# CBSE Test Paper 03
## Chapter 9 Differential Equations

1. Find the particular solution for $2xy + y^2 - 2x^2\frac{dy}{dx} = 0;\ y = 2$ when x = 1.
   a. $y = \frac{2x}{1-\log|x|}\ (x \neq 0, x \neq e)$
   b. $y = \frac{3x}{1-\log|x|}\ (x \neq 0, x \neq e)$
   c. $y = \frac{2x}{1+\log|x|}\ (x \neq 0, x \neq e)$
   d. $y = \frac{5x}{1+\log|x|}\ (x \neq 0, x \neq e)$
2. General solution of $x\frac{dy}{dx} + 2y = x^2 \log x$ is
   a. $y = \frac{x^2}{16}(4\log|x| - 1) + Cx^{-2}$
   b. $y = \frac{x^2}{16}(4\log|x| + 1) + Cx^{-3}$
   c. $y = \frac{x^2}{16}(4\log|x| + 1) - Cx^{-2}$
   d. $y = \frac{x^2}{16}(4\log|x| + 1) + Cx^{-2}$
3. General solution of y log y dx – x dy = 0.
   a. $y = e^{-cx}$
   b. $y = e^{cx}$
   c. $y^2 = e^{cx}$
   d. $y = e^{cx} + e^{-cx}$
4. A first order linear differential equation, Is a differential equation of the form
   a. $\frac{dy}{dx} = Q$
   b. $\frac{dy}{dx} + Py = Q$
   c. $\frac{dy}{dx} + Py = 0$
   d. $\frac{dy}{dx} + Px = Q$
5. Determine order and degree (if defined) of y’ + 5y = 0.
   a. 1, 1
   b. 2, 1

c. 1, 2
d. 1, degree undefined

6. The solution of the differential equation ydx + (x + xy)dy = 0 is ________.
7. The number of arbitrary constants in a particular solution of the differential equation tan x dx + tan y dy = 0 is ________.
8. The degree of the differential equation $\frac{d^2y}{dx^2}+e^{\frac{dy}{dx}}=0$ is ________.
9. Write the differential equation obtained by eliminating the arbitrary constant C in the equation representing the family of curves xy = C cos x.
10. Write the integrating factor of the following differential equation. $\left(1+y^2\right)+(2xy-\cot y)\frac{dy}{dx}=0.$
11. Verify that the given functions is a solution of the corresponding differential equation $y=\cos x+c; y'+\sin x=0.$
12. Find the general solution: $\frac{dy}{dx}=\sqrt{4-y^2}\,(-2<y<2).$
13. Verify that the function is a sol of the corresponding diff. req. y = x sin x; $xy^1=y+x\sqrt{x^2-y^2}.$
14. Form the differential equation representing the family of ellipses having foci on x – axis and centre at the origin.
15. Solve the differential equation $x\frac{dy}{dx}+y-x+xy\cot x=0, x\neq 0$ .
16. Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation: $xy=\log y+C: y'=\frac{y^2}{1-xy}(xy\neq 1)$.
17. Verify that the given function (explicit) is a solution of the corresponding differential equation: $y=x\sin x: xy'=y+x\sqrt{x^2-y^2}(x\neq 0\ and\ x>y\,or\,x<-y)$.
18. Solve the diff. eq $\frac{dy}{dx}=\frac{x(2y-x)}{x(2y+x)}$, if y = 1 when x = 1.

