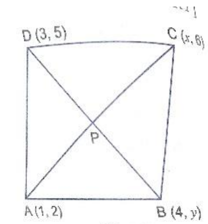

Fig. 6.10

Sol. Let $A(1,2), B(4,y), C(x,6)$ and $D(3,5)$ be the vertices of a parallelogram $ABCD$.

Since, the diagonals of a parallelogram bisect each other.

$\therefore \quad \left(\frac{x+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

$\Rightarrow \quad \frac{x+1}{2} = \frac{7}{2}$

$\Rightarrow \quad x+1=7$ or $x=6$

$\Rightarrow \quad 4 = \frac{5+y}{2} \quad 5+y=8$ or $y=8-5=3$

Hence, $x=6$ and $y=3$.

Que 11. Find the ratio in which y-axis divides the line segment joining the points $A(5,-6)$ and $B(-1,-4)$. Also find the coordinates of the point of division.

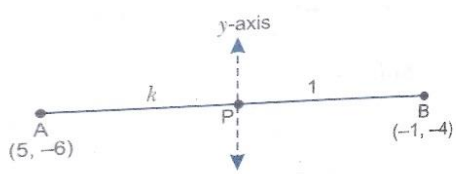

Fig. 6.11

Sol. Let the point on y-axis be $P(0,y)$ and $AP: PB = K: 1$

Therefore $\frac{5-k}{k+1} = 0$ gives $k=5$

Hence required ratio is $5:1$.

$$y = \frac{-4(5)-6}{5+1} = \frac{-13}{3}$$

Hence, point on y-axis is $P(0, \frac{-13}{3})$.

Que 12. Let P and Q be the points of trisection of the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ such that P is nearer to A. Find the coordinates of P and Q.

Fig. 6.12

Sol. $\because P$ divides AB in the ratio $1 : 2$

$\because$ Coordinates of $P = \left(\frac{1\times(-7)+2\times2}{1+2}\right), \left(\frac{1\times4+2\times-(2)}{1+2}\right)$

$$= \left(\frac{-7+4}{3}, \frac{4-4}{3}\right) = (-1,0)$$

$\therefore$ Q is the midpoint of PB

$\therefore$ Coordinates of Q = $\frac{-1+(-7)}{2}, \frac{0+4}{2}$

$$= \frac{-8}{2}, 2 = -4,2$$

Que 13. Find the ratio in which the point (-3, k) divides the line-segment joining the points (-5,-4) and (-2, 3). Also find the value of k.

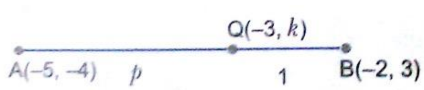

Fig. 6.13

Sol. Let Q divide AB in the ratio of p: 1

$$-3 = \frac{-2p-5}{p+1}$$

$\Rightarrow$ -3p – 3 = - 2p – 5 $\Rightarrow$ p = 2

$\therefore$ Ratio is 2: 1

$$K = \frac{2\times3-4}{2+1} = \frac{2}{3}$$

Que 14. The x-coordinate of a point p is twice its y-coordinate. If p is equidistant from Q (2,-5) and R (-3, 6), find the coordinate of p.

Sol. Let the point p be (2y, y)

PQ = PR $\Rightarrow \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$

$\Rightarrow 4y^2 + 4 - 8y + y^2 + 25 + 10y = 4y^2 + 9 + 12y + y^2 + 36 - 12y$

$\Rightarrow \quad 2y + 29 = 45 \quad \Rightarrow y = 8$

Hence, coordinates of point P are (16,8).