Chapter 13. Statistics

Ex. 13.4

Answer 1CU.

Consider the example of two data sets:

Data set 1	Data set 2
17	63
24	65
29	69
30 $\leftarrow Q_1$	73 $\leftarrow Q_1$
33	74
35	75
38	78
39 $\leftarrow Q_2$	82 $\leftarrow Q_2$
40	85

42	87
43	88
49 $\leftarrow Q_3$	89 $\leftarrow Q_3$
50	90
55	92
57	94

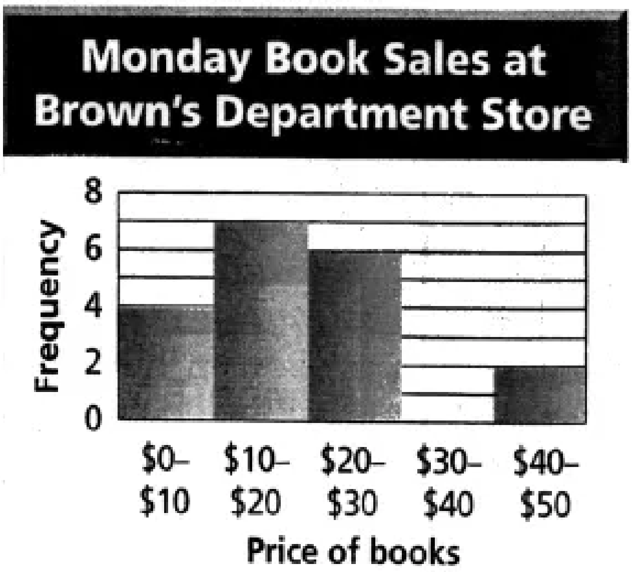
The range of data set1 is $57-17=40$ and data set 2 is $94-63=31$

IQR for data set 1 is $Q_3-Q_1=49-30=19$

IQR for data set 2 is $Q_3-Q_1=89-73=16$

Answer 1PQ.

Consider the following histogram:



From the figure, the information we obtain is:

In the range \$0-\$10, frequency of book sales is 4, in \$10-\$20 the frequency is 7, in \$20-\$30 the frequency is 6, in \$30-\$40, the frequency is 0, in \$40-\$50, the frequency is 2.

Therefore, total no of book is $4+7+6+2=19$.

So, the position of median is 10. Thus the median occurs in the range $\boxed{\$10-\$20}$.

Answer 2CU.

The mean is strongly affected by an outlier in a data set.

Consider the data set:

1,1,2,2,2,2,3,3,3,4,4,400

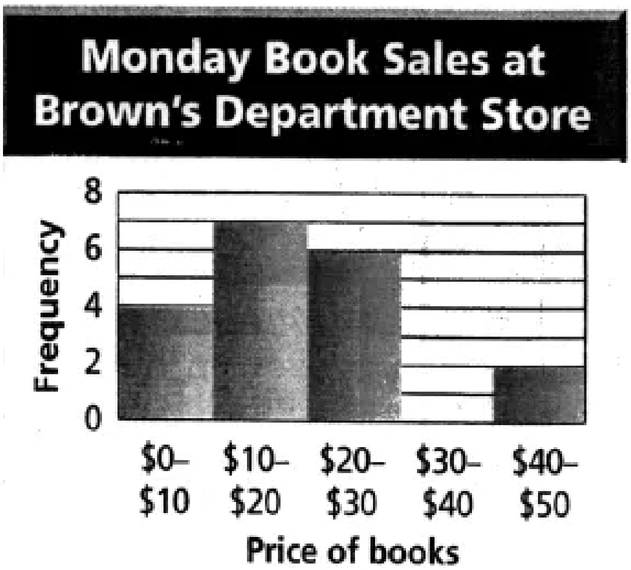
Here,

$$\text{mean} = 35{,}38$$
$$\text{median} = 2.5$$
$$\text{mode} = 2$$
$$\text{standard deviation} = 114.74$$

Thus, having outliers often has a significant effect on mean and standard deviation. Because of this we must take steps to remove outliers from our data set.

Answer 2PQ.

Consider the following histogram:



This is Right-skewed distribution. The skewed distribution is asymmetrical because a natural limit prevents outcomes on one side. The distribution's peak is off center toward the limit and a tail stretches away from it.

Answer 3CU.

The data set is:

28,30,32,36,40,41,43

Here,

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 43 - 28\\ &= 15\end{aligned}$$

Since the range is the difference between the greatest and the least values of the set so Alosco is correct.

Answer 3PQ.

Consider the following set of data:

1050, 1175, 835, 1075, 1025, 1145, 1100, 1125, 975, 1005, 1125, 1095, 1075, 1055

The data in ascending order:

835, 975, 1005, 1025, 1050, 1055, 1075, 1075, 1095, 1100, 1125, 1125, 1145, 1175

The range of the data is

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 1175 - 835\\ &= \boxed{340}\end{aligned}$$

Answer 4CU.

Consider the data set is:

85,77,58,69,62,73,25,82,67,77,59,75,69,76

List the data from least to greatest:

25,58,59,62,67,69,69,73,75,76,77,77,82,85

Here,

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 85 - 25\\ &= \boxed{60}\end{aligned}$$

$$\begin{aligned}\text{median} &= \frac{69+73}{2}\\ &= \frac{142}{2}\\ &= \boxed{71}\end{aligned}$$

$$\begin{aligned}\text{lower quartile}, Q_1 &= \frac{59+62}{2}\\ &= \frac{121}{2}\\ &= \boxed{60.5}\end{aligned}$$

$$\begin{aligned}\text{upper quartile}, Q_3 &= \frac{77+77}{2}\\ &= \frac{154}{2}\\ &= \boxed{77}\end{aligned}$$

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 77 - 60.5 \\ &= \boxed{16.5} \end{aligned}$$

An outlier must be $1.5(16.5) = 24.75$ less than the lower quartile,60.5 or $1.5(16.5) = 24.75$ greater than the upper quartile,77.

$60.5 - 24.75 = 35.75 \qquad 77 + 24.75 = 101.75$

There is value less than 35.75 that is 25 but no value greater than 101.75. Thus the outlier is $\boxed{25}$.

Answer 4PQ.

Consider the following set of data:

1050, 1175, 835, 1075, 1025, 1145, 1100, 1125, 975, 1005, 1125, 1095, 1075, 1055

The data in ascending order:

835, 975, 1005, 1025, 1050, 1055, 1075, 1075, 1095, 1100, 1125, 1125, 1145, 1175

Here,

$$\begin{aligned} \text{median} &= \frac{1075 + 1075}{2} \\ &= \frac{2150}{2} \\ &= \boxed{1075} \end{aligned}$$

$$\begin{aligned} \text{lower quartile}, Q_1 &= \frac{1005 + 1025}{2} \\ &= \frac{2030}{2} \\ &= \boxed{1015} \end{aligned}$$

$$\begin{aligned} \text{upper quartile}, Q_3 &= \frac{1125 + 1125}{2} \\ &= \frac{2250}{2} \\ &= \boxed{1125} \end{aligned}$$

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 1125 - 1015 \\ &= \boxed{110} \end{aligned}$$

Answer 5CU.

Consider the stem leaf data set is:

Stem	Leaf
7	3 7 8
8	0 0 3 5 7
9	4 6 8
10	0 1 8
11	1 9 7|3=7.3

The data can be written as:

7.3,7.7,7.8,8.0,8.0,8.3,8.5,8.7,9.4,9.6,9.8,10.0,10.1,10.8,11.1,11.9

Here,

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 11.9 - 7.3\\ &= \boxed{4.6}\end{aligned}$$

$$\begin{aligned}\text{median} &= \frac{8.7+9.4}{2}\\ &= \frac{18.1}{2}\\ &= \boxed{9.05}\end{aligned}$$

$$\text{lower quartile}, Q_1 = \boxed{8.0}$$

$$\text{upper quartile}, Q_3 = \boxed{10.1}$$

$$\begin{aligned}IQR &= Q_3 - Q_1\\ &= 10.1 - 8.0\\ &= \boxed{2.1}\end{aligned}$$

An outlier must be $1.5(2.1) = 3.15$ less than the lower quartile,8.0 or $1.5(2.1) = 3.15$ greater than the upper quartile,10.1.

$8.0 - 3.15 = 4.85$ $\qquad$ $10.1 + 3.15 = 13.25$

There is no value less than 4.85 and greater than 13.25. Thus there is no outlier.

Answer 5PQ.