## Solution

1. a. $y=\frac{2x}{1-\log|x|}\ (x\ \neq 0, x\neq e)$

   **Explanation:** Let y = vx

   $\frac{dy}{dx}=v+x\frac{dv}{dx}$

   Question becomes $v+x\frac{dv}{dx}\ =\frac{2v+v^2}{2}$

   $x\frac{dv}{dx}\ =\frac{2v+v^2}{2}-v$

   $x\frac{dv}{dx}\ =\frac{2v+v^2-2v}{2}$

   $2\int\frac{dv}{v^2}=\int\frac{dx}{x}$

   $\frac{-2}{v}=logx+c$

   When x=1 y=2 we get

   $\frac{-2x}{y}=logx+c$

   $\frac{-2}{2}=log1+c\implies\ c=-1$

   $\frac{-2x}{y}=logx-1$

   $y=\frac{2x}{1-log|x|}$

2. a. $y=\frac{x^2}{16}(4\log|x|-\ 1)+Cx^{-2}$

   **Explanation:** $\frac{dy}{dx}+\frac{2}{x}y=xlogx$

   $ye^{\int\frac{2dx}{x}}\ =\int e^{\int\frac{2dx}{x}}xlogx+C$

   $yx^2=\int x^3logxdx+c$

   $yx^2=logx\int x^3dx-\int\int x^3dx\frac{d}{dx}logx+c$

   $yx^2=logx\frac{x^4}{4}-\int\frac{x^3}{4}dx+c$

   $yx^2=\frac{x^4}{4}logx-\frac{x^4}{16}+c$

   $y=\frac{x^2}{4}logx-\frac{x^2}{16}+cx^{-2}$

   $y=\frac{x^2}{16}(4logx-1)+cx^{-2}$

3. b. $y\ =\ e^{cx}$

   **Explanation:** $y\log y\ dx=x\ dy$

   $\int\frac{1}{x}dx=\int\frac{1}{y\log y}dy$

$\log|x| = \log|\log y| + \log C$ Since $\int \frac{f'(x)dx}{f(x)} = log|f(x)| + c$ and

$\frac{1}{C} = c\ a\ new\ constant$

$\log x = \log(C\log y)$

$x = C\log y$

$\log y = \frac{1}{C}x$

$\log y = cx$

$y = e^{cx}$

4. b. $\frac{dy}{dx} + Py = Q$

   **Explanation:** Here the degree and order of the equation is 1 and also is of the form $\frac{dy}{dx} + Py = Q$ hence it is linear differential equation in first order

5. a. 1, 1

   **Explanation:** Order = 1, degree = 1. Since the equation has the highest derivative as y' and its power is 1

6. xy = Ae$^{-y}$
7. Zero
8. not defined
9. Given Equation of family of curves is xy = C cos x. ... (i)

   On differentiating both sides w.r.t. x, we get

   $1 \cdot y + x\frac{dy}{dx} = C(-\sin x)$

   $\Rightarrow \quad y + x\frac{dy}{dx} = -\left(\frac{xy}{\cos x}\right)\sin x$ [from Eq. (i)]

   $\therefore \quad y + x\frac{dy}{dx} + xy\ \tan x = 0$

10. Given differential equation is

    $\left(1+y^2\right) + (2xy - \cot y)\frac{dy}{dx} = 0.$

    The above equation can be rewritten as

    $(\cot y - 2xy)\frac{dy}{dx} = 1 + y^2$

    $\Rightarrow \quad \frac{\cot y - 2xy}{(1+y^2)} = \frac{dx}{dy}$

    $\Rightarrow \quad \frac{dx}{dy} = \frac{\cot y}{1+y^2} - \frac{2xy}{1+y^2}$

    $\Rightarrow \quad \frac{dx}{dy} + \frac{2y}{1+y^2} \cdot x = \frac{\cot y}{1+y^2}$

    which is a linear differential equation of the form

    $\frac{dx}{dy} + Px = Q$, here $P = \frac{2y}{1+y^2}$ and $Q = \frac{\cot y}{1+y^2}$ .

Now, integrating factor $= e^{\int pdy} = e^{\int \frac{2y}{1+y^2} dy}$

Put 1 + $y^2$ = t

$\Rightarrow$ 2ydy = dt

$\therefore \quad \text{IF} = e^{\int \frac{dt}{t}} = e^{\log|t|} = t = 1 + y^2$

11. $y = \cos x + c$

$\Rightarrow y' = -\sin x$

$\Rightarrow y' + \sin x = 0$

Hence verified.

