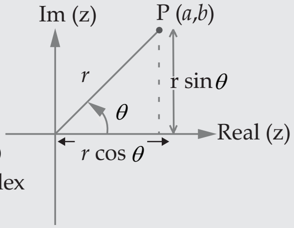
MIND MAP : LEARNING MADE SIMPLE CHAPTER - 5

Complex Number & Quadratic Equations

Polar Representation

Let $a = r\cos\theta$
$b = r\sin\theta$
where,
$r = |z|$
and $\theta = \arg(z)$
$\therefore z = a + ib = r(\cos\theta + i\sin\theta)$
The argument 'θ' of complex number z = a+ib is called principal argument of z if
$-\pi < \theta \le \pi$.

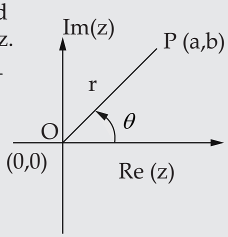


Modulus & Argument of Complex Number

If z= a+ib is a complex number
(i) Distance of z from origin is called as modulus of complex number z.

It is denoted by $r = |z| = \sqrt{a^2 + b^2}$

(ii) Angle θ made by OP with +ve direction of X-axis is called argument of z.



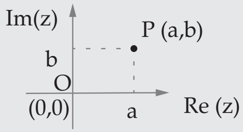
Powers of 'i'

$i = \sqrt{-1}, i^2 = -1$

In general, $i^{4k+r} = \begin{cases} 1; r = 0 \\ i; r = 1 \\ -1; r = 2 \\ -i; r = 3 \end{cases}$

Argand Plane

A complex number z=a+ib can be represented by a unique point P(a,b) in the argand plane



z=a+ib is represented by a point P (a,b)

Square root of Complex Number

Let $x + iy = \sqrt{a + ib}$, squaring both sides, we get $(x+iy)^2 = a+ib$ i.e. $x^2-y^2=a$, $2xy=b$ solving these equations, we get square root of z.

Multiplicative Inverse of complex Number

For a non-zero complex number z=a+ib$(a \neq 0, b \neq 0)$,

there exists a complex number $\frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2} i$

denoted by $\frac{1}{z}$ or z^{-1}, called multiplicative inverse of Z

Such that: $(a+ib)\left(\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}\right) = 1 + 0i = 1$

Algebra of Complex Numbers

Let: $z_1 = a+ib$ and $z_2 = c+id$ be two complex numbers, where a,b,c,d ∈ R and $i = \sqrt{-1}$

1. Addition: $z_1 + z_2 = (a+ib) + (c+id) = (a+c) + i(b+d)$

2. Subtraction: $z_1 - z_2 = (a+ib) - (c+id) = (a-c) + (b-d)i$

3. Multiplication: $z_1 \cdot z_2 = (a+ib)(c+id)$
$= a(c+id) + ib(c+id)$
$= (ac-bd) + (ad+bc)i$
$(\because i^2 = -1)$

4. Division: $\frac{z_1}{z_2} = \frac{a+ib}{c+id} = \frac{a+ib}{c+id} \cdot \frac{c-id}{c-id}$
$= \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$

Note: If a+ib = c+id
⇔ a = c & b = d

Solution of Quadratic Equation

• General form of quadratic equation in x is $ax^2+bx+c=0$,
Where a,b,c ∈R & a≠0
The solutions of given quadratic equation are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$\because b^2-4ac<0$

Note: • A polynomial equation has atleast one root.
• A polynomial equation of degree n has n roots.

Definition of Complex Numbers

A number of the form a+ib, where a,b ∈R and $i = \sqrt{-1}$ is called a complex number and denoted by 'z'.

z= a+ib → ib: Imaginary part; a: Real part

Conjugate of a complex number: For a given complex number z=a+ib, its conjugate is defined as $\bar{z}$= a – ib