Consider the following set of data:

1050, 1175, 835, 1075, 1025, 1145, 1100, 1125, 975, 1005, 1125, 1095, 1075, 1055

The data in ascending order:

835, 975, 1005, 1025, 1050, 1055, 1075, 1075, 1095, 1100, 1125, 1125, 1145, 1175

Here,

$$Q_1 = 1015, Q_3 = 1125, IQR = 110$$

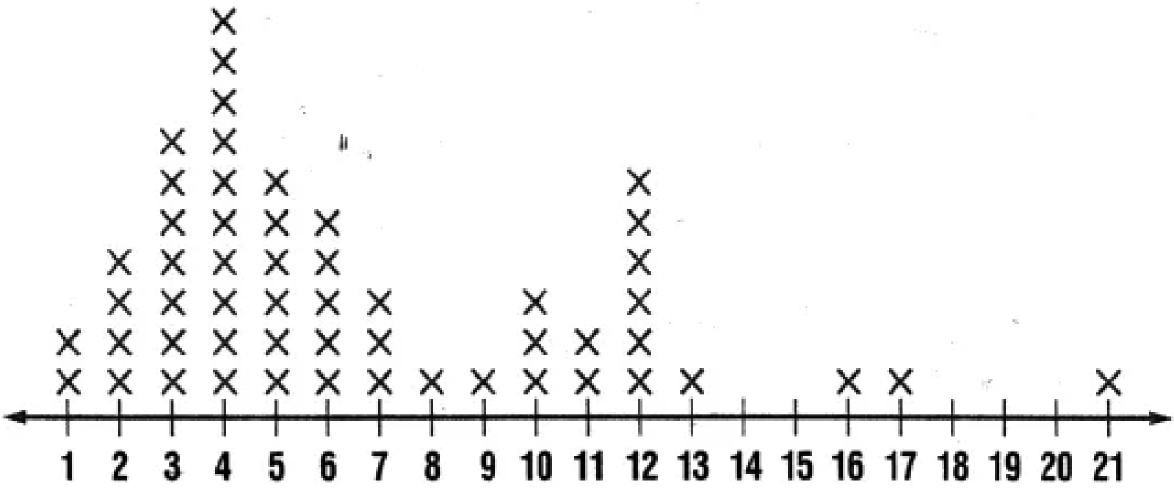
An outlier must be $1.5(110) = 165$ less than the lower quartile or $1.5(110) = 165$ greater than the upper quartile.

$$1015 - 165 = 850$$
$$1125 + 165 = 1290$$

There is a value less than 850 that is 835 but no value greater than 1290. Thus the outlier is $\boxed{835}$.

Answer 6CU.

Consider the figure:



The data can be written as:

2, 4, 7, 10, 6, 5 ,3 ,1 ,1 ,3, 2, 6, 1, 0, 0, 1, 1, 0, 0, 0, 1

List the data from least to greatest:

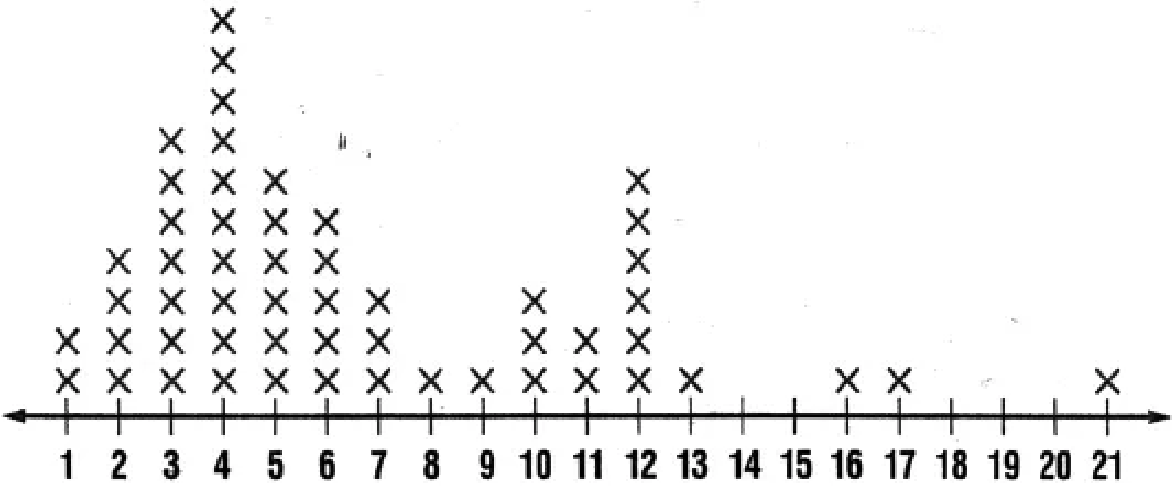
0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 3, 3, 4, 5, 6, 6, 7, 10

The range of the data is:

$$\begin{aligned} \text{range} &= \text{greatest value} - \text{lowest value} \\ &= 10 - 0 \\ &= \boxed{10} \end{aligned}$$

Answer 7CU.

Consider the figure:



The data can be written as:

2, 4, 7, 10, 6, 5 ,3 ,1 ,1 ,3, 2, 6, 1, 0, 0, 1, 1, 0, 0, 0, 1

List the data from least to greatest:

median
↓

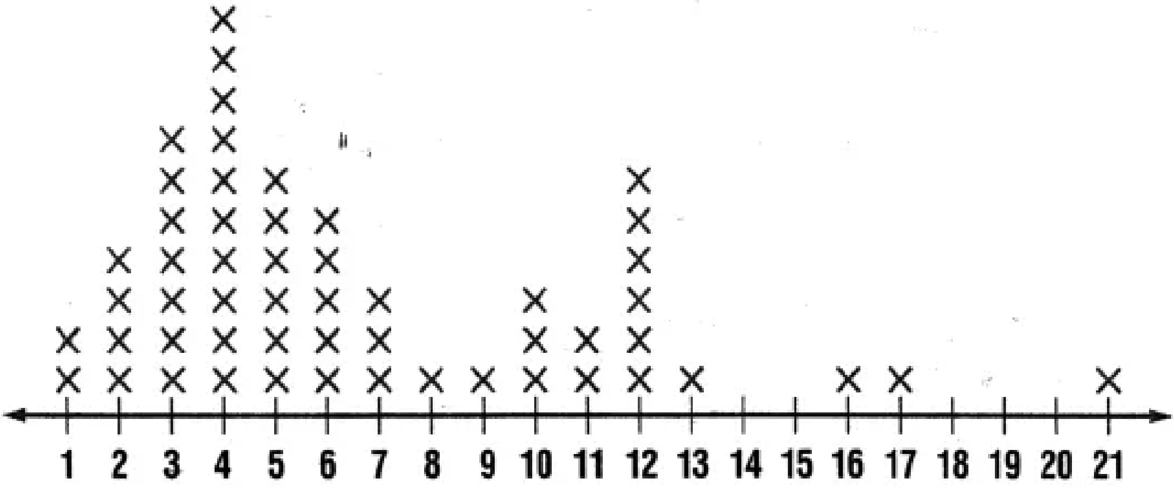
0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 3, 3, 4, 5, 6, 6, 7, 10

The median of the data is:

$\text{median} = \boxed{1}$

Answer 8CU.

Consider the figure:



The data can be written as:

2, 4, 7, 10, 6, 5 ,3 ,1 ,1 ,3, 2, 6, 1, 0, 0, 1, 1, 0, 0, 0, 1

List the data from least to greatest:

median

↓

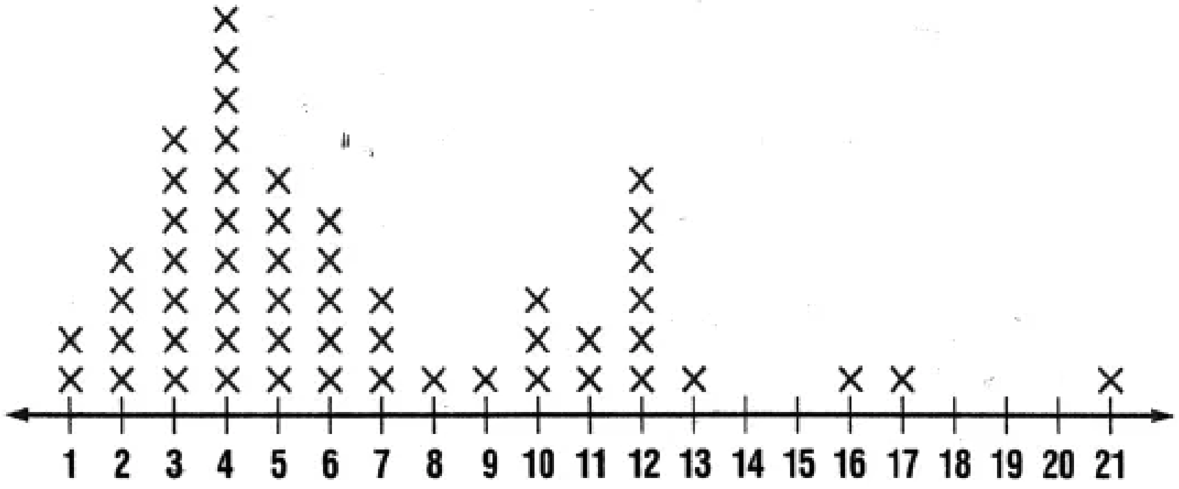
0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 3, 3, 4, 5, 6, 6, 7, 10

$$\text{lower quartile}, Q_1 = \frac{0+1}{2} = \frac{1}{2} = \boxed{0.5}$$

$$\text{upper quartile}, Q_3 = \frac{4+5}{2} = \frac{9}{2} = \boxed{4.5}$$

Answer 9CU.