12. Given: Differential equation $\frac{dy}{dx} = \sqrt{4 - y^2}$

$\Rightarrow dy = \sqrt{4 - y^2} dx$

$\Rightarrow \frac{dy}{\sqrt{4-y^2}} = dx$

Integrating both sides,

$\Rightarrow \int \frac{dy}{\sqrt{2^2 - y^2}} dy = \int 1 dx$

$\Rightarrow \sin^{-1} \frac{y}{2} = x + c \quad \left[\because \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}\right]$

$\Rightarrow \frac{y}{2} = \sin(x + c)$

$\Rightarrow y = 2\sin(x + c)$

13. y = x.sin x .......(i)

$y^1$ = x.cos x + sin x.1

$\Rightarrow xy^1 = x^2 \cos x + x.\sin x$

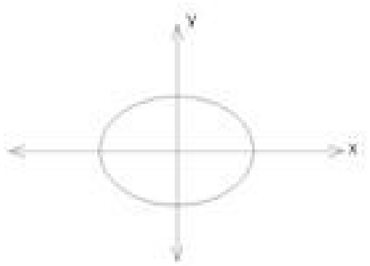
$xy^1 = x^2 \sqrt{1 - \sin^2 x} + x.\sin x$

$xy^1 = x^2 \sqrt{1 - \left(\frac{y}{x}\right)^2} + x.\sin x \quad \left[\because \frac{y}{x} = \sin x\right]$

$xy^1 = x^2 \sqrt{\frac{x^2 - y^2}{x^2}} + x.\sin x$

$xy^1 = x\sqrt{x^2 - y^2} + y$

Hence proved.

14. 

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(1)

diff eq (1) w. r. t. x,we get,

$\frac{2x}{a^2}+\frac{2y}{b^2}\cdot\frac{dy}{dx}=0$

$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx}=\frac{-2x}{a^2}$

$\Rightarrow \frac{y}{x}\cdot\frac{dy}{dx}=\frac{-b^2}{a^2}$

diff w. r. t. x,we get,

$\left(\frac{y}{x}\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(\frac{x\frac{dy}{dx}-y}{x^2}\right)=0$

$\Rightarrow xy\left(\frac{d^2y}{dx^2}\right)+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

15. According to the question,

Given differential equation is,

$x\frac{dy}{dx}+y-x+xy\cot x=0$

Above equation can be written as

$x\frac{dy}{dx}+y(1+x\cot x)=x$

On dividing both sides with x, we get

$\frac{dy}{dx}+y\left(\frac{1+x\cot x}{x}\right)=1$

$\Rightarrow \quad \frac{dy}{dx}+y\left(\frac{1}{x}+\cot x\right)=1$

which is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$,

where $P=\frac{1}{x}+\cot x$ and $Q=1$.

we know that ,

$\text{IF}=e^{\int Pdx}=e^{\int\left(\frac{1}{x}+\cot x\right)dx}=e^{\log|x|+\log\sin x}$

$\left[\because\int\frac{1}{x}dx=\log|x| \text{ and } \int\cot x dx=\log|\sin x|\right]$

$=e^{\log|x\sin x|}[\because \log m+\log n=\log mn]$

$\Rightarrow$ IF = x sin x

$y\times IF=\int(Q\times IF)dx+C$

$\therefore \quad y\times x\sin x=\int 1\times x\sin x dx+C$

$\Rightarrow \quad yx\sin x=\int \underset{I}{x}\underset{II}{\sin} x dx+C$

$\Rightarrow \quad y\cdot x\sin x=x\int\sin x dx-\int\left(\frac{d}{dx}(x)\int\sin x dx\right)dx+C$ [using integration by parts]

$\Rightarrow yx\sin x=-x\cos x-\int 1(-\cos x)dx+C$

$\Rightarrow yx\sin x = -x\cos x + \int \cos x dx + C$
$\Rightarrow yx\sin x = -x\cos x + \sin x + C$
On dividing both sides by x sin x, we get
$y = \frac{-x\cos x + \sin x + C}{x\sin x}$
$\therefore \quad y = -\cot x + \frac{1}{x} + \frac{C}{x\sin x}$
which is the required solution.