Consider the figure:



The data can be written as:

2, 4, 7, 10, 6, 5 ,3 ,1 ,1 ,3, 2, 6, 1, 0, 0, 1, 1, 0, 0, 0, 1

List the data from least to greatest:

median
↓

0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 3, 3, 4, 5, 6, 6, 7, 10

Here,

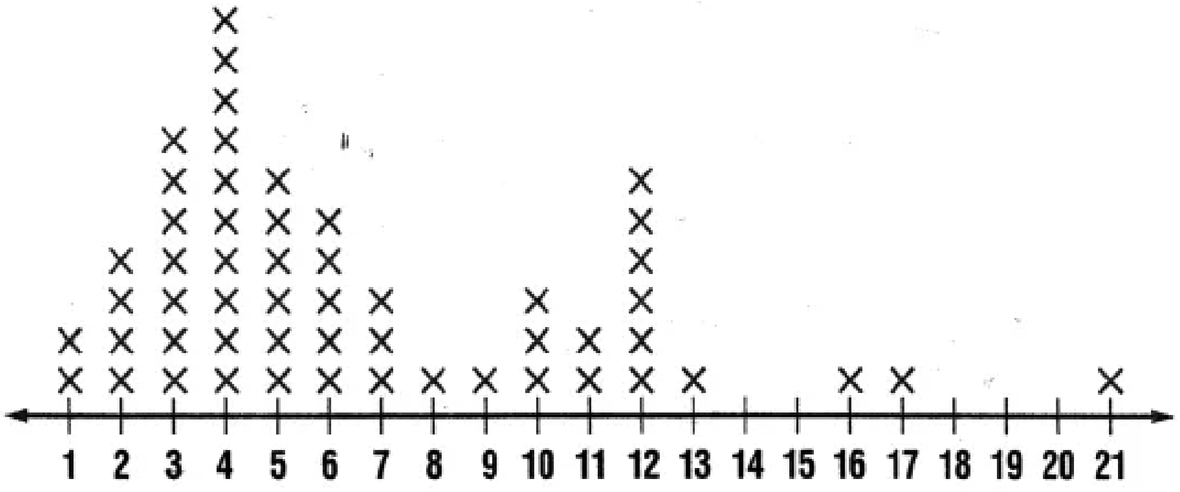
lower quartile, $Q_1 = 0.5$ and upper quartile, $Q_3 = 4.5$

Therefore,

$$\begin{aligned} \text{Interquartile range, } IQR &= Q_3 - Q_1 \\ &= 4.5 - 0.5 \\ &= \boxed{4} \end{aligned}$$

Answer 10CU.

Consider the figure:



The data can be written as:

2, 4, 7, 10, 6, 5, 3 ,1 ,1 ,3, 2, 6, 1, 0, 0, 1, 1, 0, 0, 0, 1

List the data from least to greatest:

median

$\downarrow$

0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 3, 3, 4, 5, 6, 6, 7, 10

Here,

$\text{lower quartile}, Q_1 = 0.5$ and $\text{upper quartile}, Q_3 = 4.5$

$\text{Interquartile range}, IQR = 4$

An outlier must be $1.5(4) = 6$ less than the lower quartile, 0.5 or $1.5(4) = 6$ greater than the upper quartile, 4.5.

$0.5 - 6 = -5.5 \qquad 4.5 + 6 = 10.5$

There is no value less than -5.5 and greater than 10.5. Thus there is no outlier.

Answer 11PA.

Consider the data set is:

85,77,58,69,62,73,55,82,67,77,59,92,75

List the data from least to greatest:

$$\begin{array}{l}\text{median}\\ \quad\downarrow\end{array}$$

55,58,59,62,67,69,73,75,77,77,82,85,92

Here,

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 92 - 55\\ &= \boxed{37}\end{aligned}$$

$$\text{median} = \boxed{73}$$

$$\begin{aligned}\text{lower quartile}, Q_1 &= \frac{59+62}{2}\\ &= \frac{121}{2}\\ &= \boxed{60.5}\end{aligned}$$

$$\begin{aligned}\text{upper quartile}, Q_3 &= \frac{77+82}{2}\\ &= \frac{159}{2}\\ &= \boxed{79.5}\end{aligned}$$

$$\begin{aligned}IQR &= Q_3 - Q_1\\ &= 79.5 - 60.5\\ &= \boxed{19}\end{aligned}$$

An outlier must be $1.5(19) = 28.5$ less than the lower quartile,60.5 or $1.5(19) = 28.5$ greater than the upper quartile,79.5.

$60.5 - 28.5 = 32$ $\qquad\qquad$ $79.5 + 28.5 = 108$

There is no value less than 32 and greater than 108. Thus there is no outlier.

Answer 12PA.

Consider the data set is:

28, 42, 37, 31, 34, 29, 44, 28, 38, 40, 39, 42, 30

List the data from least to greatest:

$$\begin{array}{l}\text{median}\\ \quad\downarrow\end{array}$$

28, 28, 29, 30, 31, 34, 37, 38, 39, 40, 42, 42, 44

Here,

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 44 - 28\\ &= \boxed{16}\end{aligned}$$

$$\text{median} = \boxed{37}$$

$$\begin{aligned}\text{lower quartile}, Q_1 &= \frac{29+30}{2}\\ &= \frac{59}{2}\\ &= \boxed{29.5}\end{aligned}$$

$$\begin{aligned}\text{upper quartile}, Q_3 &= \frac{40+42}{2}\\ &= \frac{82}{2}\\ &= \boxed{41}\end{aligned}$$

$$\begin{aligned}IQR &= Q_3 - Q_1\\ &= 41 - 29.5\\ &= \boxed{11.5}\end{aligned}$$

An outlier must be $1.5(11.5) = 17.25$ less than the lower quartile,29.5 or $1.5(11.5) = 17.25$ greater than the upper quartile,41.

$29.5 - 17.25 = 12.25$ $\qquad$ $41 + 17.25 = 58.25$

There is no value less than 12.25 and greater than 58.25. Thus there is no outlier.

Answer 13PA.

Consider the data set:

30.8, 29.9, 30.0, 31.0, 30.5, 30.7, 31.0

List the data from least to greatest:

$$\begin{array}{l}\text{median}\\ \quad\downarrow\end{array}$$

29.9, 30.0, 30.5, 30.7, 30.8, 31.0, 31.0

Here,

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 31.0 - 29.9\\ &= \boxed{1.1}\end{aligned}$$

$$\text{median} = \boxed{30.7}$$

$$\text{lower quartile}, Q_1 = \boxed{30.0}$$

$$\text{upper quartile}, Q_3 = \boxed{31.0}$$

$$\begin{aligned}IQR &= Q_3 - Q_1\\ &= 31.0 - 30.0\\ &= \boxed{1.0}\end{aligned}$$

An outlier must be $1.5(1.0) = 1.5$ less than the lower quartile, 30.0 or $1.5(1.0) = 1.5$ greater than the upper quartile, 31.0.

$30.0 - 1.5 = 28.5$ $\qquad\qquad$ $31.0 + 1.5 = 32.5$

There is no value less than 28.5 and greater than 32.5. Thus there is no outlier.

Answer 14PA.

Consider the data set:

2, 3.4, 5.3, 3, 1, 3.2, 4.9, 2.3

List the data from least to greatest:

1, 2, 2.3, 3, 3.2, 3.4, 4.9, 5.3

Here,

$$\begin{aligned} \text{range} &= \text{greatest value} - \text{lowest value} \\ &= 5.3 - 1 \\ &= \boxed{4.3} \end{aligned}$$

$$\begin{aligned} \text{median} &= \frac{3+3.2}{2} \\ &= \frac{6.2}{2} \\ &= \boxed{3.1} \end{aligned}$$

$$\text{lower quartile}, Q_1 = \boxed{2}$$

$$\text{upper quartile}, Q_3 = \boxed{4.9}$$

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 4.9 - 2 \\ &= \boxed{2.9} \end{aligned}$$

An outlier must be $1.5(2.9) = 4.35$ less than the lower quartile or $1.5(2.9) = 4.35$ greater than the upper quartile.

$2 - 4.35 = -2.35$ $\qquad$ $4.9 + 4.35 = 9.25$

There is no value less than -2.35 and greater than 9.25. Thus there is no outlier.

Answer 15PA.

Consider the stem leaf data set is:

Stem	Leaf
5	3 6 8
6	5 8
7	0 3 7 7 9
8	1 4 8 8 9
9	9 5|3=53

The data can be written as:

53, 56, 58, 65, 68, 70, 73, 77, 77, 79, 81, 84, 88, 88, 89, 99

Here,

$$\text{range} = \text{greatest value} - \text{lowest value}$$
$$= 99 - 53$$
$$= \boxed{46}$$

$$\text{median} = \frac{77 + 77}{2}$$
$$= \frac{154}{2}$$
$$= \boxed{77}$$

$$\text{lower quartile}, Q_1 = \boxed{65}$$

$$\text{upper quartile}, Q_3 = \boxed{88}$$

$$IQR = Q_3 - Q_1$$
$$= 88 - 65$$
$$= \boxed{23}$$

An outlier must be $1.5(23) = 34.5$ less than the lower quartile, 65 or $1.5(23) = 34.5$ greater than the upper quartile, 88.

$65 - 34.5 = 30.5$ $\quad\quad\quad$ $88 + 34.5 = 122.5$

There is no value less than 30.5 and greater than 122.5. Thus there is no outlier.

Answer 16PA.

Consider the stem leaf data set:

Stem	Leaf
19	3 5 5
20	2 2 5 8
21	5 8 8 9 9 9
22	0 1 7 8 9
23	2 19|3=193

The data can be written as:

193, 195, 195, 202, 202, 205, 208, 215, 218, 218, 219, 219, 219, 220, 221, 227, 228, 229, 232

Here,

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 232 - 193\\ &= \boxed{39}\end{aligned}$$

$$\text{median} = \boxed{218}$$

$$\text{lower quartile}, Q_1 = \boxed{202}$$

$$\text{upper quartile}, Q_3 = \boxed{221}$$

$$\begin{aligned}IQR &= Q_3 - Q_1\\ &= 221 - 202\\ &= \boxed{19}\end{aligned}$$

An outlier must be $1.5(19) = 28.5$ less than the lower quartile, 202 or $1.5(19) = 28.5$ greater than the upper quartile, 221.

$202 - 28.5 = 173.5$ $\qquad$ $221 + 28.5 = 249.5$

There is no value less than 173.5 and greater than 249.5. Thus there is no outlier.

Answer 17PA.

Consider the stem leaf data set:

Stem	Leaf
5	0 3 7 9
6	1 3 4 5 5 6
7	1 5 6 6 9
8	1 2 3 5 8
9	2 5 6 9
10	
11	7 5|0=5.0

The data can be written as:

5.0, 5.3, 5.7, 5.9, 6.1, 6.3, 6.4, 6.5, 6.5, 6.6, 7.1, 7.5, 7.6, 7.6, 7.9, 8.1, 8.2, 8.3, 8.5, 8.8, 9.2, 9.5, 9.6, 9.9, 11.7

Here,

$$\text{range} = \text{greatest value} - \text{lowest value}$$
$$= 11.7 - 5.0$$
$$= \boxed{6.7}$$

$$\text{median} = \boxed{7.6}$$

$$\text{lower quartile}, Q_1 = \frac{6.3 + 6.4}{2}$$
$$= \frac{12.7}{2}$$
$$= \boxed{6.35}$$

$$\text{upper quartile}, Q_3 = \frac{8.5 + 8.8}{2}$$
$$= \frac{17.3}{2}$$
$$= \boxed{8.65}$$

$$IQR = Q_3 - Q_1$$
$$= 8.65 - 6.35$$
$$= \boxed{2.3}$$

An outlier must be $1.5(2.3) = 3.45$ less than the lower quartile, 6.35 or $1.5(2.3) = 3.45$ greater than the upper quartile, 8.65.

$6.35 - 3.45 = 2.9$ $\quad\quad$ $8.65 + 3.45 = 12.1$

There is no value less than 2.9 and greater than 12.1. Thus there is no outlier.

Answer 18PA.

Consider the stem leaf data set:

Stem	Leaf
0	0 2 3
1	1 7 9
2	2 3 5 6
3	3 4 4 5 9
4	0 7 8 8
5	
6	8 0|2=0.2

The data can be written as:

0.0, 0.2, 0.3, 1.1, 1.7, 1.9, 2.2, 2.3, 2.5, 2.6, 3.3, 3.4, 3.4, 3.5, 3.9, 4.0, 4.7, 4.8, 4.8, 6.8

Here,

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 6.8 - 0.0\\ &= \boxed{6.8}\end{aligned}$$

$$\begin{aligned}\text{median} &= \frac{2.6+3.3}{2}\\ &= \frac{5.9}{2}\\ &= \boxed{2.95}\end{aligned}$$

$$\text{lower quartile}, Q_1 = \boxed{1.7}$$

$$\text{upper quartile}, Q_3 = \boxed{4.0}$$

$$\begin{aligned}IQR &= Q_3 - Q_1\\ &= 4.0 - 1.7\\ &= \boxed{2.3}\end{aligned}$$

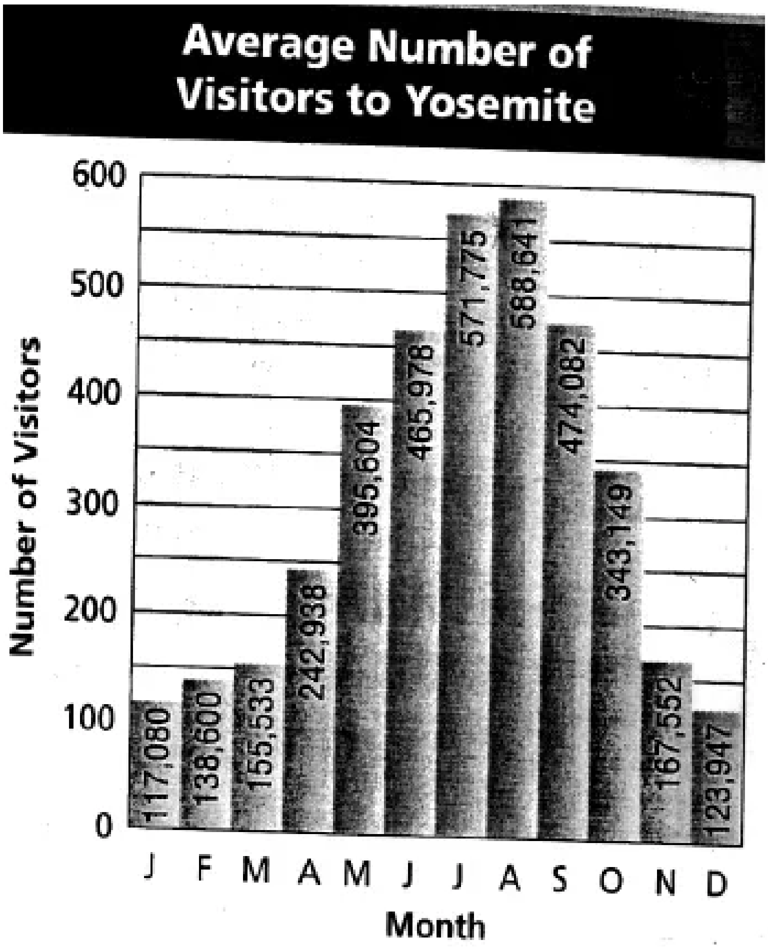
An outlier must be $1.5(2.3) = 3.45$ less than the lower quartile, 1.7 or $1.5(2.3) = 3.45$ greater than the upper quartile, 4.0.

$1.7 - 3.45 = -1.75$ $\quad\quad$ $4.0 + 3.45 = 7.45$

There is no value less than 2.9 and greater than 12.1. Thus there is no outlier.

Answer 19PA.