16. Given: xy = log y + C ...(i)

    To prove:y given by eq. (i) is a solution of differential equation $y' = \frac{y^2}{1-xy}$ ....(ii)

    Proof: Differentiating both sides of eq. (i) w.r.t x, we have

    $xy' + y(1) = \frac{1}{y}y' + 0$
    $\Rightarrow xy' - \frac{y'}{y} = -y$
    $\Rightarrow y'\left(x - \frac{1}{y}\right) = -y$
    $\Rightarrow y'\left(\frac{xy-1}{y}\right) = -y$
    $\Rightarrow y'(xy-1) = -y^2$
    $\Rightarrow y' = \frac{-y^2}{xy-1}$
    $\Rightarrow y' = \frac{-y^2}{-(1-xy)} = \frac{y^2}{1-xy}$

    Hence, function (implicit) given by eq. (i) is a solution of $y' = \frac{y^2}{1-xy}$.

17. Given: y = x sin x ...(i)

    To prove:y given by eq. (i) is a solution of differential equation
    $xy' = y + x\sqrt{x^2 - y^2}$ ...(ii)

    Proof: From eq. (i),
    $\frac{dy}{dx}(= y') = x\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x$
    = x cos x + sin x

    L.H.S. of eq. (ii)

    = xy' = x(x cos x + sin x) = $x^2$ cos x + x sin x

    R.H.S. of eq. (ii)
    $= y + x\sqrt{x^2 - y^2}$
    $= x\sin x + x\sqrt{x^2 - x^2\sin^2 x}$ [from eq. (i)]
    $= x\sin x + x\sqrt{x^2\left(1 - \sin^2 x\right)}$
    $= x\sin x + x\sqrt{x^2\cos^2 x}$

$= x\sin x + x.x\cos x$

$= x\sin x + x^2\cos x$

$= x^2\cos x + x\sin x$

$\therefore$ L.H.S. = R.H.S

Hence, y given by eq. (i) is a solution of $xy' = y + x\sqrt{x^2 - y^2}$.

18. $\frac{dy}{dx} = \frac{x(2y-x)}{x(2y+x)}$ (i)

If y = 1, when x = 1

Let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Put $\frac{dy}{dx}$ in eq(i)

$v + x\frac{dv}{dx} = \frac{(2vx-x)}{(2vx+x)}$

$v + x\frac{dv}{dx} = \frac{2v-1}{2v+1}$

$x\frac{dv}{dx} = \frac{2v-1}{2v+1} - v$

$x\frac{dv}{dx} = \frac{-2v^2+v-1}{2v+1}$

$\int \frac{2v+1}{2v^2-v+1}dv = \int -\frac{dx}{x}$

$\frac{1}{2}\int \frac{4v+2}{2v^2-v+1}dv = -\int \frac{dx}{x}$

$\frac{1}{2}\int \frac{4v-1}{2v^2-v+1}dv + \frac{1}{2}\int \frac{3}{2v^2-v+1}dv = \int -\frac{dx}{x}$

$\frac{1}{2}\int \frac{4v-1}{2v^2-v+1}dv + \frac{3}{4}\int \frac{dv}{\left(v-\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2} = \int -\frac{dx}{x}$

$\frac{1}{2}\log(2v^2 - v + 1) + \frac{3}{4}.\frac{4}{\sqrt{7}}\tan^{-1}\left(\frac{v-\frac{1}{4}}{\frac{\sqrt{7}}{4}}\right) = -\log x + \log c$

$\frac{1}{2}\log\left(2\frac{y^2}{x^2} - \frac{y}{x} + 1\right) + \frac{3}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{\sqrt{7}x}\right) = -\log x + \log c$

put x = 1, y = 1

$\log c = \frac{1}{2}\log 2 + \frac{3}{\sqrt{7}}\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$ [as log 1 = 0]

Therefore, solution is,

$\frac{1}{2}\log\left(\frac{2y^2-xy+x^2}{x^2}\right) + \frac{3}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{\sqrt{7}x}\right)$

$= -\log x + \frac{1}{2}\log 2 + \frac{3}{\sqrt{7}}\tan^{-1}\frac{3}{\sqrt{7}}$