Consider the graph:



The data from lower to higher can be written as:

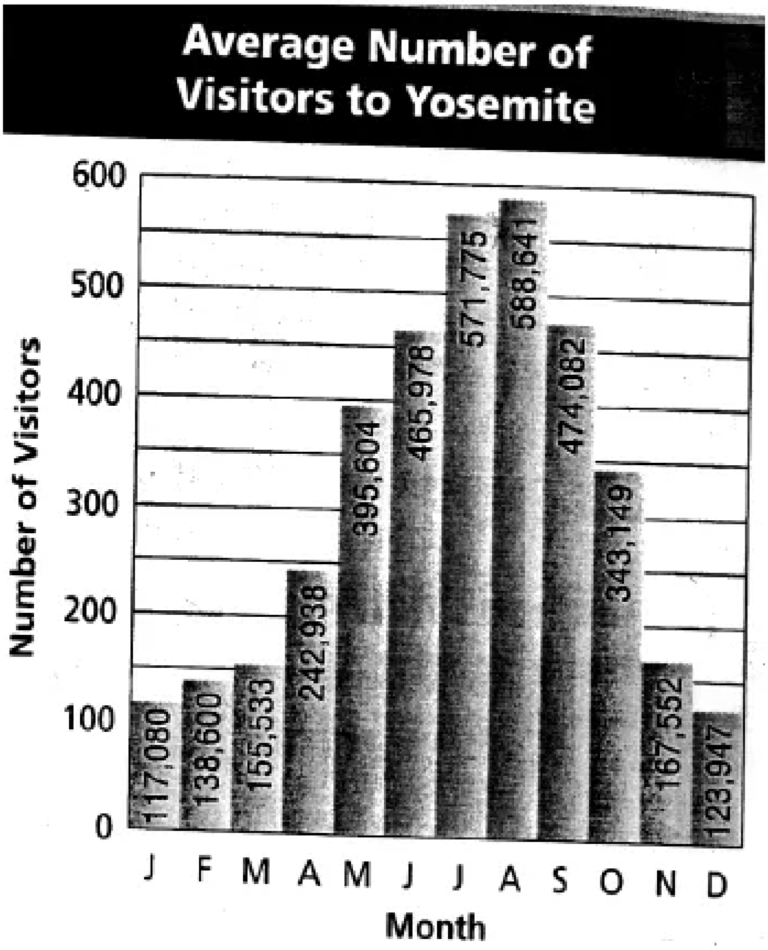
117080, 123947, 138600, 155533, 167552, 242938, 343149, 395604, 465978, 474082, 571775, 588641

The range of visitors per month is

$$\begin{aligned} \text{range} &= \text{greatest value} - \text{lowest value} \\ &= 588641 - 117080 \\ &= \boxed{471561} \end{aligned}$$

Answer 20PA.

Consider the graph:



The data from lower to higher can be written as:

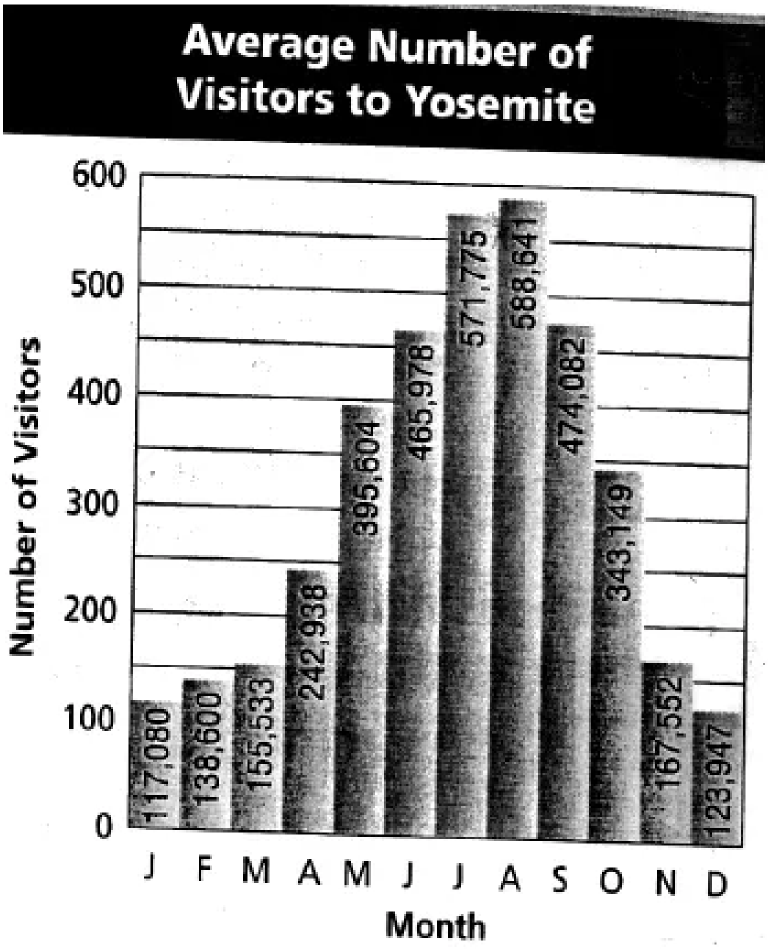
117080, 123947, 138600, 155533, 167552, 242938, 343149, 395604, 465978, 474082, 571775, 588641

The median number of visitors per month is

$$\begin{aligned}\text{median} &= \frac{242938 + 343149}{2} \\ &= \frac{586087}{2} \\ &= \boxed{293043.5}\end{aligned}$$

Answer 21PA.

Consider the graph:



The data from lower to higher can be written as:

117080, 123947, 138600, 155533, 167552, 242938, 343149, 395604, 465978, 474082, 571775, 588641

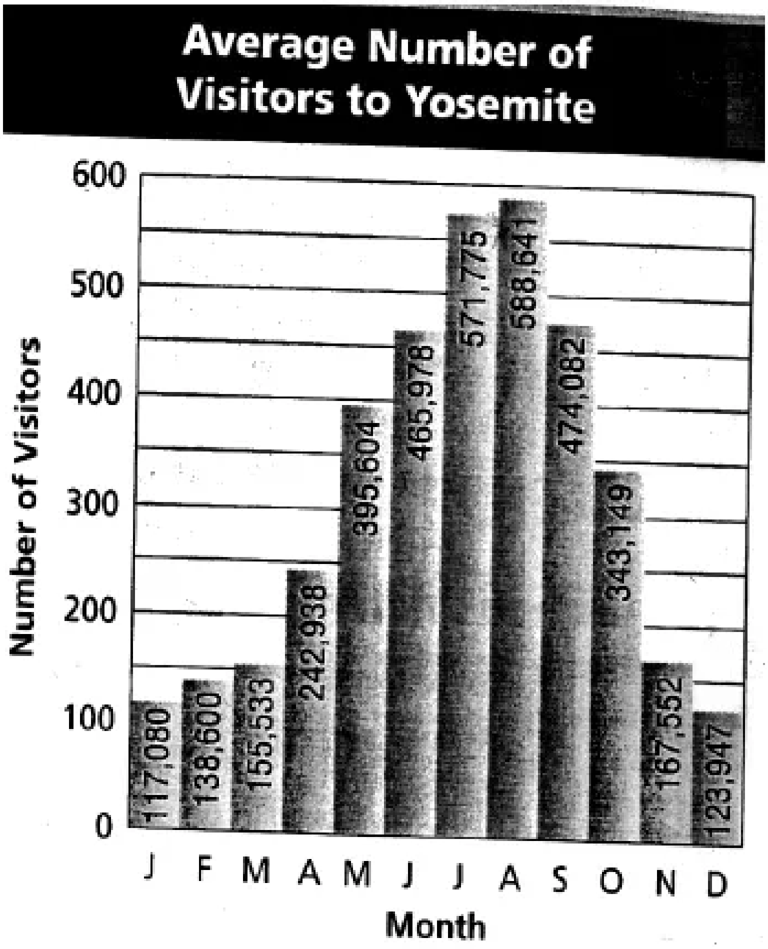
The lower and upper quartiles of the data are

$$\text{lower quartile}, Q_1 = \boxed{138600}$$

$$\text{upper quartile}, Q_3 = \boxed{474082}$$

Answer 22PA.

Consider the graph:



The data from lower to higher can be written as:

117080, 123947, 138600, 155533, 167552, 242938, 343149, 395604, 465978, 474082, 571775, 588641

The lower and upper quartiles of the data are

$$\text{lower quartile}, Q_1 = 138600$$

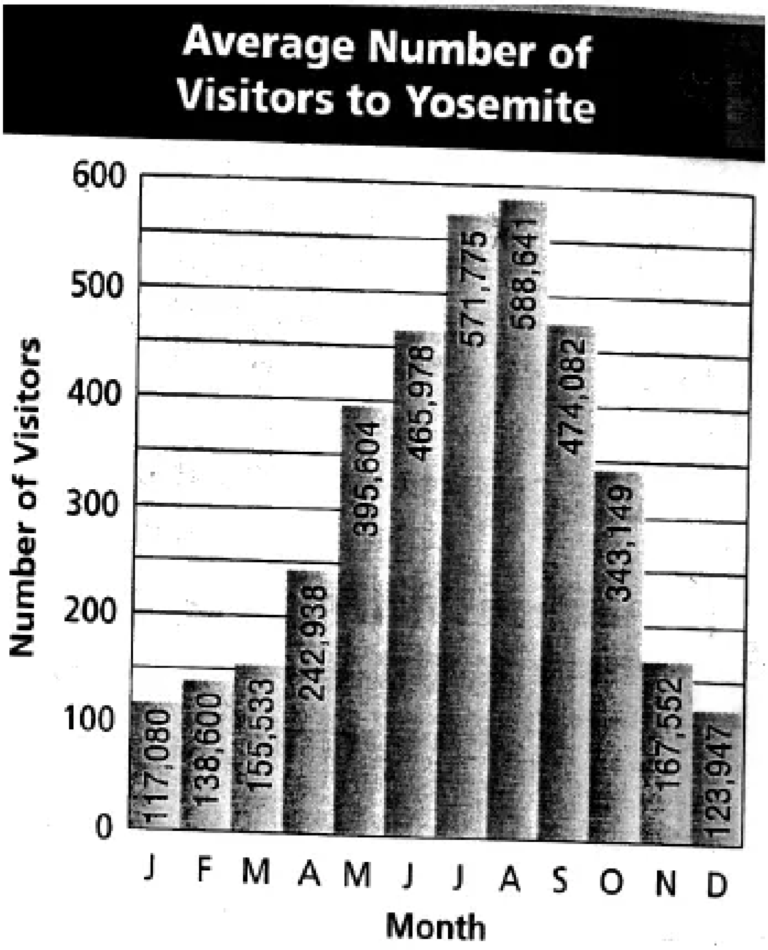
$$\text{upper quartile}, Q_3 = 474082$$

Therefore, Interquartile range of the data is

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 474082 - 138600 \\ &= \boxed{335482} \end{aligned}$$

Answer 23PA.

Consider the graph:



The data from lower to higher can be written as:

117080, 123947, 138600, 155533, 167552, 242938, 343149, 395604, 465978, 474082, 571775, 588641

Interquartile range of the data is 335482

An outlier must be $1.5(335482) = 503223$ less than the lower quartile, 138600 or $1.5(335482) = 503223$ greater than the upper quartile, 474082.

$$138600 - 503223 = -364623$$
$$474082 + 503223 = 977305$$

There is no value less than -364623 and greater than 977305. Thus there is no outlier.

Answer 24PA.

Consider the graph:

Calories for One Serving of Vegetables					
Vegetable	Calories	Vegetable	Calories	Vegetable	Calories
Asparagus	14	Carrots	28	Lettuce	9
Avocado	304	Cauliflower	10	Onion	60
Bell pepper	20	Celery	17	Potato	89
Broccoli	25	Corn	66	Spinach	9
Brussels sprouts	60	Green beans	30	Tomato	35
Cabbage	17	Jalapeno peppers	13	Zucchini	17

The data from lower to higher can be written as:

9, 9, 10, 13, 14, 17, 17, 17, 20, 25, 28, 30, 35, 60, 60, 66, 89, 304

The range of the data is

$$\begin{aligned} \text{range} &= \text{greatest value} - \text{lowest value} \\ &= 304 - 9 \\ &= \boxed{295} \end{aligned}$$

Answer 25PA.

Consider the graph:

Calories for One Serving of Vegetables					
Vegetable	Calories	Vegetable	Calories	Vegetable	Calories
Asparagus	14	Carrots	28	Lettuce	9
Avocado	304	Cauliflower	10	Onion	60
Bell pepper	20	Celery	17	Potato	89
Broccoli	25	Corn	66	Spinach	9
Brussels sprouts	60	Green beans	30	Tomato	35
Cabbage	17	Jalapeno peppers	13	Zucchini	17

The data from lower to higher can be written as:

9, 9, 10, 13, 14, 17, 17, 17, 20, 25, 28, 30, 35, 60, 60, 66, 89, 304

The median of the data is

$$\text{median} = \frac{20+25}{2}$$
$$= \frac{45}{2}$$
$$= \boxed{22.5}$$

Answer 26PA.

Consider the graph:

Calories for One Serving of Vegetables					
Vegetable	Calories	Vegetable	Calories	Vegetable	Calories
Asparagus	14	Carrots	28	Lettuce	9
Avocado	304	Cauliflower	10	Onion	60
Bell pepper	20	Celery	17	Potato	89
Broccoli	25	Corn	66	Spinach	9
Brussels sprouts	60	Green beans	30	Tomato	35
Cabbage	17	Jalapeno peppers	13	Zucchini	17

The data from lower to higher can be written as:

9, 9, 10, 13, 14, 17, 17, 17, 20, 25, 28, 30, 35, 60, 60, 66, 89, 304

The lower and upper quartiles of the data are

$$\text{lower quartile}, Q_1 = \frac{13+14}{2}$$
$$= \frac{27}{2}$$
$$= \boxed{13.5}$$

$$\text{upper quartile}, Q_3 = \frac{60+60}{2}$$
$$= \frac{120}{2}$$
$$= \boxed{60}$$

Answer 27PA.

Consider the graph:

Calories for One Serving of Vegetables					
Vegetable	Calories	Vegetable	Calories	Vegetable	Calories
Asparagus	14	Carrots	28	Lettuce	9
Avocado	304	Cauliflower	10	Onion	60
Bell pepper	20	Celery	17	Potato	89
Broccoli	25	Corn	66	Spinach	9
Brussels sprouts	60	Green beans	30	Tomato	35
Cabbage	17	Jalapeno peppers	13	Zucchini	17

The data from lower to higher can be written as:

9, 9, 10, 13, 14, 17, 17, 17, 20, 25, 28, 30, 35, 60, 60, 66, 89, 304

The lower and upper quartiles of the data are

lower quartile, $Q_1 = 13.5$

upper quartile, $Q_3 = 60$

Therefore, the interquartile range of the data is

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 60 - 13.5 \\ &= \boxed{46.5} \end{aligned}$$

Answer 28PA.

Consider the graph:

Calories for One Serving of Vegetables					
Vegetable	Calories	Vegetable	Calories	Vegetable	Calories
Asparagus	14	Carrots	28	Lettuce	9
Avocado	304	Cauliflower	10	Onion	60
Bell pepper	20	Celery	17	Potato	89
Broccoli	25	Corn	66	Spinach	9
Brussels sprouts	60	Green beans	30	Tomato	35
Cabbage	17	Jalapeno peppers	13	Zucchini	17

The data from lower to higher can be written as:

9, 9, 10, 13, 14, 17, 17, 17, 20, 25, 28, 30, 35, 60, 60, 66, 89, 304

The interquartile range of the data is 46.5

An outlier must be $1.5(46.5)=69.75$ less than the lower quartile, 13.5 or $1.5(46.5)=69.75$ greater than the upper quartile, 60.

$$13.5-69.75=-56.25$$
$$60+69.75=129.75$$

There is no value less than -56.25 but there is a value greater than 129.75 that is 304.
Therefore the outlier is $\boxed{304}$.

Answer 29PA.

Consider the stem leaf plot:

Cable-Stayed	Stem	Steel-Arch
6 4 3	6	
9 8 6 5 1 1	7	3 8 8
0	8	0 0 2 3 4
5 2	9	0 1 1 8 8 9
8 2 0 0	10	0 3 8
2	11	0 0
8 2 0	12	0 6
5 2 0	13	
9 0	14	
	15	
3	16	5
3|6 = 630 feet	17	0 7|3 = 730 feet

The data for cable-stayed bridges can be written as:

630, 640, 660, 710, 710, 750, 760, 780, 790, 800, 920, 950, 1000, 1000, 1020, 1080, 1120, 1200, 1220, 1280, 1300, 1320, 1350, 1400, 1490, 1630.

The data for steel-arch bridges can be written as:

730, 780, 780, 800, 800, 820, 830, 840, 900, 910, 910, 980, 980, 990, 1000, 1030, 1080, 1100, 1100, 1200, 1260, 1650, 1700

The range of the data for cable-stayed bridges is

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 1630 - 630\\ &= \boxed{1000}\end{aligned}$$

The range of the data for steel-arch bridges is

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 1700 - 730\\ &= \boxed{970}\end{aligned}$$

Answer 30PA.

Consider the stem leaf plot:

Cable-Stayed	Stem	Steel-Arch
6 4 3	6	
9 8 6 5 1 1	7	3 8 8
0	8	0 0 2 3 4
5 2	9	0 1 1 8 8 9
8 2 0 0	10	0 3 8
2	11	0 0
8 2 0	12	0 6
5 2 0	13	
9 0	14	
	15	
3	16	5
	17	0

3 | 6 = 630 feet *7 | 3 = 730 feet*

The data for cable-stayed bridges can be written as:

630, 640, 660, 710, 710, 750, 760, 780, 790, 800, 920, 950, 1000, 1000, 1020, 1080, 1120, 1200, 1220, 1280, 1300, 1320, 1350, 1400, 1490, 1630.

The data for steel-arch bridges can be written as:

730, 780, 780, 800, 800, 820, 830, 840, 900, 910, 910, 980, 980, 990, 1000, 1030, 1080, 1100, 1100, 1200, 1260, 1650, 1700

The lower and upper quartiles for the data for cable-stayed bridges is

$$\text{lower quartile}, Q_1 = \frac{750+760}{2} = \frac{1510}{2} = \boxed{755}$$

$$\text{upper quartile}, Q_3 = \frac{1280+1300}{2} = \frac{2580}{2} = \boxed{1290}$$

The lower and upper quartiles for the data for steel-arch bridges is

$$\text{lower quartile}, Q_1 = \boxed{820}$$

$$\text{upper quartile}, Q_3 = \boxed{1100}$$

Answer 31PA.

Consider the stem leaf plot:

Cable-Stayed	Stem	Steel-Arch
6 4 3	6	
9 8 6 5 1 1	7	3 8 8
0	8	0 0 2 3 4
5 2	9	0 1 1 8 8 9
8 2 0 0	10	0 3 8
2	11	0 0
8 2 0	12	0 6
5 2 0	13	
9 0	14	
	15	
3	16	5
3|6 = 630 feet	17	0 7|3 = 730 feet

The data for cable-stayed bridges can be written as:

630, 640, 660, 710, 710, 750, 760, 780, 790, 800, 920, 950, 1000, 1000, 1020, 1080, 1120, 1200, 1220, 1280, 1300, 1320, 1350, 1400, 1490, 1630.

The data for steel-arch bridges can be written as:

730, 780, 780, 800, 800, 820, 830, 840, 900, 910, 910, 980, 980, 990, 1000, 1030, 1080, 1100, 1100, 1200, 1260, 1650, 1700

The lower and upper quartiles for the data for cable-stayed bridges is

$$\text{lower quartile}, Q_1 = 755$$

$$\text{upper quartile}, Q_3 = 1290$$

Therefore, the interquartile range of the data is

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 1290 - 755 \\ &= \boxed{535} \end{aligned}$$

The lower and upper quartiles for the data for steel-arch bridges is

$$\text{lower quartile}, Q_1 = 820$$

$$\text{upper quartile}, Q_3 = 1100$$

Therefore, the interquartile range of the data is

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 1100 - 820 \\ &= \boxed{280} \end{aligned}$$

Answer 32PA.

Consider the stem leaf plot:

Cable-Stayed	Stem	Steel-Arch
6 4 3	6	
9 8 6 5 1 1	7	3 8 8
0	8	0 0 2 3 4
5 2	9	0 1 1 8 8 9
8 2 0 0	10	0 3 8
2	11	0 0
8 2 0	12	0 6
5 2 0	13	
9 0	14	
	15	
3	16	5
3|6 = 630 feet	17	0 7|3 = 730 feet

The data for cable-stayed bridges can be written as:

630, 640, 660, 710, 710, 750, 760, 780, 790, 800, 920, 950, 1000, 1000, 1020, 1080, 1120, 1200, 1220, 1280, 1300, 1320, 1350, 1400, 1490, 1630.

The data for steel-arch bridges can be written as:

730, 780, 780, 800, 800, 820, 830, 840, 900, 910, 910, 980, 980, 990, 1000, 1030, 1080, 1100, 1100, 1200, 1260, 1650, 1700

Therefore, the interquartile range for cable-stayed bridges is 535

An outlier must be $1.5(535)=802.5$ less than the lower quartile or $1.5(535)=802.5$ greater than the upper quartile.

$$755-802.5=-47.5$$
$$1290+802.5=2092.5$$

There is no value less than -47.5 and greater than 2092.5. Thus, there is no outlier.

Therefore, the interquartile range for steel-arch bridges is 280

An outlier must be $1.5(280)=420$ less than the lower quartile or $1.5(280)=420$ greater than the upper quartile.

$$820-420=400$$
$$1100+420=1520$$

There is no value less than 400 but there are two values greater than 1520 that is 1650 and 1700. Therefore the outliers are $\boxed{1650 \text{ and } 1700}$.

Answer 33PA.

Consider the stem leaf plot:

Cable-Stayed	Stem	Steel-Arch
6 4 3	6	
9 8 6 5 1 1	7	3 8 8
0	8	0 0 2 3 4
5 2	9	0 1 1 8 8 9
8 2 0 0	10	0 3 8
2	11	0 0
8 2 0	12	0 6
5 2 0	13	
9 0	14	
	15	
3	16	5
	17	0

3 | 6 = 630 feet *7 | 3 = 730 feet*

The data for cable-stayed bridges can be written as:

630, 640, 660, 710, 710, 750, 760, 780, 790, 800, 920, 950, 1000, 1000, 1020, 1080, 1120, 1200, 1220, 1280, 1300, 1320, 1350, 1400, 1490, 1630.

The data for steel-arch bridges can be written as:

730, 780, 780, 800, 800, 820, 830, 840, 900, 910, 910, 980, 980, 990, 1000, 1030, 1080, 1100, 1100, 1200, 1260, 1650, 1700

The range of the data for cable-stayed bridges is 1000 and for steel-arch bridges is 970

Therefore, the interquartile range for cable-stayed bridges is 535 and for steel-arch bridges is 280.

From these data, we observe that the ranges of the two bridges are almost close values but the interquartile ranges of the bridges have a large difference.

Answer 34PA.

All the values he measured were 2 inches greater than the actual values. So he should subtract 2 from each value he calculated. Then only he will get the actual values for range, median, lower quartile, upper quartile and interquartile ranges.

Answer 35PA.

The average monthly temperatures for three U.S. cities are given bellow:

Average Monthly High Temperatures (°F)			
Month	Buffalo	Honolulu	Tampa
January	30.2	80.1	69.8
February	31.6	80.5	71.4
March	41.7	81.6	76.6
April	54.2	82.8	81.7
May	66.1	84.7	87.2
June	75.3	86.5	89.5
July	80.2	87.5	90.2
August	77.9	88.7	90.2
September	70.8	88.5	89.0
October	59.4	86.9	84.3
November	47.1	84.1	77.7
December	35.3	81.2	72.1

The data in ascending order is

Buffalo: 30.2, 31.6, 35.3, 41.7, 47.1, 54.2, 59.4, 66.1, 70.8, 75.3, 77.9, 80.2

Honolulu: 80.1, 80.5, 81.2, 81.6, 82.8, 84.1, 84.7, 86.5, 86.9, 87.5, 88.5, 88.7

Tampa: 69.8, 71.4, 72.1, 76.6, 77.7, 81.7, 84.3, 87.2, 89.0, 89.5, 90.2, 90.2

We find the difference between the greatest and least values in each data set that is the ranges:

Buffalo: $80.2-30.2=50.0$

Honolulu: $88.7-80.1=8.6$

Tampa: $90.2-69.8=20.4$

The range of Buffalo is highest. Thus it shows the greatest chance in monthly highs.
Quartiles and interquartile ranges for the three cities:

Buffalo:

Lower quartile: 35.3 upper quartile: 75.3

$IQR=75.3-35.3=40.0$

Honolulu:

Lower quartile: 81.2 upper quartile: 87.5

$IQR=87.5-81.2=6.3$

Tampa:

Lower quartile: 72.1 upper quartile: 89.5

$IQR=89.5-72.1=17.4$

Answer 36PA.

Consider the set of data:

53, 57, 62, 48, 45, 65, 40, 42, 55

The data in ascending order:

40, 42, 45, 48, 53, 55, 57, 62, 65

The range of the data is

$$\begin{aligned}\text{range} &= \text{greatest value} - \text{lowest value}\\ &= 65-40\\ &= 25\end{aligned}$$

The correct option is $\boxed{B}$.

Answer 37PA.

Consider the set of data:

7,8,14,3,2,1,24,18,9,15

The data in ascending order:

1,2,3,7,8,9,14,15,18,24

The median of the data is

$$\begin{aligned}\text{median} &= \frac{8+9}{2}\\ &= \frac{17}{2}\\ &= 8.5\end{aligned}$$

The correct option is $\boxed{A}$.

Answer 38MYS.

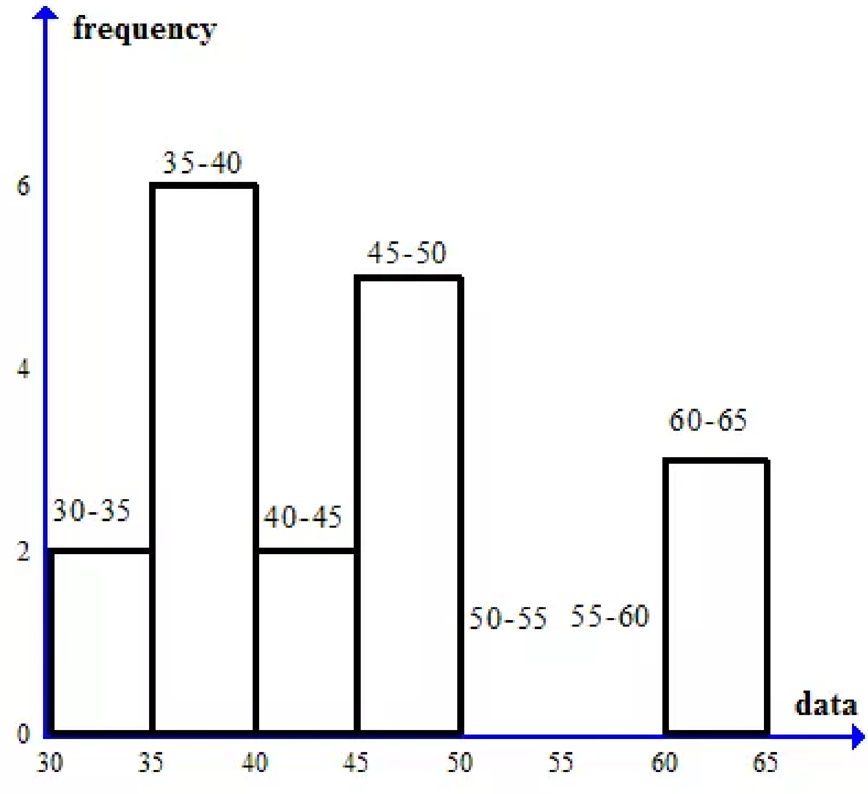
Consider the following set of data:

36, 43, 61, 45, 37, 41, 32, 46, 60, 38, 35, 64, 46, 47, 30, 38, 48,39

The frequency table is as bellow:

Interval	Tally	Frequency
$30 \le d < 35$	||	2
$35 \le d < 40$	~~||||~~ |	6
$40 \le d < 45$	||	2
$45 \le d < 50$	~~||||~~	5
$50 \le d < 55$	Nil	Nil
$55 \le d < 60$	Nil	Nil
$60 \le d < 65$	|||	3

The Histogram for these data is as bellow:



Answer 39MYS.

Consider the matrix:

$$A = \begin{bmatrix} (5) & -3 & 6 \end{bmatrix}$$

The number of rows and columns of a matrix, written in the form $\text{rows} \times \text{columns}$.

The above matrix has 1 row and 3 columns. Therefore the matrix has dimension $\boxed{1\times 3}$.

A general matrix *A* with the element's position is as $A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ where *m* is the number of rows and *n* is the number of columns.

Therefore, the position of the circled element is $\boxed{a_{11}}$.

Answer 40MYS.

Consider the matrix:

$$A=\begin{bmatrix} 3 & 1 \\ 2 & 9 \\ 4 & (3) \end{bmatrix}$$

The number of rows and columns of a matrix, written in the form $\text{rows} \times \text{columns}$.

The above matrix has 3 rows and 2 columns. Therefore the matrix has dimension $\boxed{3\times 2}$.

A general matrix A with the element's position is as $A_{m\times n}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ where m is the number of rows and n is the number of columns.

Therefore, the position of the circled element is $\boxed{a_{32}}$.

Answer 41MYS.

Consider the matrix:

$$A=\begin{bmatrix} 4 & 2 & -1 & 3 \\ 5 & (9) & 0 & 2 \end{bmatrix}$$

The number of rows and columns of a matrix, written in the form $\text{rows} \times \text{columns}$.

The above matrix has 2 rows and 4 columns. Therefore the matrix has dimension $\boxed{2\times 4}$.

A general matrix A with the element's position is as $A_{m\times n}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ where m is the number of rows and n is the number of columns.

Therefore, the position of the circled element is $\boxed{a_{22}}$.

Answer 42MYS.

Consider the expression:

$$\frac{15a}{39a^2}$$

To simplify the expression, we have

$$\frac{15a}{39a^2} = \frac{3\times5\times a}{3\times13\times a\times a} \quad \begin{bmatrix}\text{Since } 15 = 3\times5, 39 = 3\times13 \\ \text{and } a^2 = a\times a\end{bmatrix}$$

$$= \frac{\not{3}\times5\times\not{a}}{\not{3}\times13\times\not{a}\times a} \quad \begin{bmatrix}\text{Cancel same terms in} \\ \text{numerator and denominator}\end{bmatrix}$$

$$= \boxed{\frac{5}{13a}} \quad \begin{bmatrix}\text{Multiply remaining terms in} \\ \text{numerator and denominator}\end{bmatrix}$$

Answer 43MYS.

Consider the expression:

$$\frac{t-3}{t^2-7t+12}$$

To simplify the expression, we have

$$\frac{t-3}{t^2-7t+12} = \frac{t-3}{t^2-3t-4t+12} \quad \begin{bmatrix}\text{Quadratic equation in} \\ \text{denominator}\end{bmatrix}$$

$$= \frac{t-3}{t(t-3)-4(t-3)} \quad [\text{Taking common term}]$$

$$= \frac{\cancel{(t-3)}}{\cancel{(t-3)}(t-4)} \quad [\text{Cancel same term}]$$

$$= \boxed{\frac{1}{t-4}}$$

Answer 44MYS.

Consider the expression:

$$\frac{m-3}{m^2-9}$$

To simplify the expression, we have

$$\frac{m-3}{m^2-9} = \frac{m-3}{m^2-3^2} \qquad \left[\text{Since } 3^2 = 9\right]$$

$$= \frac{m-3}{(m-3)(m+3)} \quad \left[\text{Since } a^2 - b^2 = (a-b)(a+b)\right]$$

$$= \frac{\cancel{(m-3)}}{\cancel{(m-3)}(m+3)} \quad [\text{Cancel same term}]$$

$$= \boxed{\frac{1}{m+3}}$$

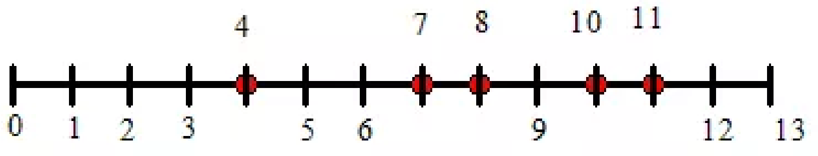
Answer 45MYS.

Consider the numbers:

$$\{4, 7, 8, 10, 11\}$$

A number line shows you a picture view of what a number is. You can visually see where your numbers are, and what other numbers are nearby.

The number line is as bellow:



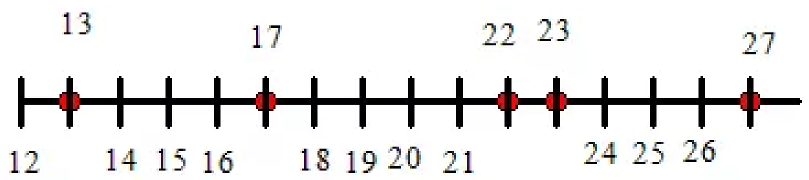
Answer 46MYS.

Consider the numbers:

$\{13, 17, 22, 23, 27\}$

A number line shows you a picture view of what a number is. You can visually see where your numbers are, and what other numbers are nearby.

The number line is as bellow:



Answer 47MYS.

Consider the numbers:

$\{30, 35, 40, 50, 55\}$

A number line shows you a picture view of what a number is. You can visually see where your numbers are, and what other numbers are nearby.

The number line is as bellow: